Chapter 2
Systemic Risk and Complex Systems: A Graph-Theory Analysis

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Abstract This chapter summarizes several empirical studies in finance, undertaken through the prism of the graph theory. In these studies, we built graphs in order to investigate integration and systemic risk in derivative markets. Several classes of underlying assets (i.e. energy products, metals, financial assets, agricultural products) are considered, on a twelve-year period. In such a high dimensional analysis, the graph theory enables us to understand the dynamic behavior of our price system. The dimension of the fully connected graph being high, we rely on a specific type of graphs: Minimum Spanning Trees (MSTs). Such a tree is especially interesting for the study of systemic risk: it can be assimilated into the shortest and most probable path for the propagation of a price shock. We first examine the topology of the MSTs. Then, given the time dependency of our correlation-based graphs, we study their evolution over time and their stability.

2.1 Introduction

This chapter summarizes several empirical works in finance undertaken since 2009: [8, 10, 11]. These works share two common points: first, they all focus on systemic risk and the integration in organized derivative markets. Second, they provide for a large-scale analysis and rely on the graph theory.

Integration and systemic risk are linked: the former is a favorable condition for the second to appear in. Concerns about systemic risk have recently grown in derivative markets, notably among energy commodities. These markets are supposed to be more and more integrated, both in regard to each other and to other markets. For some months now, fluctuations in the prices of energy products have often been

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invoked to explain corresponding fluctuations in soft commodities like soy, corn, or wheat. Furthermore, because commodities are nowadays considered a new class of assets, investors use them for diversification purposes. Therefore, the price fluctuations recorded in commodity markets might be, at least partially, explained by external events like the fall in stock prices or in interest rates [2, 4, 7].

In our studies, we propose a holistic approach for systemic risk, which examines it simultaneously in three dimensions: space, time, and the maturity of the transactions. Such an analysis accounts for the eventuality that a price shock that occurs on a specific asset's physical market can spread, not only through its own futures market, but also into other physical and/or paper markets, and *vice versa*.

A full comprehension of systemic risk can only be made through a large scale analysis that requires the manipulation of a huge amount of data. In our most extensive study [10], we work on the basis of 14 derivative markets (six energy commodities, four agricultural commodities, and four financial assets), over a 12-year period. This leads us to setup a database containing more than $750,000$ futures prices. To perform such large scale analyses, we rely on methods initially designed for statistical physics, aiming at understanding the behavior of complex systems. They incite us to consider all prices, quoted in different places and with different maturities, as a complex dynamic system. Moreover, this consideration leads us to a set of tools that proves very useful for the study of systemic risk: graph theory.

Through this prism, the nodes in our graph are the daily price's returns and the links stand for distances, the latter being computed as a function of the correlations between the returns [12, 14, 15]. This representation allows us to analyze the linkages between the markets and their evolution, thanks to the structure of the connections between the futures contracts. What is especially interesting here is that we can consider, simultaneously, all possible pairs of assets.

The size of the fully connected graph being high, we rely, most of the time, on a specific type of graphs: Minimum Spanning Trees (MSTs). A MST provides a way to extract the most important information contained in the initial graph. It is unique and corresponds to the shortest path covering all the nodes of the graph without loops. Such a tree is thus especially interesting for the study of systemic risk: it can be assimilated into the shortest and most probable path for the propagation of the price shock. To the best of our knowledge, it is the first time that this tool has been used this way.

The visualization of the MST and the computation of some specific measures, like allometric coefficients, make possible the analyzation of the organization of the trees. Two extreme configurations are used as references. A chain-like organization signifies that, when it appears at one extremity of the price system, only one way exists for the price shock to propagate: before reaching the other extremity of the graph, the shock will have to cross each node. On the other hand, in a star-like organization, the paths for the transmission of fluctuations are less easy to predict. Here, the node located at the center of the star is of crucial importance: whenever a shock arises at this point, it might disseminate to the whole system! We first examine the MST according to these two ideal types of organizations. Then, given the time dependency of correlation-based graphs, we study their evolution over time and their stability.

Our first main results lie in the economic meaningfulness of the graphs. In the spatial as well as in the 3-D analyses, the trees are organized into sub-trees corresponding to the three sectors of activity under examination: energy commodities, agricultural products, and financial assets. In the maturity dimension, as a result of arbitrage operations, the trees are ordered according to the maturity of the contracts. The second set of results, interesting for regulatory purposes, shows that energy products promote the connection between the different sectors. Moreover, crude oil stands at the center of the energy complex. A third category of results concerns the evolution of integration over time. In commodity markets, both spatial and maturity dimensions tend to be more integrated. Thus, the conditions for the appearance of systemic risk increase.

Section 2.2 of this chapter explains the data. Section 2.3 gives insights into the methodology retained. In Sect. 2.4, we present the empirical results: we summarize the main conclusions reached in previous studies (more particularly in [10]), and we add some other results, still unpublished until now. Section 2.5 concludes.[1]

2.2 Data

We select futures markets corresponding to three sectors: energy, agriculture, and financial assets. On the basis of the Futures Industry Association's reports, we retain those contracts whose characteristics are large transaction volumes over long time periods. We rearrange the futures prices in order to reconstitute daily term structures, that is, the relation linking, at a specific date, several futures contracts with different delivery dates. In order to obtain continuous time series, we remove some maturities from the database. We also take away all observation dates that are not shared by all markets. Once these selections carried out, our database still contains more than $750,000$ prices. Table 2.1 summarizes the characteristics of this database.[2]

2.3 The Graph Theory: From Full Connected Graphs to Minimum Spanning Trees (MST)

Among the different tools provided by the graph theory, we select those that allow us to analyze market integration and systemic risk by using a 3-D approach. We first focus on the synchronous correlations of price returns. Having transformed these correlations into distances, we are able to draw a fully connected graph of the price system, where the nodes (vertices) of the graph represent the time series of futures prices. In order to filter the information contained in the graph, we then rely on

[1] The review of the literature related to this chapter can be find in [10].

[2] Another study, including the freight rate, can also be find in [8].

Table 2.1 Main characteristics of the collected data: nature of the underlying asset, trading place of the futures contract, localization of the exchange, time period, longest maturity (in months) and number of records per maturity. CME stands for Chicago Mercantile Exchange, ICE for Inter Continental Exchange, US stands for United States and Eu for Europe

Underlying assets	Exchange–Zone	Period	Maturities	Records
Light crude	CME–US	1998–2011	up to 84	3343
Brent crude	ICE–Eu	2000–2011	up to 18	2923
Gasoil	ICE–Eu	2000–2011	up to 12	2950
Nat. gas (US)	CME–US	1998–2011	up to 36	3336
Nat. gas (Eu)	ICE–Eu	1997–2011	up to 9	3698
Wheat	CME–US	1998–2011	up to 15	3412
Soy bean	CME–US	1998–2011	up to 14	3370
Soy oil	CME–US	1998–2011	up to 15	3447
Corn	CME–US	1998–2011	up to 25	2960
Eurodollar	CME–US	1997–2011	up to 120	3689
Gold	CME–US	1998–2011	up to 60	3060
FX rate USD/EUR	CME–US	1999–2011	up to 12	3239
Mini SP500	CME–US	1997–2011	up to 6	3611

Minimum Spanning Trees [12]. Such a tree can be defined as the one providing the best arrangement of the network's different nodes.[3]

2.3.1 Synchronous Correlation Coefficients of Prices Returns

The synchronous correlation coefficients of price returns are defined as follows:

$$\rho_{ij}(t) = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{(\langle r_i^2 \rangle - \langle r_i \rangle^2)(\langle r_j^2 \rangle - \langle r_j \rangle^2)}}, \tag{2.1}$$

where i and j correspond to two different time series of futures returns. The daily logarithm price differential stands for price returns r_i, with $r_i = (\ln F_i(t) - \ln F_i(t - \Delta t))/\Delta t$, where $F_i(t)$ is the price of the futures contract at t. Δt is the lag between two consecutive trading days.

For a given time period and a given set of data, we thus compute the matrix of $N \times N$ correlation coefficients C for all the pairs ij. C is symmetric with $\rho_{ij} = 1$ when $i = j$. Thus, $N(N-1)/2$ coefficients characterize C.

[3]For more details on the methodology used and on the graph theory, please refer to [11].

2.3.2 From Correlations to Distances

In order to use graph theory, we need to introduce a metric. The correlation coefficient ρ_{ij} cannot be used as a distance d_{ij} between i and j, because it does not fulfill the three axioms that define a metric [6]: (1) $d_{ij} = 0$ if and only if $i = j$; (2) $d_{ij} = d_{ji}$, and (3) $d_{ij} \leq d_{ik} + d_{kj}$.

A metric d_{ij} can be extracted from the correlation coefficients through the following non linear transformation:

$$d_{ij} = \sqrt{2(1 - \rho_{ij})}. \tag{2.2}$$

A distance matrix D is thus extracted from the correlation matrix C according to (2.2). Both, C and D are $N \times N$ dimensional. Whereas the coefficients ρ_{ij} can be positive for correlated returns or negative for anti-correlated returns, the quantity d_{ij} that represents the distance between price returns is always positive. This distance matrix corresponds to the fully connected graph: it represents all the possible connections in the price system.

2.3.3 From Fully Connected Graphs to Minimum Spanning Trees

A simple connected graph represents all the possible connections between N points with $N(N-1)/2$ links (or edges). The graph can be weighted in order to represent the different intensities of the links and/or nodes. In our case, these weights represent the distances between the nodes. For a weighted graph, the MST is the one spanning all the nodes of the graph without loops. This MST also has less weight than any other tree.

Through a filtering procedure that reduces the information space from $N(N-1)/2$ to $N-1$, the MST highlights the most relevant connections in the system. In our study, the MST provides the shortest path to linking all nodes and discloses the underlying mechanisms of systemic risk. Thus, because this tree is unique, it can be considered the easiest path for the transmission of a price shock.

2.4 The Topology of the Trees

The first information that a Minimum Spanning Tree provides is the kind of arrangement that exists between the vertices: its topology. We thus focus on this topology and its consequences for systemic risk. We present the results obtained with static MST, (i.e. we consider the whole time period—or some sub-sets of this period—as a single window, and we perform a static analysis).

The first step in studying the topologies lies in the visualization of the trees in the three dimensions under consideration. In a second step, we separate the whole time

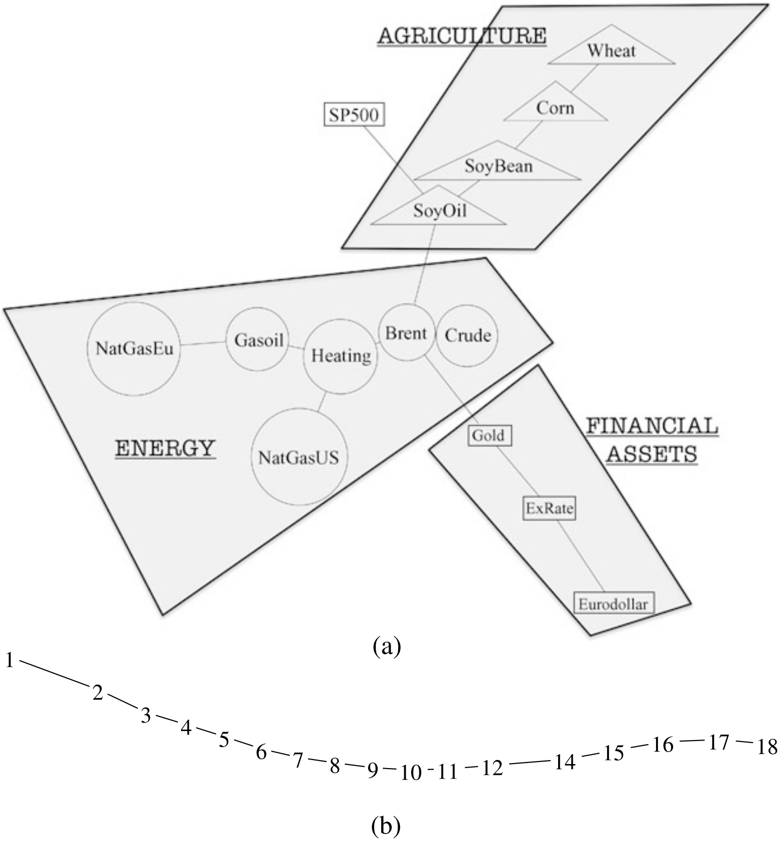

Fig. 2.1 Static Minimum Spanning Trees built from the correlation coefficients of the prices returns. (**a**) MST in the spatial dimension (April 2001–April 2011). (**b**) MST of the Brent crude in the maturity dimension (April 2000–April 2011). The curvature only eases the visualization

period into three sub-periods and show how the topology evolves in the maturity dimension. Finally, we use allometric coefficients to determine whether the MST are totally organized, totally random, or are situated somewhere between these two extreme topologies.

2.4.1 The Emerging Taxonomy in the Three Dimensions

Figure 2.1 presents the MST obtained for the spatial dimension. As far as the spatial dimension is concerned, all three sectors can be identified. Energy comprises American as well as European markets and is situated between agriculture (on the top) and financial assets (on the bottom). The most connected node in the graph is the Brent, which makes it the best candidate for the transmission of price fluctuations in the tree (actually, the same could have been said for the Crude (Light crude), as the distance between these products is very short). Further, the energy sector is the most integrated of the three sectors because the distances between the nodes are short. The link between the energy and agricultural products passes through soy oil, which can be used for fuel. The link between commodities and financial assets passes through gold, which can be seen as a commodity but also as a reserve of value. The only surprising link comes from the S&P500, which is more correlated to soy oil than to financial assets.

Such a star-like organization leads to specific conclusions regarding systemic risk. A price move in the energy markets, situated at the heart of the price system, will have more impact than a fluctuation affecting peripheral markets such as interest rates or wheat.

Things are totally different in the maturity dimension. The results are illustrated by the example of the Brent crude, depicted by Fig. 2.1b. For all contracts, the MSTs are linear and the maturities are regularly ordered from the first to the last delivery dates.

The results obtained in the maturity dimension give rise to three remarks. Firstly, the linear topology is meaningful from an economic point of view, as it reflects the presence of the Samuelson effect [17]. In derivative markets, the movements in the prices of the prompt contracts are larger than the other ones. This difference results in a decreasing pattern of volatilities along the price curve and leads to higher correlations between the maturities that are the closest to each other. Secondly, this type of organization impacts the possible transmission of price shocks. The most likely path for a shock is indeed unique and passes through each maturity, one after the other. Thirdly, the short part of the curves are less correlated with the other parts. This phenomenon can result from price shocks emerging in the physical market with the most nearby price being the most affected; it could also reflect noises introduced on the first maturity by investors in the derivative market.

Figure 2.2 represents the 3-D static MST. Its shape brings to mind the spatial dimension. However, it is enhanced by the presence of the different maturities available for each market. These maturities have a clear, linear organization. Again, the tree shows a clear separation between the sectors. Three energy contracts, the crude oil (Light crude), the Brent and the Heating oil, are at the center of the graph. They are the three closest nodes in the graph. Whereas the maturities of each market primarily have a linear organization, the American natural gas behaves differently and displays an atypical topology with numerous ramifications.

It is interesting to see which maturities connect two markets or sectors. Economic reasoning suggests that two kinds of connections should exist: with the shortest and/or with the longest part of the curves. In the first case, the price system would be essentially driven by underlying assets; in the second, it would be dominated by derivative markets. However, a closer analysis of the 3-D trees does not provide evidence of either kind. Furthermore, the analysis of the trees at different periods does not lead to the conclusion that there is something like a pattern in the way connections occur.

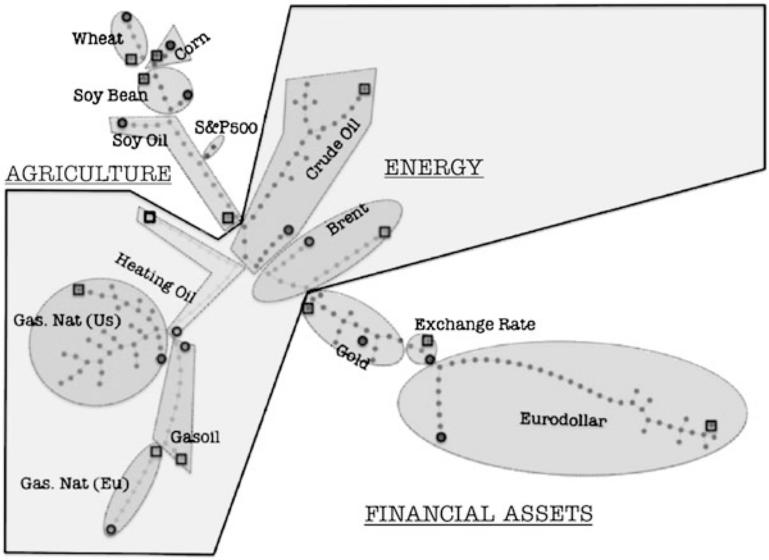

Fig. 2.2 Static MST in 3-D (2000–2011). Each futures contract is enclosed in a shaded area with its name. The first and last maturities are respectively represented by a bold circle and a bold square. The distance between the nodes is set to unity

2.4.2 The Evolution of the MSTs Topology Through Time, in the Maturity Dimension

As far as the topology in the maturity dimension is concerned, we observed, through different studies, that the linear topology exhibited by Fig. 2.1b is the result of a maturation process of the derivative market [13]. In such a market, indeed, the maturities of the futures contracts usually rise through time: the growth in the transaction volumes indeed results in the introduction of new delivery dates and extends the time horizon of arbitrage operations.

When we examine the topologies of the MST in the maturity dimension in a dynamical way, we find an illustration of this maturation process in the evolution of the topologies of the trees. Figure 2.3 gives an example of such a process through the case of the Eurodollar futures contract: in the beginning of the study period, the market is less integrated and the tree is not perfectly linear: there are branches on the latest maturities. This result is consistent with economic intuition, as the latter are always characterized by lower transaction volumes. As time goes on, however, the linearization progresses, reflecting the intensification of arbitrage operations towards longer time horizon. Moreover, the shortest maturities become perfectly ordered. As far as this maturation process is concerned, we found only one exception among the 14 markets under scrutiny: the American market for natural gas. Here, conversely to what can been observed on other markets, the MST becomes less and less linear as time goes on. More precisely, as shown by Fig. 2.4, a cyclical pattern emerges. As

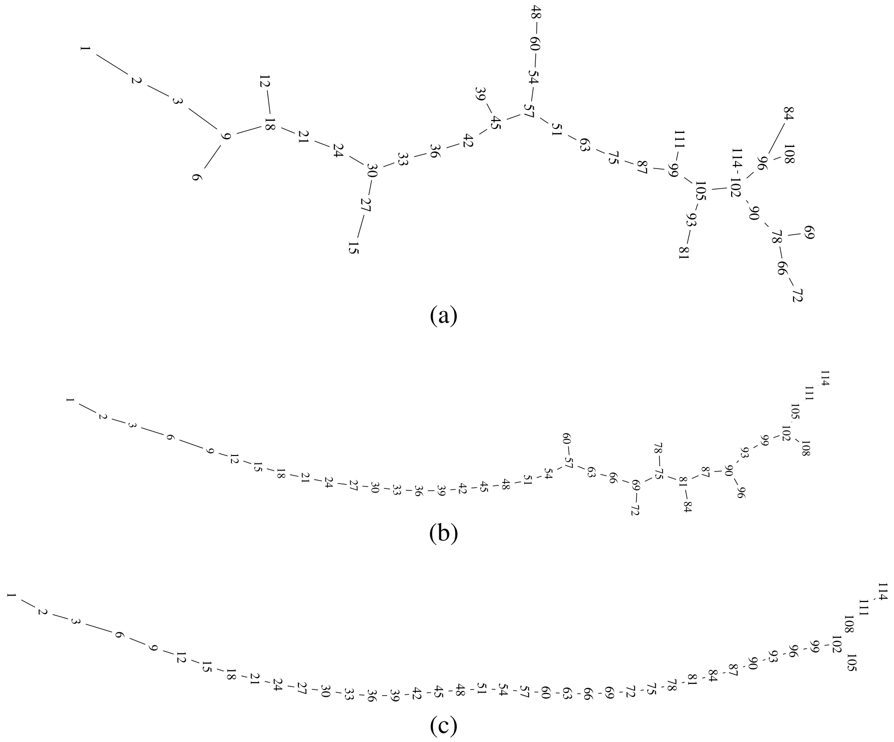

Fig. 2.3 Time evolution of the Eurodollar futures contract. (**a**) 1998–2001. (**b**) 2001–2004. (**c**) 2005–2009

the natural gas market is characterized by a strong seasonality, we first thought that this pattern reflects this seasonality. The maturities of connections between the futures contracts, however, do not make sense with such an interpretation. Moreover, the European natural gas market does not behave similarly. A possible answer lies in a possible disorganization of the market after the difficulties encountered in 2006 with the hedge fund Amaranth. We leave such investigation for later studies.

2.4.3 Allometric Properties of the MST

The computation of the allometric coefficients of a MST provides a means of quantifying where this tree stands between two asymptotic topologies: star-like trees that are symptomatic of a random organization, and chain-like trees that show a strong ordering in the underlying structure.

[1] developed the first model for the allometric scaling of a spanning tree. The first step of the procedure consists of initializing each node of the tree with the value of one. Then the root or central vertex of the tree must be identified. In what follows,

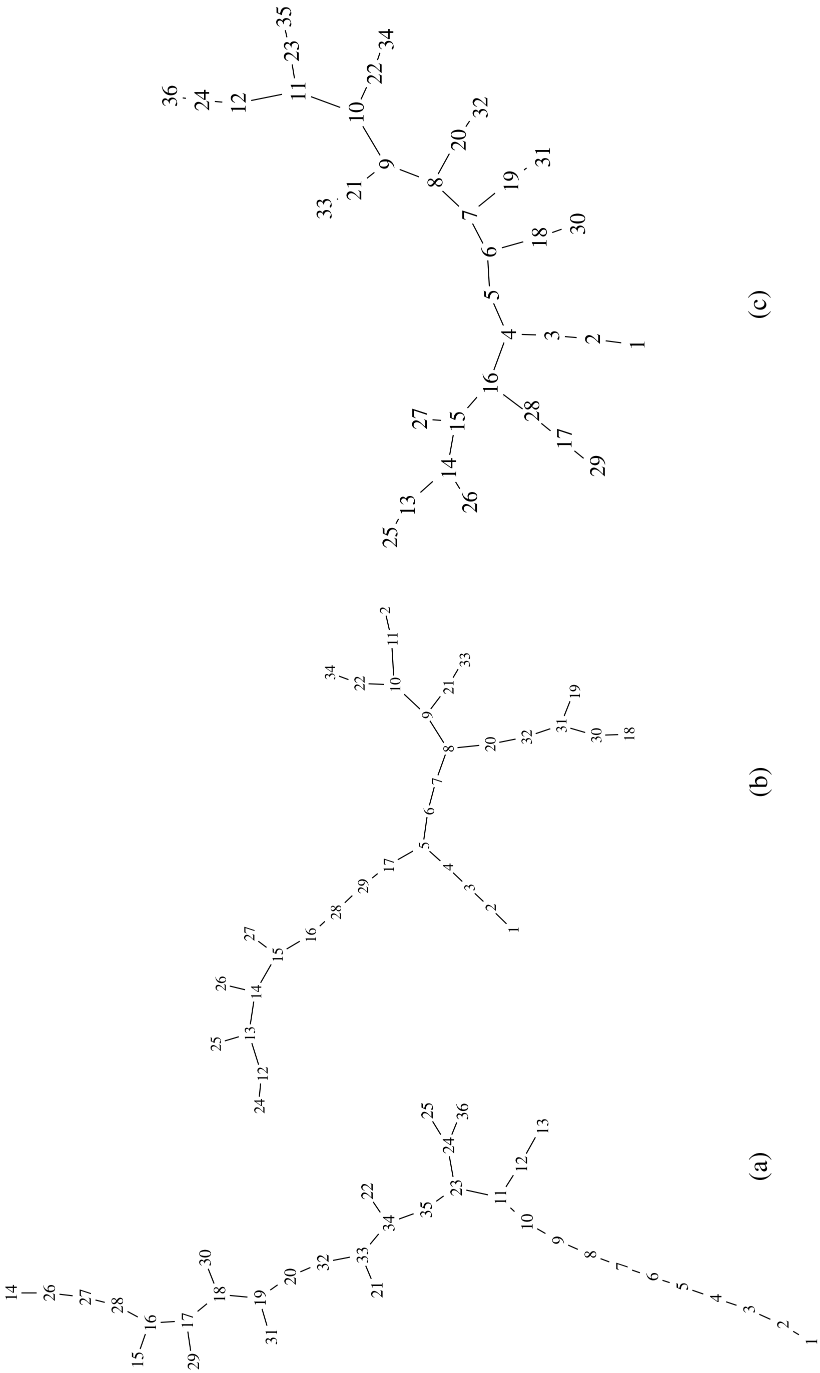

Fig. 2.4 Time evolution of the American Natural Gas futures contract. (**a**) 1998–2001. (**b**) 2001–2004. (**c**) 2004–2009

the root is defined as the node that has the highest number of links attached to it. Starting from this root, the method consists of assigning two coefficients A_i and B_i to each node i of the tree:

$$A_i = \sum_j A_j + 1 \quad \text{and} \quad B_i = \sum_j B_j + A_i, \tag{2.3}$$

where j stands for all the nodes connected to i in the MST. The definition of the allometric scaling relation is the relation between A_i and B_i:

$$B \sim A^{\eta}, \tag{2.4}$$

where η is the allometric exponent estimated after removing the leaf nodes $A = C = 1$ [5]. It represents the degree or complexity of the tree and stands between two extreme values: 1^+ for star-like trees and 2^- for chain-like trees [16].

The main results obtained with this measure are the following:[4] within the maturity dimension, the coefficients tend towards their asymptotic value: $\eta = 2^-$, for all markets under investigation. However, they are a bit smaller than 2, due to finite size effects (there is a finite number of maturities). Such a result is probably due to arbitrage operations. When performed on the basis of contracts having the same underlying asset, such operations are easy and rapidly undertaken, thus resulting in a perfect ordering of the maturity dates. Such a scaling is appealing from an empirical as well as a theoretical point of view and suggests a possible universal behavior of the topologies of derivatives networks. Additionally arbitrage operations have also a deep impact on the prices dynamic leading to ubiquitous statistical properties of price returns along the term structure [9].

With concern for the spatial dimension, the exponents indicate that even if Fig. 2.1a exhibits a star-like organization, the shape of the MST is rather complex and stands exactly between the two asymptotic topologies. There is an ordering of the tree, which is well illustrated by the agricultural sector, which forms a regular branch. Finally, even if the topologies of the spatial and 3-D trees seem similar, they are quantitatively different. The allometric exponent for the 3-D tree is higher: the best fit from our data gives an exponent close to 1.757 as compared to the value of 1.493 in the spatial case. Thus, the topology in 3-D merges the organization in sectors induced by the spatial dimension and the chain-like organization arising from the maturity dimension.

2.5 Dynamical Analysis

Because it is based on correlation coefficients, our study of market integration is intrinsically time dependent. On the basis of the fully connected graph, we first examine the dynamic properties thanks to the node's strength, which provides in-

[4] For more detailed results, please see [10].

formation on how close a given node is to all others. We then turn to the MSTs. In order to study the robustness of the topologies, we compute the length of the MST, that shows the state of the system at a specific time. Survival ratios also indicate how the topology evolves over time. Finally, these survival ratios allow us a deeper investigation into the connections between markets in the 3-D analysis.

In what follows, we retain a rolling time window of $\Delta T = 480$ consecutive trading days.

2.5.1 The Nodes Strength in the Fully Connected Graphs

In order to examine the time evolution of our system, we investigate the nodes strength in the fully connected graph. This quantity, calculated for each node i, indicates the closeness of one node i to all others. It is defined as follows:

$$S_i = \sum_{i \neq j} \frac{1}{d_{ij}}. \tag{2.5}$$

In our case, the node strength provides information on the intensity of the correlations linking a given node to the others. When S_i is high, the node is close to all others. For the sake of simplicity, we use this measure in the spatial dimension only. As far as the maturity dimension is concerned, it was indeed not easy to represent the nodes strength for all futures contracts.[5] Figure 2.5 represents the time evolution of the nodes strength in the spatial dimension. The figure has been separated into four panels: the energy sector is at the top, with American products on the left and European ones on the right; the agricultural sector is at the bottom left and financial assets are at the bottom right.

Figure 2.5 shows that, at the end of the period, out of all the assets studied, the two crude oils and Heating oil show the greatest nodes strength. However, since 2010, the American node strength has decreased, which indicates a difference in the connectivity of the two crude oils. This is an interesting result, as there are, indeed, delivery problems on the American crude oil since that date. These problems raise the question of the relevance of the Light crude oil as a worldwide benchmark. The petroleum products are followed by soy oil, other agricultural assets, the S&P500 contract, gold, the exchange rate USD/EUR, and the gasoil. A remarkable evolution is the sharp rise in the equity connectivity in the post-Lehman period, as opposed to 2001–2007. This finding corroborates those of [3] and [18]. Finally, the more distant nodes are those representing the Eurodollar and the two natural gases.

As far as the time evolution of the node strength is concerned, the sectors exhibit different patterns: the integration movement, characterized by an increase in this measure, emerges earlier for the energy sector than for the agricultural sector. However, it decreases for energy at the end of the period (especially for the Light crude

[5] For more information on these results, please see [8].

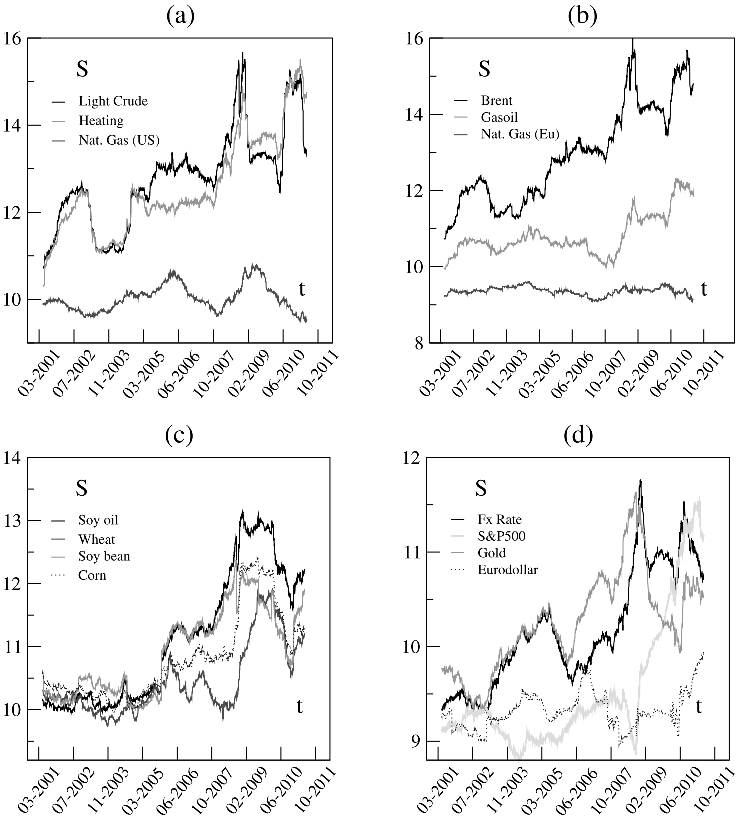

Fig. 2.5 Nodes strength of the markets in the spatial dimension 2001–2011. (**a**) American energy products. (**b**) European energy products. (**c**) Agricultural products. (**d**) Financial assets

oil). The nodes strength of the agricultural products is characterized by a plateau from the middle of 2009 to the beginning of 2010, followed by a drawdown until the Fall 2010. Last but not least, most of the products exhibit a strong increase, except for natural gas and interest rate contracts. Thus, whereas the core of the graph becomes more and more integrated, the peripheral assets do not follow this movement.

2.5.2 The Length of the Minimum Spanning Trees

The normalized tree's length can be defined as the sum of the lengths of the edges belonging to the MST:

$$\mathscr{L}(t) = \frac{1}{N-1} \sum_{(i,j) \in MST} d_{ij}, \tag{2.6}$$

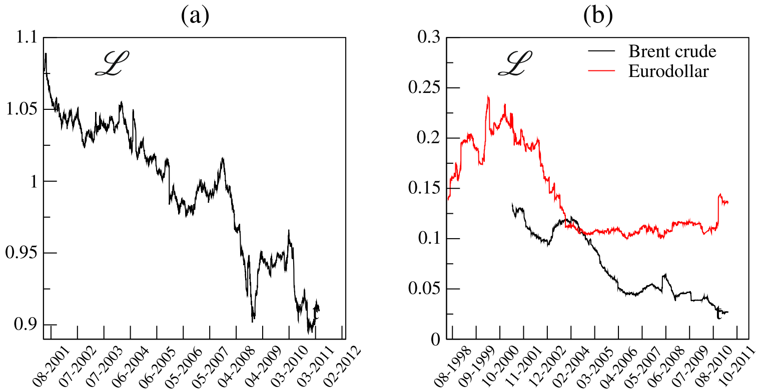

Fig. 2.6 Time evolution of the normalized tree length. (**a**) Spatial dimension. (**b**) Maturity dimension for the Brent crude (*black line*) and the Eurodollar (*gray line*)

where t denotes the date of the construction of the tree and $N - 1$ is the number of edges. The length of a tree is longer as the distances increase, and consequently when correlations are low. Thus, the more the length shortens, the more integrated the system is. On the contrary, in the case of random co-movements, the length of the tree is equal to $\sqrt{2}$.

Figure 2.6 represents the dynamic behavior of the normalized length of the MSTs in the spatial and in the maturity dimensions. In the spatial dimension, the general pattern is that the length decreases, which reflects the integration of the system. This information confirms what was observed in the fully connected graph on the basis of the nodes strength. In addition, it shows that the most efficient transmission path for price fluctuations becomes shorter as times goes on. A more in-depth examination of the figure also shows a very important decrease between October 2006 and October 2008, as well as significant fluctuations in September and October 2008. We leave the analysis of such events for future studies.

In the maturity dimension, as integration increases, the normalized tree's length also diminishes. Figure 2.6 illustrates this phenomenon by representing the evolutions recorded for the Eurodollar contract and Brent crude. As far as the interest rate contract is concerned, the tree's length first increases, then in mid-2001 it drops sharply and remains fairly stable after that date. For crude oil, the decrease is constant and steady, except for a few surges.

2.5.3 Survival Ratios, the Stability of the Prices System and the Interconnections Between Markets

The robustness of the MSTs over time is examined by computing the single-step survival ratio of the links, S_R. This quantity refers to the fraction of edges in the

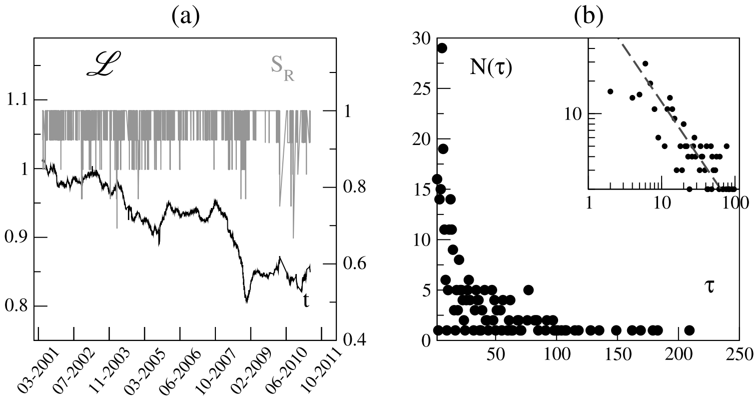

Fig. 2.7 Properties of the pruned trees. (**a**) Survival ratios and pruned tree length. (**b**) Number of occurrences of stable periods of length τ. *Inset*: same as in (**b**), but in log–log scale. The *dashed line* corresponds to τ^{-1}

MST, that survives between two consecutive trading days [14]:

$$S_R(t) = \frac{1}{N-1}\left|E(t) \cap E(t-1)\right|. \tag{2.7}$$

In this equation, $E(t)$ refers to the set of the tree's edges at date t, $\cap$ is the intersection operator, and $|.|$ gives the number of elements contained in the set. The survival ratios are very important for our study. Under normal circumstances, the topology of the trees should be very stable and the value of the survival ratio around one.

Concerning the stability of the trees, especially in 3-D, when focusing on the whole system, it is interesting to distinguish between reorganizations occurring in a specific market (i.e., between different delivery dates of the same contract) and reorganizations that change the nature of the links between two markets or even between two sectors. However, (2.7) gives the same weight to every kind of reorganization, whatever its nature. The trouble is, a change in intra-maturity links does not have the same meaning, from an economic point of view, as a movement affecting the relation between two markets or sectors. Because we are interested in the strong events that affect the markets, inter-market and inter-sector reorganizations are more relevant.[6] Thus, in order to distinguish between these categories of displacements, we "prune" the 3-D trees, that is, we only consider the links between markets, whatever the maturity considered. This pruning does not mean that maturity is removed from the analysis, but that the information on the specific maturity that is responsible for the connection between markets is no longer identified.

Pruned trees enable us to compute the length and the survival ratios on the sole basis of market links. As shown by Fig. 2.7a, most of the time, the survival ratios remain constant, with a value greater than 0.9. Thus, the topology of the trees is

[6] One can find all the results concerning the survival ratios in [10].

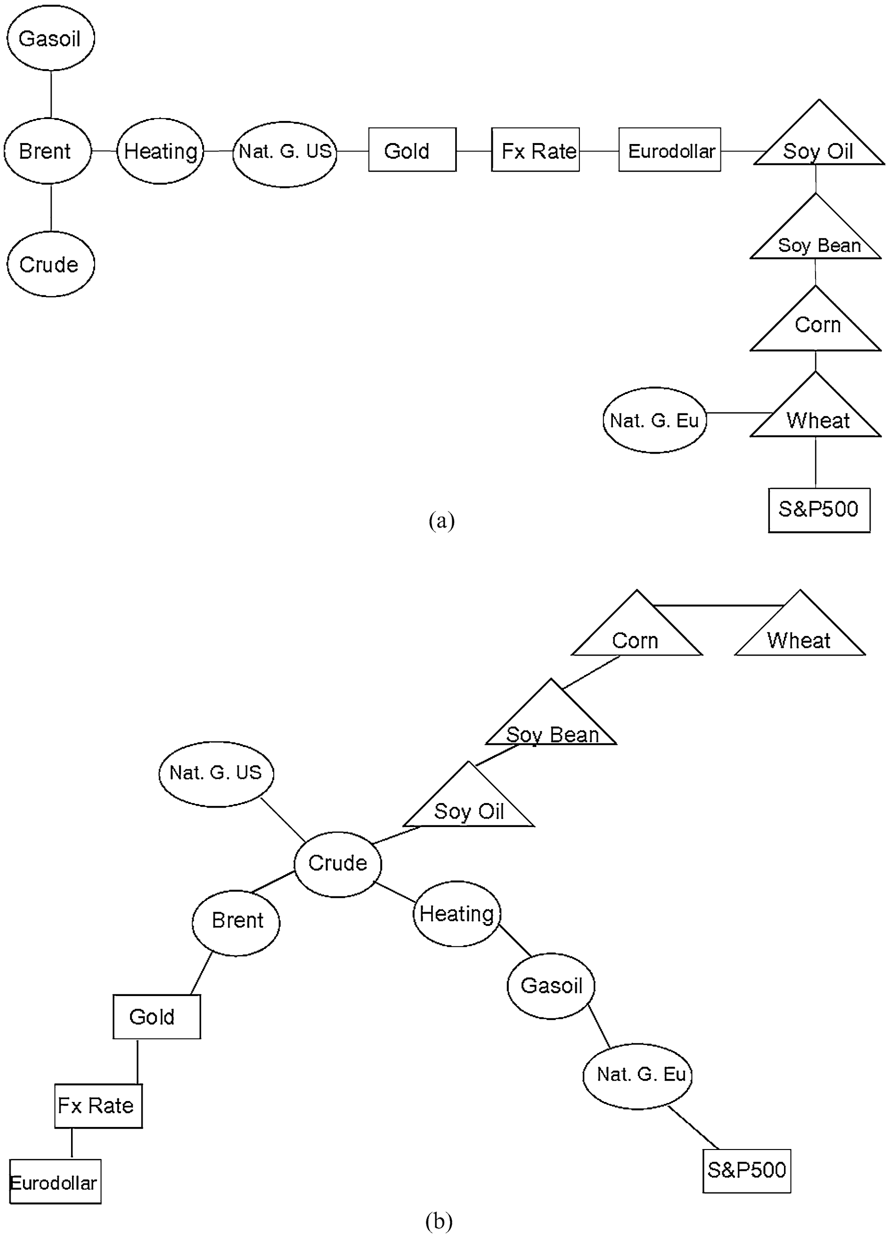

Fig. 2.8 Pruned MST of the events 09/02/2004 (**a**) and 16/09/2008 (**b**)

very stable: the shape of the most efficient path for the transmission of price shocks does not change much over time.

Another interesting characteristic of the pruned survival ratios is that they provide information on the lifetime of a configuration of such trees. In what follows, we measure the length of time τ between two different consecutive configurations and compute the occurrences $N(\tau)$ of these periods. Figure 2.7b displays our results. It shows that $N(\tau)$ decreases quickly with τ. The dashed line in the inset (in log–log scale) suggests that $N(\tau)$ is roughly proportional to τ^{-1}. Such a scaling behavior indicates that there is neither a typical nor an average lifetime of a new configuration of the MST.

On Fig. 2.7a, it is possible to identify several events which caused a significant re-arrangement of the tree. This is the case, for example, for two specific dates, namely 02/09/04 and 09/16/08, where 30 % of the edges has been shuffled. As illustrated by Fig. 2.8, a focus on these two dates shows that the trees are totally rearranged. In 2004, the MST becomes highly linear, the financial assets sector is at the center of the graph, and commodities appear mainly at the periphery of the system. Conversely, in 2008, the tree has a typical star-like shape showing an organization based on the different sectors studied.

2.6 Conclusions

We study systemic risk in energy derivative markets based on two choices. First, we focus on market integration, which is a favorable condition for the propagation of a price shock. Second, based on the fact that previous studies mainly focus on the spatio-temporal dimension of integration, we introduce the maturity dimension and perform a three-dimensional analysis.

In the context of empirical studies aiming to understand the organization and the dynamic behavior of a highly dimensional price system, our methodology, based on graph-theory, proves very useful. Moreover, Minimum Spanning Trees are particularly interesting in our framework, as they are filtered networks enabling us to identify the most probable and the shortest path for the transmission of a price shock.

We show that the topology of the MSTs tends towards a star-like organization in the spatial dimension, whereas the universal linear topology characterizes the maturity dimension. These two topologies merge in the 3-D analysis, and all of them are very stable. The star-like organization reproduces the three different sectors studied (energy, agriculture, and finance), and the chain-like structure reflects the presence of a Samuelson effect. The reasoning behind these findings is very important: the robustness of our methodology is embedded in these topologies.

Another contribution is to show that the American and European crude oils are both at the center of our large scale system; furthermore, they provide the links with the subsets of agricultural products and financial assets. Thus, crude oil is the best candidate for the transmission of price shocks. If such a shock appears at the periphery of the graph, it will necessarily pass through crude oil before spreading to other energy products and sectors. Moreover, a shock will have an impact on the whole system that will be all the greater the closer it is to the heart of the system.

Another important conclusion is that integration increases significantly on both the spatial and maturity dimensions. Such an increase can be observed in the whole price system. It is even more evident in the energy sector (with the exception of the natural gas markets), which becomes highly integrated at the end of our period. Thus, as time goes on, the heart of the price system becomes stronger whereas the peripheral assets do not change significantly. Moreover, the level of integration is higher in the maturity than in the spatial dimension: arbitrage operations are easier with standardized futures contracts written on the same underlying asset.

These results have very important consequences for regulatory as well as for diversification and hedging purposes.

Whereas the move towards integration started some time ago (and there is probably no way to refrain it), knowledge of its characteristics remains poor, especially from a holistic perspective. On the basis of this study, regulation authorities can see that their actions against systemic risk will not have the same impact depending on the market they are addressing. They should pay particular attention to the heart of the system: this is the place where price shocks spread more easily to other markets.

As far as diversification is concerned, portfolio managers must concentrate their positions on the most stable parts of the graph. More precisely, the benefits associated with diversification that rely on sub-indexes and focus on specific sectors of activity (agricultural products for example) should be more recurrent than those associated with large scale indexes.

Lastly, one important concern for hedging is the information conveyed by futures prices and its meaning. The increasing integration of derivative markets is probably not a problem for hedging purposes, until a price shock appears somewhere in the system. In such a case, the information related to the transmission path of the shock is important, as prices might temporarily become irrelevant.

These results call for further work. First, as previously exhibited, survival ratios make it possible to identify a few events leading to important reconfigurations of the trees. A thorough analysis of such phenomenon can provide the regulating authorities with a battery of stylized facts about the different possible manifestations of prices shocks and the signs announcing a future shock. Second, now that we have defined the paths for shock transmission, it is important to obtain directed graphs to determine the direction of the propagation of price movements. Third, a focus on the gas market, which exhibits a striking pattern of cross-maturity connections, can be of interest for energy specialists.

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References

1. Banavar J, Maritan A, Rinaldo A (1999) Size and form in efficient transportation networks. Nature 399:130
2. Buyukşahin B, Haigh MS, Robe MA (2010) Commodities and equities: ever a "market of one"? J Altern Invest 76–95
3. Buyuksahin B, Robe MA (2010) Speculators, commodities and cross-market linkages. Working Paper, American University, Nov 2010

4. Chong J, Miffre J (2010) Conditional returns correlations between commodity futures and traditional assets. J Altern Invest 12:61–75
5. Garlaschelli D, Caldarelli G, Pietronero L (2003) Universal scaling relations in food webs. Nature 423:165–168
6. Gower JC (1966) Some distance properties of latent root and vector methods used in multivariate analysis. Biometrika 53(3/4)
7. Korniotis GM (2009) Does speculation affect spot prices levels? The case of metals with and without futures markets. Finance and Economics. Discussion Series Working Paper 2009-29. Board of Governors of the Federal Reserve
8. Lautier D, Raynaud F (2011) Energy derivative markets and systemic risk. Technical report, French Energy Council
9. Lautier D, Raynaud F (2011) Statistical properties of derivatives: a journey in term structures. Phys A, Stat Mech Appl 390(11):2009–2019
10. Lautier D, Raynaud F (2012) Systemic risk in energy derivative markets: a graph theory analysis. Energy J 33(3):217–242 (forthcoming)
11. Lautier D, Raynaud F (2012) High dimensionality in finance: the advantages of the graph-theory. In: Batten J, Wagner N (eds) Derivatives securities pricing and modelling, Emerald Publishing, June
12. Mantegna RN (1999) Hierarchical structure in financial markets. Eur Phys J B 11
13. Milonas N (1986) Price variability and the maturity effect. J Futures Mark 6:443
14. Onnela J-P, Chakraborti A, Kaski K, Kertész J (2003) Dynamic asset trees and black monday. Physica A 324
15. Onnela J-P, Chakraborti A, Kaski K, Kertész J, Kanto A (2003) Dynamics of market correlations: taxonomy and portfolio analysis. Phys Rev E 68(5):056110
16. Qian M-C, Jiang Z-Q, Zhou W-X (2010) Universal and nonuniversal allometric scaling behaviors in the visibility graphs of world stock market indices. J Phys A, Math Theor 43(33):335002
17. Samuelson PA (1965) Proof that properly anticipated prices fluctuate randomly. Ind Manage Rev 6
18. Tang K, Xiong W (2011) Index investing and the financialization of commodities. NBER, Working Paper No. 16325

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