Week 7

Lecture 13 - Quantum Supremacy
11.1 Boson Sampling
11.2 IQP Problem
11.3 Google's pseudorandom circuits

Lecture 14 - Errors
12.1 Quantum errors: unitary and stochastic errors
12.2 Randomized Benchmarking
12.3 Purity

Lab 7
Quantum Supremacy and Errors

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Quantum Supremacy

Physics 90045
Lecture 13

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Determining supremacy?

Gary Kasparov

vs

Deep Blue

On February 10, 1996, Deep Blue beat Kasparov under tournament regulations. In the subsequent 1997 rematch, Deep Blue won the series.

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Quantum supremacy in the news

WIRED Google, Alibaba Spar Over Timeline for 'Quantu

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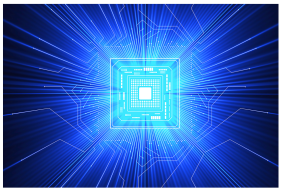
TOM SIMONITE BUSINESS 05.10.18 07:00 AM

GOOGLE, ALIBABA SPAR OVER TIMELINE FOR 'QUANTUM SUPREMACY'

https://www.wired.com/story/google-alibaba-spar-over-timeline-for-quantum-supremacy/

NewScientist

Google just made it much harder to build a serious quantum computer



Is a quantum revolution near? ALFRED PASIEKA/SCIENCE PHOTO LIBRARY

By **Chelsea Whyte**

Google is racing to create the first quantum computer capable of solving a problem ordinary computers cannot – and it has just made that challenge much harder.

Achieving "quantum supremacy", as it is known, involves building a device that can solve a problem faster than any non-quantum computer.

https://www.newscientist.com/article/2176575-google-just-made-it-much-harder-to-build-a-serious-quantum-computer/

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What is quantum supremacy?

Quantum supremacy is using a quantum computer to solve a problem which classical computers **practically** cannot.

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Algorithms for quantum supremacy

The race is on to build a quantum computer which will achieve quantum supremacy. Implementing large scale factoring would demonstrate quantum supremacy, but that would require a very large (potentially millions of qubits) quantum computer. In the short term we will only have access to NISQ devices.

Noisy
Intermediate **S**cale (50-100 qubits)
Quantum devices

Three quantum algorithms that might be able to demonstrate quantum supremacy with 50-100 qubits:

- Boson Sampling
- Instantaneous Quantum Polynomial-Time circuits (IQP)
- Pseudorandom circuits

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HOWTO quantum supremacy

Pick a problem which is:

- ❑ As easy as possible for a quantum computer
- ❑ As hard as possible for a classical computer to simulate

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Boson Sampling

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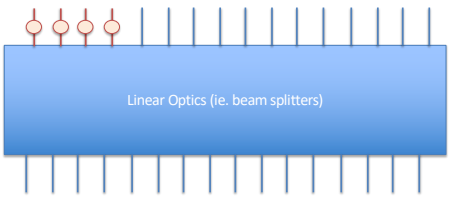
A little physics experiment…

n single photon sources



You won't need to know the physics of this device.

n^2 output modes.

Can a classical computer produce **samples** from the output which mimic the quantum device?

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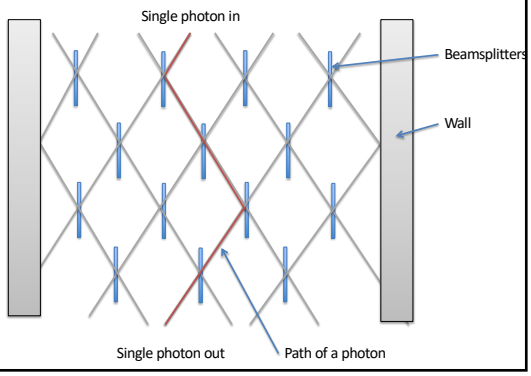
Quantum Pachinko



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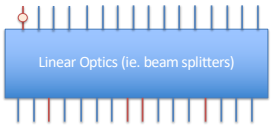
One single path

We can write out a matrix: If a single photon enters in mode x, what is the amplitude it will have at output y?



Input modes

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

Amplitudes of output modes

Unitary matrix

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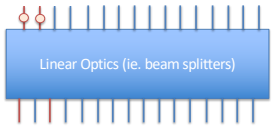
Many paths

If we have two input photons:



What is the amplitude associated with being detected in output modes 1 and 3?

Input modes

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

Amplitudes of output modes

Amplitude of photons in locations 1,3:

$$a_{11}a_{32} + a_{12}a_{31}$$

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Many paths

In general take the submatrix corresponding to a particular input and output, and find its **permanent:**

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix}$$

Submatrix defined by the input modes and output modes

The resulting amplitude is the **permanent** of the submatrix:

$$\mathrm{perm}(A) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i,\sigma(i)}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \mathrm{perm}(A) = ad + bc$$

Same as determinant, but with no subtraction, all addition.

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Complexity of finding permanent

Unlike determinants, finding a permanent of a matrix is a *surprisingly difficult* computational problem.

Finding the permanent is a #P complete problem.

#P is the set of counting problems associated with decision problems in NP.

NP: Is there as a satisfying assignment of variables to this 3SAT problem?
#P: How many satisfying assignments of variables are there to this 3SAT problem?

NP: Is there a travelling salesman path with distance less than *d*?
#P: How many travelling salesman paths are there with a distance less than d.

Calculating amplitudes and probabilities for Boson sampling is a hard classical problem!

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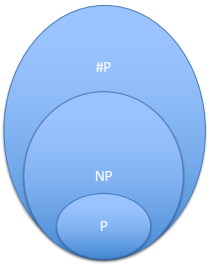
Some classical complexity classes



Informally:

P: Problems which can be solved in polynomial time

NP: Problems which can be checked in polynomial time (ie. they have an efficiently verifiable proof)

#P: Problems which count the number of solutions in NP

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The polynomial hierarchy

Very quick introduction: Given an **oracle** in some complexity class which evaluates instantly, what problems can we now evaluate in polynomial time?

$$P^P = P$$

Polynomial time algorithm
With access to an oracle which can instantaneously evaluate functions in P

$$NP^P = NP$$

But a polynomial time algorithm with an NP oracle appears to be more powerful than both P and NP:

$$P^{NP}$$

We can recursively define complexity classes this way, with oracles which increase in strength at each level. This whole hierarchy is known as the Polynomial Hierarchy, **PH**.

If, at some level, providing the oracle didn't lead to a superset of problems, the polynomial hierarchy would "collapse". Computer scientists don't think this happens.

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Sampling is also hard to simulate

Calculating amplitudes and probabilities for Boson sampling is a hard classical problem!

Calculate the **permanent** of the submatrix:

$$\mathrm{perm}(A) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i,\sigma(i)} \qquad \mathrm{perm}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad + bc$$

We don't technically have to calculate the probabilities explicitly. Maybe we can *sample* from the probability distribution?

No – this would result in a collapse of the Polynomial Hierarchy.

Not proven, but like P=NP, computer scientists generally don't expect the polynomial hierarchy collapses.

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HOWTO quantum supremacy

Boson Sampling is a problem which:

- ❑ "Easy" to implement using linear optics
- ❑ Hard for a classical computer to simulate –
  Polynomial Hierarchy would collapse

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IQP Circuits
Instantaneous Quantum Polynomial-Time

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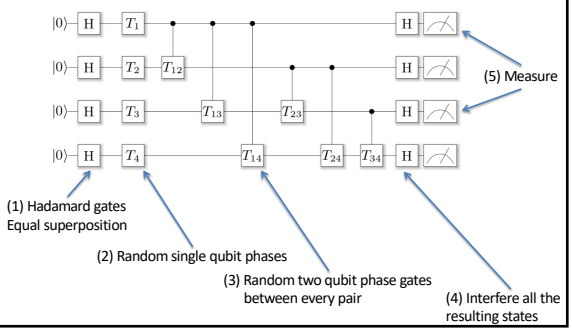
IQP Circuits



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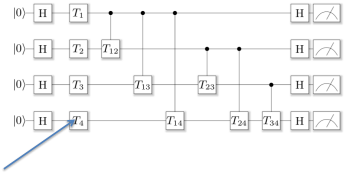
Random Phases



Each of these T_m gates is a rotation (around z) by a multiple of π/4:

$$T_m = \cos\left(\frac{k_m\pi}{8}\right) I + i \sin\left(\frac{k_m\pi}{8}\right) Z_m$$

$$T_m = R_z\left(-\frac{k_m\pi}{4}\right)$$ on the m^{th} qubit

Where k_m is an integer chosen uniformly at random between 0 and 7. This is equivalent (up to a global phase) of applying a T gate k_m times.

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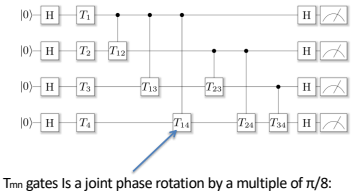
Random Joint Phases



Each of these T_{mn} gates Is a joint phase rotation by a multiple of π/8:

$$T_{mn} = \cos\left(\frac{k_{mn}\pi}{8}\right) I + i \sin\left(\frac{k_{mn}\pi}{8}\right) Z_m Z_n$$

Where k_m is an integer chosen uniformly at random between 0 and 7.

In the lab we can implement a similar algorithm with controlled T_{mn} gates.

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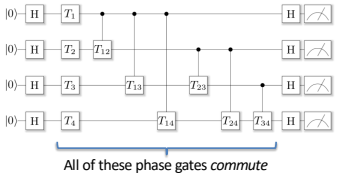
"Instantaneous"



All of these phase gates *commute*

The order which you apply the single and two qubit phase gates doesn't matter. They commute with each other, so can be applied in any order.

Eg.

$$ZT_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$

$$T_2 Z = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$

Diagonal gates commute

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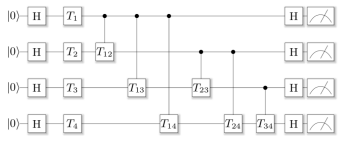
Collapse of the polynomial hierarchy



Aim: To sample from the output of this circuit. Easy for a quantum computer.

If this could be done efficiently using a classical computer, it would imply the collapse of the polynomial hierarchy (and so isn't expected to be possible).

Practically, classical simulations are limited to <50-70 qubits (for low error rates).

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Pseudorandom Circuits

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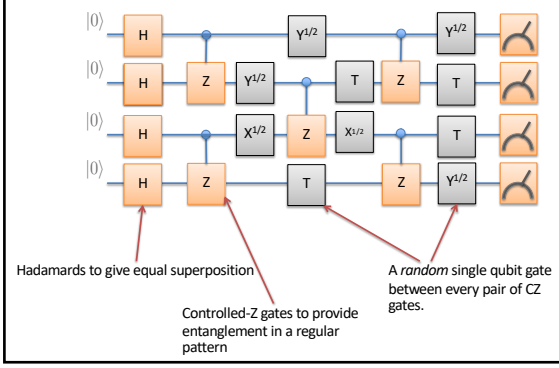
The circuit



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Square Root X and Y

In the previous slide, we simply have that

$$X^{1/2} = R_x\left(\frac{\pi}{2}\right)$$

and similarly,

$$Y^{1/2} = R_y\left(\frac{\pi}{2}\right)$$

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Schedule of CZ

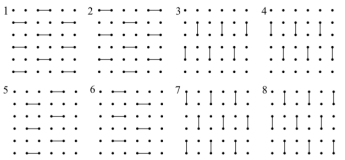

FIG. 6. Layouts of CZ gates in a 6×6 qubit lattice. It is currently not possible to perform two CZ gates simultaneously in two neighboring superconducting qubits [33, 34, 49, 52]. We iterate over these arrangements sequentially, from 1 to 8.

From Boixo et al, https://arxiv.org/pdf/1608.00263.pdf, 2016.

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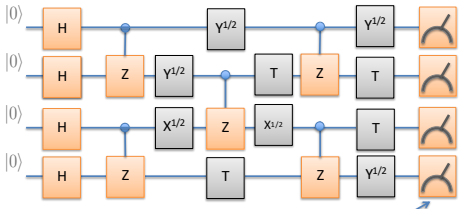
Sampling is hard



Once again, the aim of the algorithm is to sample from the measured values.

If this were possible to do efficently classically, it would imply a collapse of the polynomial hierarchy.

In practice, simulating ~45 qubits for this problem is hard (note: need high depth circuit)

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What do you need for quantum supremacy?

- Many qubits (>~50)
- Large depth circuit (>~100)
- Entanglement, high T gate count
- Low error rates (<1%)

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60 Qubit Simulations of Shor's algorithm

Using MPS, Aidan Dang wrote parallel code were able to do large scale simulations of Shor's algorithm.

l	r	α	β	n_{node}	t_U	t_{meas}	t_{QFT}	t_{total}
16	28140	2	7035	2	1538	353	4290	6181
17	57516	2	14379	24	1694	406	4544	6644
20	479568	4	29973	216	4271	1496	20236	26003

Table 3.2: Further QCMPS benchmarks, this time across multiple nodes of a supercomputer. Each node has 24 cores and 64 GB of RAM. With n_{node} nodes, we simulated the three cases $l = 16$, $N = 56759$, $a = 2$; $l = 17$, $N = 124631$, $a = 2$; and also $l = 20$, $N = 961307$, $a = 5$.

Source: Dang, thesis, 2017

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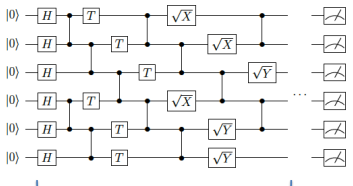
Simulating Google's Pseudo-Random Circuits



Depth of the circuit is equivalent to the number of timesteps

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Importance of Depth

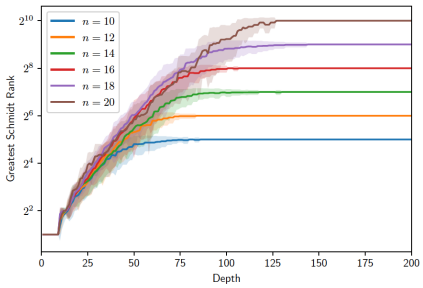

Figure 4.3: Mean maximum Schmidt ranks in the MPS as a function of depth. Sampled over 100 instances, the shading represents one standard deviation.

Source: Dang, thesis, 2017

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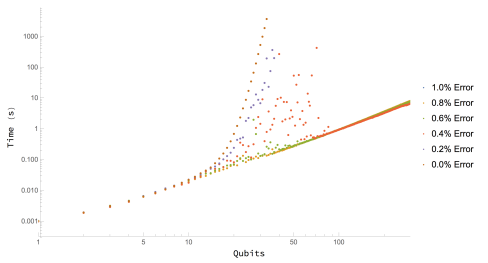
Effects of Errors on IQP simulation runtime



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Lab 7
Quantum Supremacy and Errors

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