

PHYC90045 Introduction to Quantum Computing

Week 10



Lecture 21
Adiabatic Quantum Computing

Lecture 22
Optimisation

Lab 10
Adiabatic quantum computation

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Adiabatic Quantum Computation

Physics 90045
Lecture 21

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Overview



In this lecture we will cover/review

- Quantum Adiabatic Processes
- The problem Hamiltonian
- Avoided crossings and energy gap
- The adiabatic theorem

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D-Wave systems



Image: D-Wave Systems

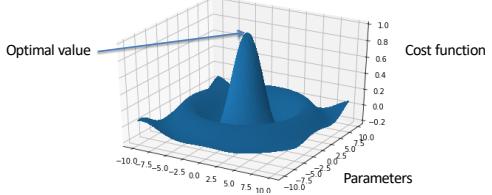
- 2000 qubit quantum "annealers"
- Sold quantum computers to Lockheed Martin, Google, NASA, Los Alamos National Laboratory (\$15 million each)

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Review: Optimization Problems

Given some cost-function or "objective function" we would like to maximize/minimize. Often the inputs/parameters are constrained.



We have seen in previous lecture how to map these onto QUBO problem Hamiltonians.

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Adiabatic Processes

- Start in the known ground state of a simple Hamiltonian
- Slowly change the Hamiltonian to the problem Hamiltonian
- Provided the change has been "slowly enough" the system will remain in the ground state, and we will have found the ground state of the problem Hamiltonian.

$$H(s) = (1 - s)H_x + sH_p \quad 0 \leq s \leq 1$$

$s = t/T$

Bad analogy: Moving a glass full of water slowly enough means that you keep the water in the glass. If you move it too fast all the water slops out.

Note: Quantum adiabatic process is not the same as thermodynamic adiabatic process ($Q=0$)

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Hamiltonian of transverse field

$H_x \propto B_x \sum_i X_i$

Transverse field Hamiltonian

Eg. Three electrons in a transverse field:

$$H_x = g\mu_B B_x (X_1 + X_2 + X_3)$$

We call this the transverse Hamiltonian, H_x .

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Problem Hamiltonian

We have seen many examples of problem Hamiltonians in our discussions of QUBO Problems. Two-body because that's what nature gives us.

Eg. Number partitioning for the set {1, 2, 3}:

$$H = 4Z_1Z_2 + 6Z_1Z_3 + 12Z_2Z_3 + 14I$$

Finding minimum energy state will solve the problem!

We call this the problem Hamiltonian, H_p .

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Quantum “Annealing”

Definition according to Google:

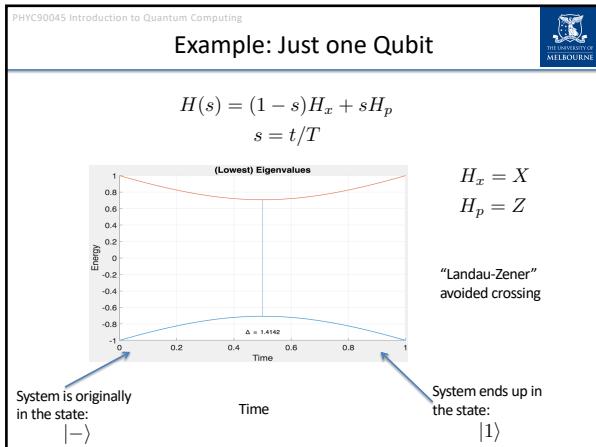
Annealing: heat (metal or glass) and allow it to cool slowly, in order to remove internal stresses and toughen it. “Copper tubes must be annealed after bending or they will be brittle”

$$H(s) = (1 - s)H_x + sH_p$$

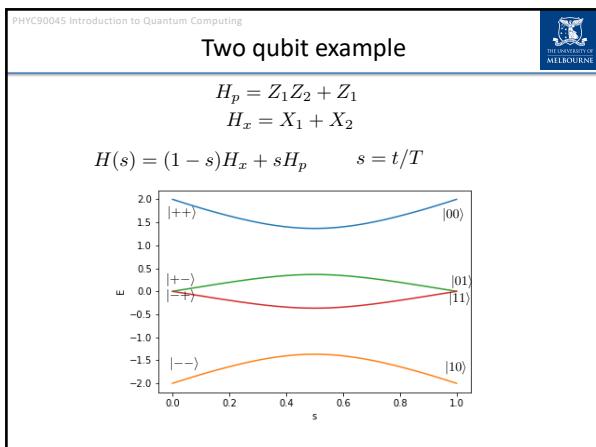
Transverse field plays the role of temperature. Causes excitations, strength is slowly being lowered.

Problem Hamiltonian defines the energy landscape of the problem we want to solve.

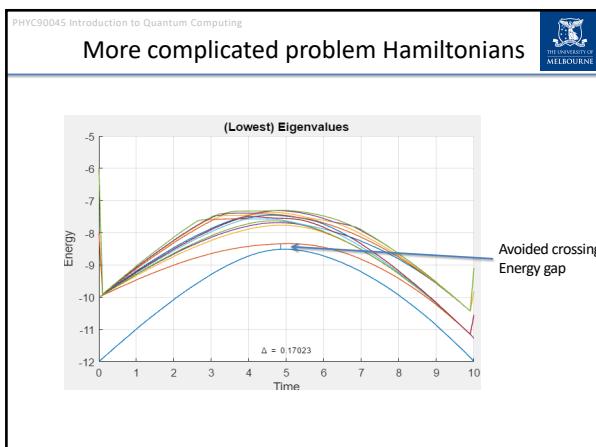
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Adiabatic Theorem

How slowly is slowly enough? Adiabatic criterion.

Time derivative of Hamiltonian

m, n^{th} element

$$\sum_{m \neq n} \frac{\hbar |\langle m | \dot{H} | n \rangle|}{|E_n - E_m|^2} = \sum_{m \neq n} \left| \frac{\hbar \langle m | \dot{n} \rangle}{E_n - E_m} \right| \ll 1$$

Energy of eigenstates:
 n is the ground state,
 m is every other state

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Roughly speaking...

$$\sum_{m \neq n} \frac{\hbar |\langle m | \hat{H} | n \rangle|}{|E_n - E_m|^2} = \sum_{m \neq n} \left| \frac{\hbar \langle m | \hat{n} \rangle}{E_n - E_m} \right| \ll 1$$

For ground state, largest contribution from the smallest two eigenvalues. $E_n - E_m$

$$|E_n - E_m| > \Delta$$

Energy gap between
ground state and first
excited Eigenstate

For our linear schedule:

$$\dot{H}_{mn} \approx \frac{H_{mn}}{T}$$

Time required scales inversely proportional to the gap:

$$T \propto \frac{1}{\Delta}$$

For a given problem: How big is the gap? Difficult to work out in general!

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Energy gap

(Lowest) Eigenvalues

$\Delta = 0.17023$

Energy gap

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Quantum Tunneling

When might AQC give an advantage?

Quantum annealing has been shown to outperform classical annealing in the case where the barriers are high, but thin.

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Equivalent of Grover's algorithm

Roland and Cerf demonstrated that AQC could be used to implement an unordered search:

$$H_x = I - 2|\phi\rangle\langle\phi|$$

$$H_p = I - 2|m\rangle\langle m|$$

$$|\phi\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle$$

Marked state, m

Optimization, since the energy spectrum is known:
 Change Hamiltonian faster when the gap is larger, faster when it is smaller.

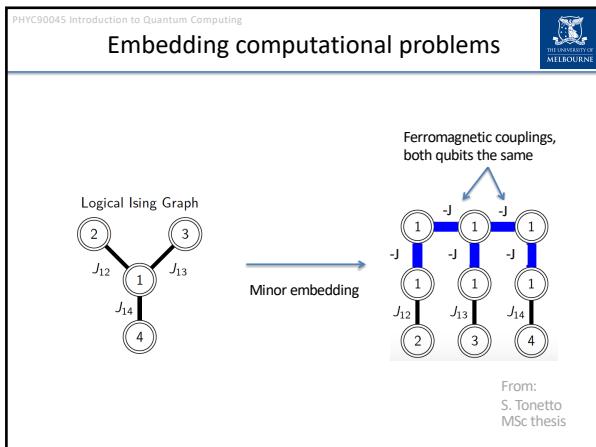
This achieves the same $O(\sqrt{N})$ speedup as Grover's algorithm.

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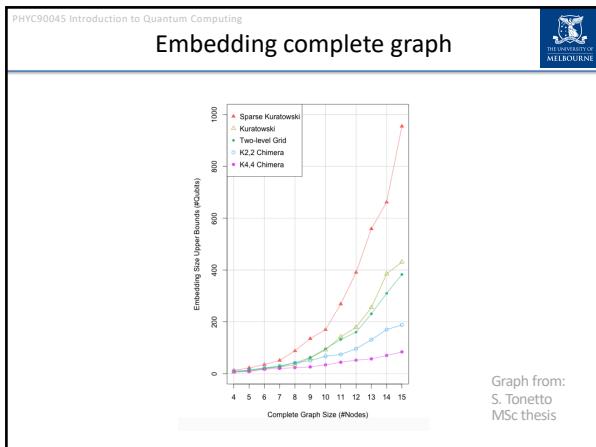
The figure displays five different graph-based subgraph structures used in quantum computing:

- K_{4,4} and K_{2,2} Chimera Subgraphs:** Two adjacent Chimera blocks. The left block is a K_{4,4} grid where each node is connected to its four neighbors. The right block is a K_{2,2} grid where nodes are arranged in two columns and two rows, with connections forming a diamond-like pattern.
- Two-level Grid Subgraph:** A square grid of nodes connected by a grid of lines, representing a two-level grid structure.
- Kuratowski Subgraph:** A complex graph structure composed of several interconnected cycles and nodes, representing a Kuratowski subgraph.
- Sparse Kuratowski Subgraph:** A simplified version of the Kuratowski subgraph, showing fewer nodes and connections while maintaining its characteristic structure.

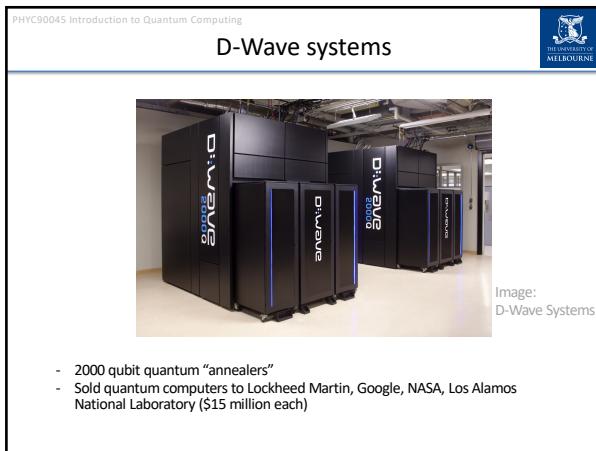
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Adiabatic from Start to Finish

(1) Map computational problem to QUBO/Hamiltonian
 (2) Embed problem on physical architecture
 (3) Execute the adiabatic algorithm
 (4) Read the ground state configuration for the answer to the problem

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MAX-Cut Problem

Partition the nodes into two disjoint subsets (not necessarily with equal numbers of nodes in each!) so that there is the maximum number of edges between the two subsets.

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MAX-Cut Problem

Partition the nodes into two disjoint subsets (not necessarily with equal numbers of nodes in each!) so that there is the maximum number of edges between the two subsets.

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Graph Partitioning to QUBO

Qubits are $|0\rangle$ if they're in subset 0, $|1\rangle$ if they're in subset 1

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Map Problem to QUBO/Hamiltonian

Hamiltonian which counts the edges between subsets:

$$H = \sum_{i,j \in E} \frac{Z_i Z_j - I}{2}$$

Score -1 if edges are in different subsets
Score 0 if the edges are in the same subset

Most edges between subsets will have the minimum energy.
Ground state gives the answer to the MAX-CUT problem.

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Map QUBO to Hamiltonian

$$H = \sum_{i,j \in E} \frac{Z_i Z_j - I}{2}$$

In our case:

$$H = \frac{Z_1 Z_2 - I}{2} + \frac{Z_2 Z_3 - I}{2} + \frac{Z_3 Z_4 - I}{2} + \frac{Z_4 Z_5 - I}{2} + \frac{Z_5 Z_1 - I}{2} + \frac{Z_2 Z_4 - I}{2}$$

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Map QUBO to Hamiltonian

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The figure illustrates the embedding of a quantum circuit into a larger physical architecture. On the left, labeled "Actual architecture", is a 4x4 grid of blue circles representing qubits. On the right, labeled "Problem we want to embed", is a 5-qubit chain represented by blue circles numbered 3 through 5. The chain starts at circle 3, which is connected to circle 4. Circle 4 is connected to both circles 3 and 5. Circles 3, 4, and 5 each have a vertical edge connecting them to a horizontal row above them, with edge weights labeled as $\frac{1}{2}$. This indicates that each of these three qubits is part of a two-qubit interaction with the qubit directly above it.

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Embed on Physical Architecture

Let's imagine that our quantum computing architecture was made up of a square pattern:

The diagram shows two quantum circuit representations. The left side, labeled "Actual architecture", shows a 5-qubit chain with horizontal connections between adjacent qubits. Qubits are numbered 1 to 5. The connections are as follows: qubit 1 to 2 (top), qubit 2 to 3 (top), qubit 3 to 4 (top), qubit 4 to 5 (top), and qubit 3 to 1 (bottom). Each connection has a weight of $\frac{1}{2}$. There is also a vertical connection from qubit 3 to 2 with a weight of -2. The right side, labeled "Problem we want to embed", shows a 5-qubit chain with connections between qubits 1 and 2, 2 and 3, 3 and 4, 4 and 5, and 3 and 1, all with weight $\frac{1}{2}$. A red arrow points from the problem circuit to the actual architecture.

Actual architecture

Problem we want to embed

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Execute algorithm

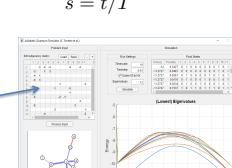
At this point you would program the couplings into your physical quantum computer, and physically perform the annealing schedule:

$$H(s) = (1 - s)H_x + sH_p$$

$$0 \leq s \leq 1$$

$$s = t/T$$

In our case, we will enter the couplings in our MATLAB environment



The figure shows a screenshot of a quantum computing software interface. On the left, there is a 'Quantum Circuit Designer' window with a grid of qubits and various gates. In the center, there are two tables: 'Parameter' and 'Initial State'. The 'Parameter' table includes fields for 'Number of qubits' (set to 8), 'Number of readout qubits' (set to 1), 'Number of readout bits' (set to 1), 'Number of readout fusions' (set to 1), and 'Number of readout fusions' (set to 1). The 'Initial State' table lists states for each qubit from 0 to 7. On the right, there is a plot titled 'Current Eigenspectrum' showing Energy (Hz) on the y-axis (ranging from 0 to 4) versus Time (ns) on the x-axis (ranging from 0 to 10). Multiple curves represent different energy levels over time.

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Energy for our MAX-CUT example

Prepare in $|-\rangle^{\otimes 5}$ state

In ground state,
solving computational problem

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Read Ground State configuration

After the adiabatic evolution we read the state of the quantum computer. Provided we have changed our Hamiltonian slowly enough, we will be in the ground state:

$$|01001\rangle$$

or

$$|10110\rangle$$

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Adiabatic from Start to Finish

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The screenshot shows the MATLAB environment with a quantum computing simulation window. The window title is "MATLAB environment in the lab". The main area displays a quantum circuit diagram with two qubits (q1 and q2) and various gates (H, CNOT, SWAP). Below the circuit is a plot titled "(Lowest) Eigenvalues" showing Energy vs Time, with multiple curves peaking around Time = 5. At the bottom, tabs for "Final Histogram", "Eigenstate probabilities", and "Schedule" are visible.

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Week 10

The logo of The University of Melbourne, featuring a crest with a lion and a unicorn flanking a shield, with the university's name in a scroll below.

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