Definition (Proper coloring)

A **proper coloring** of a graph is an assignment of colors to the vertices of the graph so that no two adjacent vertices have the same color.

Definition (Proper coloring)

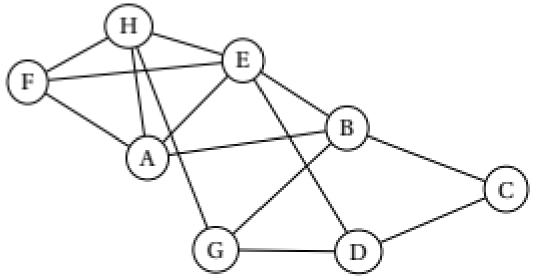
Let $G = (V, E)$ be an undirected graph and C a set of colors. A coloring of G is a mapping c from V to C. c is said a **proper coloring** if and only if $\forall (v, v') \in E \Rightarrow c\ (c) \neq c\ (c')$

Definition (k-coloring)

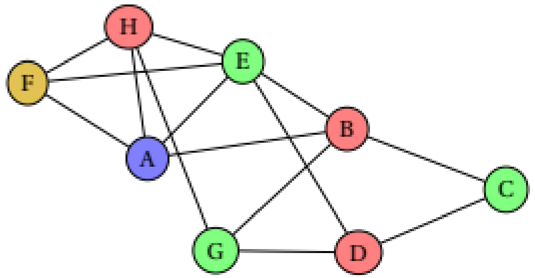
Let $G = (V, E)$ be an undirected graph and k is a non-zero integer. A k-coloring of G is a coloring c of G such that $card(c(V)) = k$. G is said k-colorable if and only if there is a proper k-coloring of G.

Example

Consider the following graph :



A coloring of the above graph may be :



Definition (Chromatic number)

The chromatic number of a graph G is the minimum number of colors required in a proper coloring; it is denoted $\chi(G)$ and is defined as follows :

$$\chi(G)=\min\{card(c(V), c \text{ is a proper coloring of } G\}$$

Definition (Optimal coloring)

Let G=(V,E) be an undirected graph. An optimal proper coloring c of G is $\chi(G)$-coloring of G.

Searching of coloring

Exact algorithm

- In such algorithm, each of the k^n assignments of k colors to n vertices is considered and checks for each if it is proper.

- Algorithm used only for smallest input graphs.

Searching of coloring

WELSH and POWELL algorithm

This class is intended to implement the Welsh-Powell algorithm for the problem of graph coloring. It provides a greedy algorithm that runs on a static graph.

This is an iterative greedy algorithm:

Step 1: All vertices are sorted according to the decreasing value of their degree in a list V.

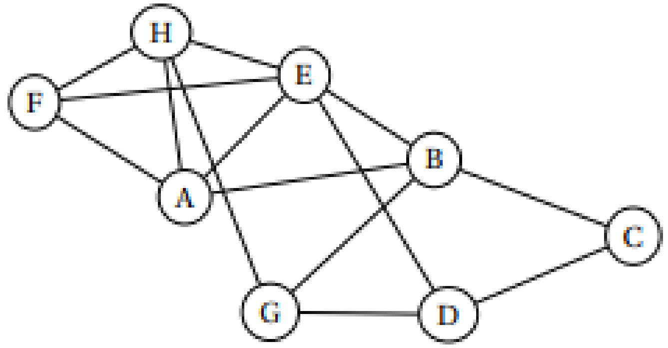
Step 2: Colors are ordered in a list C.

Step 3: The first non colored vertex v in V is colored with the first available color in C (*available* means a color that was not previously used by the algorithm).

Step 4: The remaining part of the ordered list V is traversed and the same color is allocated to every vertex for which no adjacent vertex has the same color.

Step 5: Steps 3 and 4 are applied iteratively until all the vertices have been colored.

Application : Welsh et Powell

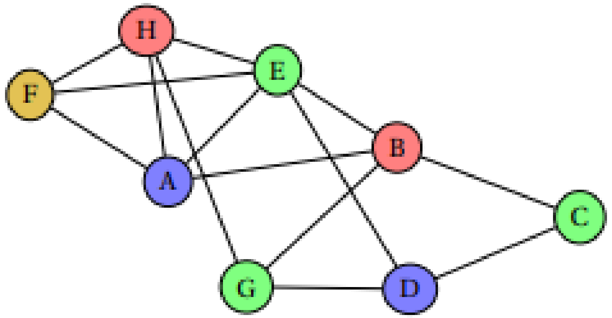


Exemple. On reprend l'exemple page précédente et on trie les sommets par degré décroissant :

v	E	A	B	H	D	F	G	C
$d(v)$	5		4			3		2

L'application de l'algorithme de WELSH et POWELL donne la coloration suivante.

- Couleur nº 1 : E, G puis C
- Couleur nº 2 : A puis D
- Couleur nº 3 : B puis H
- Couleur nº 4 : F



Proposition (Majorant for the chromatic number)

Let $G = (V, E)$ be a undirected graph and et Δ = max{d(v)|v $\in$ V}. Then $\chi(G)$ <= Δ+1

Proposition (Chromatic number of a full graph)

Let n be a non-zero integer. Then $\chi(K_n) = n$.

Proposition (Increasing chromatic number)

Let $G = (V, E)$ be a undirected graph and $G' = (V', E'), avec\ V' \subseteq V et V' \subseteq V$, a sub-graph of G. Then $\chi(G') \leq \chi(G)$.

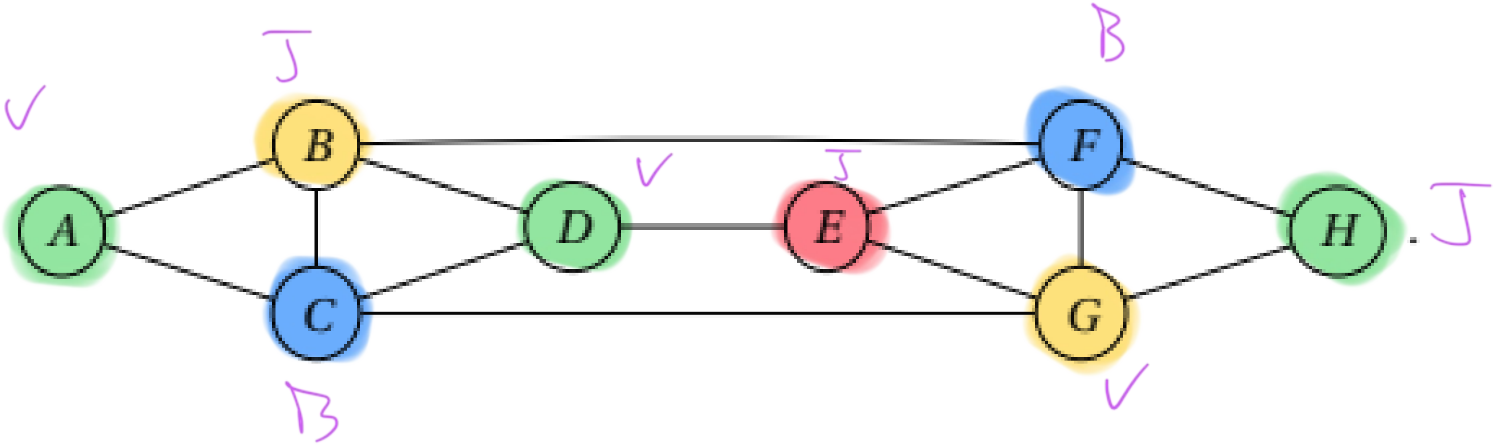
Proposition(Minorant for the chromatic number)

Let $G = (V, E)$ be a undirected graph. If G contains a full sub-graph with p vertices, $\chi(G) \geq p$.

Exercise

1. Give a coloring of the below graph by applying WELSH *et* POWELL algorithm
2. Give a 3-coloring of the graph
3. Show that $\chi(G)$=3
4. Conclude

3 couleurs

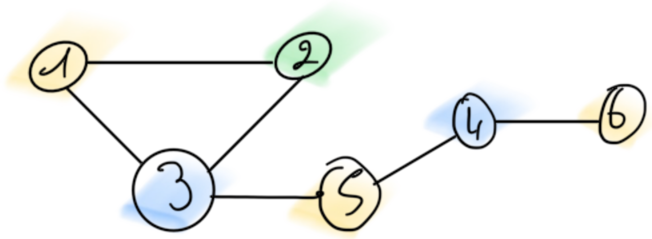


Exercise

We want to plan the end of session exams in ING3. As there are subjects common to several options, we want to pool the tests.

— *Option 1 contains modules m1, m2 and m3.*
— *Option 2 contains modules m4 and m6.*
— *Option 3 contains modules m4 and m5.*
— *Option 4 contient modules m3 and m5.*

Knowing that each exam lasts three hours, what is the minimum number of hours necessary for all exams



⇒ Il faut au mini 3 créneaux de 3H.