PHYC90045 Introduction to Quantum Computing

Assignment 2

Due: 5pm Thursday 30th October, 2020

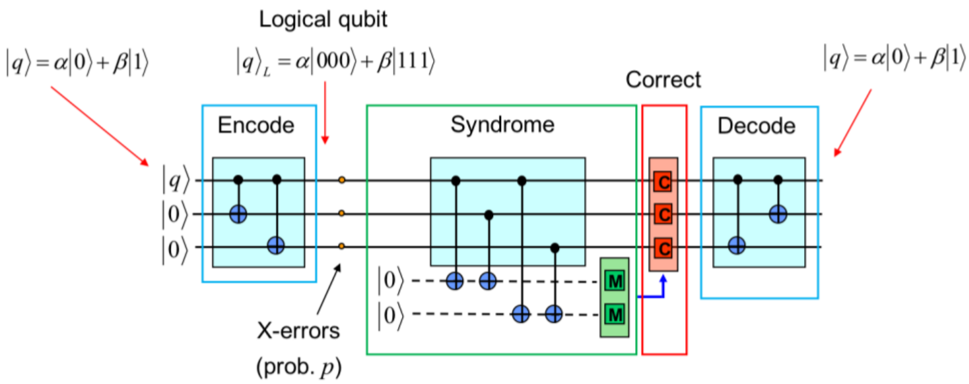
Welcome to Assignment 2 for PHYC90045 *Introduction to Quantum Computing*.

Instructions: Work on your own, attempt all questions. Hand in your completed written work on or before the due date as per instructions above (standard late penalties will apply). The QUI circuits you create for this assignment must be shared (to do this you must enable sharing through the file menu) and saved with the indicated filenames. Please submit your assignment online via LMS. To do this convert your answers to a PDF

Total marks = 40

(1) Phase Flip Quantum Error Correction [7 marks = 2, 1, 4]

(a) Show how you can convert the 3 qubit bit-flip quantum error correction code (shown below) into a phase-flip (i.e. corrects a Z error) by introducing H gates.



Program the phase-flip error correction circuit into the QUI, together with a Toffoli-based correction circuit. Implement your circuit to encode (and decode) the state $|q\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$. Save your file as "Student number_Assignment-2_Q1".

(b) What are the stabilizers of the three qubit phase flip code?

(c) Implement your circuit using an IBM Q device. What is the probability that you measure the encoded state? What is the probability that no error is detected? Does this circuit actually protect against errors – briefly discuss, giving reasons why or why not.

(2) Toffoli gate on IBM Q device. [8 marks = 3, 2, 3]

We have seen that many quantum algorithms rely on the Toffoli gate. In practice, most quantum computer architectures do not implement a 3-qubit gate directly and so the Toffoli gate must be constructed from (compiled into) a decomposition involving 1 and 2-qubit gates.

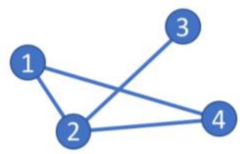
a) Using any IBM 5-qubit quantum computing device, implement the best Toffoli gate you can (averaging over 1000 instances). Provide full details of the including your Qiskit code, device and circuit image, and a summary of the results in the for each input in the computational basis. Your circuit must work on an arbitrary input state, and you will be judged on the circuit optimisation and final fidelity obtained.

b) Assume the main source of error in a) is an effective Y-rotation error. Using the QUI, find the continuous error model (including mean and standard deviation) that most closely matches the distribution of probabilities most closely matching your results in a) for the 110 input state. Save your circuit as "Student number_Assignment-2_Q2b".

c) The IBM 15-qubit QC *Melbourne* has a different connection architecture. Using qubits of your choice, program a Toffoli gate using Qiskit and compile and run on the hardware (averaging over 1000 instances, start in 110 state). Your circuit must work on an arbitrary state, and you will be judged on the final fidelity obtained for the 110 case. Compare the fidelity with that obtained in a) and explain the difference. Provide full details of the program (text code and compiled circuit image), and the results (probability histogram).

(3) QAOA on IBM Q devices [8 marks = 2, 1, 3, 2]

(a) Write out the Hamiltonian resulting from the MAX-CUT problem resulting from the following graph:



problem graph

(b) What are the solution(s) you would expect for this problem?

(c) Using QUI, write out a corresponding QAOA circuit (using just p=1 iterations), find the optimal angles, and probability of successfully finding one of the solution states. Save your circuit as "Student number_Assignment-2_Q3". What is the probability of finding the solution state(s)?

(d) Implement the same circuit on an IBM Q device, show the circuit and measurement results, and hence determine the probability of success using an IBM quantum device. Briefly comment on your results.

(4) Variational Quantum Eigensolver [7 marks = 2, 2, 3]

Consider a problem for which the Hamiltonian can be written as:

$$\mathrm{H} = \mathrm{c}_0\mathrm{I} + \mathrm{c}_1\mathrm{Z}_1 + \mathrm{c}_2\mathrm{Z}_2 + \mathrm{c}_3\mathrm{Z}_1\mathrm{Z}_2 + \mathrm{c}_4\mathrm{Y}_1\mathrm{Y}_2 + \mathrm{c}_5\mathrm{X}_1\mathrm{X}_2 + \mathrm{c}_6\mathrm{X}_1\mathrm{Y}_2$$

The trial wavefunction, from which we will start our search for the lowest energy configuration, can be shown to have the form

$$|\varphi(\theta)\rangle = (\cos(\theta)\, I - i\sin(\theta)\, X_1 Y_2)|01\rangle$$

a) The circuit to construct $|\varphi(\theta)\rangle$ differs from the circuit to get ZZ-couplings in the QAOA applications we have considered so far. Based on the circuit for a ZZ coupling (e.g. shown in 5d), and the fact that $X = H\,Z\,H$ and $Y = \left(\sqrt{X}\right)^{\dagger} Z\sqrt{X}$ (where H are Hadamard operations), construct a circuit in the QUI that produces $|\varphi(\theta)\rangle$. Save your circuit as "Student number_Assignment-2_Q4".

b) In order to minimize the energy of the Hamiltonian, θ must be varied to minimize

$$\langle H\rangle = c_0 + c_1\langle Z_1\rangle + c_2\langle Z_2\rangle + c_3\langle Z_1 Z_2\rangle + c_4\langle Y_1 Y_2\rangle + c_5\langle X_1 X_2\rangle + c_6\langle X_1 Y_2\rangle$$

where for each observable $\mathcal{O}$ we must measure the quantity $\langle\mathcal{O}\rangle = \langle\varphi(\theta)|\mathcal{O}|\,\varphi(\theta)\rangle$. Note that measuring in the X or Y bases is equivalent to measuring in Z with a suitable basis-change gate. Modify your circuit to measure the expectation values required to determine $\langle H\rangle$ for a given value of θ. Rather than submit all the circuits, describe how you are programming the measurement of each term in $\langle H\rangle$.

c) Consider the case where the c's are given by:

$c_0 = -0.470$
$c_1 = 0.342$
$c_2 = -0.446$
$c_3 = 0.573$
$c_4 = c_5 = 0.091$
$c_6 = 0.010$

By trying out different values of θ, find the minimum energy for the Hamiltonian.

(5) Three qubit K-state [10 marks = 4, 3, 1, 2]

A three-qubit K-state given by

$$|K\rangle = \frac{\sqrt{3}|100\rangle - e^{i\pi/4}|010\rangle + \sqrt{2}|001\rangle}{\sqrt{6}}$$

(a) Using QUI, implement a circuit to create the three-qubit K-state. Save your circuit as "<Student_number>_Assignment-2_Q5" and briefly explain the reasoning you used to create this circuit.

(b) On an IBM-Q device of your choice, make a choice of qubits and implement this circuit, attempting to achieve the highest fidelity state possible. Describe which device, and which qubits you chose, and any additional optimization which you made.

(c) Measure each of the qubits in the Z-basis, and record the results. Plot the measurement results. Comment on the results you obtained, including any errors.

(d) Apply a Hadamard gate to every qubit directly before the measurement and measure the result. What is the resulting probability of measuring the $|000\rangle$ state. What should the probability be ideally? Show your working.