Week 12

Lecture 23

Quantum Computing architectures and quantum complexity classes

Lecture 24

Quantum Computing Review

Lab 12

HHL algorithm using the QUI

Quantum Computing Review

Lecture 24

Review
(Selected Highlights)

Linear Algebra and Dirac notation

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle$$

For qubits we can use column vectors to represent a convenient basis for kets:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Computational basis states

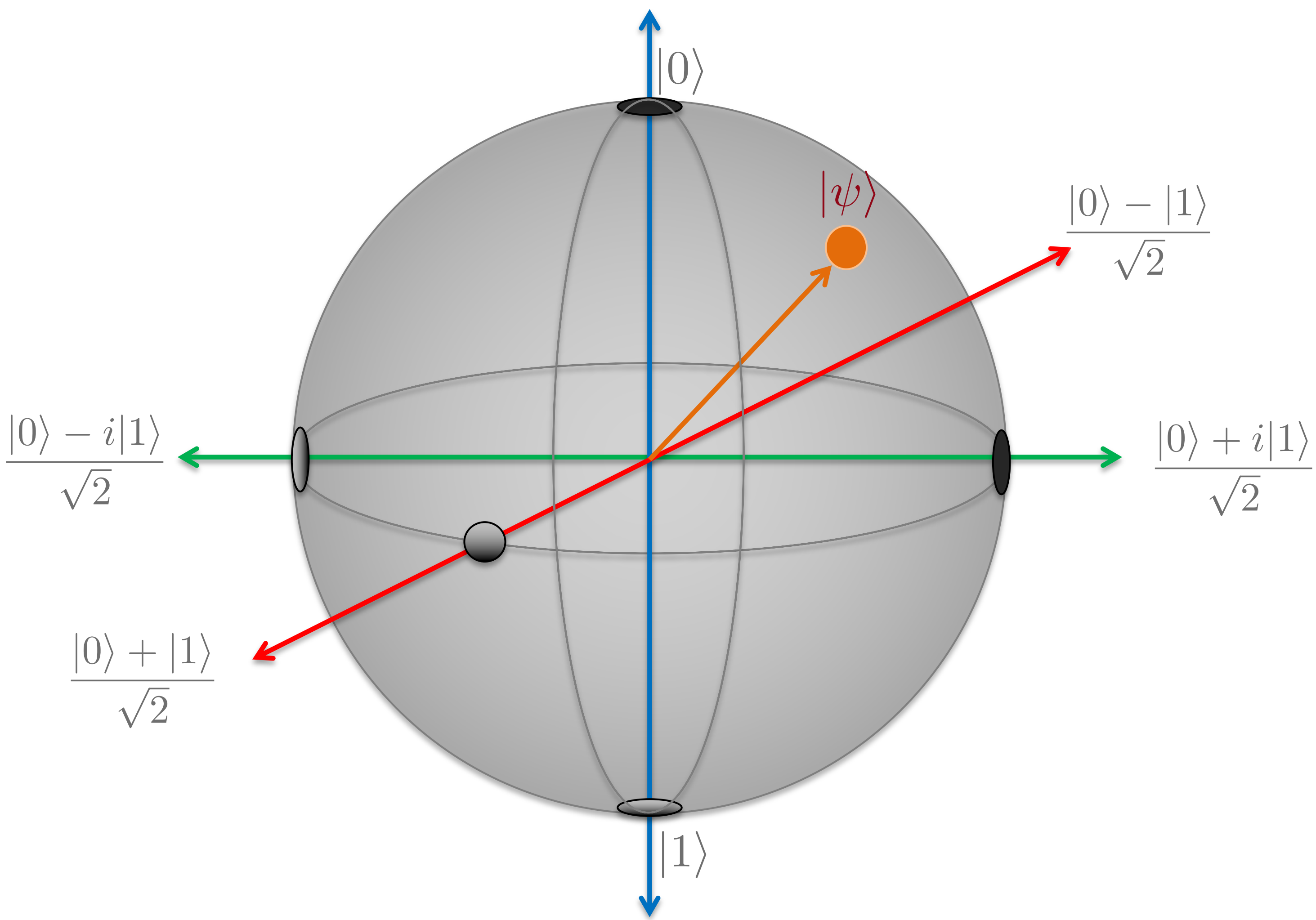
$$a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$a, b \in \mathbb{C}$$

General qubit state
a, b are “amplitudes”

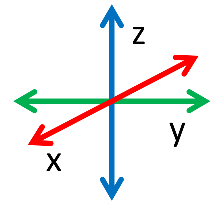
The Bloch Sphere

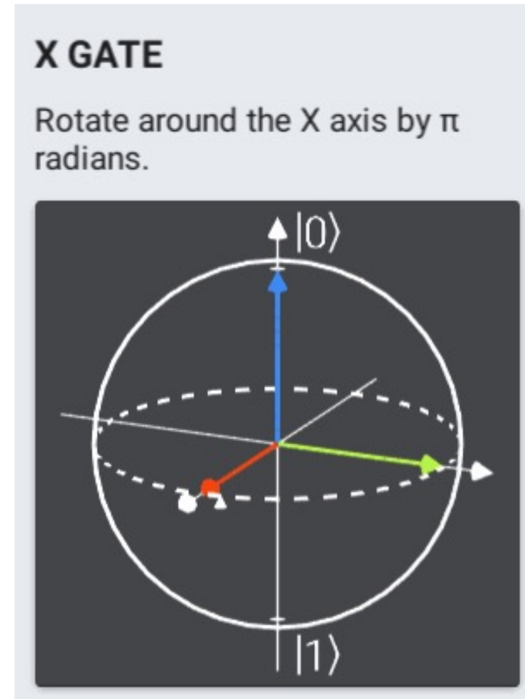
A convenient geometric representation of single qubit states is the Bloch sphere:



Single Qubit Gates

Circuit symbol:



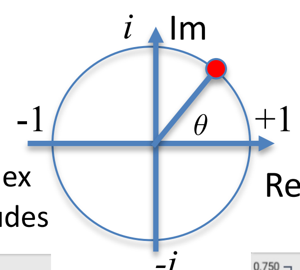


Matrix representation: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

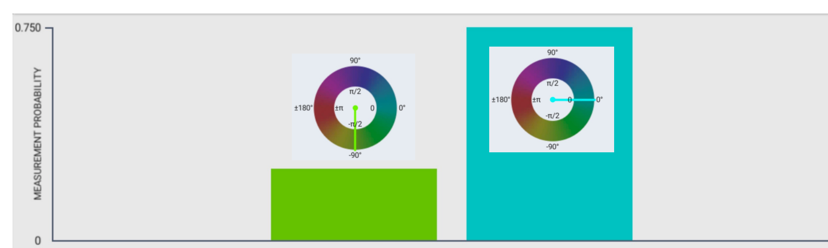
Action on ket states: $\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$

QUI example:

$\frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle$



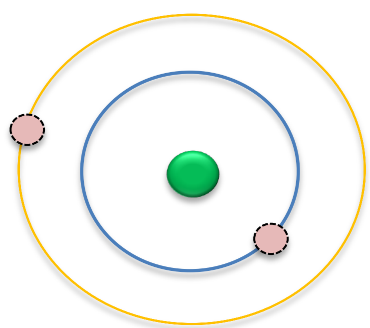
$|0\rangle$ $|1\rangle$



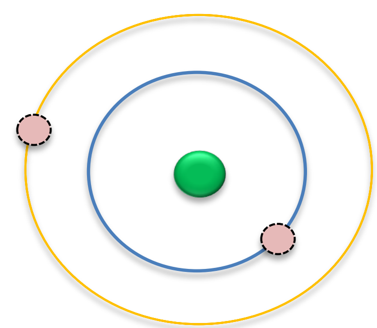
$|0\rangle$ $|1\rangle$

Multiple Qubit States

$$\begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac \\ ad \\ bc \\ bd \end{bmatrix} \begin{array}{l} \text{00 amplitude} \\ \text{01 amplitude} \\ \text{10 amplitude} \\ \text{11 amplitude} \end{array}$$

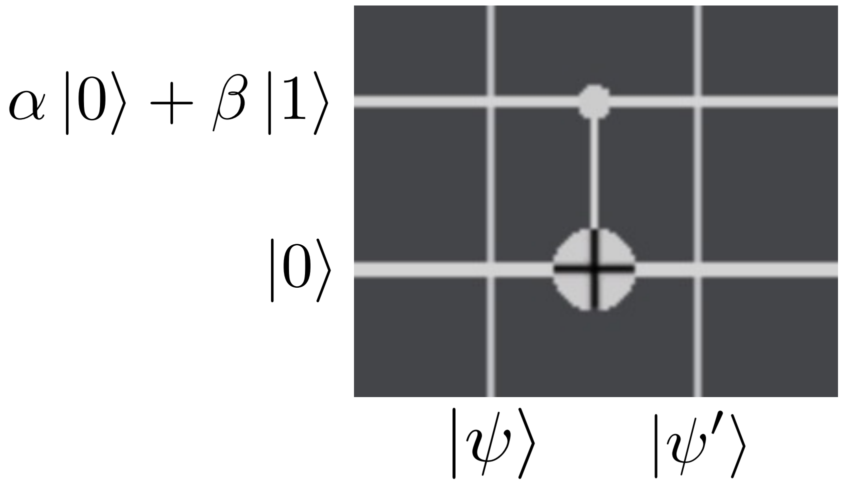


$$|\psi\rangle = a|0\rangle + b|1\rangle$$



$$|\phi\rangle = c|0\rangle + d|1\rangle$$

Two qubit operations



Before the CNOT, the state is:

$$|\psi\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle = \alpha|00\rangle + \beta|10\rangle$$

After the CNOT, the state is:

$$|\psi'\rangle = \alpha|00\rangle + \beta|11\rangle$$

Using the QUI

In labs you will learn to use the "Quantum User Interface" (QUI) to construct circuits. Your first lab will be all about single qubit rotations.

Entanglement

We can never find a, b, c, d, s.t.

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} \neq (a\,|0\rangle + b\,|1\rangle) \otimes (c\,|0\rangle + d\,|1\rangle)$$

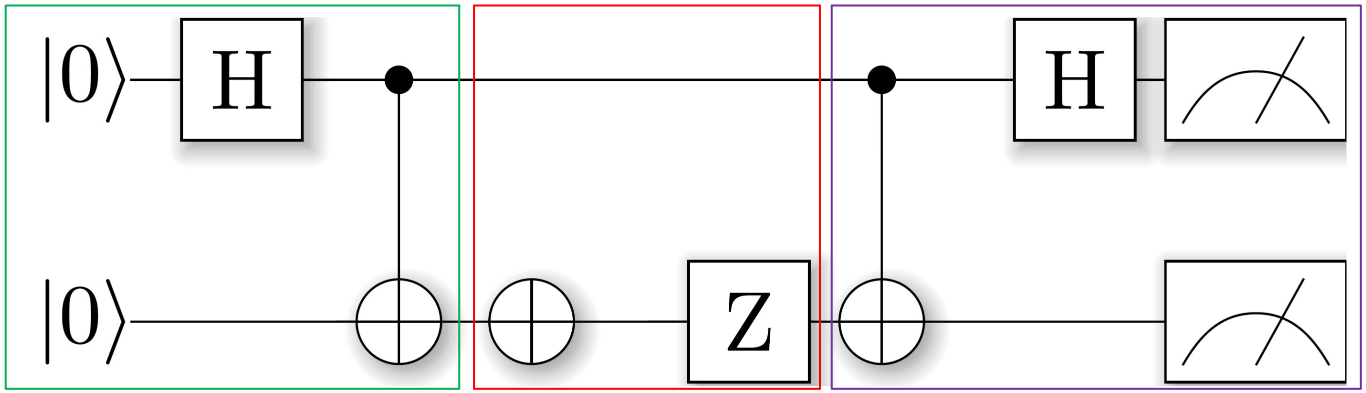
A state which is not separable is called an **entangled** state.

Entanglement is a uniquely quantum mechanical property, with no direct classical analogue.

Dense Coding Circuit

Bob

Alice



Bell state preparation:

Bell state preparation

Alice encoding

Bob's measurement

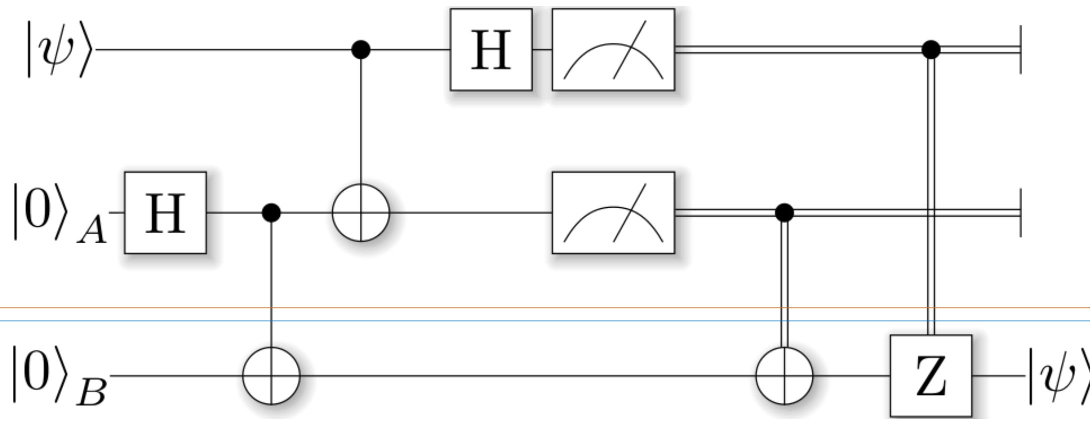
$$|00\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}|0\rangle \rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$\begin{array}{lll}
0,0 & \frac{|00\rangle + |11\rangle}{\sqrt{2}} \rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}} \\
1,0 & X_2\frac{|00\rangle + |11\rangle}{\sqrt{2}} \rightarrow \frac{|01\rangle + |10\rangle}{\sqrt{2}} \\
0,1 & Z_2\frac{|00\rangle + |11\rangle}{\sqrt{2}} \rightarrow \frac{|00\rangle - |11\rangle}{\sqrt{2}} \\
1,1 & X_2Z_2\frac{|00\rangle + |11\rangle}{\sqrt{2}} \rightarrow \frac{|01\rangle - |10\rangle}{\sqrt{2}}
\end{array}$$

Alice applies one of four different operations to her qubit, based on the classical information she would like to send.

Teleportation

$\alpha\,|0\rangle + \beta\,|1\rangle$



$\alpha\,|0\rangle + \beta\,|1\rangle$

Alice measures	Bob's qubit
$0, 0$	$\alpha\|0\rangle + \beta\|1\rangle$
$0, 1$	$\alpha\|1\rangle + \beta\|0\rangle$
$1, 0$	$\alpha\|0\rangle - \beta\|1\rangle$
$1, 1$	$\alpha\|1\rangle - \beta\|0\rangle$

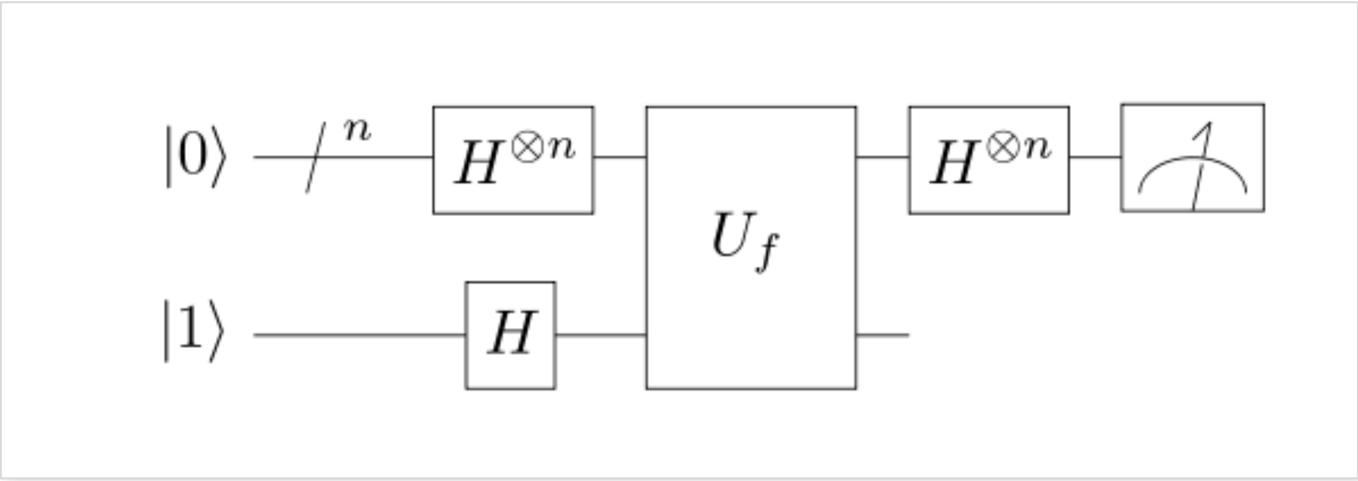
i.e. after correction Bob has successfully reconstructed Alice's original state.

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle + \beta|1\rangle$$
$$X(\alpha|1\rangle + \beta|0\rangle) \rightarrow \alpha|0\rangle + \beta|1\rangle$$
$$Z(\alpha|0\rangle - \beta|1\rangle) \rightarrow \alpha|0\rangle + \beta|1\rangle$$
$$ZX(\alpha|1\rangle - \beta|0\rangle) \rightarrow \alpha|0\rangle + \beta|1\rangle$$

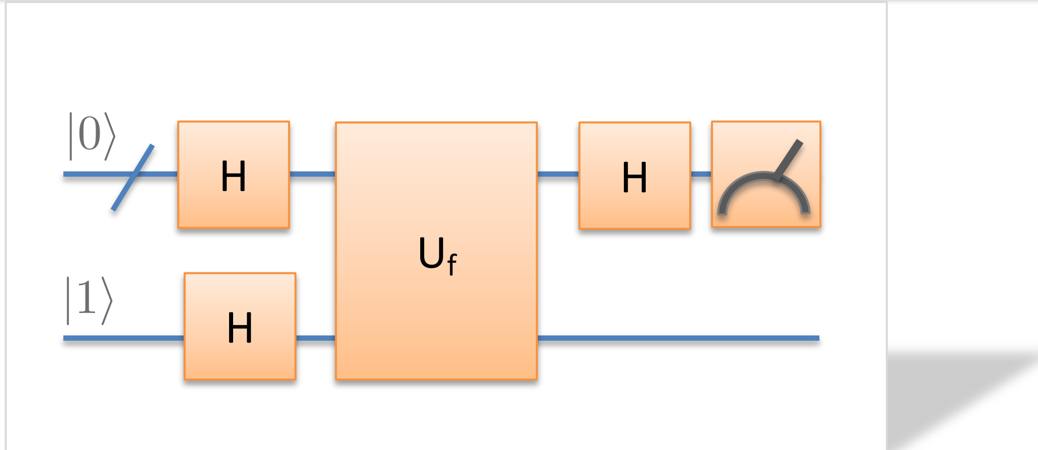
Deutsch-Josza algorithm

- Given a boolean function, ***f***, determine if:
  f is constant (always gives the same result), or
  f is balanced (gives equal numbers of 0s and 1s)

- **Classical algorithm** *(worst case) needs $2^n/2+1$ queries*
- ***Quantum algorithm** needs just 1 query.*



Bernstein-Vazirani Algorithm



Given a Boolean function, f:

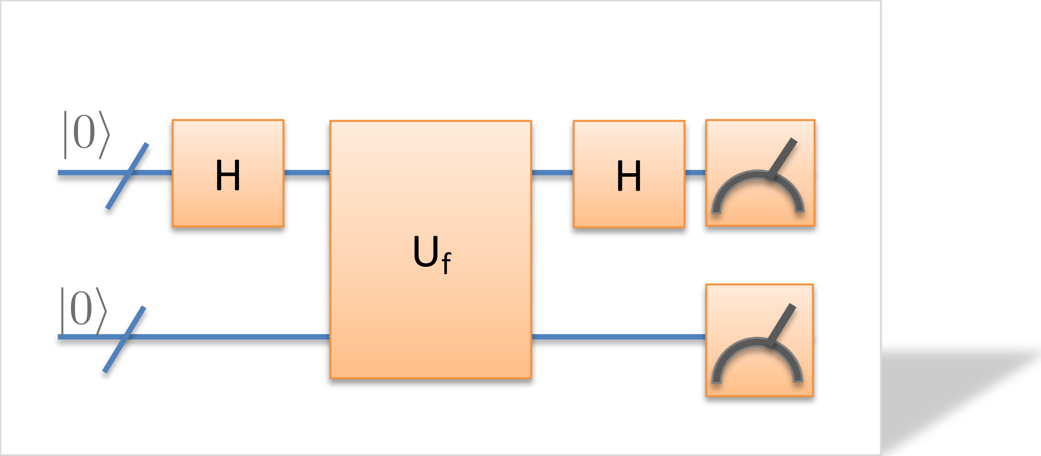
$$f(x) = x \cdot s \mod 2$$

find s.

$$x \cdot s = \sum_i x_i s_i$$

- **Classical algorithm** *needs n queries*
- ***Quantum algorithm*** *needs just 1 query.*

Simon's Algorithm



Given a 2-to-1 function, f, such that

$$f(x) = f(x \oplus a)$$

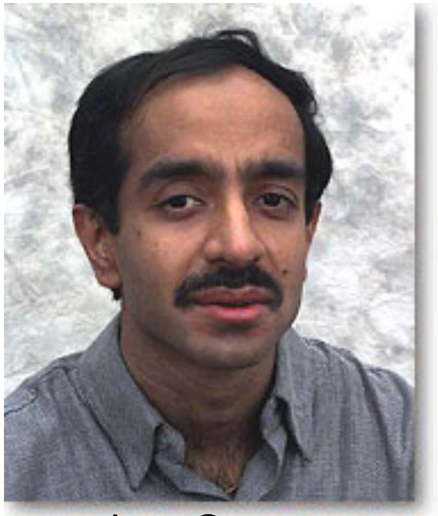
Find a.

Classical algorithm: $O(\sqrt{N})$ Queries to the oracle (probabilistically)

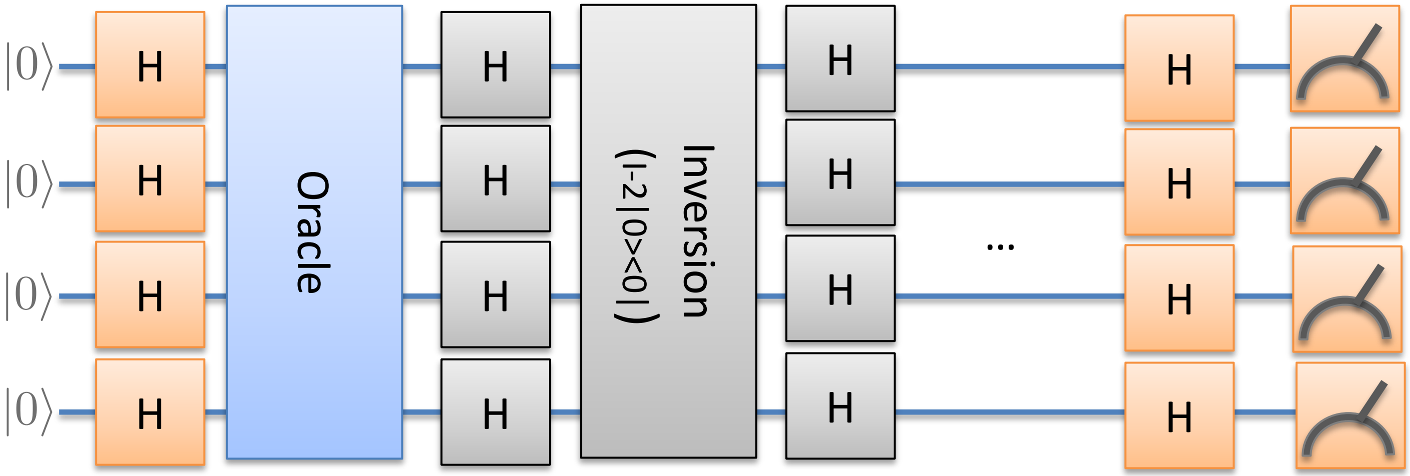
Quantum algorithm: $O(n)$ Queries to the oracle

Grover's Algorithm

- Unordered search, find one marked item among many
- Classically, this requires N/2 uses of the oracle
- Quantum mechanically, requires only O(√N).



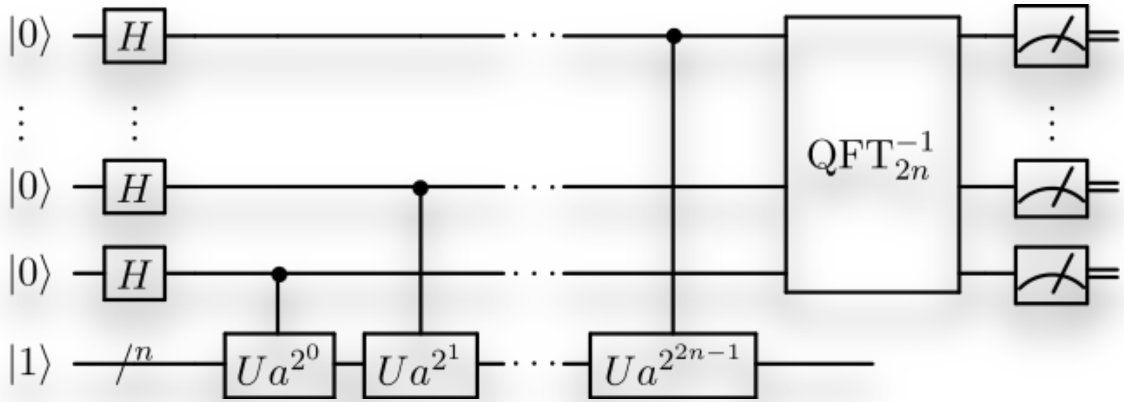
Lou Grover



Shor's algorithm

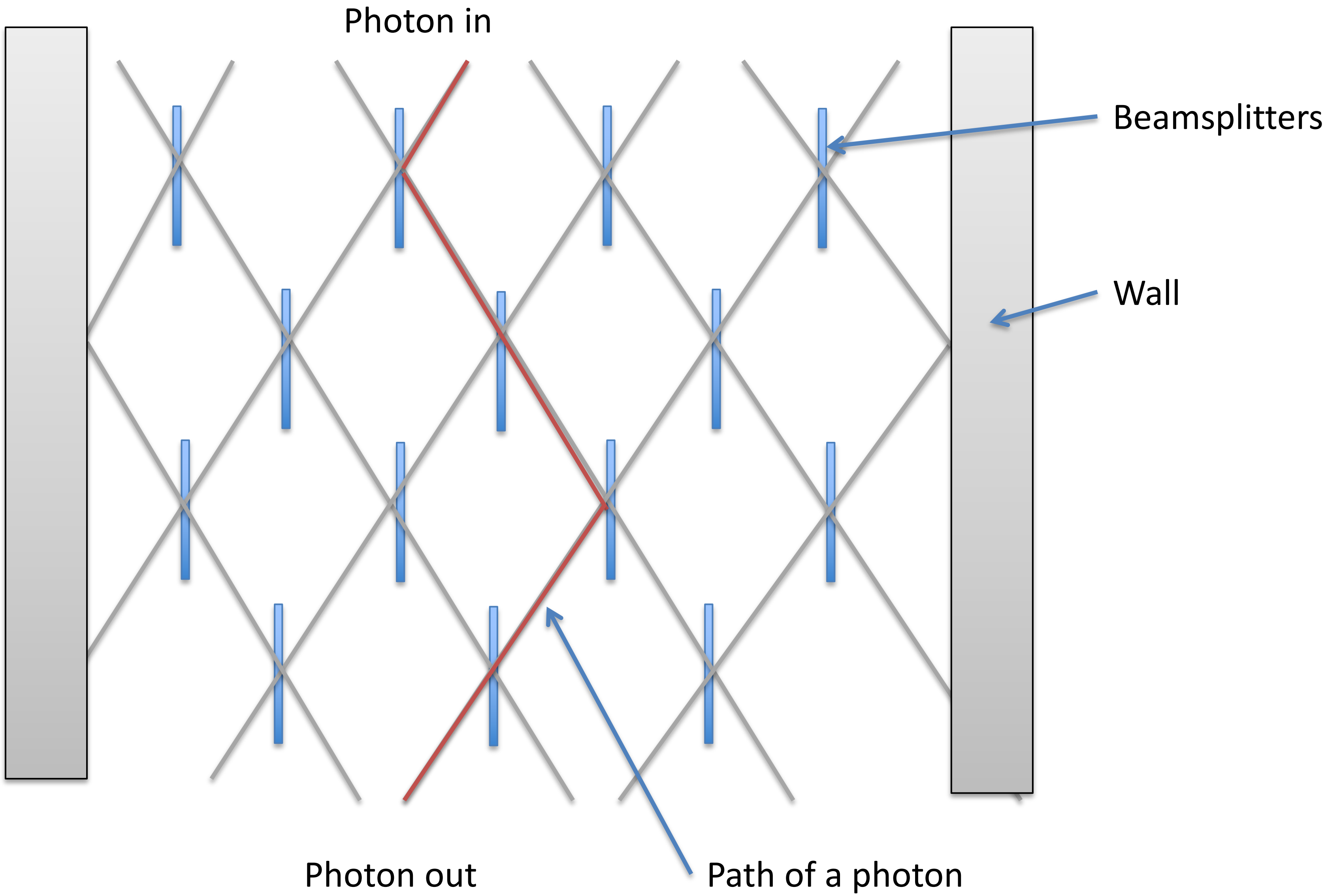
- Efficient quantum algorithms for **factoring** semiprime numbers
- Best known classical algorithm is number field sieve (exponential in bit-length).
- Underpins the RSA cryptosystem
- Hidden Subgroup Problems (eg. Discrete logarithm) similar.

Peter Shor

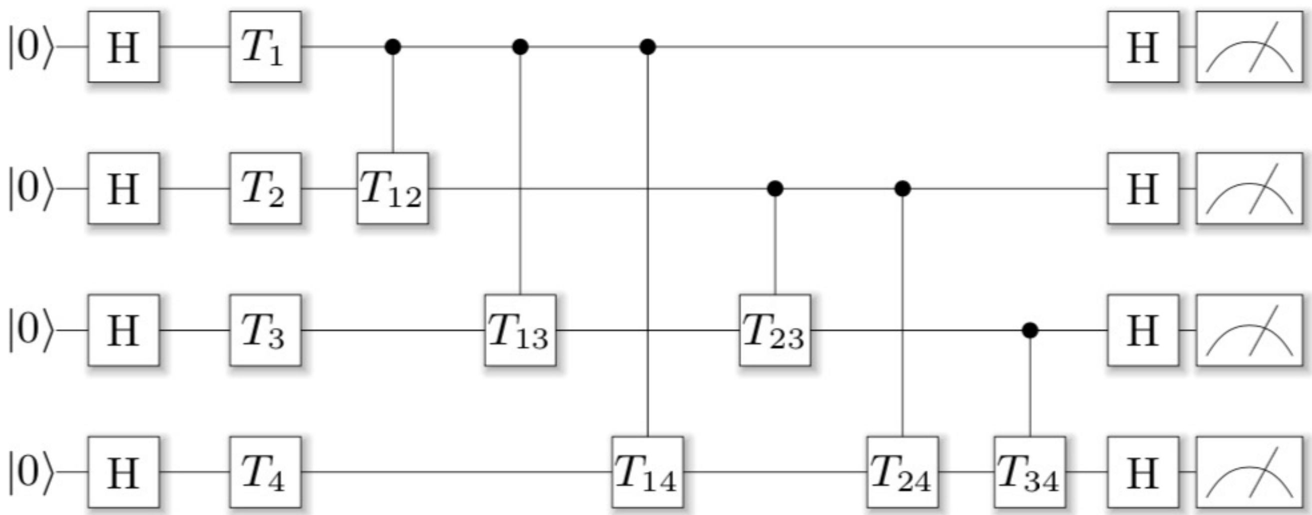


Shor, Proc 35^{th} Ann Symp of Comp Sci, 26, (1995)

Quantum Supremacy: Boson Sampling



IQP Circuits



(5) Measure

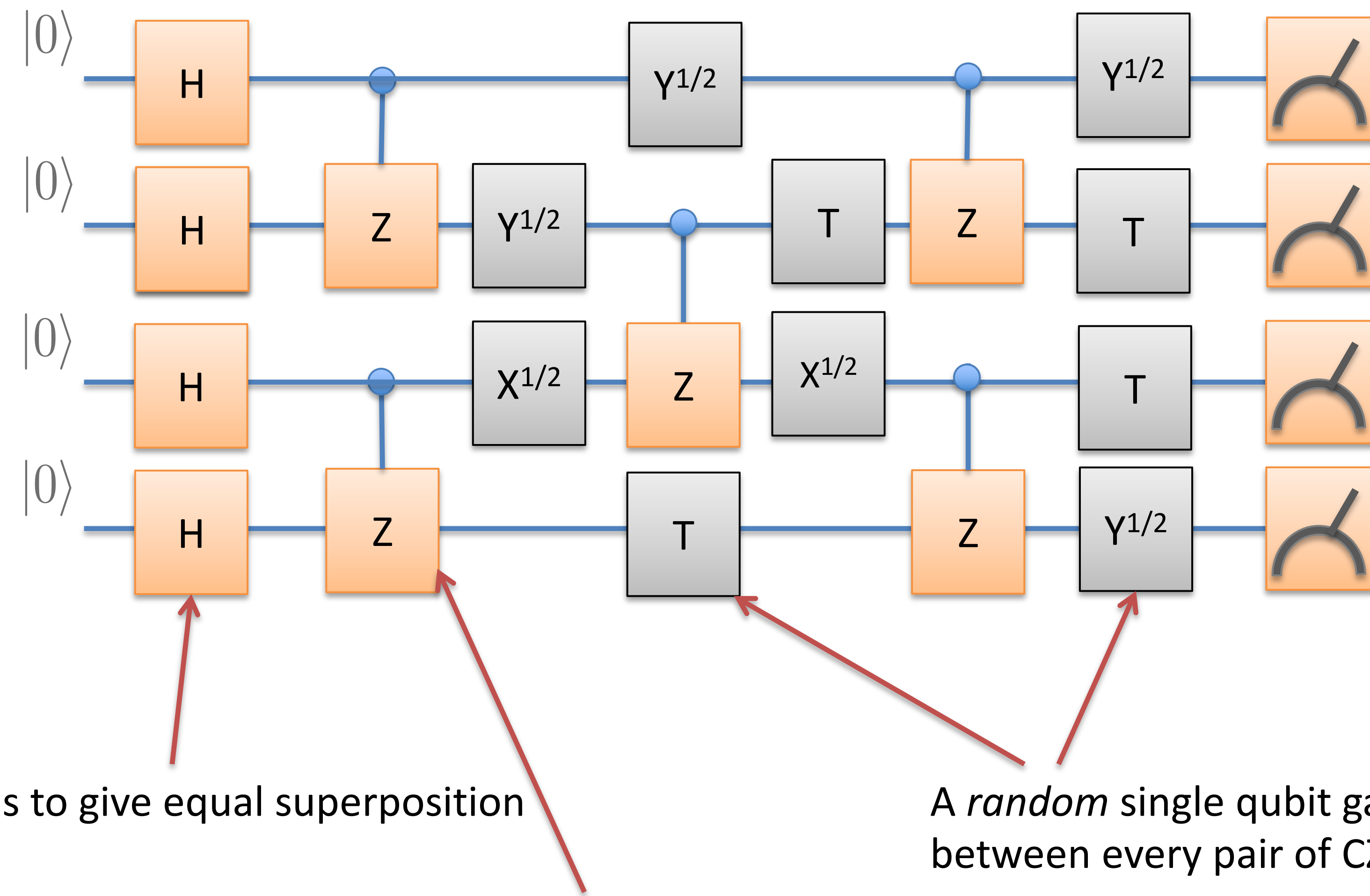
(1) Hadamard gates
Equal superposition

(2) Random single qubit phases

(3) Random two qubit phase gates
between every pair

(4) Interfere all the
resulting states

Pseudo-random Circuits



Hadamards to give equal superposition

Controlled-Z gates to provide entanglement in a regular pattern

A *random* single qubit gate between every pair of CZ gates.

:(

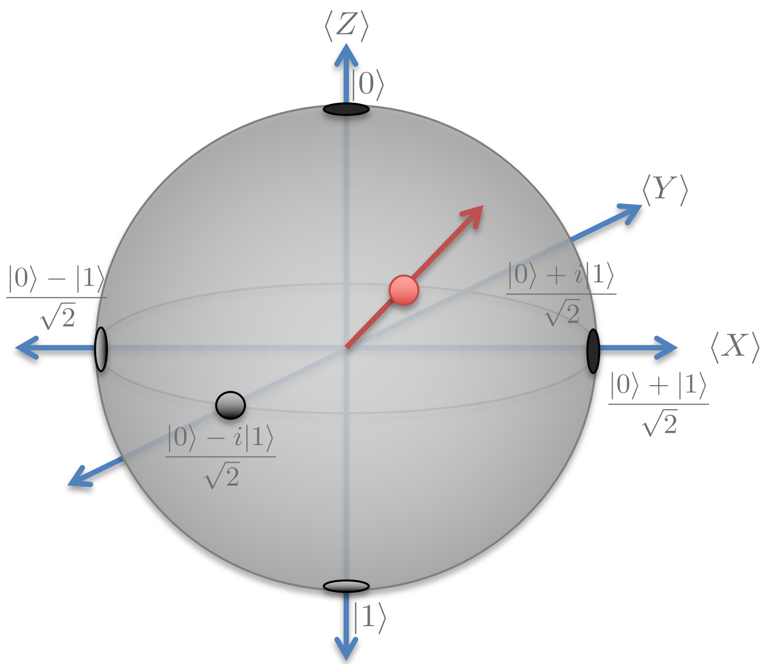
Your PC ran into a problem and needs to restart. We're just collecting some error info, and then we'll restart for you.

55% complete

For more information about this issue and possible fixes, visit http://windows.com/stopcode

If you call a support person, give them this info:
Stop code: CRITICAL_PROCESS_DIED

Purity for one qubit



If the distance from the origin to the state is measured to be r, the purity is:

$$P = \frac{1 + r^2}{2}$$

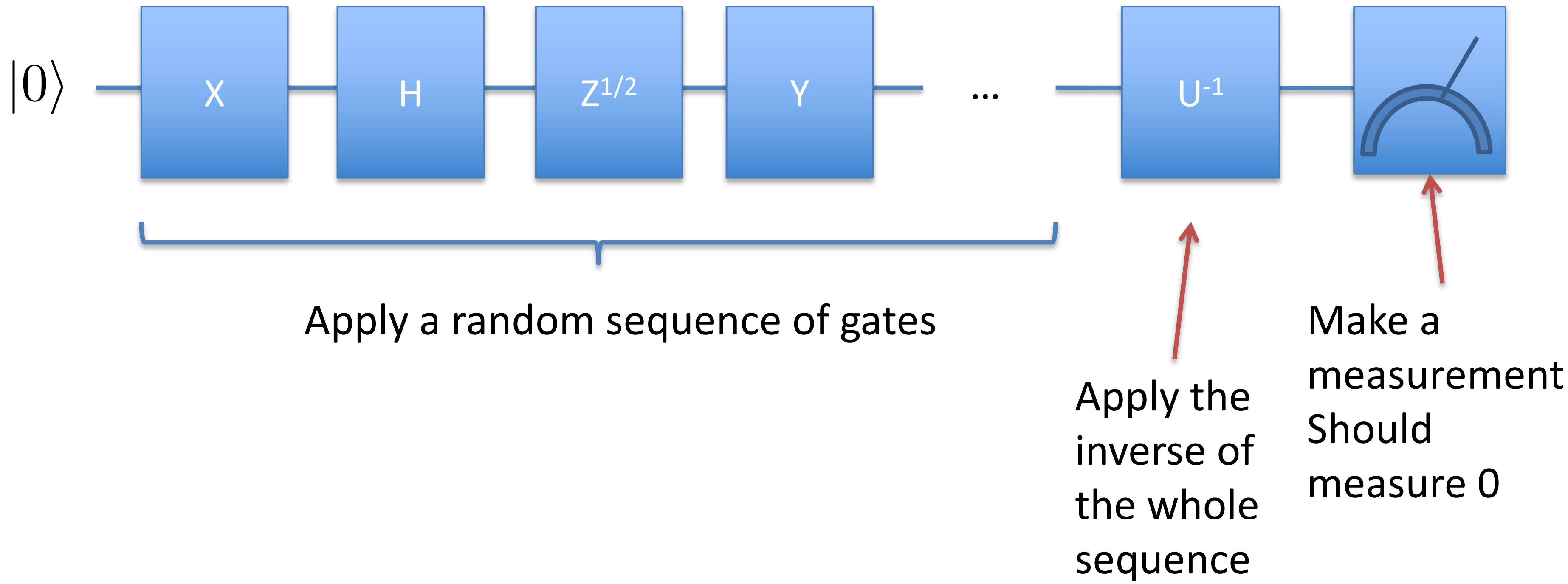
Maximum purity of 1 for all pure states.

Minimum purity of ½ for a completely mixed state.

Note: There's a more technical definition of purity in terms of density matrices, which we won't cover in this course.

Randomized Benchmarking

How good are our gates individual gate? We want a number for how much error doing each operation is. One way of determining this is to perform **randomized benchmarking**.



Quantum Error Correction

Similar to classical error correction codes, we can have a quantum repetition code:

$$|0\rangle \to |000\rangle \qquad \text{“Logical 0”}$$

$$|1\rangle \to |111\rangle \qquad \text{“Logical 1”}$$

In particular, a quantum superposition would be encoded as:

$$\alpha\,|0\rangle + \beta\,|1\rangle \to \alpha\,|000\rangle + \beta\,|111\rangle$$

Two key differences between quantum and classical error correction codes:

1. Cannot measure the codewords directly; would collapse the state
2. Phase errors

The Surface Code

- A topological code suited to solid state

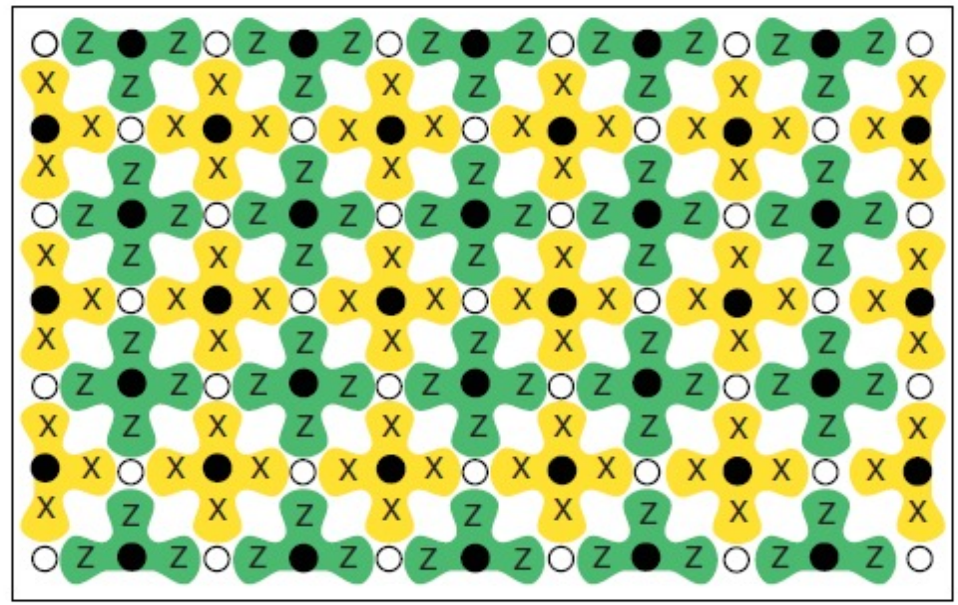
  (Kitaev 1997,

  Raussendorff 2007)

- A remarkably high threshold of ~1%

  (Wang et al, 2011)

- Parallel and synchronous control required



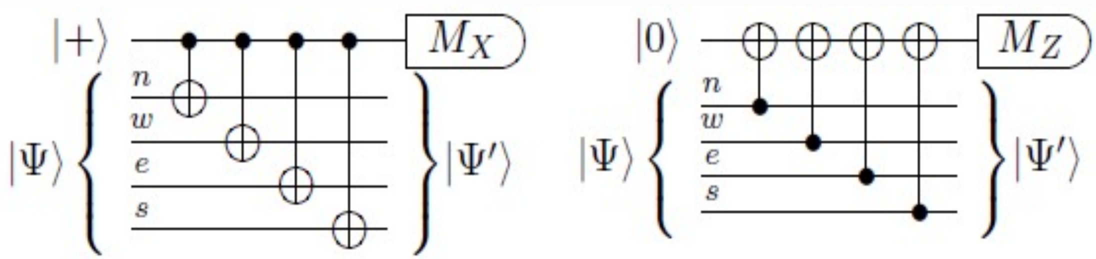
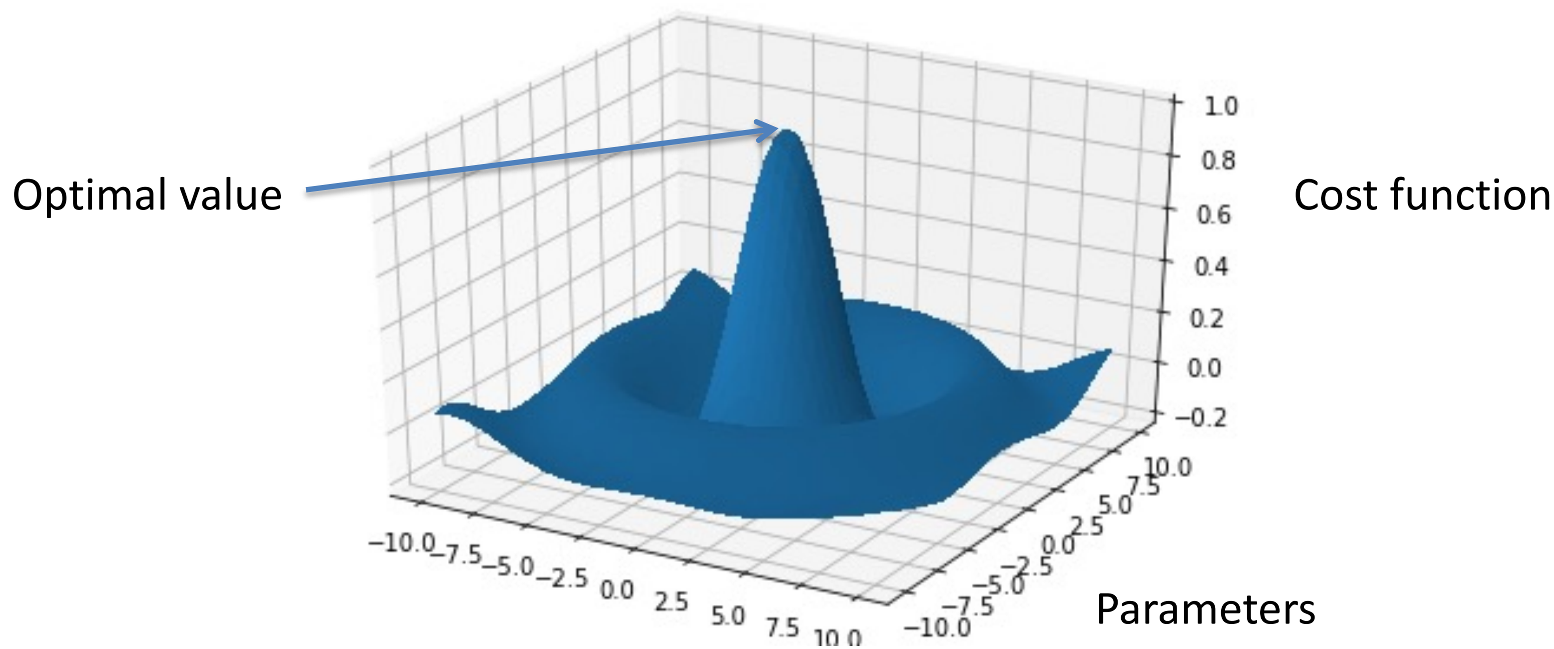


Fig. 5. Circuits used to measure X-stabilizers (left) and Z-stabilizers (right).

Optimization Problems

Given some cost-function or "objective function" we would like to maximize/minimize. Often the inputs/parameters are constrained.



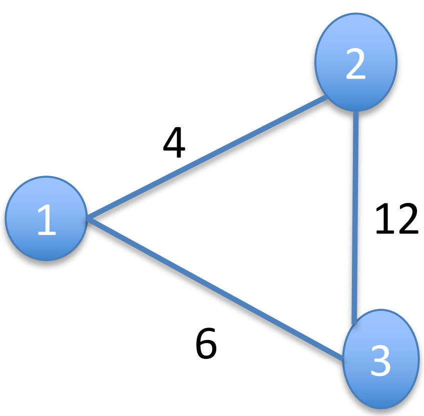
Number partitioning as a QUBO problem

But if we square, we should get a positive solution (or zero). We want to find the assignment of spins which has the minimum energy (ie. closest to zero):

$$H = \left(\sum_i w_i Z_i\right)^2 = \sum_{i \neq j} 2 w_i w_j Z_i Z_j + \sum_i w_i^2 I$$

Coupling is the product of numbers

Eg. For the set **{1, 2, 3}**:



$$H = 4Z_1Z_2 + 6Z_1Z_3 + 12Z_2Z_3 + 14I$$

Finding minimum energy state will solve the problem!

VQE/QAOA

QUI/Quantum Computer

(1) Prepare a trial state $|\psi(\theta)\rangle$

on the quantum computer, where θ can be any adjustable gate parameter.

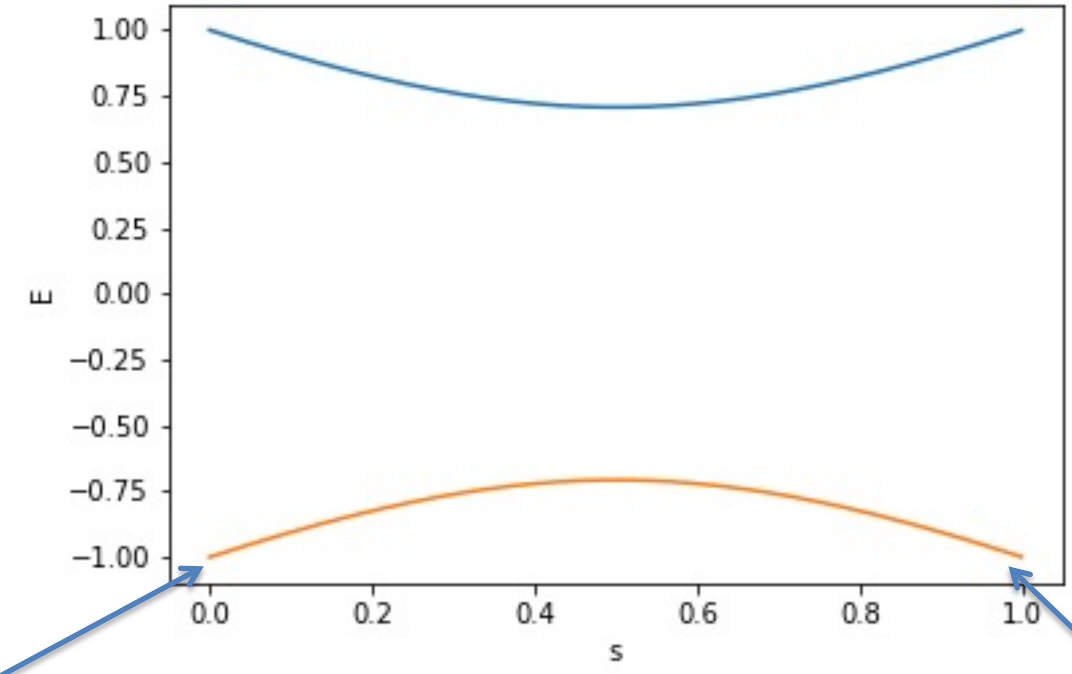
(2) **Measure** the expectation value of the energy, E.

(3) Use a **classical optimization** technique such as the Nelder–Mead simplex method, determine new values of θ that decrease E.

Repeat these steps until the value of the energy converges

Adiabatic Quantum Computation

$$H(s) = (1-s)H_x + sH_p$$
$$s = t/T$$



$$H_x = X$$
$$H_p = Z$$

"Landau-Zener" avoided crossing

System is originally in the state:

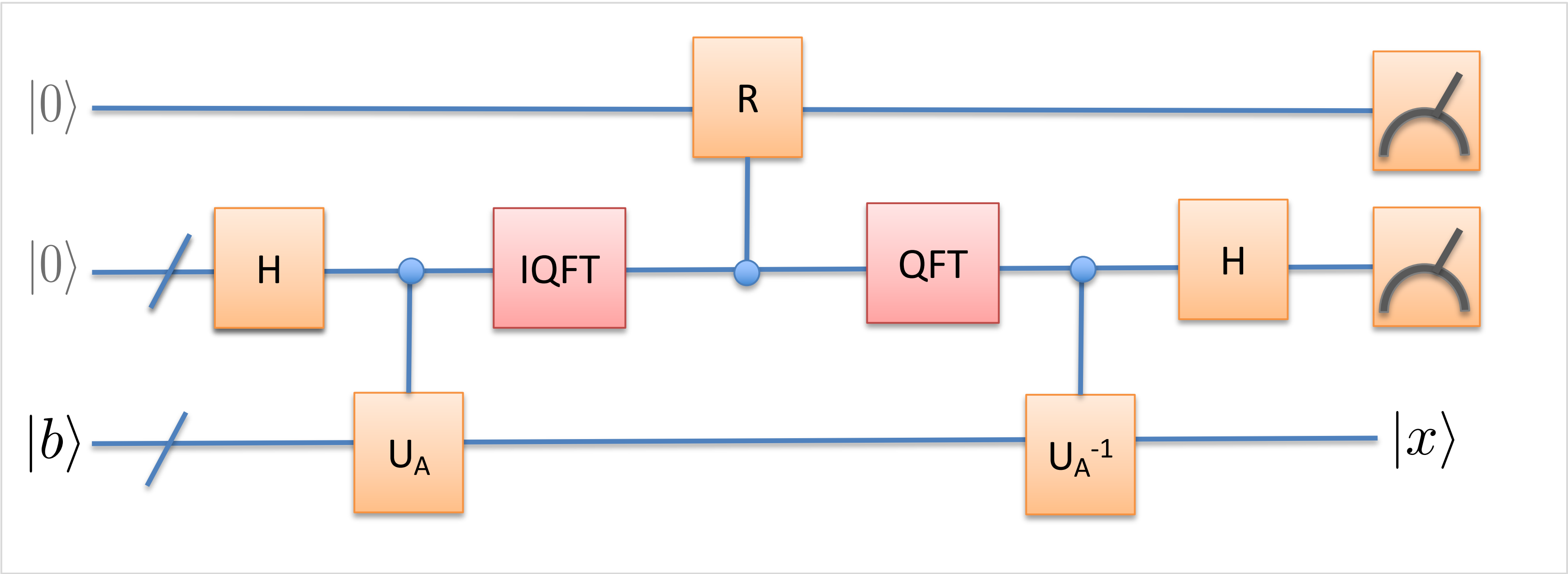
$$|-\rangle$$

Time

System ends up in the state:

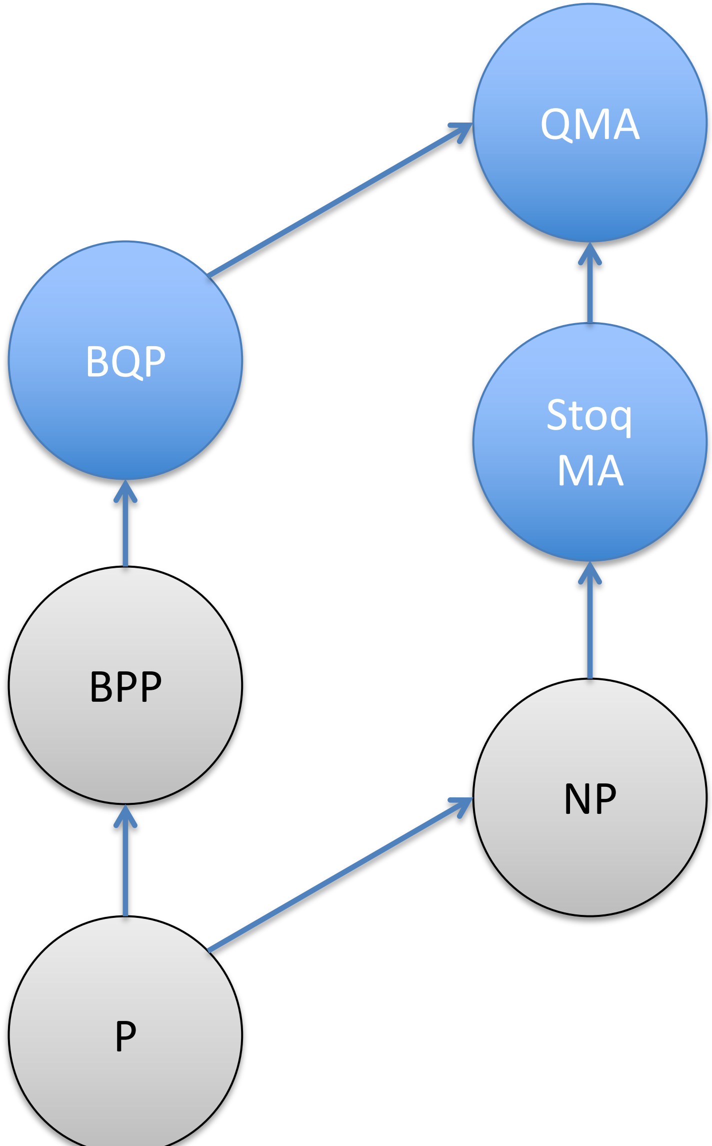
$$|1\rangle$$

HHL Algorithm



- HHL algorithm solves systems of linear equations
- Need a method for emulating *exp(iAt)*
- Can be used as a quantum subroutine
- Has a runtime, exponentially faster than classical versions.

Quantum Complexity Classes



Quantum Complexity Classes

Classical Complexity Classes

Questions?

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