

PHYC90045 Introduction to Quantum Computing

Week 7

Lecture 13 - Quantum Supremacy

- 11.1 Boson Sampling
- 11.2 IQP Problem
- 11.3 Google's pseudorandom circuits

Lecture 14 - Errors

- 12.1 Quantum errors: unitary and stochastic errors
- 12.2 Purity
- 12.3 Tomography
- 12.4 Randomized Benchmarking

Lab 7

Quantum Supremacy and Errors

1

:(
Your PC ran into a problem and needs to restart. We're just collecting some error info, and then we'll restart for you.
55% complete



For more information about this issue and possible fixes, visit
<http://windows.com/stopcode>

If you call a support person, give them this info:
Stop code: CRITICAL_PROCESS_DIED

2

PHYC90045 Introduction to Quantum Computing

Real quantum devices

a Scanning electron micrograph (SEM) showing a nanowire structure with dimensions: 100 nm width, 100 nm height, and 100 nm depth. Labels include B_1 , B_2 , D , A , E , F , G , H , I , J , K , L , M , N , O , P , Q , R , S , T , U , V , W , X , Y , Z .

b Schematic diagram of a quantum device structure. It shows two donor regions with SET Drains (Source-Drain Transistors). A central region is labeled "Spin control".

c Timeline diagram showing the sequence of Read/Initiate and Spin control pulses over time.

d State diagram showing transitions between $|1\rangle$ (Electron spin), $|1\rangle\rangle$ (Electron spin + Nuclear spin), $|2\rangle$ (Electron spin), and $|2\rangle\rangle$ (Electron spin + Nuclear spin).

e Timeline diagram showing the sequence of Read/Initiate pulses over time.

f Timeline diagram showing the sequence of Read/Initiate pulses over time.

g Plot of current (I_{SD} in nA) versus time (ms). The plot shows periodic pulses corresponding to the Read/Initiate pulses.

Pin et al., Nature, 2012

3

PHYC90045 Introduction to Quantum Computing


THE UNIVERSITY OF
MELBOURNE

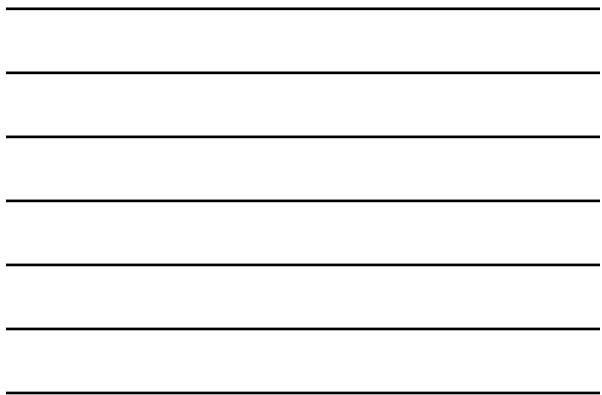
Two types of errors

Quantum computers are extremely fragile, and vulnerable to noise and errors. While errors occur in classical computing too, we're accustomed to very low error rates – our hard drives rarely forget what they store.

Two types of error:

- (1) **Systematic unitary errors** – eg. Control pulse error
- (2) **Random noise** – eg. Decoherence

4



PHYC90045 Introduction to Quantum Computing

Control Errors

The logo of The University of Melbourne, featuring a crest with a shield containing a book and a laurel wreath, with the university's name in a circular border.

Control of qubits requires high precision, and errors can sneak in. For example:

- Variations in magnetic fields across the sample, or variations in material properties.
- Stray electric fields, charge traps, strain.
- Applying a microwave pulse where the strength of the pulse is slightly too strong or too weak causes a systematic over-rotation or under-rotation.
- Cross-talk between gates.
- Unwanted interaction between qubits.

A close-up photograph of a quantum computing chip, showing a complex grid of metal lines and pads on a light-colored substrate. The image has a blue tint.

IBM image, Flickr

5



PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Systematic Errors in the QUI

The QUI is effectively a pristine qubit environment, but we can introduce such effects systematically and investigate how quantum gate errors affect the output of quantum circuits.

We will consider rotation errors around the cartesian axes in the QUI using the R-gate. For example, a Z-rotation error (or just “Z-error”) is a gate δZ defined as:

$$\delta Z \equiv \begin{pmatrix} e^{-i\epsilon/2} & 0 \\ 0 & e^{i\epsilon/2} \end{pmatrix}$$

where the level of error is governed by the angle ϵ (assumed to be small). Similarly, we could consider small rotations around other axes:

$$\delta X = R_X(\epsilon), \quad \delta Y = R_Y(\epsilon), \quad \delta Z = R_Z(\epsilon)$$

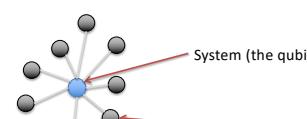
In the lab on Friday, we will consider the effect of these errors on the success of quantum circuits.

6

PHYC90045 Introduction to Quantum Computing

THE UNIVERSITY OF
MELBOURNE

Decoherence



In realistic quantum systems there will always be (unwanted) interaction between the qubits and the environment (electrons, spins, phonons, charge traps).

This causes a type of noise on the system (ie. qubits we want to protect) which we call decoherence.

7

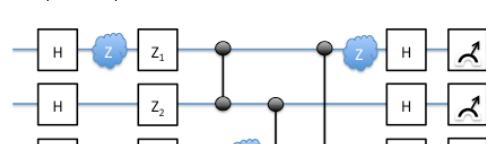
PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Modeling Stochastic Errors

Stochastic errors can be modeled in the QUI by randomly applying bit and phase flips to the qubits.



```

graph LR
    subgraph Q1 [Q1]
        H1[ ] --> Z1[ ]
        Z1 --> N1(( ))
        N1 --> H2[ ]
        H2 --> M1["Measurement"]
    end
    subgraph Q2 [Q2]
        H3[ ] --> Z2[ ]
        Z2 --> N2(( ))
        N2 --> H4[ ]
        H4 --> M2["Measurement"]
    end
    subgraph Q3 [Q3]
        H5[ ] --> Z3[ ]
        Z3 --> N3(( ))
        N3 --> H6[ ]
        H6 --> M3["Measurement"]
    end
  
```

Dephasing noise: Apply Z gate with some probability p .

Depolarizing noise: Apply either X , Y or Z gates, each with probability $p/3$.

Perform many “Monte Carlo” simulations, where errors are placed randomly on each run, and average the measurement results.

8

PHYC90045 Introduction to Quantum Computing

Pure and Mixed States

The logo of The University of Melbourne, featuring a shield with a figure holding a book and a staff, with the text "THE UNIVERSITY OF MELBOURNE" below it.

Pure states have no errors, and are perfectly coherent.

For example, the state

$$|\psi\rangle = |0\rangle$$

is a coherent, quantum state.

Mixed states may have errors, and are less coherent.

Imagine we took system in a pure quantum state, and noise – with say 20% probability- flipped the qubit around the X axis. That state would then be a “mixed” state.

9

PHYC90045 Introduction to Quantum Computing

Superposition vs mixed states



THE UNIVERSITY OF
MELBOURNE

Consider the pure state,

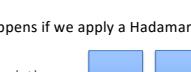
$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

and a mixed state which is

$$50\% |0\rangle \quad 50\% |1\rangle$$

Is it possible to tell these two states apart in experiment?

Consider what happens if we apply a Hadamard gate, then measure:



A quantum circuit diagram illustrating the effect of a Hadamard gate. On the left, a blue box contains the symbol $|\psi\rangle$, representing an input state. A horizontal blue line extends from this box to the right, representing the path of the qubit. In the center of this line is a blue square containing the letter 'H', representing a Hadamard gate. To the right of the gate is another blue box containing a curved arrow pointing upwards, representing the measurement of the qubit's state.

10

PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Superposition vs mixed states

For the mixed state if $|0\rangle$ were prepared :

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

50% of the time we will measure 0
 50% of the time we will measure 1

For the mixed state if $|1\rangle$ were prepared :

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

50% of the time we will measure 0
 50% of the time we will measure 1

So if the mixed state is prepared:

50% of the time we will measure 0
 50% of the time we will measure 1

For the pure state: $H|+\rangle = |0\rangle$ so **100%** of the time we will measure 0.

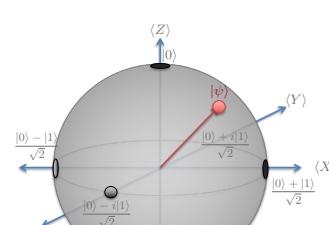
11

PHYS90045 Introduction to Quantum Computing

Purity on the Bloch Sphere



THE UNIVERSITY
OF MELBOURNE



Pure states lie on the surface of the Bloch Sphere

12

PHYC90045 Introduction to Quantum Computing

Mixed states on the Bloch Sphere

Mixed states lie inside the Bloch Sphere.

The closer to the origin, the more mixed.

13

PHYC90045 Introduction to Quantum Computing

Purity for one qubit

If the distance from the origin to the state is measured to be r , the purity is:

$$P = \frac{1 + r^2}{2}$$

Maximum purity of 1 for all pure states.

Minimum purity of $\frac{1}{2}$ for a completely mixed state.

Note: There's a more technical definition of purity in terms of density matrices, which we won't cover in this course.

14

PHYC90045 Introduction to Quantum Computing

Reminder: Calculating expectation values

Example of calculating an expectation value for X,

$$\begin{aligned} \langle X \rangle &= \langle \psi | X | \psi \rangle \\ &= (a^* \langle 0 | + b^* \langle 1 |) X (a | 0 \rangle + b | 1 \rangle) \\ &= a^* a \langle 0 | X | 0 \rangle + b^* b \langle 1 | X | 1 \rangle + a^* b \langle 0 | X | 1 \rangle + b^* a \langle 1 | X | 0 \rangle \\ &= a^* b + b^* a \end{aligned}$$

15

PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Example: 20% error

Eg. Consider a state $|0\rangle$ present in a noisy system. 20% of the time, a bit flip X has applied to it.

$$\begin{aligned}\langle X \rangle &= 0.80 \langle 0 | X | 0 \rangle + 0.2 \langle 1 | X | 1 \rangle \\ &= 0 + 0 \\ &= 0\end{aligned}$$

$$\begin{aligned}\langle Y \rangle &= 0.80 \langle 0 | Y | 0 \rangle + 0.2 \langle 1 | Y | 1 \rangle \\ &= 0 + 0 \\ &= 0\end{aligned}$$

$$\begin{aligned}\langle Z \rangle &= 0.80 \langle 0 | Z | 0 \rangle + 0.2 \langle 1 | Z | 1 \rangle \\ &= 0.8 - 0.2 \\ &= 0.6\end{aligned}$$

16

PHYC90045 Introduction to Quantum Computing


THE UNIVERSITY OF
MELBOURNE

The purity

The purity of this mixed state is therefore:

$$\begin{aligned}P &= \frac{1 + r^2}{2} \\&= \frac{1 + 0.6^2}{2} \\&= 0.68\end{aligned}$$

17

PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Measuring purity in the QUI

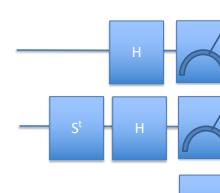
Run many trials – on each trial choosing a different random set of errors.

Measure $\langle X \rangle$

Measure $\langle Y \rangle$

Measure $\langle Z \rangle$

Then calculate the purity:

$$P = \frac{1 + \langle X \rangle^2 + \langle Y \rangle^2 + \langle Z \rangle^2}{2}$$


18

PHYC90045 Introduction to Quantum Computing

Quantum State Tomography

“Tomography” is quantum computing jargon for measuring/determining the quantum state, as well as possible. For one qubit, this is just measuring:

$$\langle X \rangle, \langle Y \rangle, \langle Z \rangle$$

For two qubits, we need to accurately measure correlations between the qubits as well. We measure the 15 parameters:

$$\begin{aligned} & \langle XX \rangle, \langle XY \rangle, \langle XZ \rangle, \langle XI \rangle \\ & \langle YX \rangle, \langle YY \rangle, \langle YZ \rangle, \langle YI \rangle \\ & \langle ZX \rangle, \langle ZY \rangle, \langle ZZ \rangle, \langle ZI \rangle \\ & \langle IX \rangle, \langle IY \rangle, \langle IZ \rangle \end{aligned}$$

Because of counting statistics the states can be unphysical (eg. radius greater than 1). However, these measurements can be used to estimate the closest (mixed) quantum state.

19

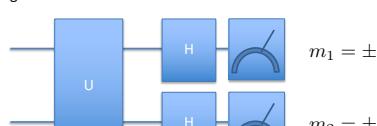
PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Two qubit example

Measuring $\langle XX \rangle$:



A quantum circuit diagram showing operations on two qubits. The circuit consists of three horizontal lines representing qubits. From left to right:

- The first qubit passes through a blue rectangular box labeled "U".
- The second qubit passes through a blue rectangular box labeled "H".
- The third qubit passes through a blue rectangular box labeled "H".
- All three qubits then enter a measurement device, represented by a blue semi-circular arc.

To the right of the circuit, the measurement results are listed as $m_1 = \pm 1$ and $m_2 = \pm 1$.

Find the product of these, $m = m_1 m_2$

Average over many runs of the experiment, with different locations/errors on each run to determine $\langle XX \rangle$.

20

Randomized Benchmarking

21

PHYC90045 Introduction to Quantum Computing

The Clifford Gates (for one qubit)

Apply a random sequence of gates

Typically this random sequence is chosen from a small gate set. One common choice is called the Clifford gates: These gates only rotate between the states which lie along +/- x, y and z axes.

$$|0\rangle, |1\rangle, \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}}, \frac{|0\rangle + i|1\rangle}{\sqrt{2}}, \frac{|0\rangle - i|1\rangle}{\sqrt{2}}$$

All of the preset gates except T are Clifford: X, Y, Z, S.

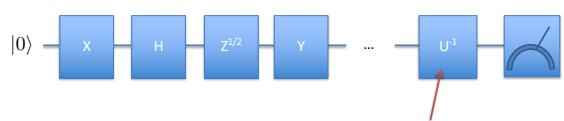
22

PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Randomized Benchmarking



The diagram shows a quantum circuit starting with the state $|0\rangle$. It consists of a sequence of operations: X , H , $Z^{1/2}$, Y , followed by an ellipsis (...), and finally U^{-1} . A red arrow points from the text "Apply the inverse of the preceding sequence" to the U^{-1} gate.

At the last step we apply the inverse of the preceding sequence. So if we started in the state

$$|0\rangle$$

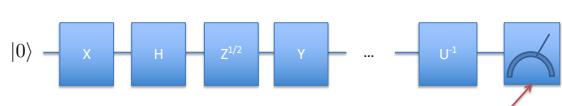
We should also end up in that state. If there were no errors, should measure 0, with certainty. However, with errors, the fidelity of the final measurement drops.

23

PHYC90045 Introduction to Quantum Computing

THE UNIVERSITY OF
MELBOURNE

The Clifford Gates (for one qubit)



A quantum circuit diagram starting with a state $|0\rangle$. The circuit consists of a sequence of blue rectangular boxes representing gates, connected by horizontal lines. The gates are labeled as follows: the first gate is X , the second is H , the third is $Z^{1/2}$, the fourth is Y , and the fifth is U^1 . After the U^1 gate, there is a vertical ellipsis \dots followed by another blue box. A red arrow points from the text "Measure" to the rightmost blue box, which has a semi-circular icon on its top-right corner.

Repeat sequences of the same length many times. For each length of sequence, average over many runs of the sequences, to work out the probability of measuring the correct result.

We can plot the fidelity (ie. the probability of getting the correct answer) against the length of sequence.

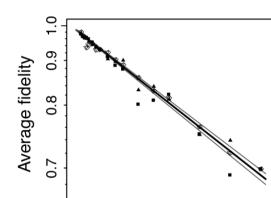
24

PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Example Plot



A scatter plot showing Average fidelity on the y-axis (ranging from 0.7 to 1.0) versus the Number of computational gates on the x-axis (ranging from 0 to 100). The data points show a decreasing trend, fitted by two parallel downward-sloping lines.

Number of computational gates	Average fidelity
0	0.98
10	0.95
20	0.92
30	0.88
40	0.82
50	0.78
60	0.75
70	0.72
80	0.68
90	0.65

Taken from Knill et al, PRA, 2007.

Fit this curve with:

$$F_m = A + Bf^m$$

where F_m is the average fidelity after m steps, and A and B and f are determined by the fit.

25

PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Randomized Benchmarking Sequence

We have found some number $f < 1$ which we can then relate to the average fidelity of a gate. If the dimension of the system is d ,

$$f = \frac{dF_{av} - 1}{d - 1}$$

In our case $d=2$, so the average fidelity is

$$F_{av} = \frac{f + 1}{2}$$

26

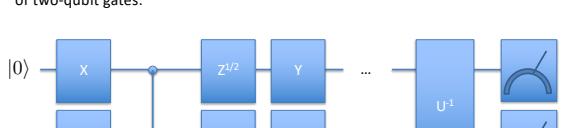
PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Different Randomized Benchmarking sequences

For two qubit gates, the set can also include gates such as CZ or CNOT. A similar method (with a different d) then can be used to determine the average fidelity of two-qubit gates.



27

PHYC90045 Introduction to Quantum Computing


THE UNIVERSITY OF
MELBOURNE

Interleaved Randomized Benchmarking

To determine the fidelity of an individual gate, interleave it throughout the randomized benchmarking. Eg. X



This gives an indication of the error in individual gates.

More advanced schemes also exist, eg. Adaptive versions of randomized benchmarking pinpoint where and what type of errors are occurring, rather than just giving a single number of average error per gate.

28

PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Quantum Process Tomography

Just as we can do tomography to determine a (mixed) quantum state, in principle we can measure what happens in a quantum process. Technically we are determining a completely positive (CP) map.

General strategy for one qubit

For each possible input states:

$$|0\rangle, |1\rangle, \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}}, \frac{|0\rangle + i|1\rangle}{\sqrt{2}}, \frac{|0\rangle - i|1\rangle}{\sqrt{2}}$$

Act the operation, U, on each input states

Do complete state tomography on each output (ie. $\langle X \rangle, \langle Y \rangle, \langle Z \rangle$)

Similar process for multiple qubits - QPT requires many measurements!

29

PHYC90045 Introduction to Quantum Computing



 THE UNIVERSITY OF
 MELBOURNE

Week 7

Lecture 13 - Quantum Supremacy

- 11.1 Boson Sampling
- 11.2 IQP Problem
- 11.3 Google's pseudorandom circuits

Lecture 14 - Errors

- 12.1 Quantum errors: unitary and stochastic errors
- 12.2 Purity
- 12.3 Tomography
- 12.4 Randomized Benchmarking

Lab 7

Quantum Supremacy and Errors

30