PHYC90045 Introduction to Quantum Computing

Assignment 1

Due: 5pm, Friday 11th September, Week 6, 2020

Assignment 1 for PHYC90045 *Introduction to Quantum Computing*.

Work on your own, attempt all questions, and hand in your completed written work on or before the due date as per instructions above. The QUI circuits you create for this project should be saved with the indicated filenames (including your student number as specified) and where you have created a QUI circuit, the circuits should be shared, and a link to the relevant circuits should be included in the written assignment. Submit this assignment online via LMS.

Total marks = 40

Question 1 [5 marks]

(a) Starting from the $|0\rangle$ state, what state do you end up in if you apply a Hadamard gate?

(b) Starting from the $|0\rangle$ state, find a unitary operation can you apply resulting in the state:

$$\frac{|0\rangle - i\,|1\rangle}{\sqrt{2}}$$

(c) Starting from the $|0\rangle$ state, find a unitary operation can you apply to result in the state,

$$\frac{|0\rangle - \sqrt{3}e^{i\frac{\pi}{4}}|1\rangle}{2}$$

(d) Find $\langle X\rangle, \langle Y\rangle$ and $\langle Z\rangle$ for the state found in part **(c)** and plot it on the Bloch sphere.

(e) Starting from the state

$$\frac{|0\rangle - \sqrt{3}e^{i\frac{\pi}{4}}|1\rangle}{2}$$

find a unitary operation which results in the state,

$$\frac{|0\rangle - i\,|1\rangle}{\sqrt{2}}$$

Question 2 [5 marks]

Consider the following entangled two-qubit state: $|\psi\rangle = cos(\theta)\,|00\rangle + sin(\theta)\,|11\rangle$.

(a) Write out the circuit containing one R-gate around a single cartesian axis (indicating the axis, rotation angle, and global phase), and one two-qubit gate that produces this state. Save your circuit as "<Student Number> Assignment 1 Q2".

(b) Using the definition of entanglement entropy (described in an aside in the lectures),

$$S = -\sum_i p_i \, log\, p_i$$

calculate the entanglement entropy in this state as a function of θ,

(c) Plot the entropy of entanglement for angles between $\theta = 0$ and $\pi/2$.

Question 3 [7 marks=2+3+2]

A GHZ state is given by:

$$|GHZ\rangle = \frac{|000\rangle + \ |111\rangle}{\sqrt{2}}$$

(a) Construct a circuit which constructs the GHZ-state, using only single qubit and two-qubit operations. Optimize the circuit as much as possible. Save this circuit as "<Student number> Assignment 1 Q3a".

A W-state is a three qubit state given by:

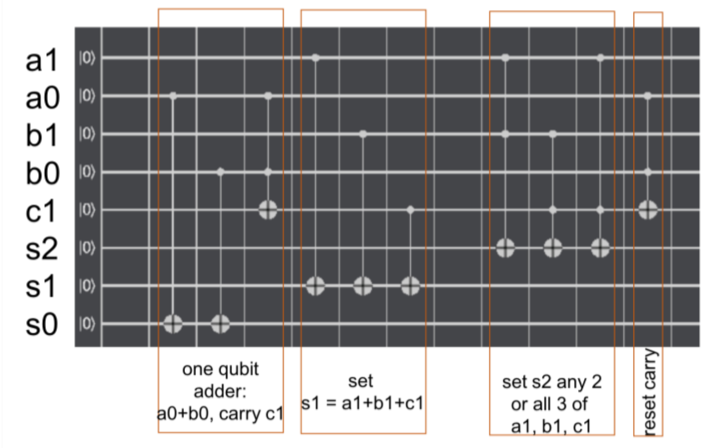
$$|W\rangle = \frac{|001\rangle + |010\rangle + |100\rangle}{\sqrt{3}}$$

(b) Construct a circuit which constructs the W-state, using only single qubit and two-qubit operations. Optimise this circuit as much as possible, and briefly describe your construction. Save this circuit as "<Student number> Assignment 1 Q3b".

(c) Is it possible to convert W-states to GHZ-states using only single qubit operations? Show your reasoning why or why not. You may use QUI for calculating entanglement entropy but include working for any other quantities calculated. *Hint:* You may use the fact that single qubit operations will neither increase nor decrease the entanglement entropy.

Question 4 [4 marks]

Consider the two-bit adder circuit we encountered in Lab 4.



Program this in the QUI using only CNOT gates and single qubit gates (no Toffoli gates!), and minimise the total number of gates as far as possible. Briefly describe any simplifications you have made. You will be marked on whether the circuit works (and gives the correct phase for each state), and also the degree of compression you have achieved.

Save the circuit as "<Student number> Assignment 1 Q4".

Question 5 [4 marks]

Construct a quantum function, which calculates the square of the input. Assume one input register of three qubits, and an output register of six qubits. You may use multiply controlled gates, and ancilla qubits – however you must reset your ancilla qubits to 0, and your function must be reversible. Briefly describe the construction of your circuit.

Save this file as "<Student Number> Assignment 1 Q5"

Question 6 [5 marks=1+1+2+1]

Alice, Bob and Charlie have been captured by quantum bandits who do not believe they know quantum computing. The bandits tell them, "We are going to separate you so that you cannot communicate, and ask you each for a value of either x, or y. You must respond with a value of +1, or -1. Out of the three questions, we will either ask for two y and one x value, for which your answers must multiply to give -1, or we will ask you all for a value of x for which your answers must multiply to give +1. If your answers don't multiply to the correct value, then a horrible fate awaits you all."

Alice, Bob and Charlie correctly write down the following equations, required to fulfil the bandits' demands:

$$x_A y_B y_C = -1$$
$$y_A x_B y_C = -1$$
$$y_A y_B x_C = -1$$
$$x_A x_B x_C = +1$$

(a) It is not possible for Alice, Bob and Charlie to classically assign values of +1 or -1 to each of x_A, x_B, x_C, y_A, y_B and y_C, so that they *always* answer the bandit's questions correctly, and avoid their horrible fate. Why not?

(b) What classical strategy can they use to have the highest probability of success? What is that probability?

Alice, Bob and Charlie realize that if they share a GHZ state that they can *always* answer correctly. Their strategy is as follows:

They each obtain one qubit of the GHZ state. If they are asked for value of x, they measure in the x-basis by applying a Hadamard gate first, and measuring their qubit. If they asked for the value of y, they measure their qubit in the y-basis by applying an $S^{\dagger}$ gate followed by a Hadamard gate. They then measure their qubit and if they measure the state "0" they reply +1, and if they obtain the result "1", they reply -1.

(c) Construct the circuits for each of the four possible choice of questions. Save these circuits as "<Student number> Assignment 1 Q6cXYY" ect.

(d) With reference to the states obtained from these circuits, show that Alice, Bob and Charlie can answer correctly 100% of the time if they follow this quantum strategy.

Question 7 [10 marks=2+3+2+3]

Qutrits are quantum states with three basis states, $|0\rangle$, $|1\rangle$, $|2\rangle$. An operation on a qubit is given by

$$U = exp\left(i\frac{\pi}{6}\lambda_4\right)$$

where

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(a) Write out the matrix representation for U. Show your working.

(b) Using two qubits you can implement the qutrit operation using the states $|00\rangle$, $|01\rangle$, $|10\rangle$. Do this in the QUI, and comment briefly on the construction of your circuit. Save this circuit as "<Student number> Assignment 1 Q7b".

A particular two-qutrit operation is given by:

$$V = exp\left(i\frac{\pi}{8}\lambda_4 \otimes \lambda_4\right)$$

(c) Write out the 9x9 matrix representation for V. Show your working.

(d) Using two qubits for each qutrit, you can implement the two-qutrit operation using four qubits. Do this in the QUI, and comment briefly on the construction of your circuit. Save this circuit as "<Student number> Assignment 1 Q7d"