PHYC90045 Introduction to Quantum Computing

Week 12

Lecture 23
Quantum Computing architectures and quantum complexity classes

Lecture 24
Quantum Computing Review

Lab 12
HHL algorithm using the QUI

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Quantum Computing Review

Physics 90045
Lecture 24

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Review
(Selected Highlights)

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Linear Algebra and Dirac notation

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle$$

For qubits we can use column vectors to represent a convenient basis for kets:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Computational basis states

$$a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$a, b \in \mathbb{C}$$

General qubit state
a, b are "amplitudes"

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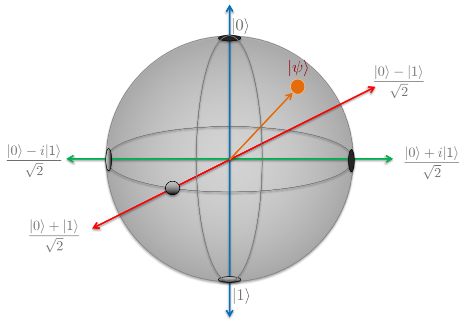
The Bloch Sphere

A convenient geometric representation of single qubit states is the Bloch sphere:



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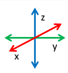
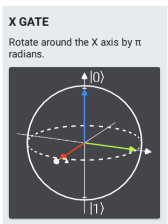
Single Qubit Gates

Circuit symbol: X



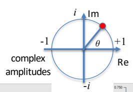


Matrix representation: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Action on ket states: $\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$

QUI example:

$\frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$



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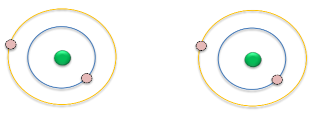
Multiple Qubit States

$$\begin{bmatrix}a\\b\end{bmatrix}\otimes\begin{bmatrix}c\\d\end{bmatrix}=\begin{bmatrix}ac\\ad\\bc\\bd\end{bmatrix}\begin{array}{l}\leftarrow \text{00 amplitude}\\ \leftarrow \text{01 amplitude}\\ \leftarrow \text{10 amplitude}\\ \leftarrow \text{11 amplitude}\end{array}$$



$|\psi\rangle = a|0\rangle + b|1\rangle$ $\quad$ $|\phi\rangle = c|0\rangle + d|1\rangle$

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Two qubit operations

$\alpha\,|0\rangle + \beta\,|1\rangle$

$|0\rangle$

$|\psi\rangle$ $\quad$ $|\psi'\rangle$

Before the CNOT, the state is:

$$|\psi\rangle = (\alpha\,|0\rangle + \beta\,|1\rangle)\otimes|0\rangle = \alpha\,|00\rangle + \beta\,|10\rangle$$

After the CNOT, the state is:

$$|\psi'\rangle = \alpha\,|00\rangle + \beta\,|11\rangle$$

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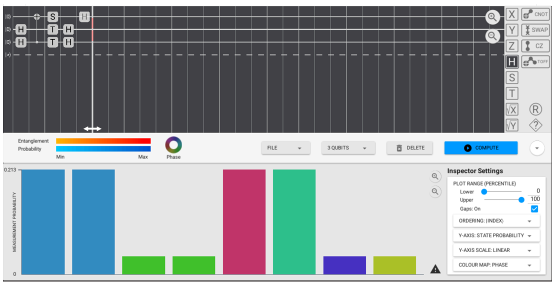
Using the QUI

In labs you will learn to use the "Quantum User Interface" (QUI) to construct circuits. Your first lab will be all about single qubit rotations.



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Entanglement

We can never find a, b, c, d, s.t.

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} \neq (a\,|0\rangle + b\,|1\rangle) \otimes (c\,|0\rangle + d\,|1\rangle)$$

A state which is not separable is called an **entangled** state.

Entanglement is a uniquely quantum mechanical property, with no direct classical analogue.

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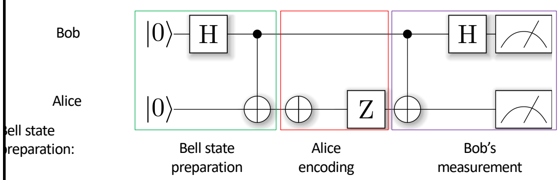
Dense Coding Circuit



$$|00\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}|0\rangle \rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$0,0$	$\frac{\lvert 00\rangle + \lvert 11\rangle}{\sqrt{2}} \rightarrow \frac{\lvert 00\rangle + \lvert 11\rangle}{\sqrt{2}}$	
$1,0$	$X_2\frac{\lvert 00\rangle + \lvert 11\rangle}{\sqrt{2}} \rightarrow \frac{\lvert 01\rangle + \lvert 10\rangle}{\sqrt{2}}$	
$0,1$	$Z_2\frac{\lvert 00\rangle + \lvert 11\rangle}{\sqrt{2}} \rightarrow \frac{\lvert 00\rangle - \lvert 11\rangle}{\sqrt{2}}$	
$1,1$	$X_2Z_2\frac{\lvert 00\rangle + \lvert 11\rangle}{\sqrt{2}} \rightarrow \frac{\lvert 01\rangle - \lvert 10\rangle}{\sqrt{2}}$	

Alice applies one of four different operations to her qubit, based on the classical information she would like to send.

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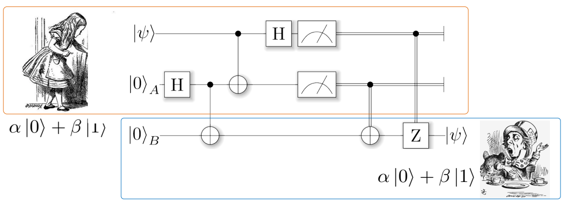
Teleportation



i.e. after correction Bob has successfully reconstructed Alice's original state.

Alice measures	Bob's qubit	
$0,0$	$\alpha\lvert 0\rangle + \beta\lvert 1\rangle$	$\alpha\lvert 0\rangle + \beta\lvert 1\rangle \rightarrow \alpha\lvert 0\rangle + \beta\lvert 1\rangle$
$0,1$	$\alpha\lvert 1\rangle + \beta\lvert 0\rangle$	$X(\alpha\lvert 1\rangle + \beta\lvert 0\rangle) \rightarrow \alpha\lvert 0\rangle + \beta\lvert 1\rangle$
$1,0$	$\alpha\lvert 0\rangle - \beta\lvert 1\rangle$	$Z(\alpha\lvert 0\rangle - \beta\lvert 1\rangle) \rightarrow \alpha\lvert 0\rangle + \beta\lvert 1\rangle$
$1,1$	$\alpha\lvert 1\rangle - \beta\lvert 0\rangle$	$ZX(\alpha\lvert 1\rangle - \beta\lvert 0\rangle) \rightarrow \alpha\lvert 0\rangle + \beta\lvert 1\rangle$

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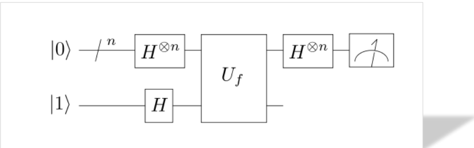
Deutsch-Josza algorithm

- Given a boolean function, ***f***, determine if:
  f is constant (always gives the same result), or
  f is balanced (gives equal numbers of 0s and 1s)

- **Classical algorithm** *(worst case) needs $2^n/2+1$ queries*
- ***Quantum algorithm*** *needs just 1 query.*



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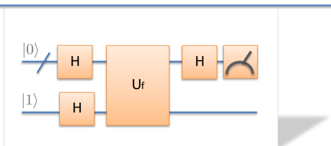
Bernstein-Vazirani Algorithm



Given a Boolean function, *f*:

$$f(x) = x \cdot s \mod 2$$

find s.

$$x \cdot s = \sum_i x_i s_i$$

- **Classical algorithm** *needs n queries*
- ***Quantum algorithm*** *needs just 1 query.*

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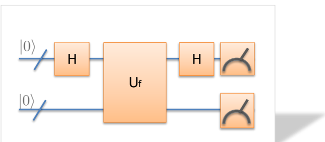
Simon's Algorithm



Given a 2-to-1 function, f, such that

$$f(x) = f(x \oplus a)$$

Find a.

Classical algorithm: $O(\sqrt{N})$ Queries to the oracle (probabilistically)

Quantum algorithm: $O(n)$ Queries to the oracle

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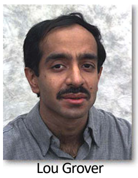
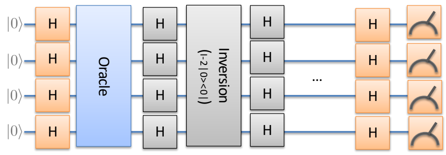
Grover's Algorithm

- Unordered search, find one marked item among many
- Classically, this requires N/2 uses of the oracle
- Quantum mechanically, requires only O(√N).



Lou Grover



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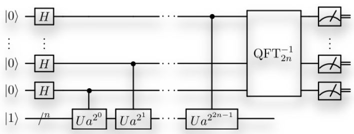
Shor's algorithm

- Efficient quantum algorithms for **factoring** semiprime numbers
- Best known classical algorithm is number field sieve (exponential in bit-length).
- Underpins the RSA cryptosystem
- Hidden Subgroup Problems (eg. Discrete logarithm) similar.

Peter Shor



Shor, Proc 35th Ann Symp of Comp Sci, 26, (1995)

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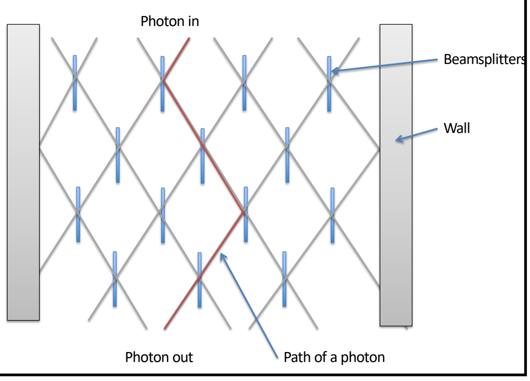
Quantum Supremacy: Boson Sampling



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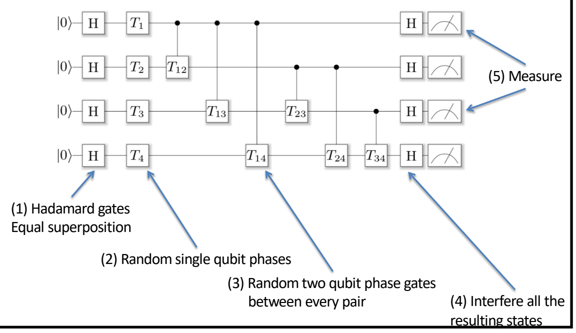
IQP Circuits



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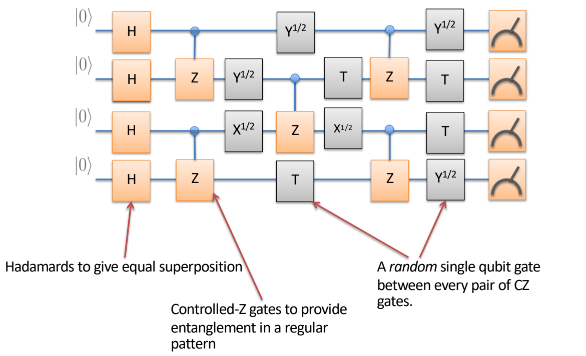
Pseudo-random Circuits



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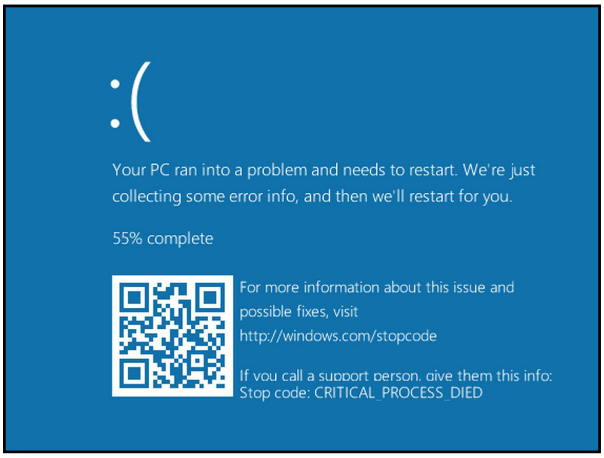

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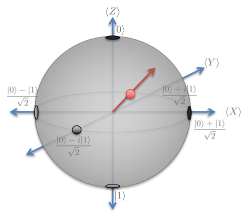
Purity for one qubit



If the distance from the origin to the state is measured to be r, the purity is:

$$P = \frac{1+r^2}{2}$$

Maximum purity of 1 for all pure states.

Minimum purity of ½ for a completely mixed state.

Note: There's a more technical definition of purity in terms of density matrices, which we won't cover in this course.

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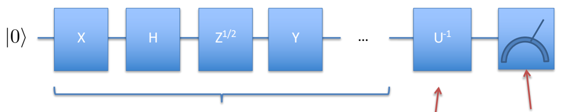
Randomized Benchmarking

How good are our gates individual gate? We want a number for how much error doing each operation is. One way of determining this is to perform **randomized benchmarking**.



Apply a random sequence of gates

Apply the inverse of the whole sequence

Make a measurement: Should measure 0

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Quantum Error Correction

Similar to classical error correction codes, we can have a quantum repetition code:

$$|0\rangle \rightarrow |000\rangle \quad \text{"Logical 0"}$$
$$|1\rangle \rightarrow |111\rangle \quad \text{"Logical 1"}$$

In particular, a quantum superposition would be encoded as:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$

Two key differences between quantum and classical error correction codes:

1. Cannot measure the codewords directly; would collapse the state
2. Phase errors

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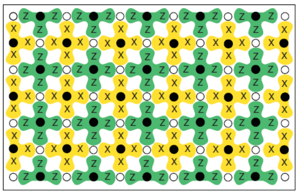
The Surface Code

- A topological code suited to solid state (Kitaev 1997, Raussendorff 2007)
- A remarkably high threshold of ~1% (Wang et al, 2011)
- Parallel and synchronous control required



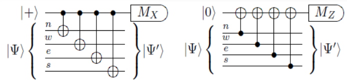


Fig. 5. Circuits used to measure X-stabilizers (left) and Z-stabilizers (right).

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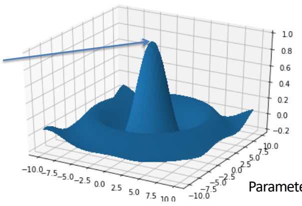
Optimization Problems

Given some cost-function or "objective function" we would like to maximize/minimize. Often the inputs/parameters are constrained.



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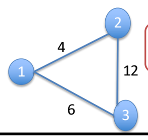
Number partitioning as a QUBO problem

But if we square, we should get a positive solution (or zero). We want to find the assignment of spins which has the minimum energy (ie. closest to zero):

$$H = \left(\sum_i w_i Z_i\right)^2 = \sum_{i \neq j} 2 w_i w_j Z_i Z_j + \sum_i w_i^2 I$$

Coupling is the product of numbers

Eg. For the set **{1, 2, 3}**:



$$H = 4Z_1Z_2 + 6Z_1Z_3 + 12Z_2Z_3 + 14I$$

Finding minimum energy state will solve the problem!

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VQE/QAOA

QUI/Quantum Computer

(1) Prepare a trial state $|\psi(\theta)\rangle$

on the quantum computer, where θ can be any adjustable gate parameter.

(2) **Measure** the expectation value of the energy, E.

(3) Use a **classical optimization** technique such as the Nelder–Mead simplex method, determine new values of θ that decrease E.

Repeat these steps until the value of the energy converges

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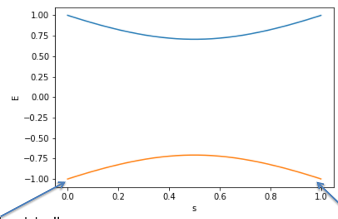
Adiabatic Quantum Computation

$$H(s) = (1-s)H_x + sH_p$$
$$s = t/T$$



$$H_x = X$$
$$H_p = Z$$

"Landau-Zener" avoided crossing

System is originally in the state: $|-\rangle$

Time

System ends up in the state: $|1\rangle$

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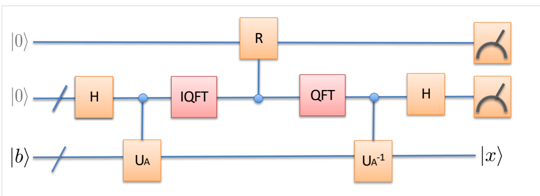
HHL Algorithm



- HHL algorithm solves systems of linear equations
- Need a method for emulating *exp(iAt)*
- Can be used as a quantum subroutine
- Has a runtime, exponentially faster than classical versions.

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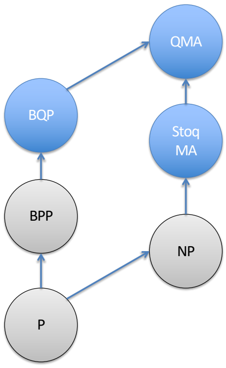
Quantum Complexity Classes



Quantum Complexity Classes

Classical Complexity Classes

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Questions?

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