The University of Melbourne
Semester One 2021

MULT90063

INTRODUCTION TO QUANTUM COMPUTING

10am, Monday 7th June, 2021.

Reading time:	**15 minutes + 30 minutes for submission**
Exam Duration:	**2 hours**
This paper has:	**5 pages (including front page)**

Instructions to students:

Attempt **ALL** questions. The total number of marks is 100.

Please start each new question on a new page. Write the question number and your student number on each page. Once you have finished the exam, scan your written work into a single PDF and submit it via LMS.

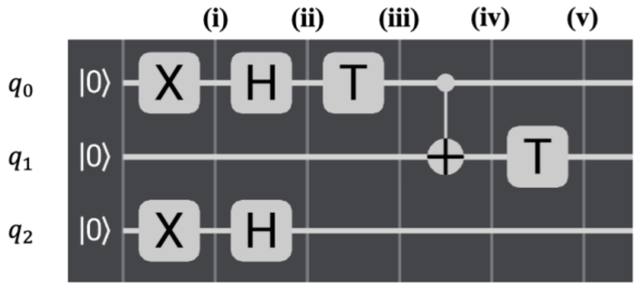
The timing of this exam is as follows –

10:00am: Download the exam from LMS and begin your reading time
10:15am: Begin work
12:15pm: Finish work and upload your exam to LMS
12:45pm: Exams received after this time will be considered late.

This is an open book examination. You may make use of textbooks, the QUI, lecture and tutorial materials on LMS, but *no* other online materials. Although you may use the QUI to verify your answers, *please show the complete working for your answers*. You may *not* consult with other people during the exam by any means (invigilators excepting).

Question 1 [3 + 5 + 2 + 2 + 2 = 14 marks]

Consider the following circuit:



(a) In terms of the ket representation for qubits, write down the action of each of the gates X, H and T separately on an arbitrary (normalised) superposition $\alpha|0\rangle + \beta|1\rangle$.

(b) Write down the state of the two-qubit system at each of the different time-points marked **(i)**, **(ii)**, **(iii)**, **(iv)** and **(v)**. Order your states, $|q_0\rangle|q_1\rangle|q_2\rangle$ and show your working.

(c) Is the state at time-point **(iv)** entangled? Explain which qubits are entangled and how you know.

(d) At the end of this circuit (time-point (v)), what is the probability of measuring the second (middle) qubit in the state '0'? Show your working.

(e) With reference to the state at (v), show how you can entangle all the qubits by adding a single gate to the circuit.

Question 2 [2 + 6 + 3 + 3 = 14 marks]

Consider an implementation of a function given by:

$$f(x \oplus a) = f(x)$$

The function *f(x)* for possible values of *x* is described as follows:

x	*f(x)*
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6

0111	7
1000	1
1001	0
1010	3
1011	2
1100	5
1101	4
1110	7
1111	6

(a) Using the data given in the table, find a.

(b) Draw the quantum circuit (oracle) implementing the function *f(x)* defined in the table above.

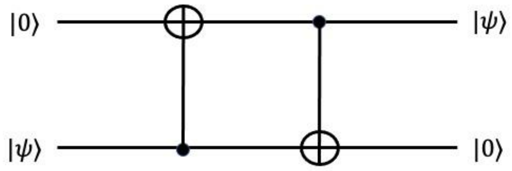
(c) Modify the above circuit to guarantee that measured values (of one of the registers), y, satisfies $a \cdot y = 0 \bmod 2$.

(d) Suppose we run the circuit and randomly measure the following values of y: 0010, 0100, 1111, and 1001. Find a by solving a set of linear equations based on these measurement outcomes. Show the complete procedure.

Question 3 [4 + 4 + 6 + 6 = 20 marks]

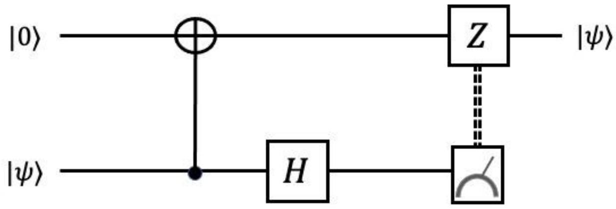
Consider the unknown state $|\psi\rangle = \alpha\,|0\rangle + \beta|1\rangle$

(a) Verify that the circuit below swaps the location of the state $|\psi\rangle$

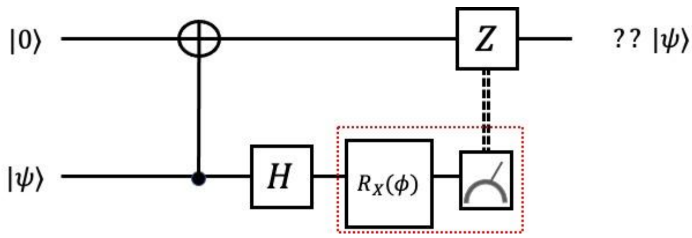


(b) Does the circuit above work as a swap on the unknown state $|\psi\rangle$ if the top register is now in the $|1\rangle$ state? How could you modify this circuit to guarantee the unknown state $|\psi\rangle$ is swapped, independent of the state of the top register?

(c) Go through each step of the circuit below (assuming the measurement is in the computational basis) and verify that it acts as a single-bit teleportation:



(d) We can use the single-bit teleportation circuit to induce a single qubit operation via choosing an appropriate measurement basis. In this question, consider changing the measurement basis by applying the single qubit gate $R_x(\phi)$ (where we take the global phase to be 0) before the measurement (assuming the computational basis), as shown in the red box. What effective single qubit gate does this apply to the unknown state $|\psi\rangle$?



Question 4 [4 + 4 + 4 + 6 = 18 marks]

At a prestigious university, the antiquated heating system has been modified over many years. Three heaters are arranged side-by-side. If turned on, heater 1 draws 20W, heater 2 draws 30W and heater 3 draws 10W. However, students found this often makes them too hot, so in between the heaters air-conditioners were installed. If heaters 1 and 2 are on, then the air-conditioner between them cools with 10W power. If heaters 2 and 3 are both on, then the air-conditioner between them cools with 15W power.

It is a very cold winter's day, and the students would like to heat themselves as much as possible.

(a) Write a Boolean equation (using variables x_1, x_2, and x_3 which are equal to 0 if the corresponding heater is off, and 1 if it is on) for the total heat provided by the heaters and air-conditioners to the students.

(b) Write a Hamiltonian whose ground state correspond to the most heating that can be obtained by any combination of heaters and air-conditioners.

(c) Determine the ground-state energy for the Hamiltonian in (b), and the state corresponding to this energy.

(d) Including a circuit diagram with appropriately labelled angles, describe how this problem might be solved using the QAOA algorithm on a quantum computer.

Question 5 [2 + 2 + 4 + 2 + 2 + 2 = 14 marks]

A [4,2,2] quantum error correction code has the following logical basis states:

$$|00\rangle_L = \frac{|0000\rangle + |1111\rangle}{\sqrt{2}}$$
$$|01\rangle_L = \frac{|0011\rangle + |1100\rangle}{\sqrt{2}}$$
$$|10\rangle_L = \frac{|0101\rangle + |1010\rangle}{\sqrt{2}}$$
$$|11\rangle_L = \frac{|0110\rangle + |1001\rangle}{\sqrt{2}}$$

where the logical states ($|00\rangle_L$, $|01\rangle_L$, $|10\rangle_L$, $|11\rangle_L$) encode two logical qubits.

(a) Show by explicit calculation (in ket notation) that XXXX stabilizes all four states.

(b) If a Z-error happens to the third qubit, what happens to the syndrome measurement of the XXXX operator?

(c) Write down a circuit for measurement of the XXXX operator (using an extra ancilla qubit).

(d) Write down another non-trivial (ie. not the identity) stabilizer of the code.

(e) Write down a circuit for the logical operation for X_L applied to the first qubit of the two logical qubits (ie. $|00\rangle_L \rightarrow |10\rangle_L$ and $|01\rangle_L \rightarrow |11\rangle_L$ etc).

(f) Is the circuit which you wrote down in part (e) fault tolerant? Why, or why not?

Question 6 [20 marks]

The next major milestone for quantum computing, beyond demonstration of quantum supremacy, is the demonstration of an algorithm of *practical importance* for which quantum computers outperform their classical counterparts. From the algorithms that we have covered in this course, which do you think is the most likely to demonstrate this type of quantum advantage, and why? Write approximately half an A4 page (200-300 words) to justify your answer.

End of examination