Week by week

(1) Introduction to quantum computing
(2) **Single qubit representation and operations**
(3) Two and more qubits
(4) Simple quantum algorithms
(5) Quantum search (Grover's algorithm)
(6) Quantum factorization (Shor's algorithm)
(7) Quantum supremacy and noise
(8) Programming real quantum computers (IBM Q)
(9) Quantum error correction (QEC)
(10) QUBO problems and Adiabatic Quantum Computation (AQC)
(11) Variational/hybrid quantum algorithms (QAOA and VQE)
(12) Solving linear equations, QC computing hardware

Week 2

Lecture 3

3.1 The Bloch Sphere representation for qubits

3.2 Quantum operations on qubits

3.3 Qubit gates in matrix form and the Pauli matrices

Lecture 4

4.1 The Pauli gates X, Y and Z and the QUI

4.2 Qubit operations around non-cartesian axes – H and R gates

4.3 Matrix exponential and arbitrary rotations

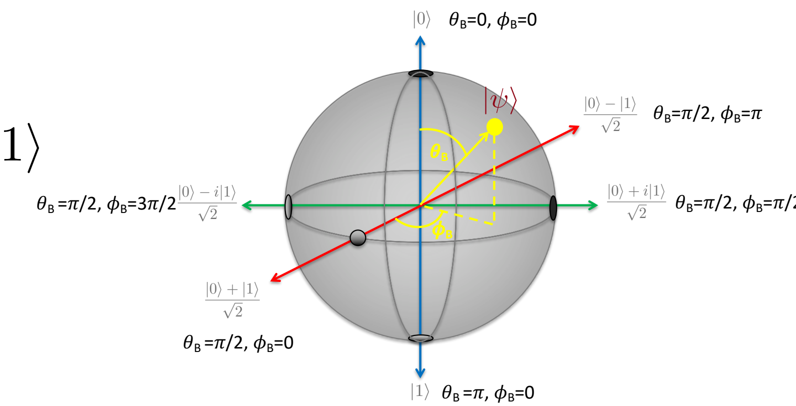
Practice class 2

Bloch sphere and single qubit logic operations on the QUI

Lecture 3 recap

Qubits on the Bloch sphere:

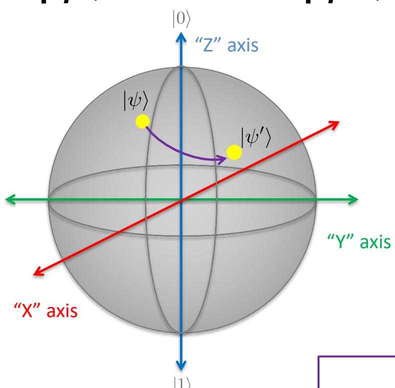
$$|\psi\rangle = a_0\,|0\rangle + a_1\,|1\rangle \rightarrow \cos\frac{\theta_B}{2}\,|0\rangle + \sin\frac{\theta_B}{2}e^{i\phi_B}\,|1\rangle$$



Operations on qubit states:

$$|\psi\rangle \xrightarrow{U} |\psi'\rangle$$

$$|\psi'\rangle = U\,|\psi\rangle$$



Ket notation: e.g. X-gate

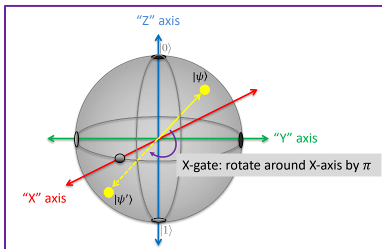
$$a_0\,|0\rangle + a_1\,|1\rangle \xrightarrow{X} a_1\,|0\rangle + a_0\,|1\rangle$$

Operations: matrix notation

$$|\psi'\rangle = U\,|\psi\rangle$$

$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} \text{2 x 2} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

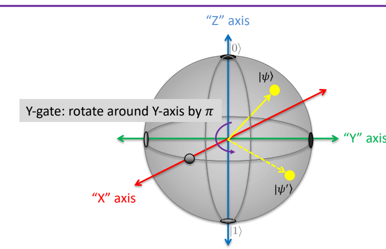
Pauli Matrices



$$|\psi'\rangle = X\,|\psi\rangle$$

$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

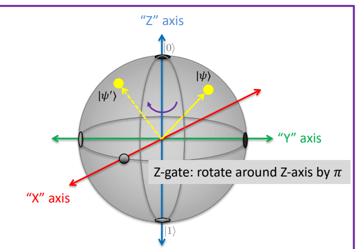
$$\boxed{X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$$



$$|\psi'\rangle = Y\,|\psi\rangle$$

$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$\boxed{Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}}$$



$$|\psi'\rangle = Z\,|\psi\rangle$$

$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

$$\boxed{Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}$$

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Recap: Linear Algebra and Dirac notation

$$|\psi\rangle = a_0\,|0\rangle + a_1\,|1\rangle \;\rightarrow |\psi\rangle = |a_0|e^{i\theta_0}\,|0\rangle + |a_1|e^{i\theta_1}\,|1\rangle$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Computational basis states

$$a_0\,|0\rangle + a_1\,|1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$a_0, a_1 \in \mathbb{C}$$

General qubit state

a_0 and a_1 are "amplitudes"

$$a_0\,|0\rangle + a_1\,|1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

Prob[measure "0"] = $|a_0|^2$

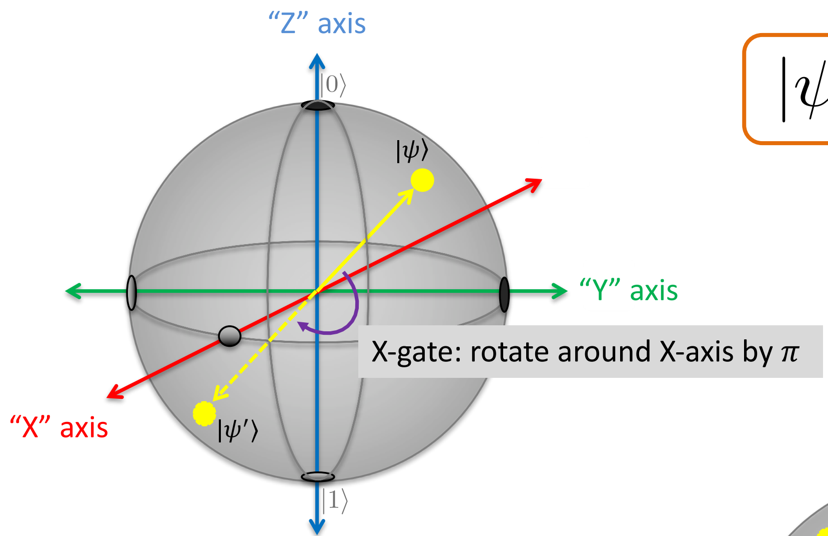
Prob[measure "1"] = $|a_1|^2$

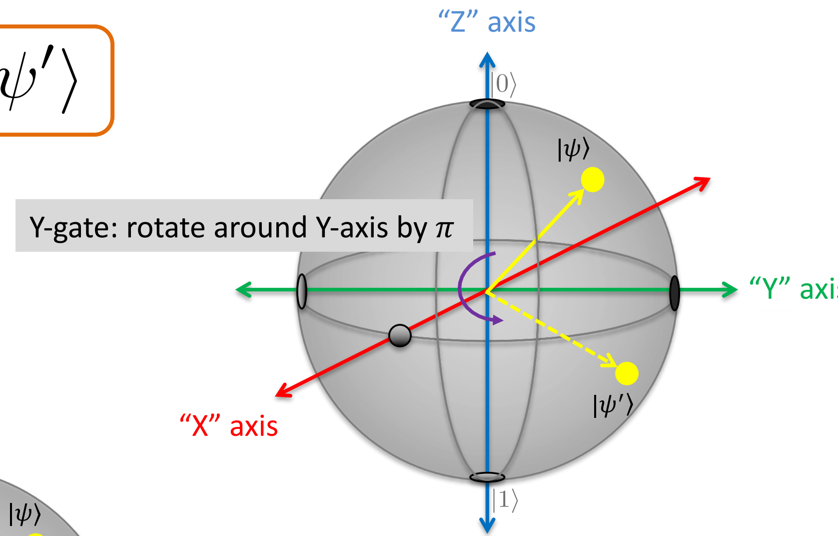
The Pauli gates X, Y and Z and the QUI

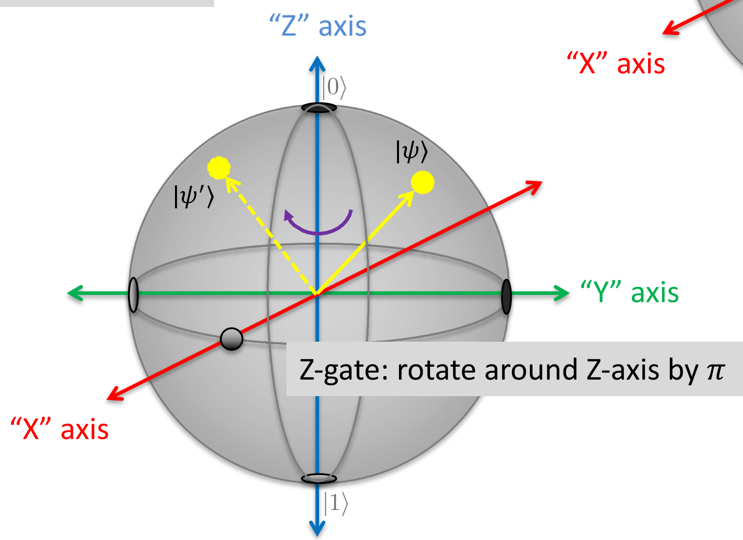
Recap: the "Cartesian" quantum operations: X, Y, Z

We can specify the state moving across the Bloch sphere in many ways, but the "Cartesian" operations are very simple – a rotation of π (180°) about any of X, Y, or Z axes:

$$|\psi\rangle \rightarrow |\psi'\rangle$$







NB. Perspectives not 100% accurate!

The Cartesian rotations are usually referred to as the "Pauli" operators X, Y, Z

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Recap: the X gate in matrix form

Action of X-gate in "ket" form: $a_0 \left|0\right\rangle + a_1 \left|1\right\rangle \xrightarrow{\text{X}} a_1 \left|0\right\rangle + a_0 \left|1\right\rangle$

What is the X-gate in "matrix" form?

Recall "matrix" notation: $\left|0\right\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\left|1\right\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $a_0 \left|0\right\rangle + a_1 \left|1\right\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$ $\left|\psi\right\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$

Operations in matrix representation:

$$|\psi'\rangle = U \; |\psi\rangle$$

$$\begin{pmatrix} a_0' \\ a_1' \end{pmatrix} = \begin{pmatrix} \text{2 x 2} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

Action of X-gate in matrix form:

$$a_0|0\rangle + a_1|1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \longrightarrow a_1|0\rangle + a_0|1\rangle = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

i.e. $\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \xrightarrow{\text{X}} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$

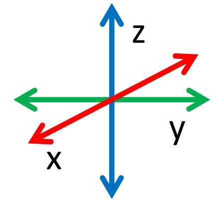
$$\boxed{X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$$

In matrix notation, in general:

$$X\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

X Gate (the X operator): π around X-axis

Circuit symbol: 



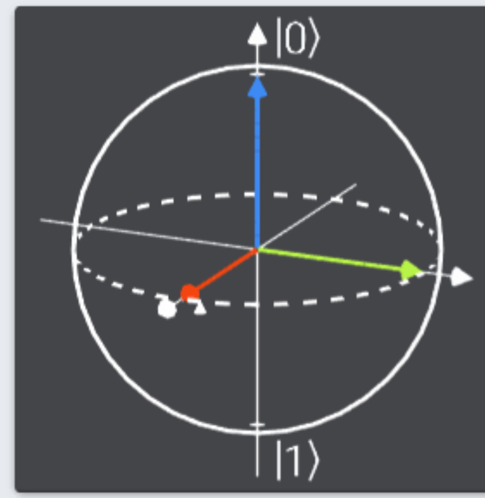
Matrix representation: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$X\left(a_0 |0\rangle + a_1 |1\rangle\right) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

$$\text{i.e.} \quad X\left(a_0 |0\rangle + a_1 |1\rangle\right) = a_1 |0\rangle + a_0 |1\rangle$$

X GATE

Rotate around the X axis by π radians.

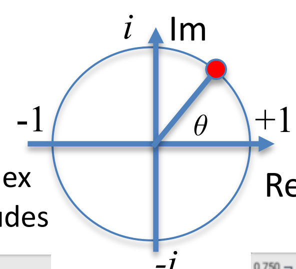


Action on ket states: $a_0 |0\rangle + a_1 |1\rangle \rightarrow a_1 |0\rangle + a_0 |1\rangle$

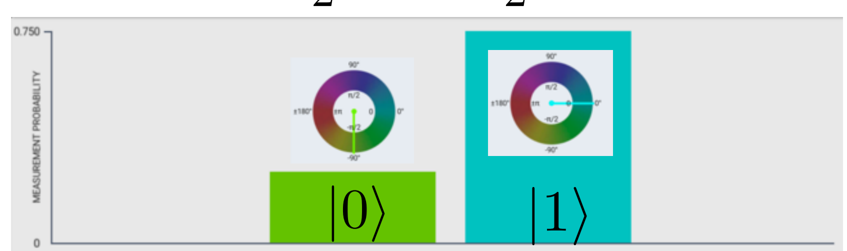
QUI example:

$\frac{\sqrt{3}}{2} |0\rangle + \frac{-i}{2} |1\rangle$

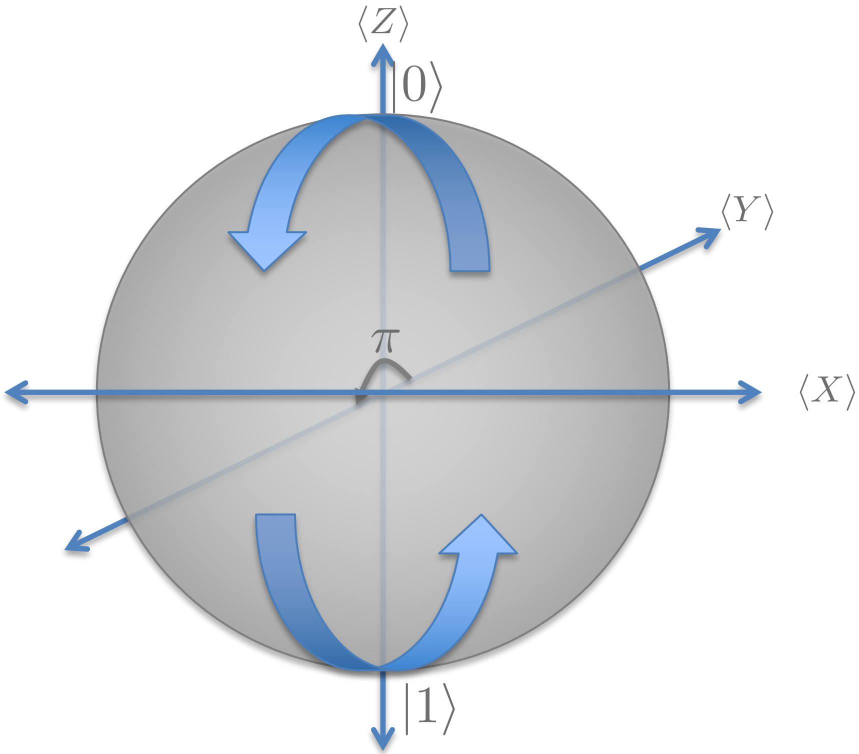
complex amplitudes



$\frac{-i}{2} |0\rangle + \frac{\sqrt{3}}{2} |1\rangle$

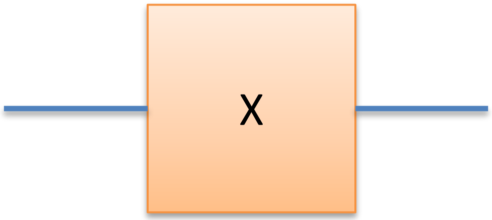


X Gate



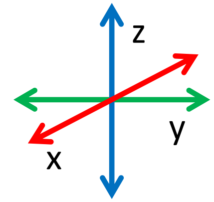
$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



Y Gate (the Y operator): π around Y-axis

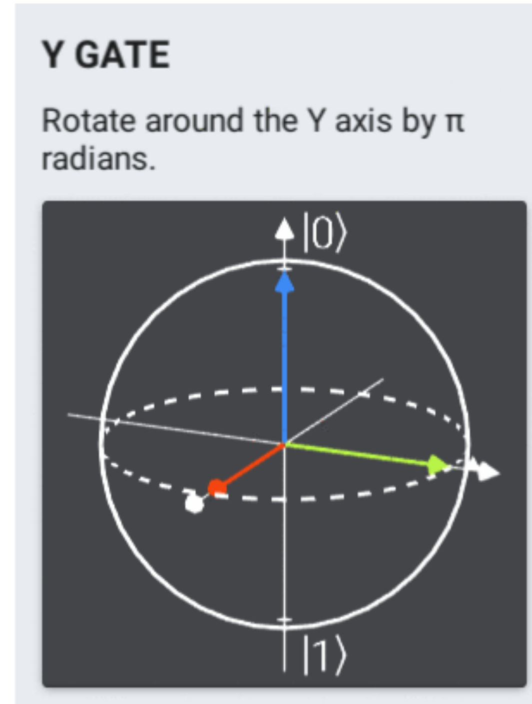
Circuit symbol: 



Matrix representation: $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$$Y\left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -ia_1 \\ ia_0 \end{bmatrix}$$

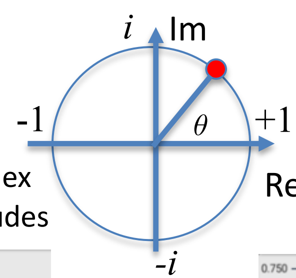
$$\text{i.e.} \quad Y\left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) = -ia_1\left|0\right\rangle + ia_0\left|1\right\rangle$$



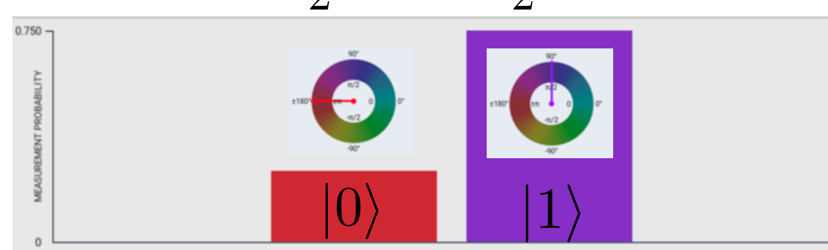
Action on ket states: $a_0\left|0\right\rangle + a_1\left|1\right\rangle \rightarrow -ia_1\left|0\right\rangle + ia_0\left|1\right\rangle$

QUI example:

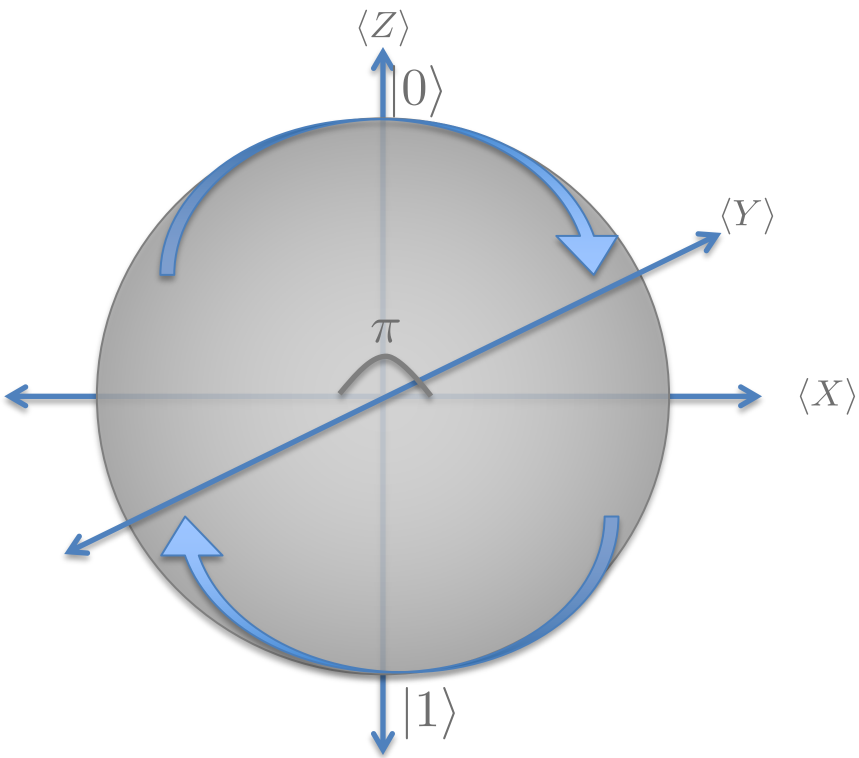
$$\frac{\sqrt{3}}{2}\left|0\right\rangle + \frac{-i}{2}\left|1\right\rangle$$



$$\frac{-1}{2}\left|0\right\rangle + \frac{i\sqrt{3}}{2}\left|1\right\rangle$$

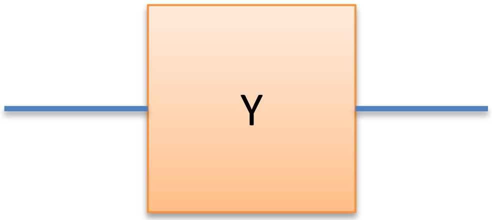


Y Gate



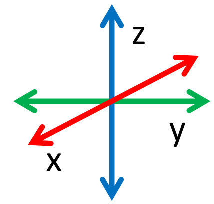
$$\alpha|0\rangle + \beta|1\rangle \rightarrow i\alpha|1\rangle - i\beta|0\rangle$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$



Z Gate (the Z operator): π around Z-axis

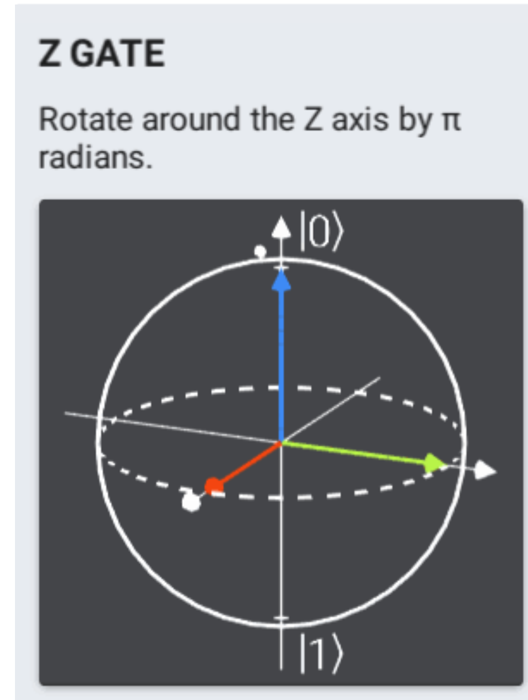
Circuit symbol: 



Matrix representation: $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$Z\left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_0 \\ -a_1 \end{bmatrix}$$

$$\text{i.e.} \quad Z\left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) = a_0\left|0\right\rangle - a_1\left|1\right\rangle$$

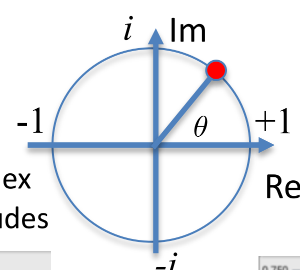


Action on ket states: $a_0\left|0\right\rangle + a_1\left|1\right\rangle \rightarrow a_0\left|0\right\rangle - a_1\left|1\right\rangle$

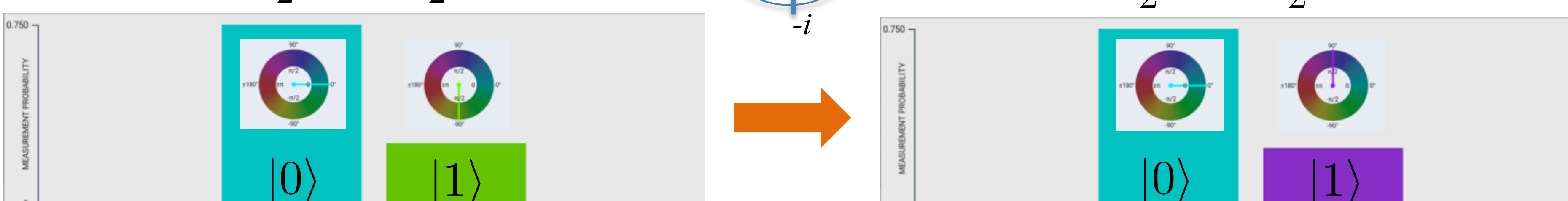
QUI example:

$$\frac{\sqrt{3}}{2}\left|0\right\rangle + \frac{-i}{2}\left|1\right\rangle$$

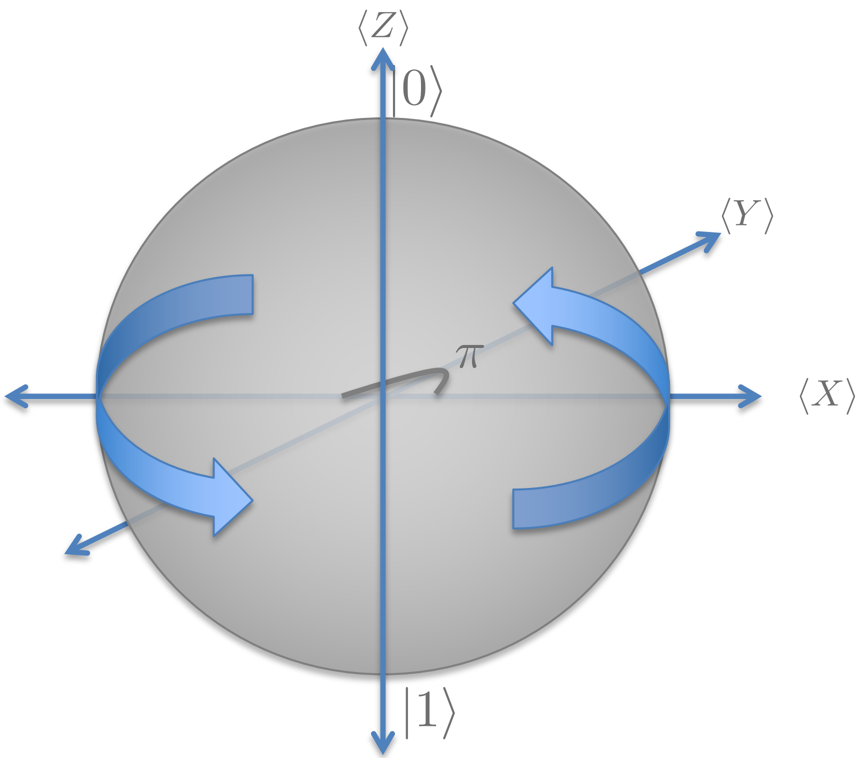
complex amplitudes



$$\frac{\sqrt{3}}{2}\left|0\right\rangle + \frac{i}{2}\left|1\right\rangle$$

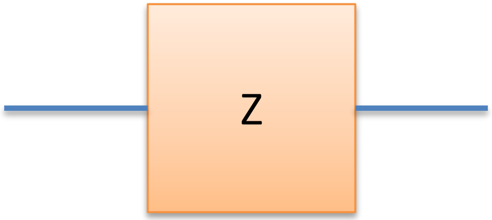


Z Gate



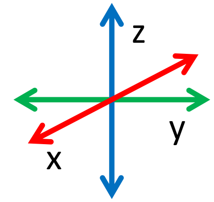
$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



S Gate (the S operator): Z-axs, $\pi/2$ rotation

Circuit symbol: 



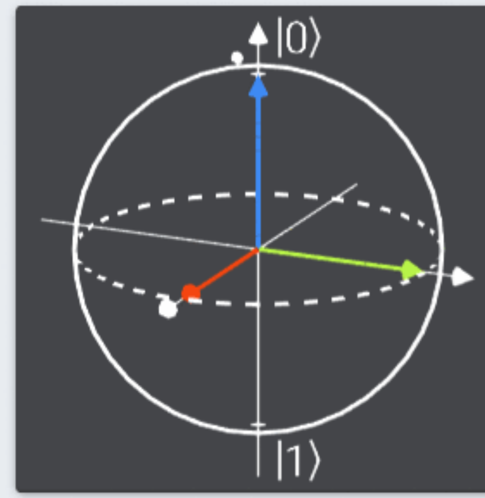
Matrix representation: $S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

$$S\left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_0 \\ ia_1 \end{bmatrix}$$

$$\text{i.e.} \quad S\left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) = a_0\left|0\right\rangle + ia_1\left|1\right\rangle$$

S GATE

Rotate around the Z axis by π / 2 radians.

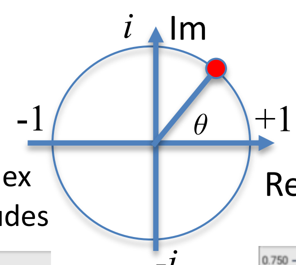


Action on ket states: $a_0\left|0\right\rangle + a_1\left|1\right\rangle \rightarrow a_0\left|0\right\rangle + ia_1\left|1\right\rangle$

QUI example:

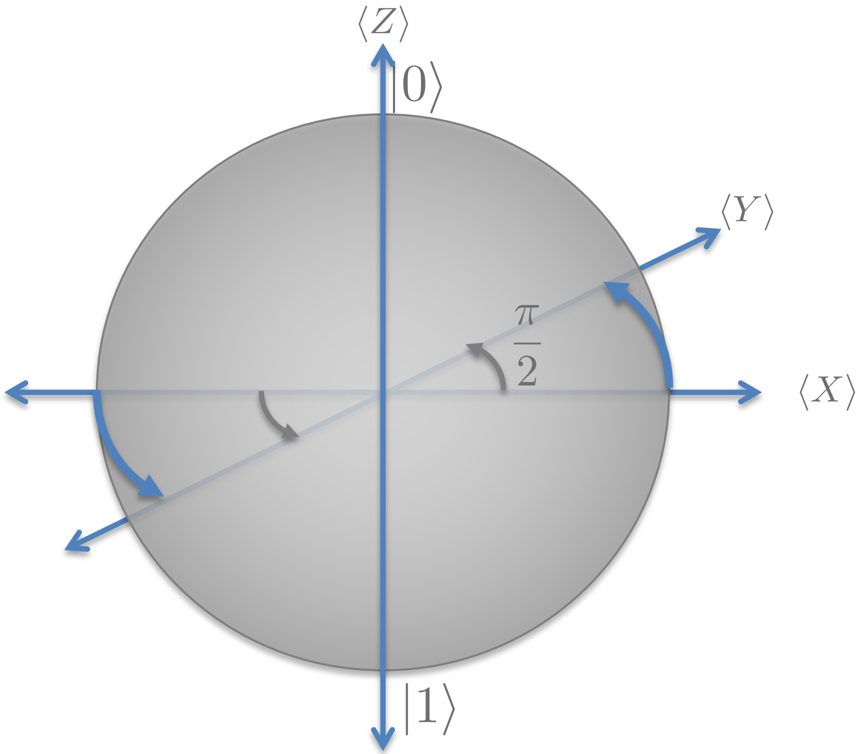
$$\frac{\sqrt{3}}{2}\left|0\right\rangle + \frac{-i}{2}\left|1\right\rangle$$

complex amplitudes



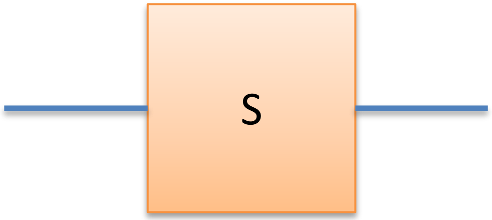
$$\frac{\sqrt{3}}{2}\left|0\right\rangle + \frac{1}{2}\left|1\right\rangle$$

Non-π rotations: S Gate



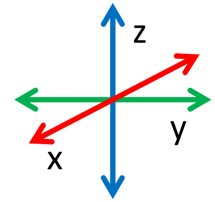
$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle + i\beta|1\rangle$$

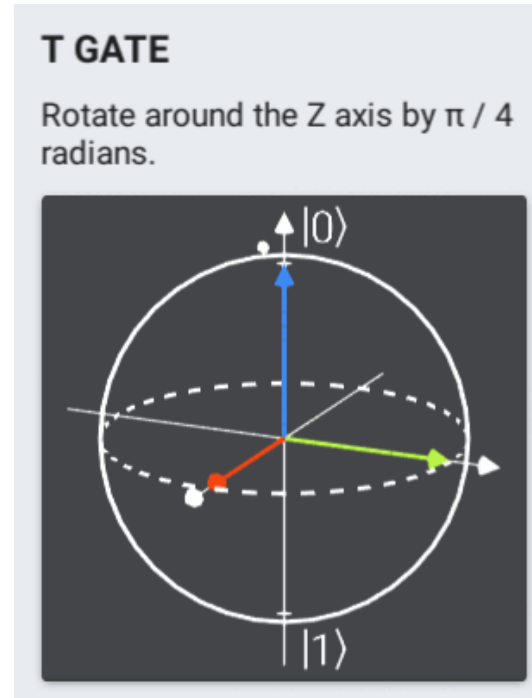
$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$



T Gate (the T operator): Z-axis, $\pi/4$ rotation

Circuit symbol: 





Matrix representation: $$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

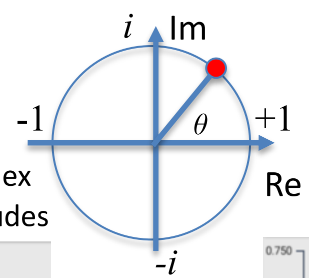
$$T\left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_0 \\ e^{i\pi/4}a_1 \end{bmatrix}$$

i.e. $$T\left(a_0\left|0\right\rangle + a_1\left|1\right\rangle\right) = a_0\left|0\right\rangle + e^{i\pi/4}a_1\left|1\right\rangle$$

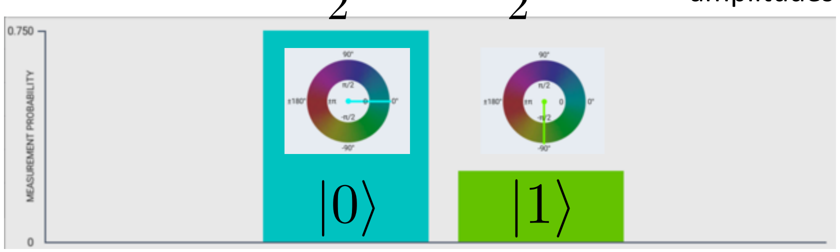
Action on ket states: $a_0\left|0\right\rangle + a_1\left|1\right\rangle \rightarrow a_0\left|0\right\rangle + e^{i\pi/4}a_1\left|1\right\rangle$

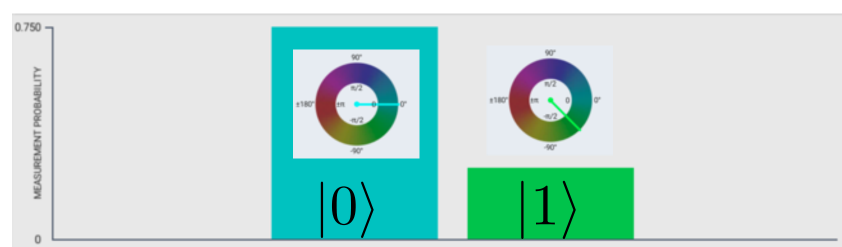
QUI example:

$$\frac{\sqrt{3}}{2}\left|0\right\rangle + \frac{-i}{2}\left|1\right\rangle$$

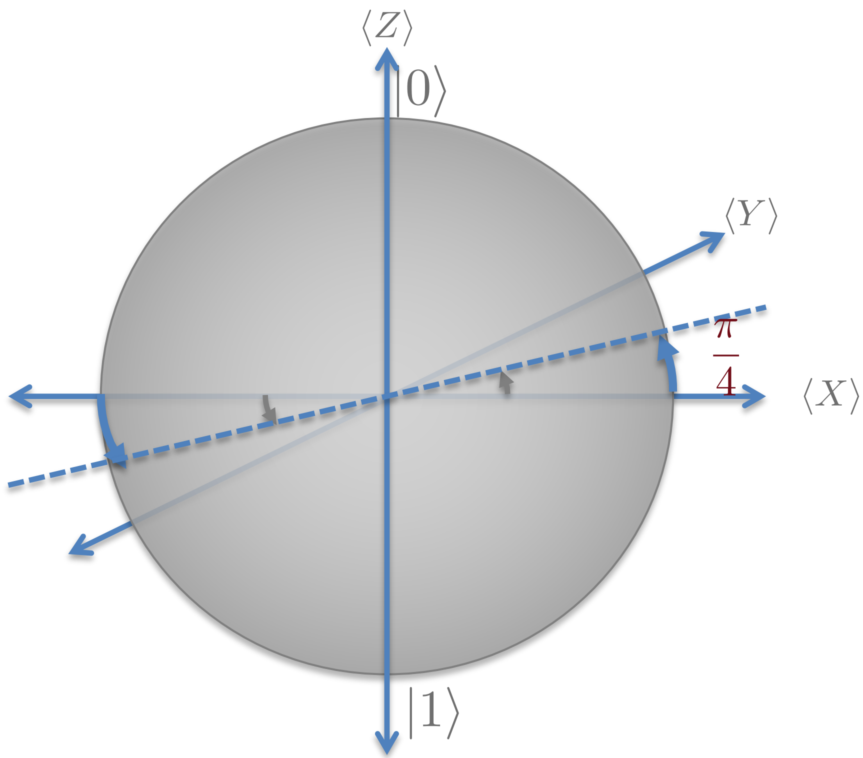


$$\frac{\sqrt{3}}{2}\left|0\right\rangle + \frac{-ie^{i\pi/4}}{2}\left|1\right\rangle$$



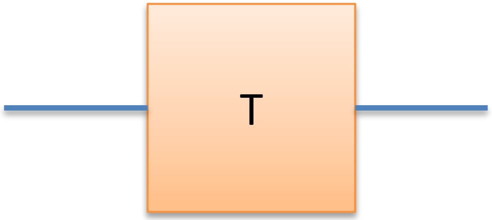


T Gate



$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle + e^{i\frac{\pi}{4}}\beta|1\rangle$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$



The first “non-Clifford” gate we’ve encountered – more difficult to perform under some error correction codes.

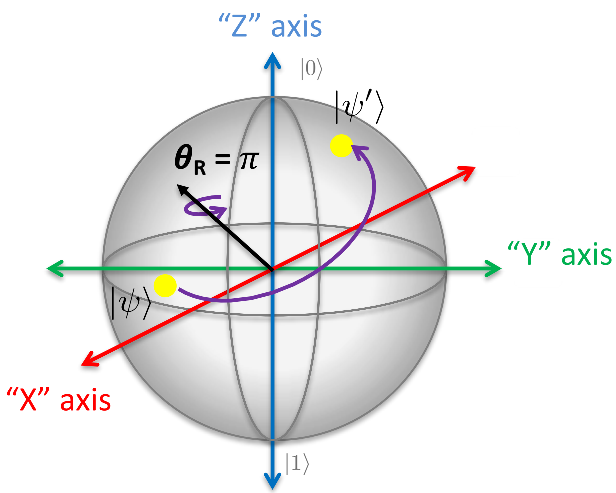
The Hadamard gate

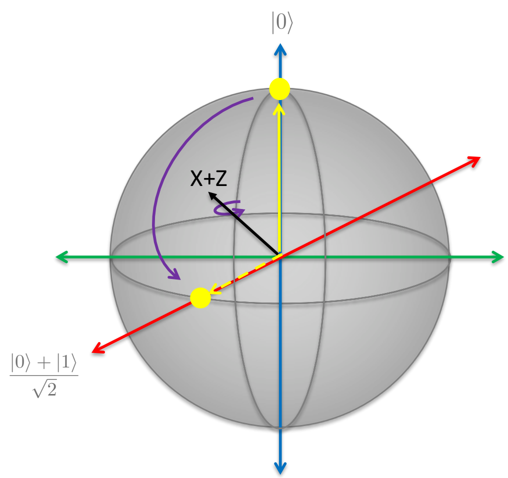
The Hadamard gate H

The Hadamard gate H is one of the most important – it generates superposition states.

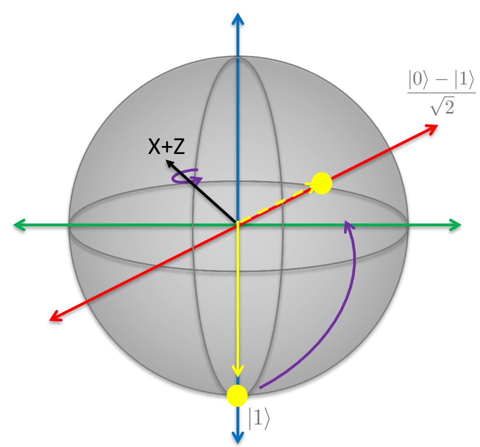
Unlike the X, Y and Z gates which rotate about one of the cartesian axes, the H-gate rotates about the X+Z axis (or unit vector) by an angle π.

Rotation about "X+Z" axis, given by $\boldsymbol{n} = (1,0,1)/\sqrt{2}$





$$|0\rangle \longrightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$$|1\rangle \longrightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

The Hadamard gate H

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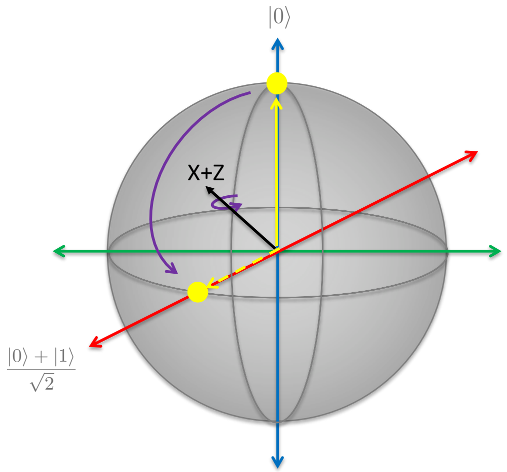
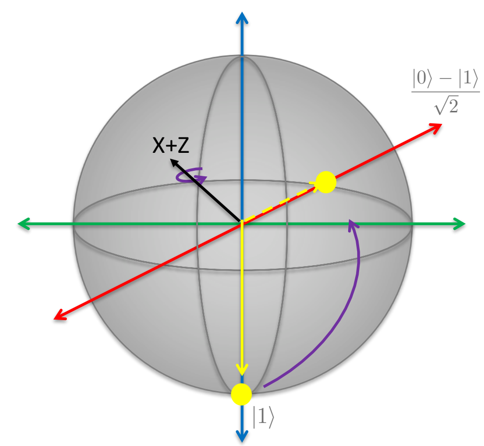
H-gate on the computational states:



$$|0\rangle \longrightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$$|1\rangle \longrightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

H-gate on a general superposition state:

$$a_0|0\rangle + a_1|1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \longrightarrow a_0 \frac{|0\rangle + |1\rangle}{\sqrt{2}} + a_1 \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$= \frac{a_0 + a_1}{\sqrt{2}}|0\rangle + \frac{a_0 - a_1}{\sqrt{2}}|1\rangle \longrightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} a_0 + a_1 \\ a_0 - a_1 \end{bmatrix}$$

$$|\psi'\rangle = H \; |\psi\rangle$$

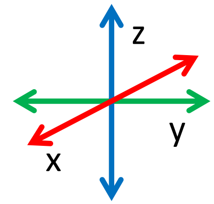
$$\begin{pmatrix} a'_0 \\ a'_1 \end{pmatrix} = \begin{pmatrix} \text{2 x 2} \\ \text{matrix} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

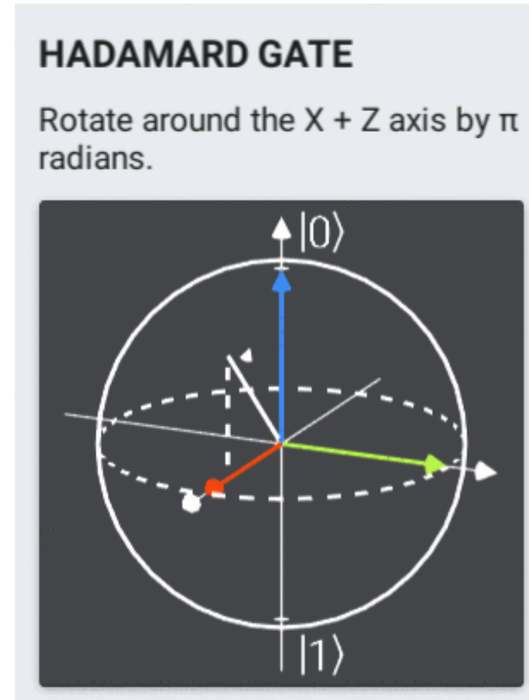
$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(Ex. check it)

H Gate (the H operator): π around X+Z-axis

Circuit symbol: 



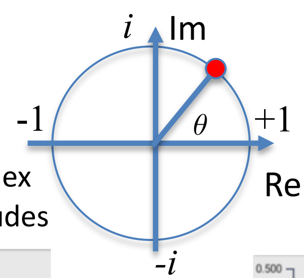


Matrix representation: $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Action on ket states: $|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ $\quad |1\rangle \rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

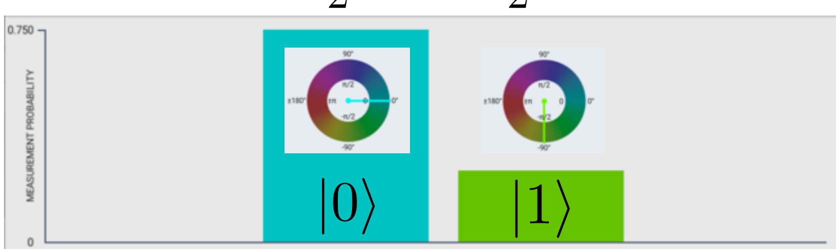
$$a_0\,|0\rangle + a_1\,|1\rangle \rightarrow \frac{a_0 + a_1}{\sqrt{2}}\,|0\rangle + \frac{a_0 - a_1}{\sqrt{2}}\,|1\rangle$$

QUI example:



complex amplitudes

$$\frac{\sqrt{3}}{2}\,|0\rangle + \frac{-i}{2}\,|1\rangle$$



$$\frac{\sqrt{3}-i}{2\sqrt{2}}\,|0\rangle + \frac{\sqrt{3}+i}{2\sqrt{2}}\,|1\rangle$$

Zoo of one-qubit gates

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

π rotation about the x-, y- and z- axes.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

"Hadamard" gate

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$\pi/2$ rotation about the z- axis.

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$\pi/4$ rotation about the z- axis.

General rotations

The Pauli Matrices

Operations transform states and are equivalent to moving around on the Bloch sphere.

For qubits, important matrices are the Pauli matrices:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

All square to identity (check):

$$X^2 = X\,X = I \qquad Y^2 = Y\,Y = I \qquad Z^2 = Z\,Z = I$$

Odd and even powers:

$$X^3 = X\,X\,X = I\,X = X \qquad X^4 = X\,X\,X\,X = I\,I = I$$ etc

Why is this important? – quantum logic gates are ultimately written as <u>exponentials</u> of Pauli operators…let's see how this works…

Exponential of a Matrix

We can define the exponential of a matrix using the power series for the exponential:

$$\exp(A) = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!} + \ldots$$

The exponential of a Pauli Matrix (eg. X) including an angle parameter:

$$\begin{aligned}
\exp(i\theta X) &= I + i\theta X - \frac{\theta^2}{2!}X^2 - \frac{i\theta^3}{3!}X^3 + \frac{\theta^4}{4!}X^4 + \ldots \\
&= I + i\theta X - \frac{\theta^2}{2!}I - \frac{i\theta^3}{3!}X + \frac{\theta^4}{4!}I + \ldots \\
&= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \ldots\right) I + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \ldots\right) X \\
&= \cos\theta I + i\sin\theta X
\end{aligned}$$

(used power series for cos and sin)

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General exponentiation of Paulis

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

We just proved: $\exp(i\,\theta\,X) = I\cos\theta + i\,X\sin\theta$

to generalise we form a 2x2 matrix Σ from the Pauli matrices as: $\Sigma = \hat{\mathbf{n}} \cdot \boldsymbol{\sigma}$

$$\boxed{\hat{n}\cdot\sigma = n_x X + n_y Y + n_z Z}$$

$$\boldsymbol{\sigma} = (X, Y, Z)$$

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{|\mathbf{n}|}$$

where $\hat{\mathbf{n}}$ is a unit spatial 3-vector and σ is a vector of Paulis, a similar proof follows, so

$$\boxed{\exp(i\,\theta\,\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}) = I\cos\theta + i\,\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}\,\sin\theta}$$

e.g. $\hat{\mathbf{n}} = (1,0,0) \rightarrow \hat{\mathbf{n}}\cdot\boldsymbol{\sigma} = (1,0,0)\cdot\boldsymbol{\sigma} = X$

Recover previous result: $\exp(i\,\theta\,X) = I\cos\theta + i\,X\sin\theta$

Rotation as an exponential

A rotation by angle θ around a (unit vector) axis $\hat{n}$, is given by:

$$R_n(\theta) = \exp\left(-i\frac{\theta}{2}\hat{n}\cdot\sigma\right)$$

You can write this exponential using this identity:

$$\exp(i\,\theta\,\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}) = I\cos\theta + i\,\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}\,\sin\theta$$

Global Phase

There is no experimental difference between

$$|\psi\rangle$$

and

$$\exp(i\theta_{global})\,|\psi\rangle$$

This means that a phase which multiplies **all** the amplitudes (and it really does have to be all is not measurable).

It also means that when you're choosing a gate, you can multiply **the entire matrix** by a global phase factor, and there won't be any measurable difference.

Note: It is common to choose the global phase so that the determinant, |U| = 1. Gates in this group are said to be the the group SU(n).

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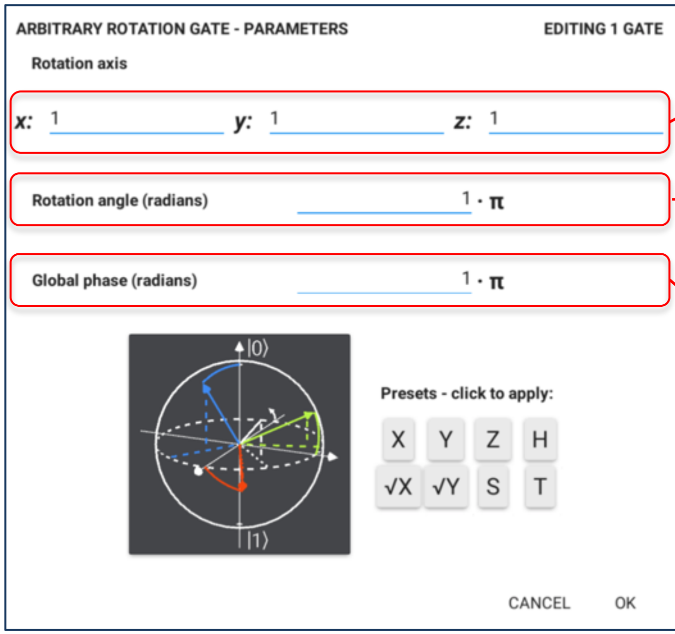
Arbitrary axis rotation – coding in QUI

Coding an arbitrary rotation gate in QUI – R-gate

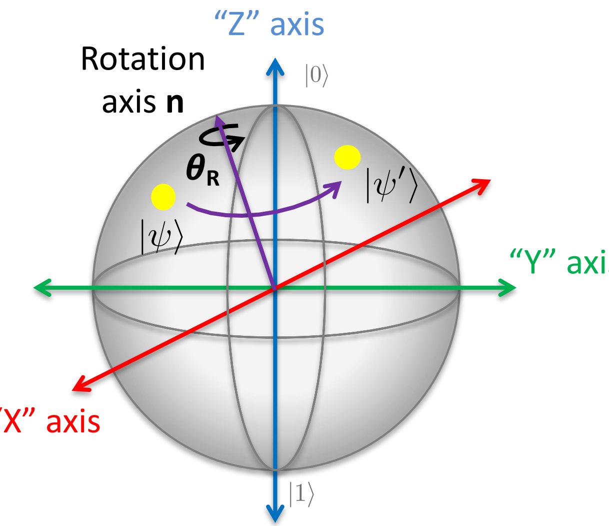
The "parameters" menu allows you to specify axis and angle:



Cartesian cords for axis of rotation **n**

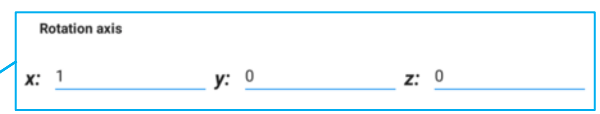
Angle of rotation θ_R about **n**

Global phase θ_g : generally set this to <u>zero</u> unless otherwise directed!

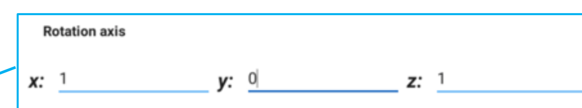


NB. QUI normalises axis for you:

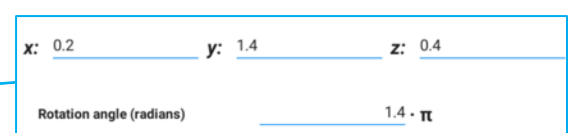
e.g. X axis, i.e. $\boldsymbol{n} = (1,0,0)$ -> enter (1,0,0)



e.g. X+Z axis, i.e. $\boldsymbol{n} = (1,0,1)/\sqrt{2}$ -> enter (1,0,1)



(entries can be decimals)



Useful facts about one qubit operations

Operations are unitary

In quantum mechanics, *unitary* operators acting on quantum states produce new quantum states. These operators can be described by *unitary* matrices.

The new state is given by: $|\psi'\rangle = U\,|\psi\rangle$

Unitary operations are ones for which: $U^\dagger U = I$

Where the dagger represents taking the transpose (t) and complex conjugate (*).

$$U^\dagger = U^{t*}$$

In quantum mechanics, all unitary operations are **reversible**.

It’s possible to efficiently express every classical computation using equivalent reversible logic gates, but there can be a cost in terms of additional bits and operations.

Operations Don't Commute!

For operators (e.g. matrices), remember

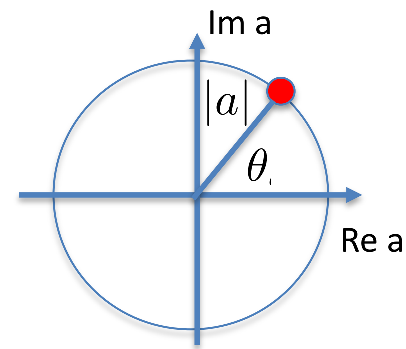
$$AB \neq BA$$

Order matters!

Note angles in context – abundant use of θ

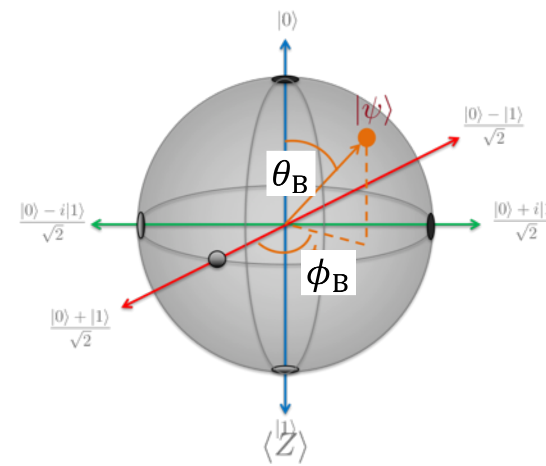
Phase angle of complex amplitudes in polar coordinates:

$$|\psi\rangle = a_0\,|0\rangle + a_1\,|1\rangle \quad \rightarrow |\psi\rangle = |a_0|e^{i\theta_0}\,|0\rangle + |a_1|e^{i\theta_1}\,|1\rangle$$



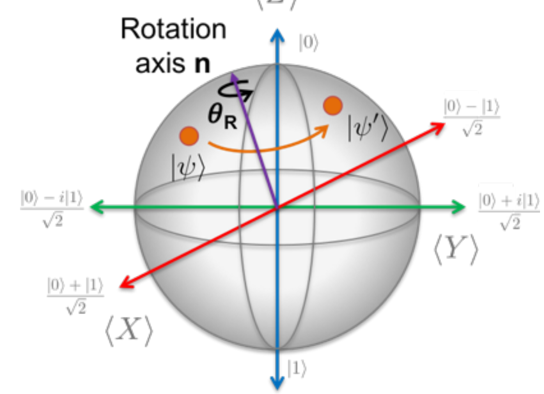
Angle specifying position on the Bloch sphere:

$$|\psi\rangle = a_0\,|0\rangle + a_1\,|a_1\rangle \rightarrow \cos\frac{\theta_{\rm B}}{2}\,|0\rangle + \sin\frac{\theta_{\rm B}}{2}e^{i\phi_{\rm B}}\,|1\rangle$$



Angle of rotation of a qubit state on the Bloch sphere about a specified axis (unit vector), **n**:

$$|\psi'\rangle = R_{\mathbf{n}}(\theta_{\rm R})\,|\psi\rangle$$



A zoo of one-qubit gates

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Week 2

Lecture 3
3.1 The Bloch Sphere representation for qubits
3.2 Quantum operations on qubits
3.3 Qubit gates in matrix form and the Pauli matrices

Lecture 4
4.1 The Pauli gates X, Y and Z and the QUI
4.2 Qubit operations around non-cartesian axes – H and R gates
4.3 Programming sequences over the qubit logic gate library
4.4 Note on the context and use of angles

Practice class 2
Bloch sphere and single qubit logic operations on the QUI

Week by week

(1) Introduction to quantum computing
(2) Single qubit representation and operations
(3) Two and more qubits
(4) Simple quantum algorithms
(5) Quantum search (Grover's algorithm)
(6) Quantum factorization (Shor's algorithm)
(7) Quantum supremacy and noise
(8) Programming real quantum computers (IBM Q)
(9) Quantum error correction (QEC)
(10) QUBO problems and Adiabatic Quantum Computation (AQC)
(11) Variational/hybrid quantum algorithms (QAOA and VQE)
(12) Solving linear equations, QC computing hardware