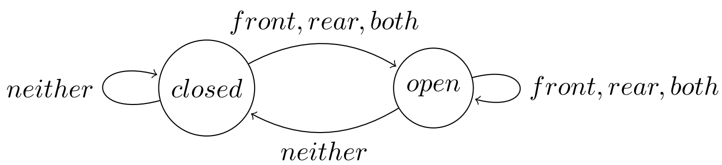
# 2 Lesson 2

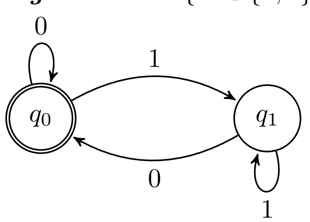
## 2.1 Finite Automation

***e.g.* 2.1.** *Automatic Door*




| | $front$ | $rear$ | $both$ | $neither$ |
|---|---|---|---|---|
| $front$ | ✓ | × | ✓ | × |
| $rear$ | × | ✓ | ✓ | × |

***e.g.* 2.2.** $L = \{w \in \{0,1\}^* | w = w_1 w_2 \cdots w_n, w_n = 0\}$




***remark*.** $q_0$ : *accepted state*

$Q = \{q_0, q_1\}, \Sigma = \{0, 1\}, \delta : Q \times \Sigma \to Q$

| | 0 | 1 |
|---|---|---|
| $q_0$ | $q_0$ | $q_1$ |
| $q_1$ | $q_0$ | $q_1$ |

***def* 2.1.** *(finite automation)*

*A* ***finite automation*** *is a 5-tuple* $(Q, \Sigma, \delta, q_0, F)$*, where:*

1. $Q$ *is a finite set called the states*
2. $\Sigma$ *is the alphabet*
3. $\delta : Q \times \Sigma \to Q$ *is the transition function*

4. $q_0$ *is the start state*

5. $F \subseteq Q$ *is the set of accept states*

***def* 2.2.** *Let* $M = (Q, \Sigma, \delta, q_0, F)$ *be a finite automation, let* $w = w_1 w_2 \cdots w_n$ *be a string, where each* $w_i \in \Sigma$.

*Then* $M$ *accept* $w$ *if there is a sequence of states* $r_0, r_1, \cdots, r_n \in Q$, *such that:*

1. $r_0 = q_0$

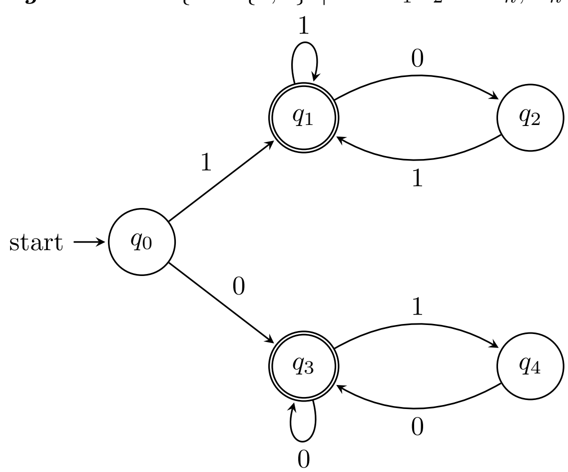
2. $\delta(r_i, w_{i+1}) = r_{i+1}$, *for* $i = 0, 1, 2, \cdots, n-1$

3. $r_n \in F$

***def* 2.3.** *If L is the set of strings that M accepts, we say L is the language of M, and write L(M) = L, we say M recognizes/decides/accepts L.*

*If M accepts no string, it recognizes one language namely, the empty language.*

***e.g.* 2.3.** $L = \{w \in \{0,1\}^* | w = w_1 w_2 \cdots w_n, w_n = w_1\}$




## 2.2 Regular Language

***def* 2.4.** *(regular language)* $L \subseteq \Sigma^*$ *is a* ***regular language*** *if there is a finite automation that accepts* $L$

*Let* $A, B \subseteq \Sigma^*$, *define:*

- *(union)* $A \cup B = \{x \in \Sigma^* | x \in A \text{ or } x \in B\}$

- *(concatenation)* $AB = \{xy | x \in A, y \in B\}$

- *(star)* $A^* = \{x_1x_2\cdots x_k | k \geq 0, x_1, x_2, \cdots, x_k \in A\}$

***thm* 2.5.** *If $A_1, A_2$ are regular languages, so is $A_1 \cup A_2$*

*Proof.* *Let $M_1 = (Q_1, \Sigma_1, \delta_1, q_{10}, F_1)$ accepts $A_1$, $M_2 = (Q_2, \Sigma_2, \delta_2, q_{20}, F_2)$ accepts $A_2$, construct $M = (Q, \Sigma, \delta, q_0, F)$:*

1. $Q = Q_1 \times Q_2 = \{(r_1, r_2) | r_1 \in Q_1, r_2 \in Q_2\}$
2. $\delta : Q \times \Sigma \to Q$ *is defined as for each* $(r_1, r_2) \in Q$*, and each* $a \in \Sigma$*, let* $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$
3. $q_0 = (q_{10}, q_{20})$
4. $F = \{(r_1, r_2) | r_1 \in F_1 \text{ or } r_2 \in F_2\}$

□

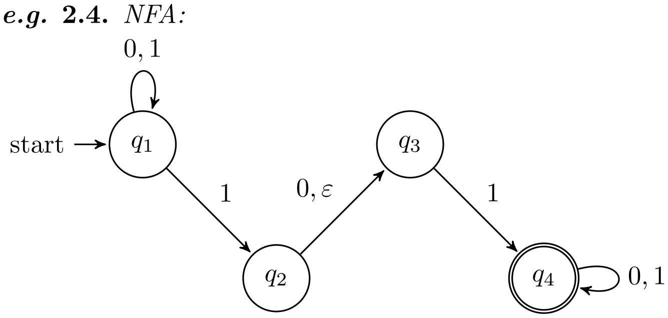
***remark.*** *so is $A \cap B$*

***thm* 2.6.** *If $A_1, A_2$ are regular languages, so is $A_1A_2$*

- ***DFA****: deterministic finite automation*
- ***NFA****: nondeterministic...*

***If at least one of these processes accepts, then the entire computation accepts.***

***e.g.* 2.4.** *NFA:*




*input:* 010110

***e.g.* 2.5.** *Let $A$ be the language consisting of all strings over $\{0, 1\}$ containing a 1 in the third position from the end (e.g., 000100 is in $A$ but 0011 is not). The following four-state NFA recognizes $A$.*

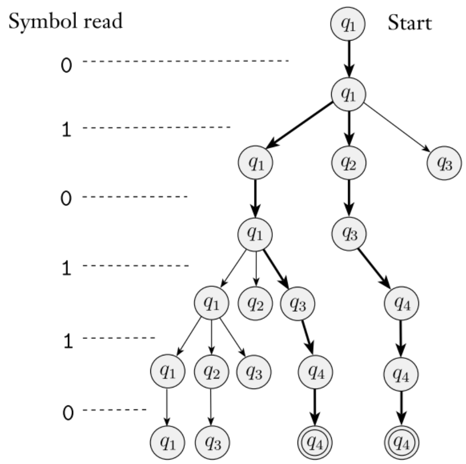



Figure 1: *The computation of NFA on input 010110*

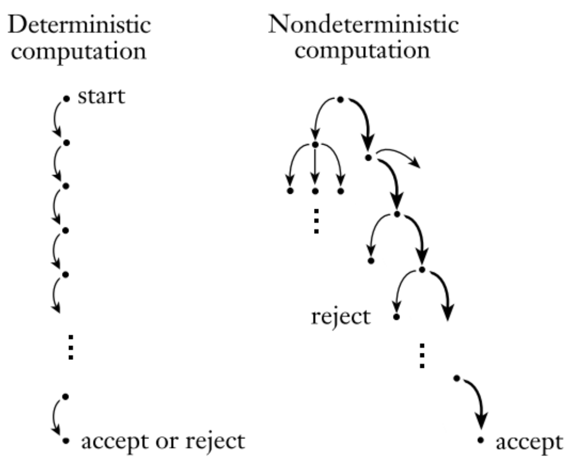
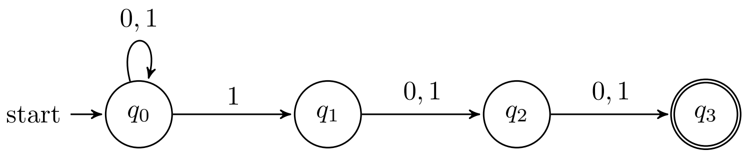
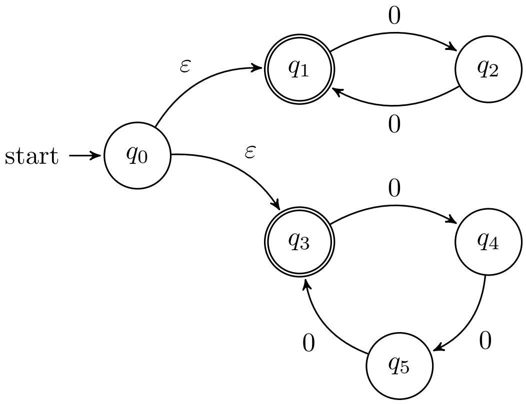



Figure 2: *Deterministic and nondeterministic computations with an accepting branch*




***e.g.* 2.6.** $L = \{0^k, 2|k \text{ or } 3|k\}$




***def* 2.7.** *(NFA)*

*An **NFA** is a 5-tuple* $(Q, \Sigma, \delta, q_0, F)$*,where:*

1. $Q$ *is a finite set of states*
2. $\Sigma$ *is the alphabet*
3. $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to P(Q)$ *is the transive funtion*
4. $a_0 \in Q$ *is the start state*
5. $F \subseteq Q$ *is the set of accept states*

***def* 2.8.**

*Let* $N = (Q, \Sigma, \delta, q_0, F)$ *be an NFA, and let* $w \in \Sigma^*$*. Say N accepts w if we can write* $w = y_1 y_2 \cdots y_m$*, where* $y_i \in \Sigma \cup \{\varepsilon\}$*, and there exist* $r_0, r_1, \cdots, r_m \in Q$*, such that:*

1. $r_0 = q_0$
2. $r_{i+1} \in \delta(r_i, y_{i+1})$ *for* $i = 0, 1, \cdots, m-1$
3. $r_m \in F$

***thm* 2.9.** *Every NFA has an equivalent DFA*

*Proof.* □