Week 7

Lecture 13 - Quantum Supremacy

11.1 Boson Sampling

11.2 IQP Problem

11.3 Google's pseudorandom circuits

Lecture 14 - Errors

12.1 Quantum errors: unitary and stochastic errors

12.2 Purity

12.3 Tomography

12.4 Randomized Benchmarking

Lab 7

Quantum Supremacy and Errors

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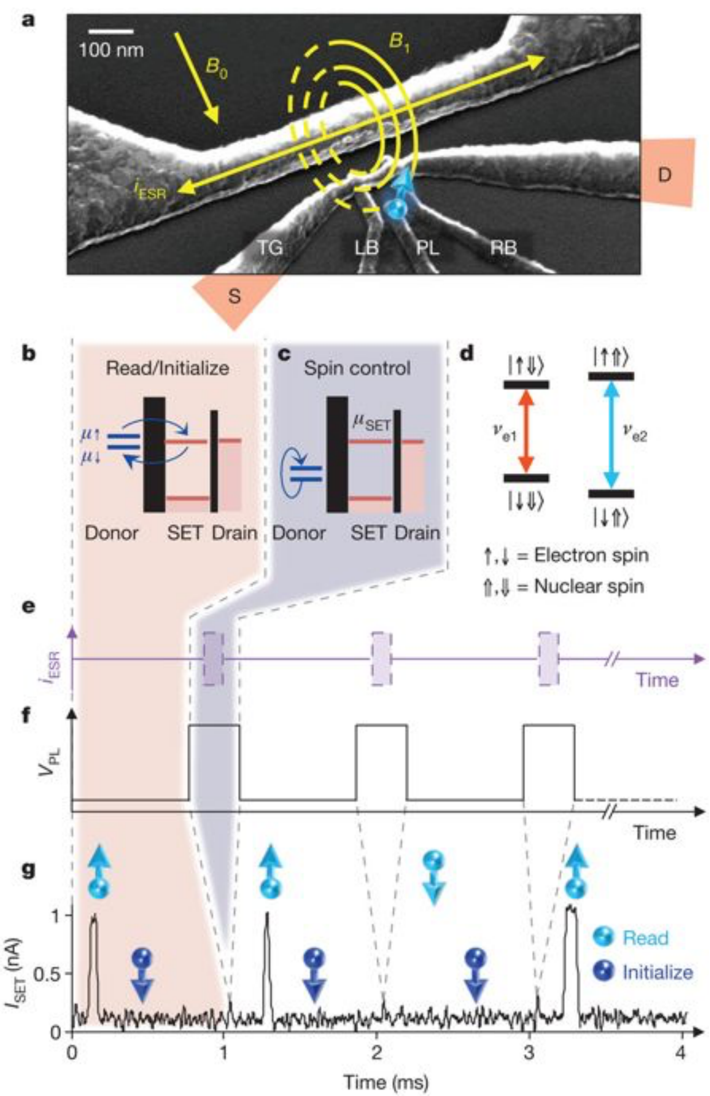
Your PC ran into a problem and needs to restart. We're just collecting some error info, and then we'll restart for you.

55% complete

For more information about this issue and possible fixes, visit http://windows.com/stopcode

If you call a support person, give them this info:
Stop code: CRITICAL_PROCESS_DIED

Real quantum devices



Pla et al, Nature, 2012

Two types of errors

Quantum computers are extremely fragile, and vulnerable to noise and errors. While errors occur in classical computing too, we’re accustomed to very low error rates – our hard drives rarely forget what they store.

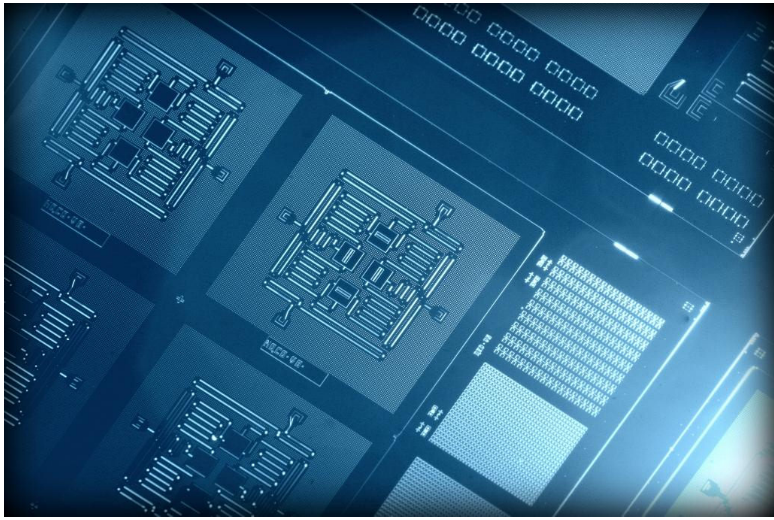
Two types of error:

(1) Systematic unitary errors – eg. Control pulse error
(2) Random noise – eg. Decoherence

Control Errors

Control of qubits requires high precision, and errors can sneak in. For example:

- Variations in magnetic fields across the sample, or variations in material properties.
- Stray electric fields, charge traps, strain.
- Applying a microwave pulse where the strength of the pulse is slightly too strong or too weak causes a systematic over-rotation or under-rotation.
- Cross-talk between gates.
- Unwanted interaction between qubits.



IBM image, Flickr

Systematic Errors in the QUI

The QUI is effectively a pristine qubit environment, but we can introduce such effects systematically and investigate how quantum gate errors affect the output of quantum circuits.

We will consider rotation errors around the cartesian axes in the QUI using the R-gate. For example, a Z-rotation error (or just "Z-error") is a gate δZ defined as:

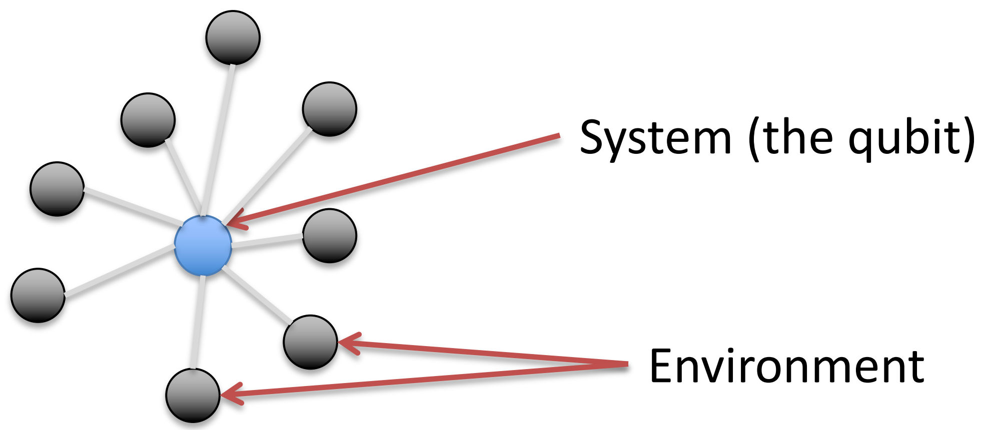
$$\delta Z \equiv \begin{pmatrix} e^{-i\epsilon/2} & 0 \\ 0 & e^{i\epsilon/2} \end{pmatrix}$$

where the level of error is governed by the angle ε (assumed to be small). Similarly, we could consider small rotations around other axes:

$$\delta X = R_X(\epsilon)\,, \quad \delta Y = R_Y(\epsilon)\,, \quad \delta Z = R_Z(\epsilon)$$

In the lab, we will consider the effect of these errors on the success of quantum circuits.

Decoherence

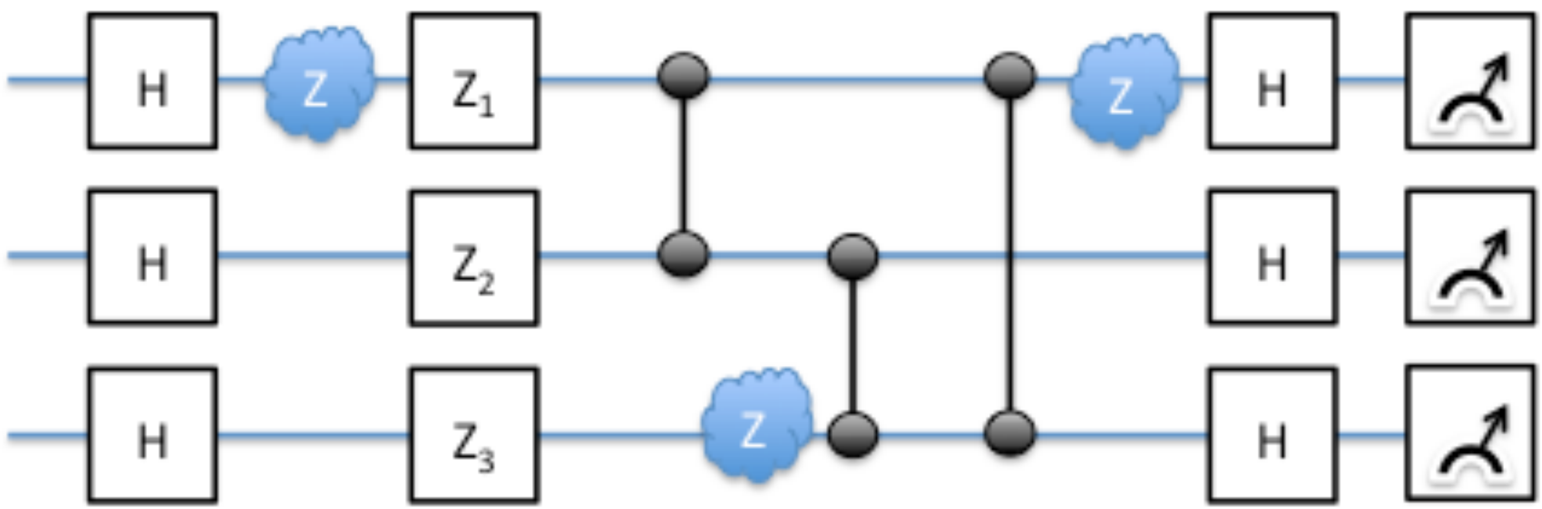


In realistic quantum systems there will always be (unwanted) interaction between the qubits and the environment (electrons, spins, phonons, charge traps).

This causes a type of noise on the system (ie. qubits we want to protect) which we call decoherence.

Modeling Stochastic Errors

Stochastic errors can be modeled in the QUI.



Dephasing noise: Apply *Z* gate with some probability *p*.
Depolarizing noise: Apply either X, Y or Z gates, each with with probability *p/3.*

Perform many "Monte Carlo" simulations, where errors are placed randomly on each run, and average the measurement results.

Pure and Mixed States

Pure states have no errors, and are perfectly coherent.

For example, the state

$$|\psi\rangle = |0\rangle$$

is a coherent, quantum state.

Mixed states may have errors, and are less coherent.

Imagine we took system in a pure quantum state, and noise – with say 20% probability- flipped the qubit around the X axis. That state would then be a “mixed” state.

Superposition vs mixed states

Consider the pure state,

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

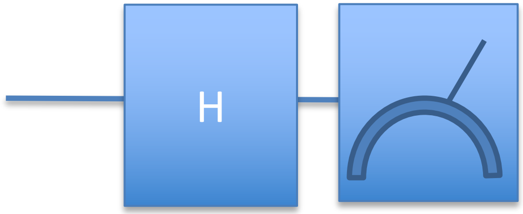
and a mixed state which is

50% $|0\rangle$ 50% $|1\rangle$

Is it possible to tell these two states apart in experiment?

Consider what happens if we apply a Hadamard gate, then measure:

$|\psi\rangle$



Superposition vs mixed states

For the mixed state if $|0\rangle$ were prepared :

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

50% of the time we will measure 0
50% of the time we will measure 1

For the mixed state if $|1\rangle$ were prepared :

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

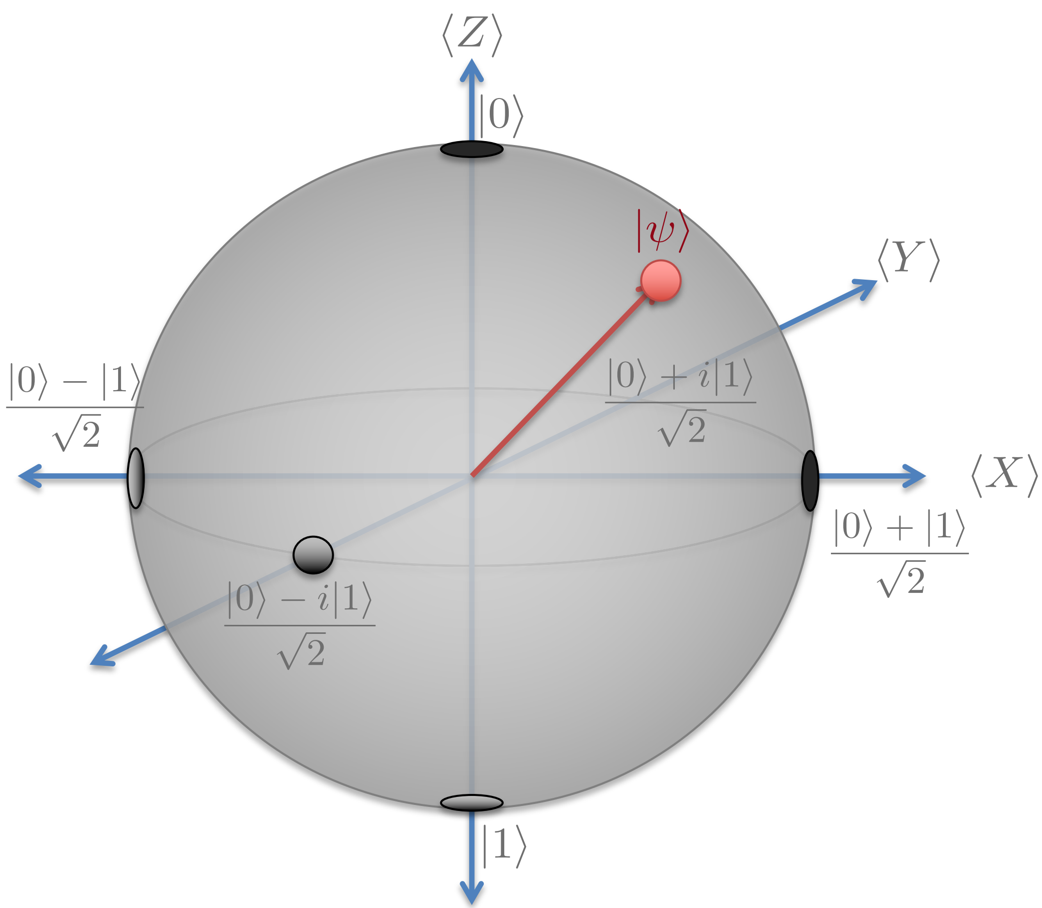
50% of the time we will measure 0
50% of the time we will measure 1

So if the mixed state is prepared:

50% of the time we will measure 0
50% of the time we will measure 1

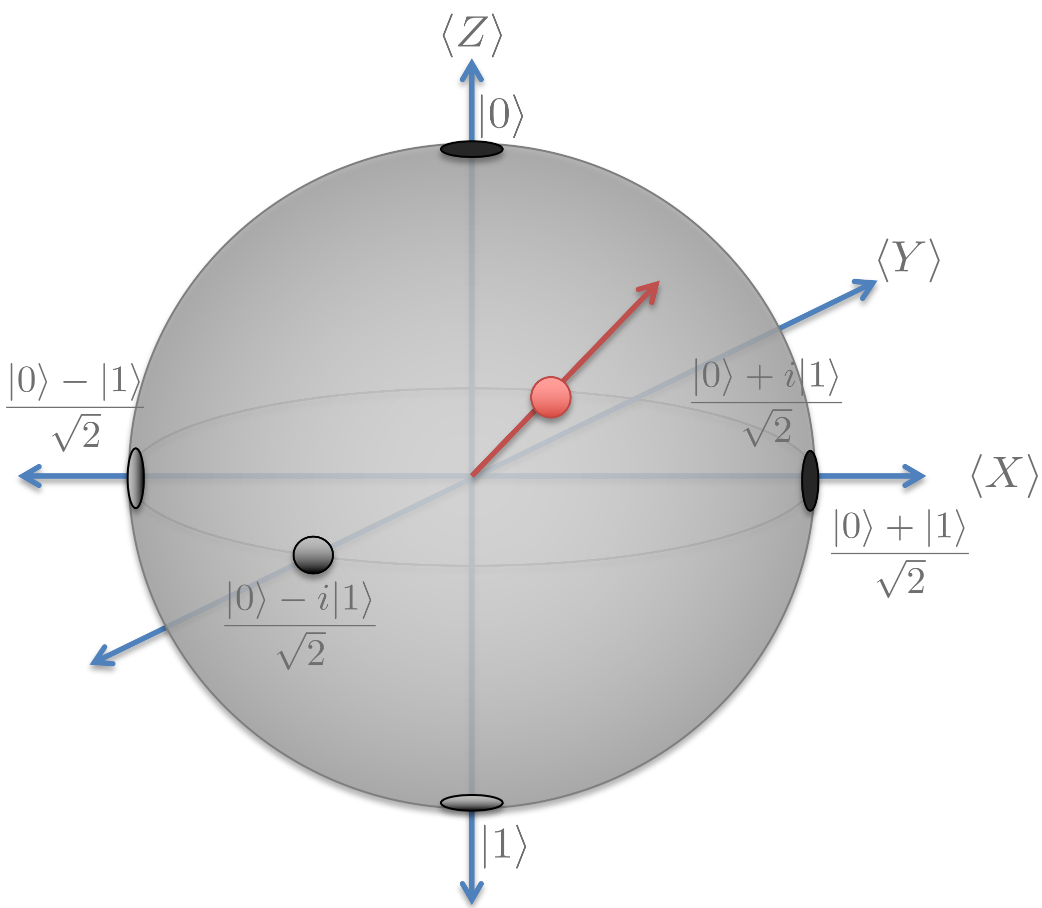
For the pure state: $H|+\rangle = |0\rangle$ so **100%** of the time we will measure 0.

Purity on the Bloch Sphere



Pure states lie on the surface of the Bloch Sphere

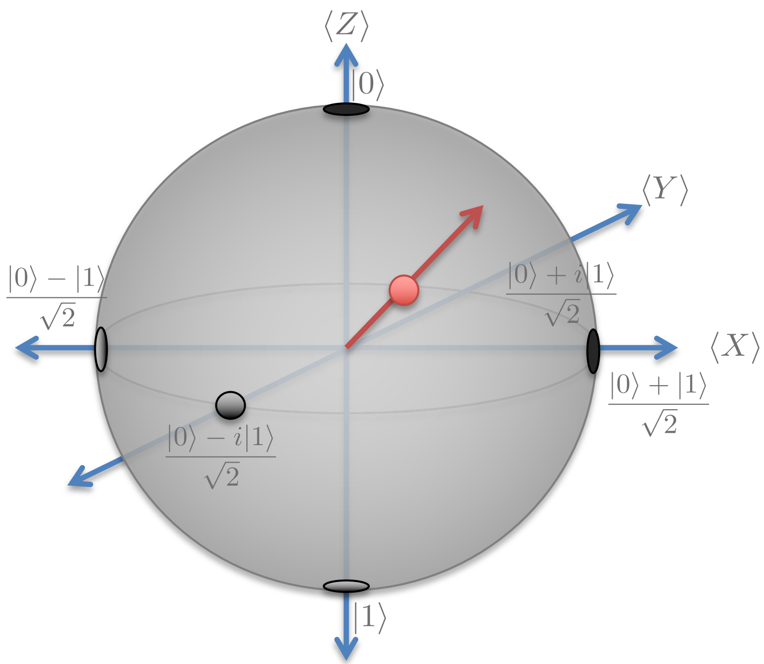
Mixed states on the Bloch Sphere



Mixed states lie inside the Bloch Sphere.

The closer to the origin, the more mixed.

Purity for one qubit



If the distance from the origin to the state is measured to be r, the purity is:

$$P = \frac{1 + r^2}{2}$$

Maximum purity of 1 for all pure states.

Minimum purity of ½ for a completely mixed state.

Note: There's a more technical definition of purity in terms of density matrices, which we won't cover in this course.

Reminder: Calculating expectation values

Example of calculating an expectation value for X,

$$
\begin{aligned}
\langle X \rangle &= \langle \psi | X | \psi \rangle \\
&= (a^* \langle 0| + b^* \langle 1|) X (a |0\rangle + b |1\rangle) \\
&= a^* a \langle 0| X |0\rangle + b^* b \langle 1| X |1\rangle + a^* b \langle 0| X |1\rangle + b^* a \langle 1| X |0\rangle \\
&= a^* b + b^* a
\end{aligned}
$$

Example: 20% error

Eg. Consider a state $|0\rangle$ present in a noisy system. 20% of the time, a bit flip X has applied to it.

$$\begin{aligned}\langle X\rangle &= 0.80\,\langle 0|\,X\,|0\rangle + 0.2\,\langle 1|\,X\,|1\rangle \\ &= 0+0 \\ &= 0\end{aligned}$$

$$\begin{aligned}\langle Y\rangle &= 0.80\,\langle 0|\,Y\,|0\rangle + 0.2\,\langle 1|\,Y\,|1\rangle \\ &= 0+0 \\ &= 0\end{aligned}$$

$$\begin{aligned}\langle Z\rangle &= 0.80\,\langle 0|\,Z\,|0\rangle + 0.2\,\langle 1|\,Z\,|1\rangle \\ &= 0.8-0.2 \\ &= 0.6\end{aligned}$$

The purity

The purity of this mixed state is therefore:

$$\begin{aligned} P &= \frac{1+r^2}{2} \\ &= \frac{1+0.6^2}{2} \\ &= 0.68 \end{aligned}$$

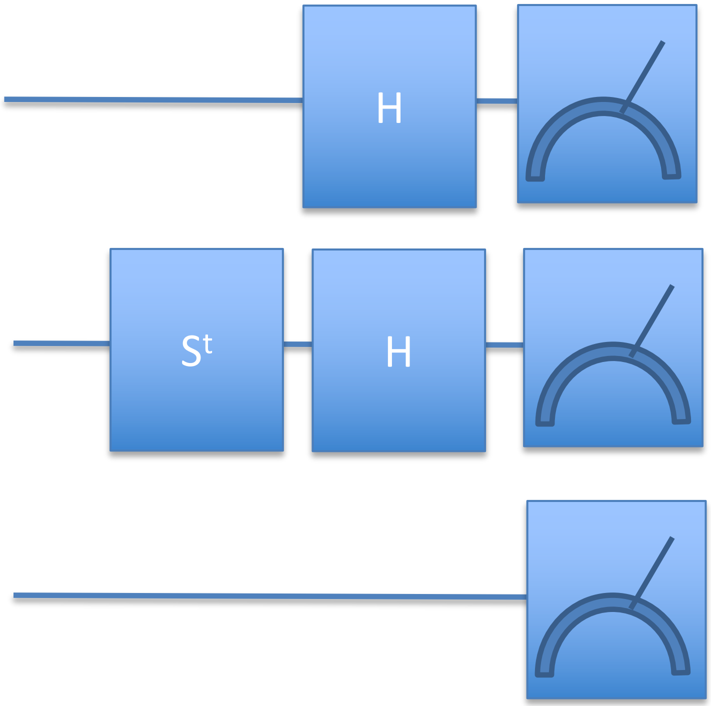
Measuring purity in the QUI

Run many trials – on each trial choosing a different random set of errors.

Measure <X>

Measure <Y>

Measure <Z>



Then calculate the purity:

$$P = \frac{1 + \langle X \rangle^2 + \langle Y \rangle^2 + \langle Z \rangle^2}{2}$$

Quantum State Tomography

"Tomography" is quantum computing jargon for measuring/determining the quantum state, as well as possible. For one qubit, this is just measuring:

$$\langle X\rangle,\ \langle Y\rangle, \langle Z\rangle$$

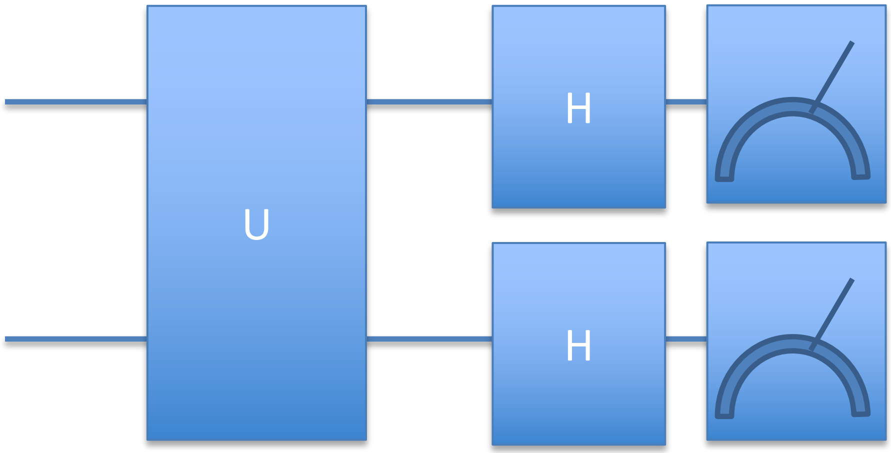
For two qubits, we need to accurately measure correlations between the qubits as well. We measure the 15 parameters:

$$\begin{array}{llll}\langle XX\rangle, & \langle XY\rangle, & \langle XZ\rangle, & \langle XI\rangle \\ \langle YX\rangle, & \langle YY\rangle, & \langle YZ\rangle, & \langle YI\rangle \\ \langle ZX\rangle, & \langle ZY\rangle, & \langle ZZ\rangle, & \langle ZI\rangle \\ \langle IX\rangle, & \langle IY\rangle, & \langle IZ\rangle & \end{array}$$

Because of counting statistics the states can be unphysical (eg. radius greater than 1). However, these measurements can be used to estimate the closest (mixed) quantum state.

Two qubit example

Measuring <XX>:



$$m_1 = \pm 1$$

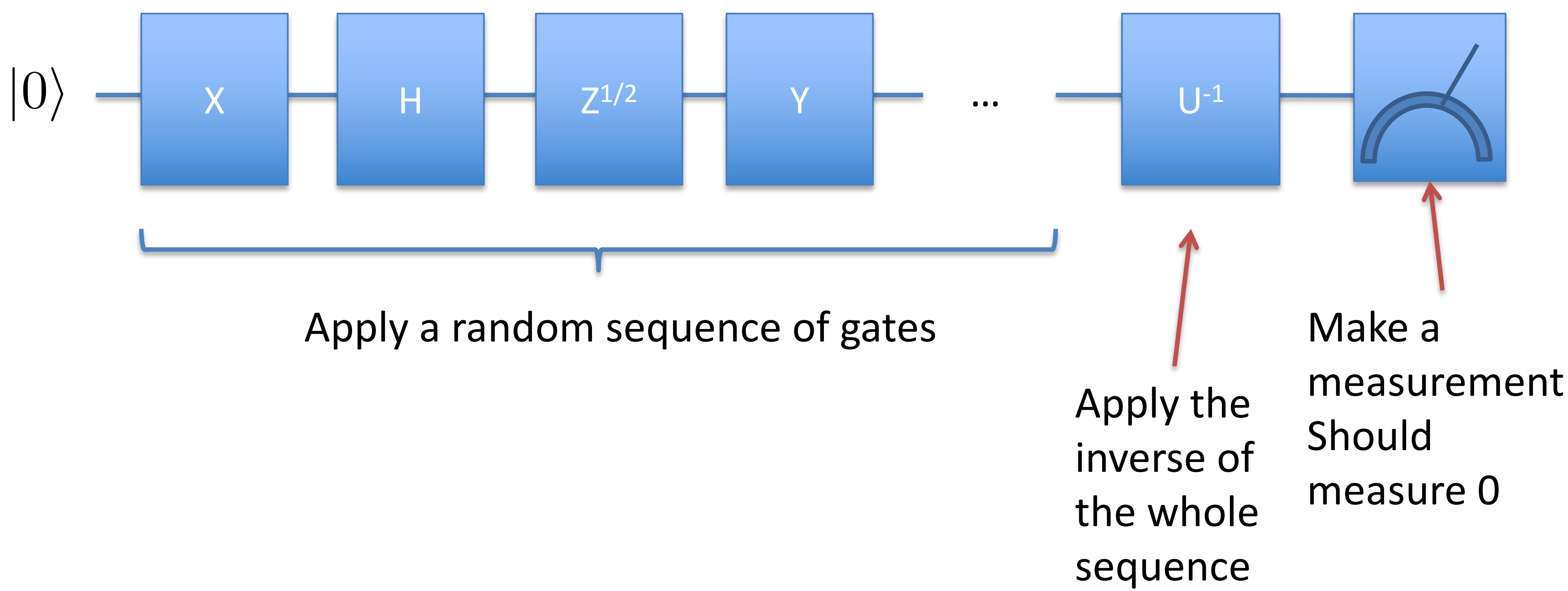
$$m_2 = \pm 1$$

Find the product of these, $m = m_1 m_2$

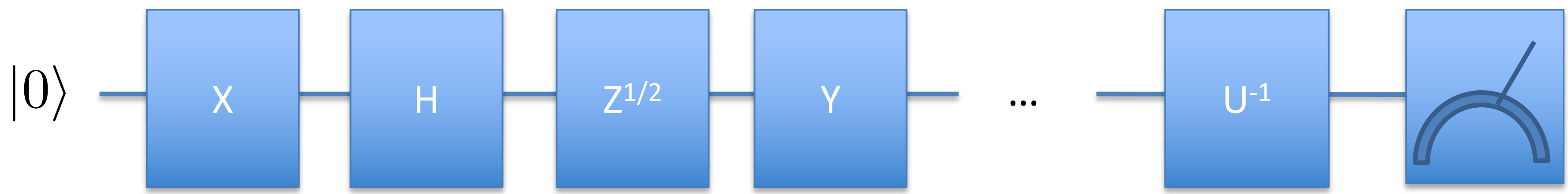
Average over many runs of the experiment, with different locations/errors on each run to determine <XX>.

Randomized Benchmarking

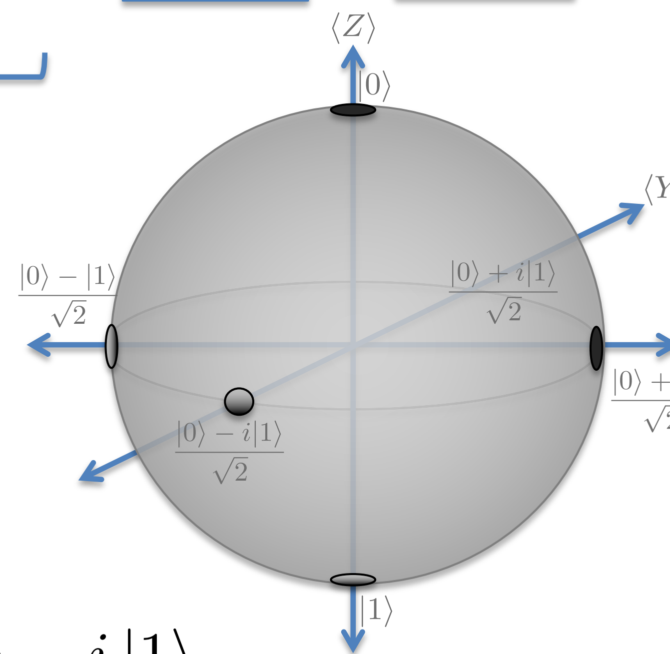
How good are our gates individual gate? We want a number for how much error doing each operation is. One way of determining this is to perform **randomized benchmarking**.



The Clifford Gates (for one qubit)



Apply a random sequence of gates

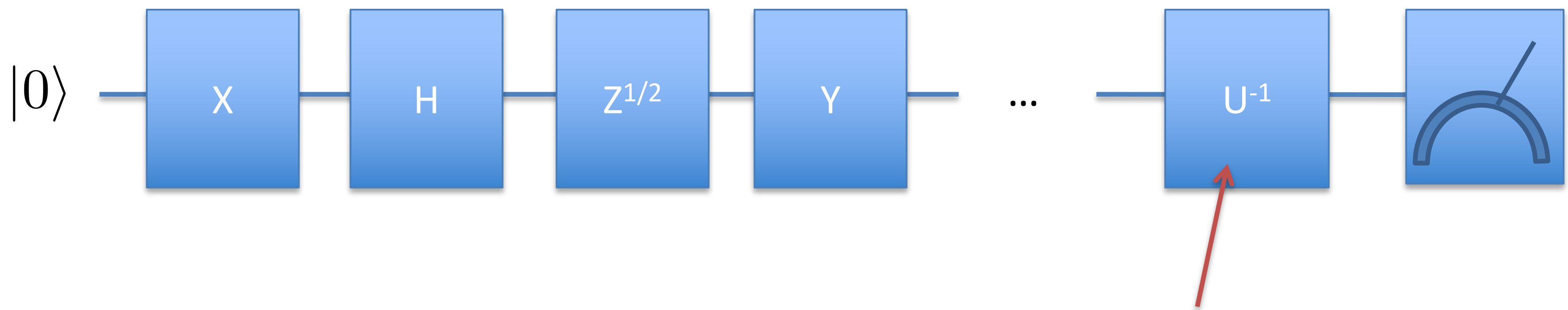


Typically this random sequence is chosen from a small gate set. One common choice is called the Clifford gates: These gates only rotate between the states which like along +/- x, y and z axes.

$$|0\rangle\,,|1\rangle\,,\frac{|0\rangle+|1\rangle}{\sqrt{2}},\frac{|0\rangle-|1\rangle}{\sqrt{2}},\frac{|0\rangle+i\,|1\rangle}{\sqrt{2}},\frac{|0\rangle-i\,|1\rangle}{\sqrt{2}}$$

All of the preset gates except T are Clifford: X, Y, Z, S.

Randomized Benchmarking



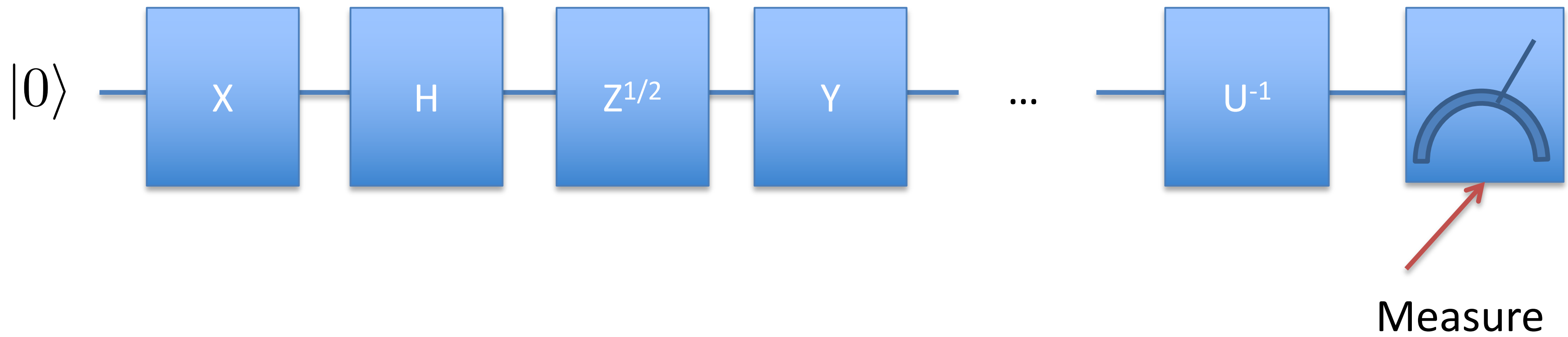
Apply the inverse of the preceding sequence

At the last step we apply the inverse of the preceding sequence. So if we started in the state

$$|0\rangle$$

We should also end up in that state. If there were no errors, should measure 0, with certainty. However, with errors, the fidelity of the final measurement drops.

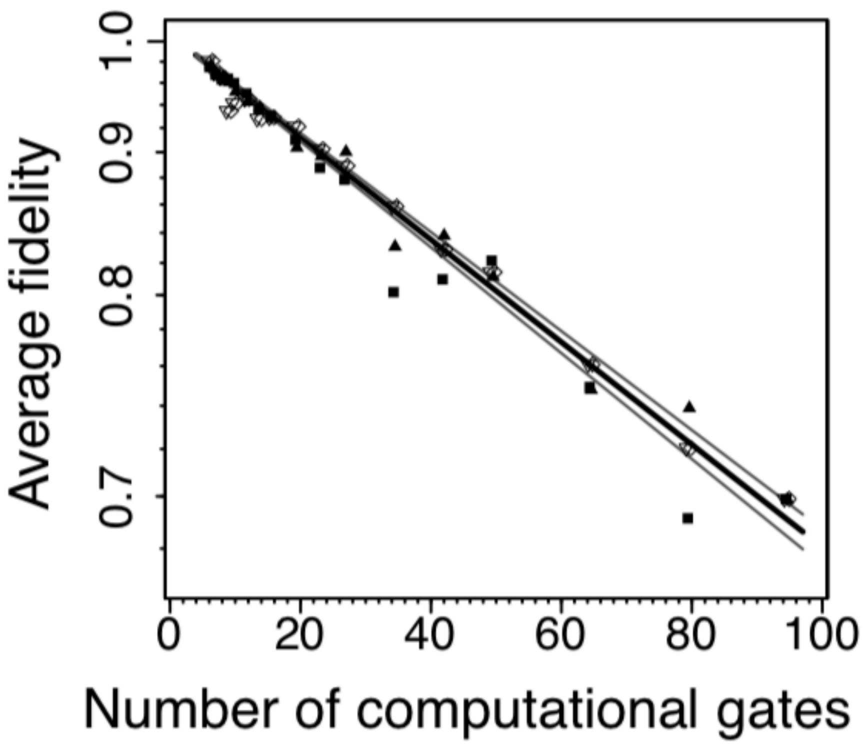
The Clifford Gates (for one qubit)



Repeat sequences of the same length many times. For each length of sequence, average over many runs of the sequences, to work out the probability of measuring the correct result.

We can plot the fidelity (ie. the probability of getting the correct answer) against the length of sequence.

Example Plot



Taken from Knill et al, PRA, 2007.

Fit this curve with:

$$F_m = A + Bf^m$$

where F_m is the average fidelity after *m* steps, and A and B and f are determined by the fit.

Randomized Benchmarking Sequence

We have found some number f < 1 which we can then relate to the average fidelity of a gate. If the dimension of the system is d,

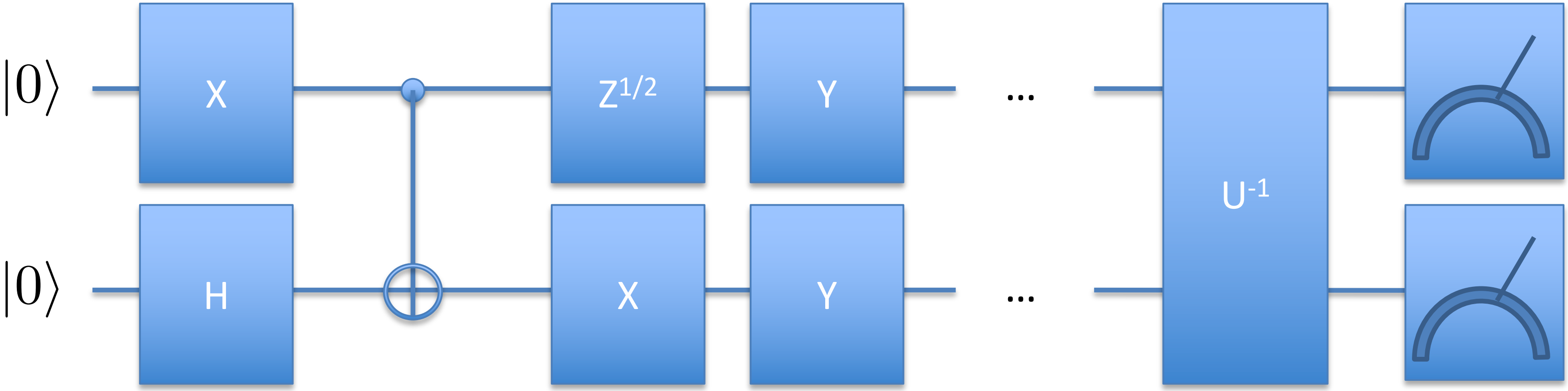
$$f = \frac{dF_{av} - 1}{d - 1}$$

In our case d=2, so the average fidelity is

$$F_{av} = \frac{f + 1}{2}$$

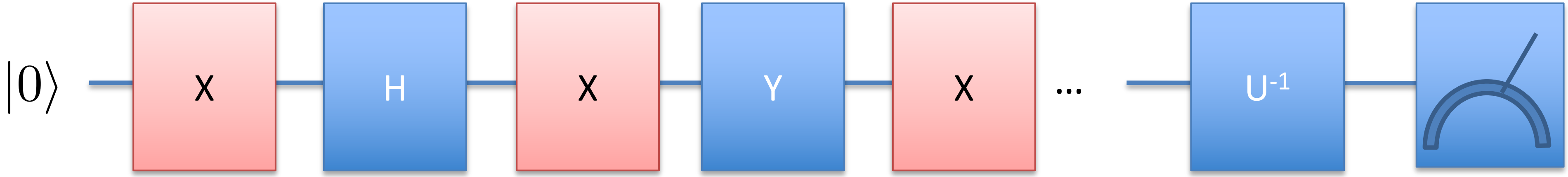
Different Randomized Benchmarking sequences

For two qubit gates, the set can also include gates such as CZ or CNOT. A similar method (with a different d) then can be used to determine the average fidelity of two-qubit gates.



Interleaved Randomized Benchmarking

To determine the fidelity of an individual gate, interleave it throughout the randomized benchmarking. Eg. X



This gives an indication of the error in individual gates.

More advanced schemes also exist, eg. Adaptive versions of randomized benchmarking pinpoint where and what type of errors are occurring, rather than just giving a single number of average error per gate.

Quantum Process Tomography

Just as we can do tomography to determine a (mixed) quantum state, in principle we can measure what happens in a quantum process. Technically we are determining a completely positive (CP) map.

General strategy for one qubit

For each possible input states:

$$|0\rangle\,,|1\rangle\,,\frac{|0\rangle+|1\rangle}{\sqrt{2}},\frac{|0\rangle-|1\rangle}{\sqrt{2}},\frac{|0\rangle+i\,|1\rangle}{\sqrt{2}},\frac{|0\rangle-i\,|1\rangle}{\sqrt{2}}$$

Act the operation, U, on each input states

Do complete state tomography on each output (ie. <X>, <Y>, <Z>)

Similar process for multiple qubits - QPT requires many measurements!

Week 7

Lecture 13 - Quantum Supremacy

11.1 Boson Sampling

11.2 IQP Problem

11.3 Google's pseudorandom circuits

Lecture 14 - Errors

12.1 Quantum errors: unitary and stochastic errors

12.2 Purity

12.3 Tomography

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Quantum Supremacy and Errors