

Week by week

- (1) Introduction to quantum computing
- (2) Single qubit representation and operations
- (3) Two and more qubits
- (4) Simple quantum algorithms
- (5) Quantum search (Grover's algorithm)
- (6) Quantum factorization (Shor's algorithm)
- (7) Quantum supremacy and noise
- (8) Programming real quantum computers (IBM Q)
- (9) Quantum error correction (QEC)
- (10) QUBO problems and Adiabatic Quantum Computation (AQC)
- (11) Variational/hybrid quantum algorithms (QAOA and VQE)
- (12) Solving linear equations, QC computing hardware

Week 2

Lecture 3

- 3.1 The Bloch Sphere representation for qubits
- 3.2 Quantum operations on qubits
- 3.3 Qubit gates in matrix form and the Pauli matrices

Lecture 4

- 4.1 The Pauli gates X, Y and Z and the QUI
- 4.2 Qubit operations around non-cartesian axes – H and R gates
- 4.3 Matrix exponential and arbitrary rotations

Practice class 2

Bloch sphere and single qubit logic operations on the QUI

Opportunity to be a student representatives

Your feedback is valuable to us.

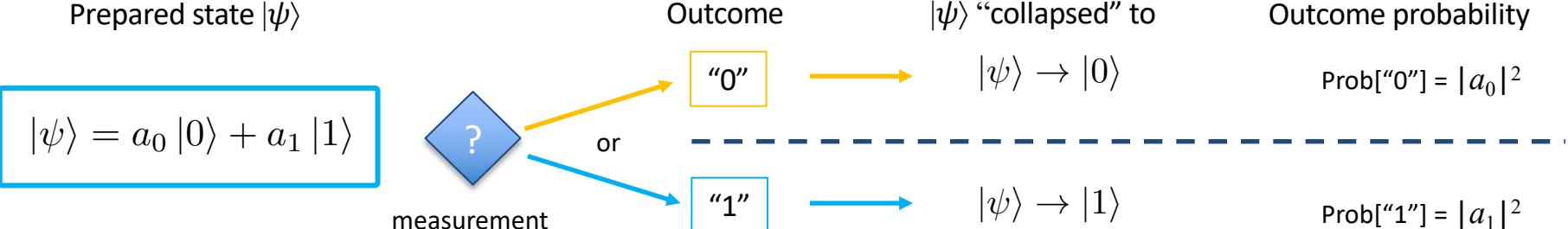
If you would like to representative for this subject, email me at
cdhill@unimelb.edu.au.

Lecture 2 recap

A qubit in “ket” notation:

$$|a_0|^2 + |a_1|^2 = 1$$

Prepared state $|\psi\rangle$



$$\rightarrow |\psi\rangle = |a_0|e^{i\theta_0} |0\rangle + |a_1|e^{i\theta_1} |1\rangle \quad \text{Amplitudes in polar notation}$$

“matrix” notation: $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad a_0 |0\rangle + a_1 |1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad |\psi\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$

dual “bra” notation: $\langle\psi| = [a_0^* \quad a_1^*]$

inner product “bra-ket”: $|\psi\rangle = \begin{bmatrix} a \\ b \end{bmatrix} \quad |\phi\rangle = \begin{bmatrix} c \\ d \end{bmatrix} \quad \langle\psi|\phi\rangle \equiv \langle\psi||\phi\rangle = [a^* \quad b^*] \begin{bmatrix} c \\ d \end{bmatrix} = a^*c + b^*d$

More detail about measurement

Projective measurement

What is the state of the system after a measurement is made?

Two useful operators, known as projectors:

$$\begin{aligned} P_0 &= |0\rangle\langle 0| \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} P_1 &= |1\rangle\langle 1| \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Can also easily construct other projectors (using outer product).

One-qubit measurement

Measurement of a state:

- (1) Apply projector into the measured state
- (2) Renormalize the state

$$|\psi'\rangle = \frac{P_m |\psi\rangle}{\sqrt{p(m)}}$$

$$|\psi\rangle = a |0\rangle + b |1\rangle$$

Apply Born's rule to work out the measurement probabilities.

If state is measured to be “0”, apply P_0 and renormalize to get the collapsed state:

If state is measured to be “1”, apply P_1 and renormalize to get the collapsed state:

$$|\psi'\rangle = \frac{a |0\rangle}{\sqrt{|a|^2}} = e^{i\theta_0} |0\rangle$$

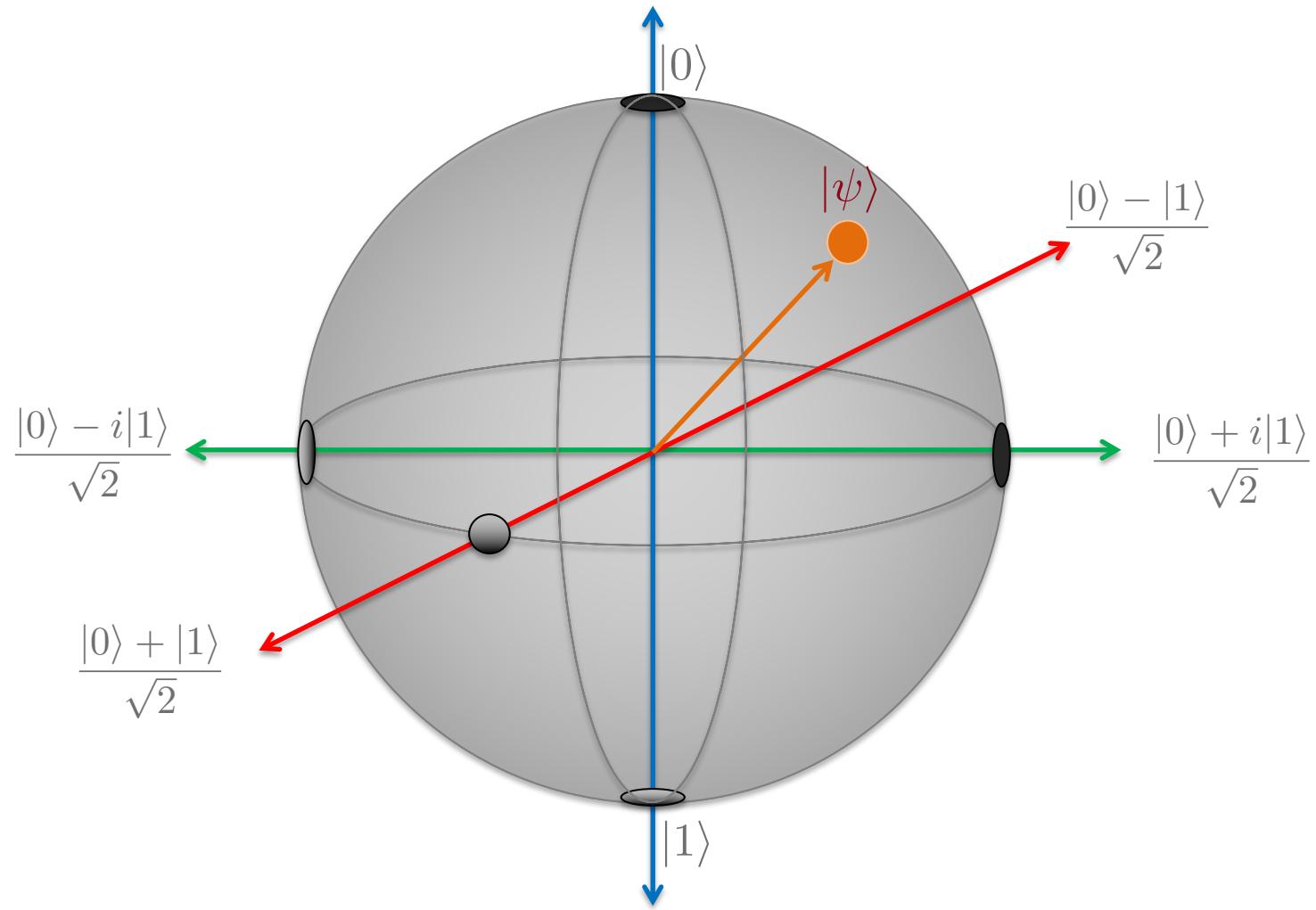
$$|\psi'\rangle = \frac{b |1\rangle}{\sqrt{|b|^2}} = e^{i\theta_1} |1\rangle$$

In quantum mechanics measurement necessarily disturbs the state you are measuring! Taking a superposition and measuring it, a superposition of many different states becomes just one. This is known as the “**collapse**” of the wavefunction.

The Bloch Sphere representation for qubits

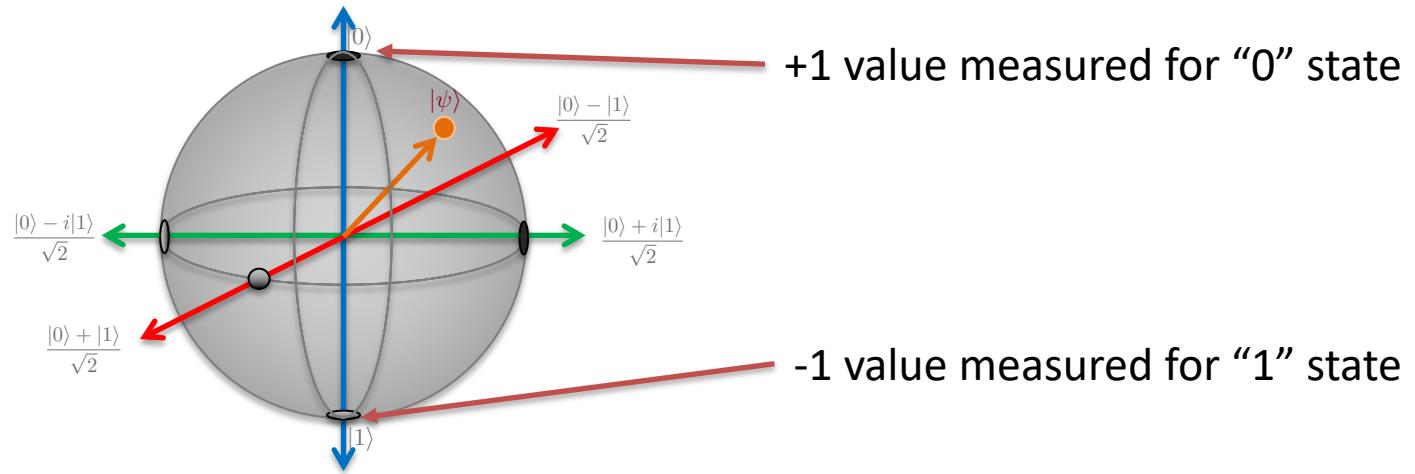
The Bloch Sphere

A convenient geometric representation of single qubit states is the Bloch sphere:



What is the z-projection?

Let's relate the Bloch sphere to states. Average, or "expectation" of the measured z component:



Average, or "expectation" value of the z (after measurement):

$$\langle Z \rangle = (+1)P_0 + (-1)P_1$$

Z-projection continued

Expressing P_0 and P_1 in terms of the state:

$$P_0 = |a_0|^2 = a_0^* a_0 = \langle \psi | 0 \rangle \langle 0 | \psi \rangle$$

$$P_1 = |a_1|^2 = a_1^* a_1 = \langle \psi | 1 \rangle \langle 1 | \psi \rangle$$

Therefore,

$$\begin{aligned} \langle Z \rangle &= (+1)P_0 + (-1)P_1 \\ &= (+1)\langle \psi | 0 \rangle \langle 0 | \psi \rangle + (-1)\langle \psi | 1 \rangle \langle 1 | \psi \rangle \\ &= \langle \psi | (|0\rangle\langle 0| - |1\rangle\langle 1|) |\psi \rangle \\ &= \langle \psi | Z | \psi \rangle \end{aligned}$$

Where

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Expectation Values

The *expectation value* of an operator is given by:

$$\langle A \rangle = \langle \psi | A | \psi \rangle$$

Hermitian operators correspond to physical observables – the expectation value then gives the average value of that quantity when measured in a given state.

For example, consider measuring the total energy of the system represented by the energy operator, i.e. the “Hamiltonian” \mathcal{H} which in matrix representation is

$$\mathcal{H} = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix}$$

Expectation values:

$$\langle 0 | \mathcal{H} | 0 \rangle = E_0 \quad \langle 1 | \mathcal{H} | 1 \rangle = E_1$$

Expectation values in a superposition state

For example, consider measuring the the total energy of the system

$$\mathcal{H} = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix}$$

For the equal superposition state:

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

The expectation value is:

$$\langle + | H | + \rangle = \frac{E_0 + E_1}{2}$$

The Pauli Matrices

Can think about the expectation along any of the cartesian axes. Equivalent to taking expectation values of any of these matrices:

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

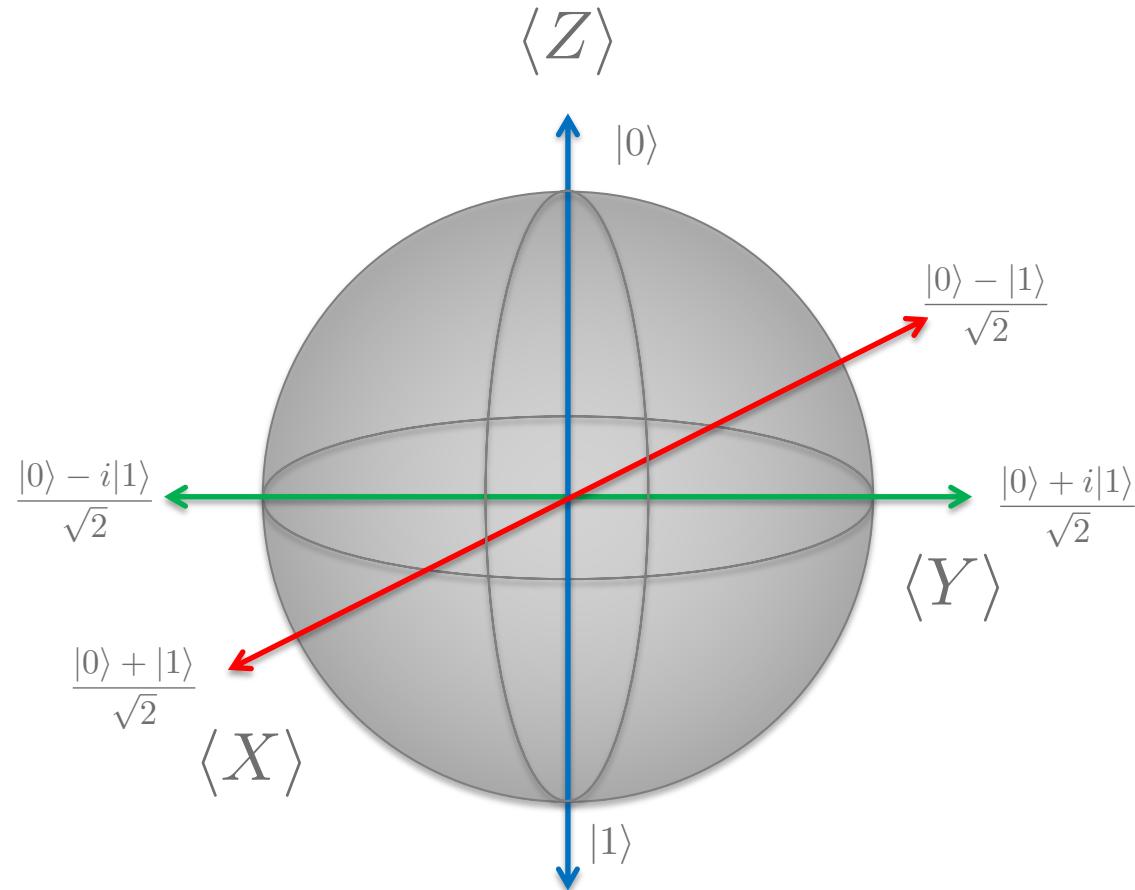
$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

These matrices are known as the Pauli matrices. Worth remembering!

Pauli matrices & Bloch sphere axes

The expectation values of the Pauli matrices (operators) define the axes on the Bloch sphere.



Expectation of Pauli Matrices

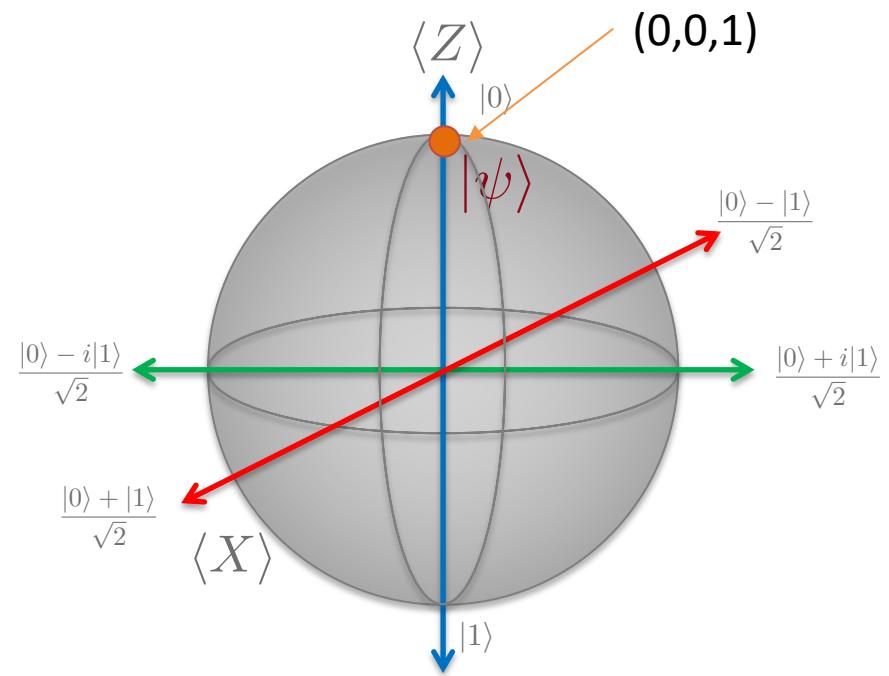
Recall: expectation value for an operator: $\langle A \rangle = \langle \psi | A | \psi \rangle$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\langle X \rangle = [1 \ 0] \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$$

$$\langle Y \rangle = [1 \ 0] \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$$

$$\langle Z \rangle = [1 \ 0] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$



Expectation of Pauli Matrices

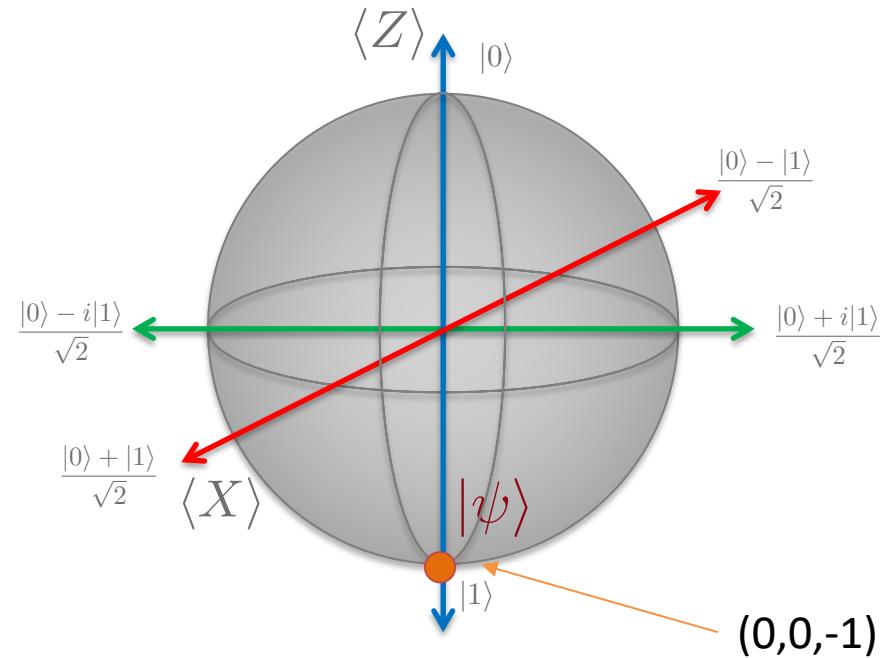
Recall: expectation value for an operator: $\langle A \rangle = \langle \psi | A | \psi \rangle$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\langle X \rangle = [\begin{array}{cc} 0 & 1 \end{array}] \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$\langle Y \rangle = [\begin{array}{cc} 0 & 1 \end{array}] \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$\langle Z \rangle = [\begin{array}{cc} 0 & 1 \end{array}] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -1$$



Expectation values of Pauli matrices

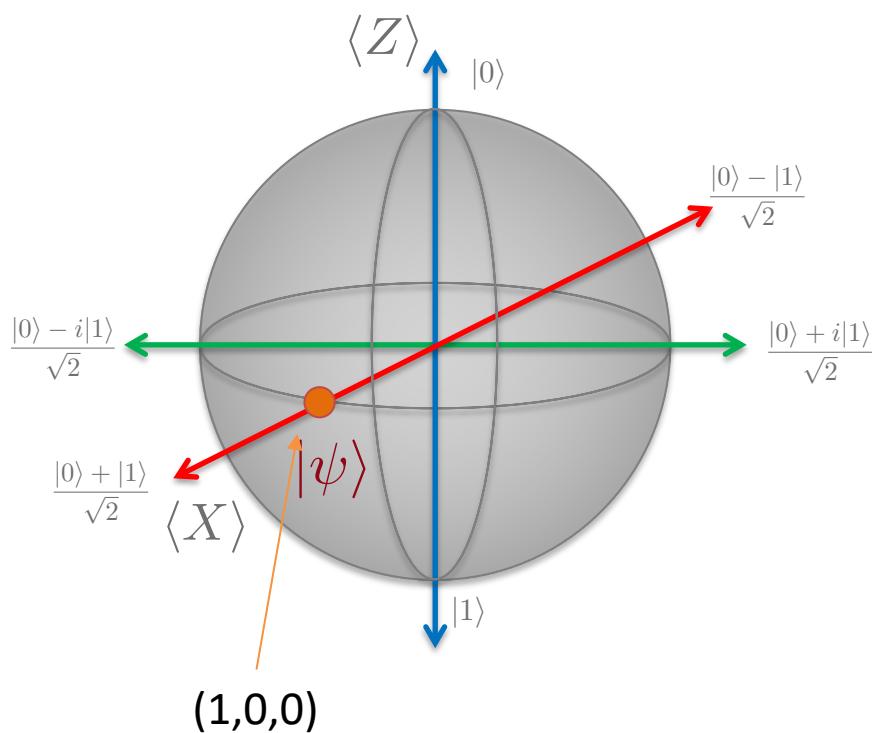
Recall: expectation value for an operator: $\langle A \rangle = \langle \psi | A | \psi \rangle$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$\langle X \rangle = \frac{1}{2} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1$$

$$\langle Y \rangle = \frac{1}{2} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

$$\langle Z \rangle = \frac{1}{2} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$



Transformation to Bloch form

Recall, arbitrary qubit state: $|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle = |a_0|e^{i\theta_0} |0\rangle + |a_1|e^{i\theta_1} |1\rangle$

Probability normalisation: $|a_0|^2 + |a_1|^2 = 1$

We can rearrange as:

$$|\psi\rangle = e^{i\theta_0} \left(|a_0| |0\rangle + |a_1| e^{i(\theta_1 - \theta_0)} |1\rangle \right) = e^{i\theta_{\text{global}}} \left(\cos \frac{\theta_B}{2} |0\rangle + \sin \frac{\theta_B}{2} e^{i\phi_B} |1\rangle \right)$$

Where: $\phi_B \equiv \theta_1 - \theta_0$

$$\left. \begin{aligned} |a_0| &\equiv \cos \frac{\theta_B}{2} \\ |a_1| &\equiv \sin \frac{\theta_B}{2} \end{aligned} \right\} \quad \begin{aligned} \text{NB: } & \left(\cos \frac{\theta_B}{2} \right)^2 + \left(\sin \frac{\theta_B}{2} \right)^2 = 1 \\ \text{i.e. } & |a_0|^2 + |a_1|^2 = 1 \end{aligned}$$

For single qubit we ignore global phase θ_{global} → state expressed via “Bloch” angles θ_B, ϕ_B

Suggests a representation on a sphere – the Bloch Sphere

Re-writing the state...

So, a qubit state (global phase set to zero) can be represented by two angles:

$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow \cos \frac{\theta_B}{2} |0\rangle + \sin \frac{\theta_B}{2} e^{i\phi_B} |1\rangle$$

The real variables θ_B and ϕ_B dictate the position of this state on the Bloch sphere.

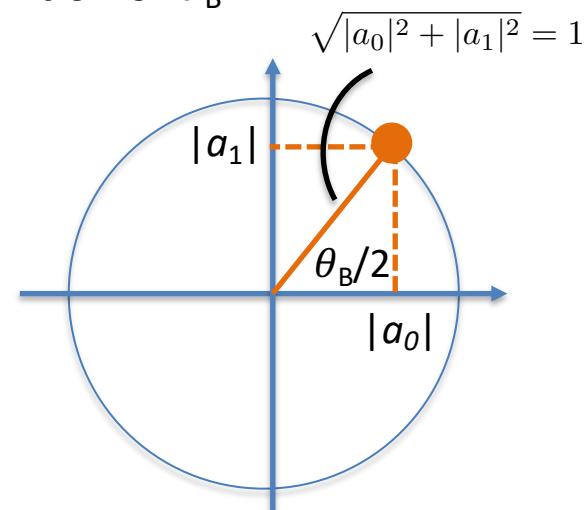
$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle$$

$$\phi_B \equiv \theta_1 - \theta_0 \quad |a_0| \equiv \cos \frac{\theta_B}{2} \quad |a_1| \equiv \sin \frac{\theta_B}{2}$$

And the azimuthal angle ϕ_B runs from 0 to 2π

NB. Bloch Sphere angles distinct from amplitude polar angles...

Definition of θ_B :



Qubit states on the Bloch sphere

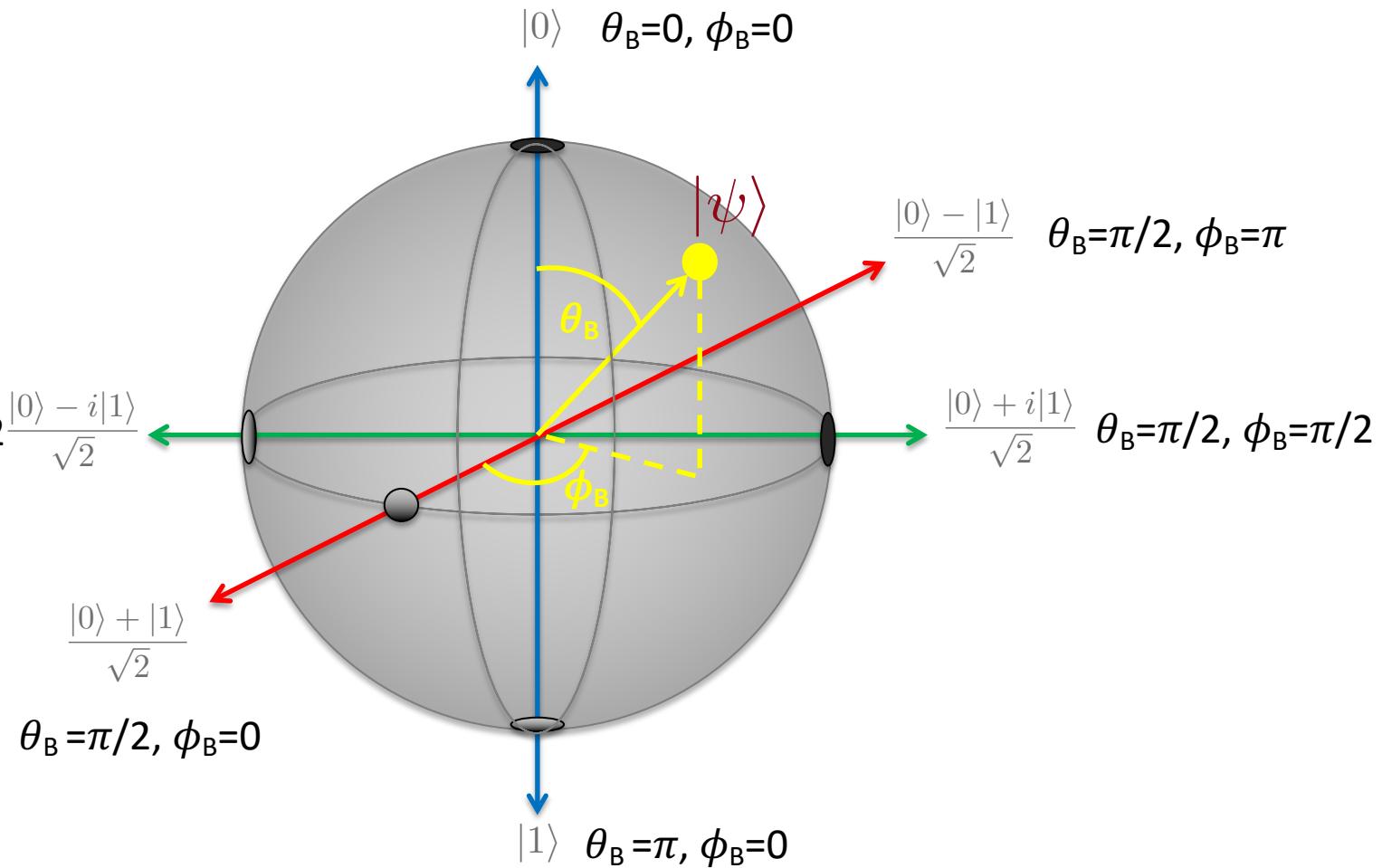
$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow \cos \frac{\theta_B}{2} |0\rangle + \sin \frac{\theta_B}{2} e^{i\phi_B} |1\rangle$$

$$\phi_B \equiv \theta_1 - \theta_0$$

$$|a_0| \equiv \cos \frac{\theta_B}{2}$$

$$|a_1| \equiv \sin \frac{\theta_B}{2}$$

$$\theta_B = \pi/2, \phi_B = 3\pi/2 \quad \frac{|0\rangle - i|1\rangle}{\sqrt{2}}$$



Polar co-ordinates and global phase

Recall, an arbitrary qubit state: $|\psi\rangle = a|0\rangle + b|1\rangle = |a|e^{i\theta_a}|0\rangle + |b|e^{i\theta_b}|1\rangle$

Which we can rearrange to be:

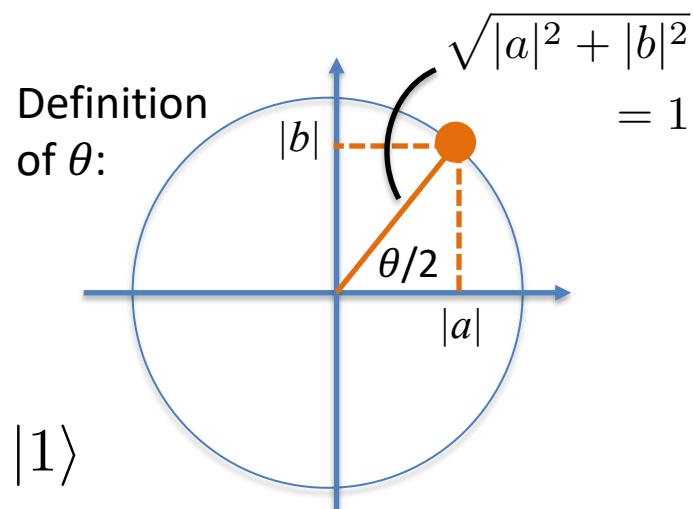
$$|\psi\rangle = e^{i\theta_a} \left(|a| |0\rangle + |b| e^{i(\theta_b - \theta_a)} |1\rangle \right) = e^{i\theta_a} \left(|a| |0\rangle + |b| e^{i(\theta_b - \theta_a)} |1\rangle \right)$$

$$|\psi\rangle = e^{i\theta_a} \left(\cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i(\theta_b - \theta_a)} |1\rangle \right) = e^{i\theta_{\text{global}}} \left(\cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle \right)$$

The global phase is unimportant, and can never be measured in experiment.

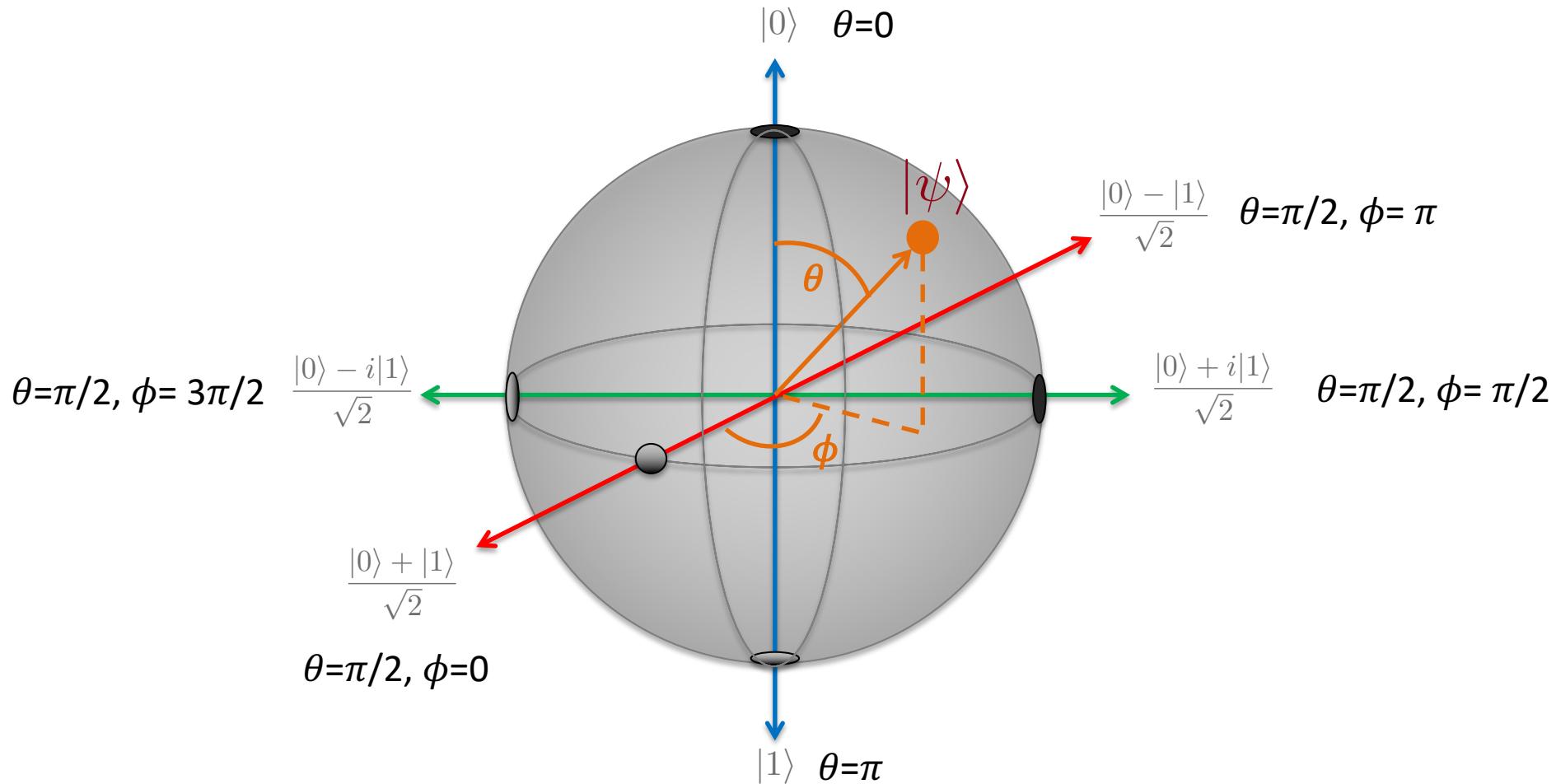
The other variables θ and ϕ relate to the position of this state on the Bloch sphere:

$$|\psi\rangle = a|0\rangle + b|1\rangle \rightarrow \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle$$



States on the Bloch sphere

$$|\psi\rangle = a|0\rangle + b|1\rangle \rightarrow \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle$$



Qubit states on the Bloch sphere – example (Prac Class 1)

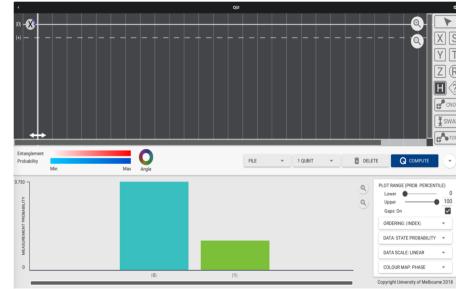
Recall, in the prac class we produced the state: $|\psi\rangle = \frac{\sqrt{3}}{2} |0\rangle + \frac{-i}{2} |1\rangle$

$$a_0 = \frac{\sqrt{3}}{2}$$

$$\text{i.e. } |a_0| = 0.866, \theta_0 = 0$$

$$a_1 = \frac{-i}{2}$$

$$\text{i.e. } |a_1| = 0.500, \theta_1 = -\pi/2$$



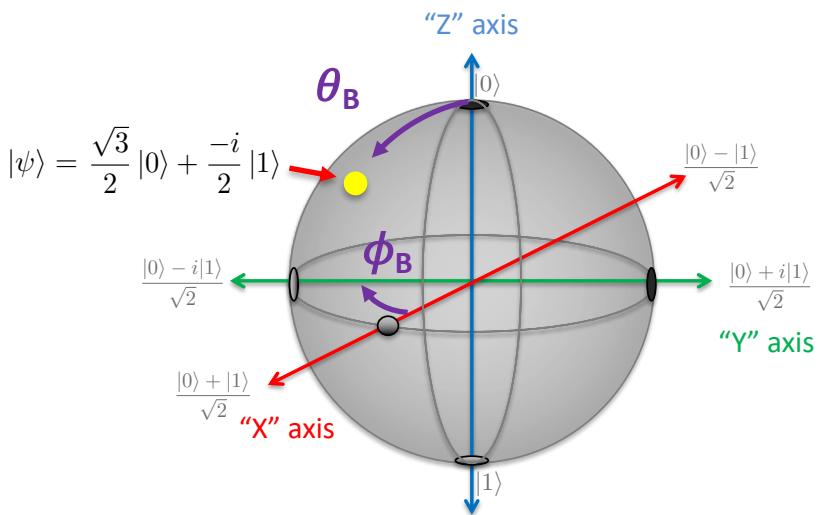
We want to express this on the Bloch Sphere...

$$\phi_B = \theta_1 - \theta_0 = -\frac{\pi}{2} = -90^\circ$$

$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow \cos \frac{\theta_B}{2} |0\rangle + \sin \frac{\theta_B}{2} e^{i\phi_B} |1\rangle$$

$$\cos \frac{\theta_B}{2} = \frac{\sqrt{3}}{2} \rightarrow \theta_B = 2\cos^{-1} \frac{\sqrt{3}}{2} = 60^\circ$$

$$\begin{aligned} \sin \frac{\theta_B}{2} &= \sin 30 = \frac{1}{2} \rightarrow e^{i\phi_B} = -i \\ \rightarrow \phi_B &= -90^\circ \end{aligned}$$



Qubit states on the Bloch sphere – example (Prac Class 1)

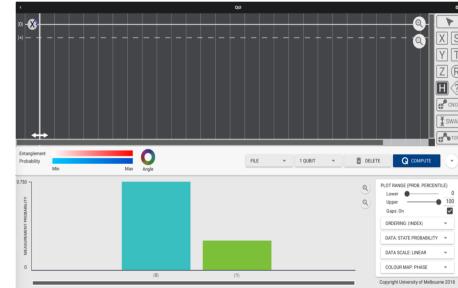
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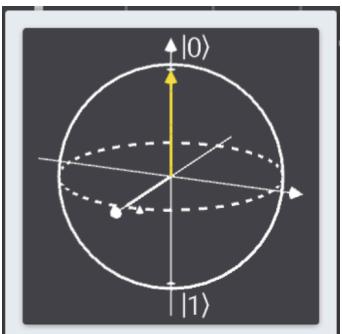


We want to express this on the Bloch Sphere...

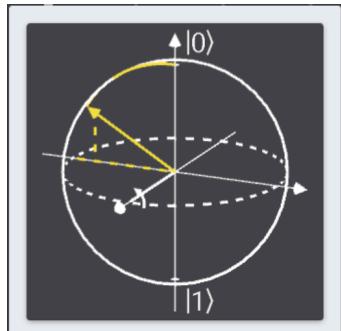
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$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow \cos \frac{\theta_B}{2} |0\rangle + \sin \frac{\theta_B}{2} e^{i\phi_B} |1\rangle$$

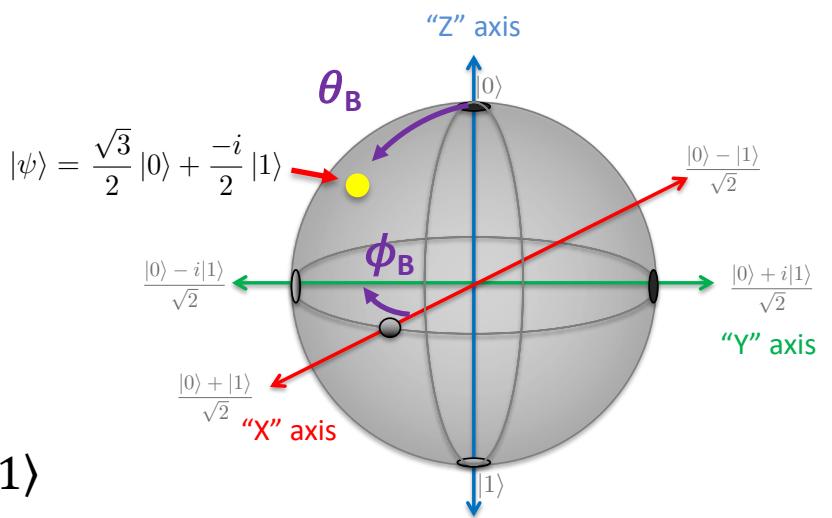
QUI Bloch spheres:



$$|\psi\rangle = |0\rangle$$



$$|\psi'\rangle = \frac{\sqrt{3}}{2} |0\rangle + \frac{-i}{2} |1\rangle$$

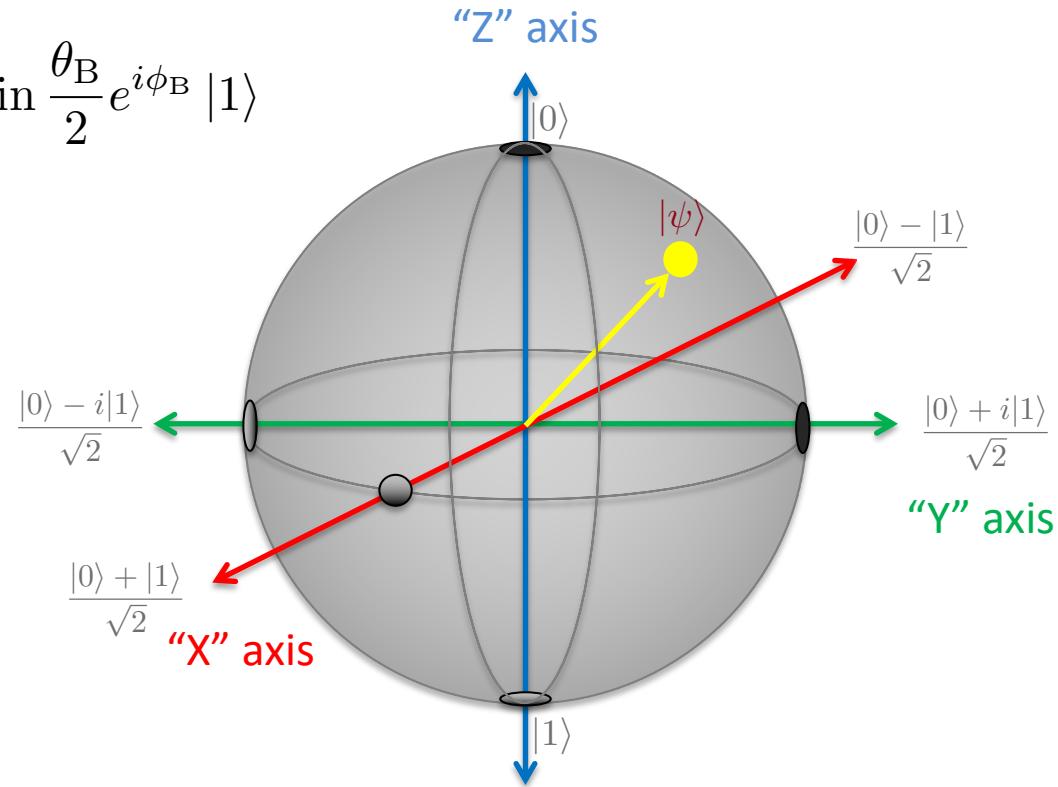
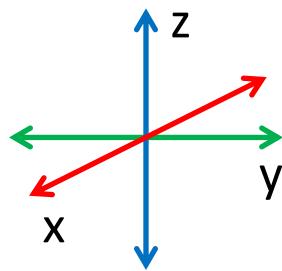


The Bloch Sphere – Cartesian Axes

Bloch sphere representation:

$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow \cos \frac{\theta_B}{2} |0\rangle + \sin \frac{\theta_B}{2} e^{i\phi_B} |1\rangle$$

C/f cartesian axes labelling:



These **X**, **Y** and **Z** axes are not our usual “real space” cartesian axes...these are a representation of the space in which the single-qubit states live...
 ...but they are used a lot in describing quantum logic operations!

Quantum operations on qubits

Quantum operations in general

A *quantum operation* “ U ” takes a state $|\psi\rangle$ and changes it to a new state $|\psi'\rangle$

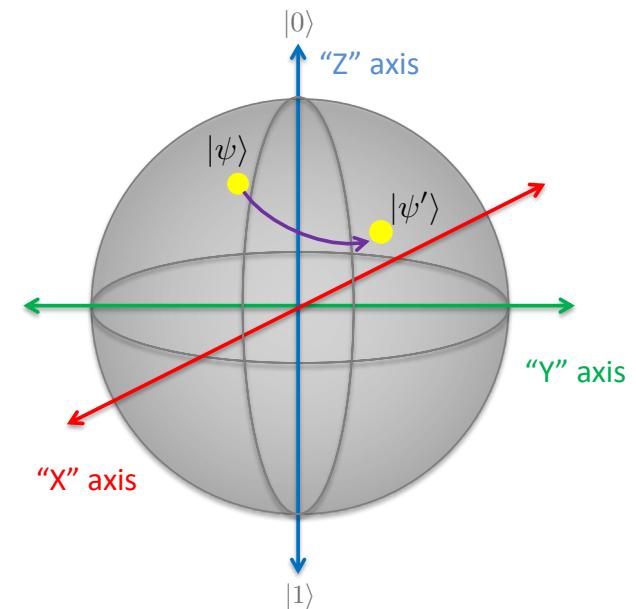
$$|\psi\rangle \xrightarrow{U} |\psi'\rangle$$

Mathematically, we write this as: $U|\psi\rangle = |\psi'\rangle$

i.e. U acting on the state $|\psi\rangle$ gives the new state $|\psi'\rangle$

$$|\psi'\rangle = U|\psi\rangle$$

The Bloch sphere is a useful way to visualise how these operations work...(and how it's done in the QUI)



The operation “ U ” moves the state across the surface of the Bloch sphere.

Qubit operations/Bloch sphere – example, (Prac Class 1)

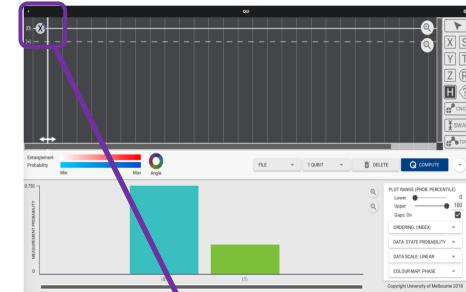
Recall, in the prac class we produced the state: $|\psi\rangle = \frac{\sqrt{3}}{2} |0\rangle + \frac{-i}{2} |1\rangle$

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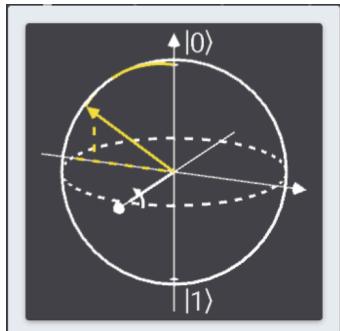
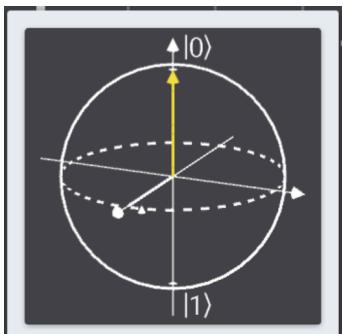
Operation U

We want to express this on the Bloch Sphere...

$$\phi_B = \theta_1 - \theta_0 = -\frac{\pi}{2} = -90^\circ$$

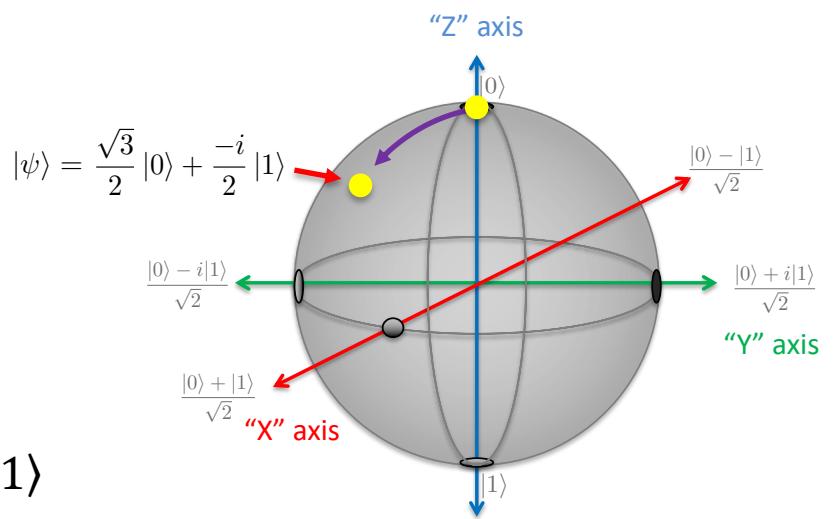
$$|\psi\rangle = a_0 |0\rangle + a_1 |1\rangle \rightarrow \cos \frac{\theta_B}{2} |0\rangle + \sin \frac{\theta_B}{2} e^{i\phi_B} |1\rangle$$

QUI animations:



$$|\psi\rangle = |0\rangle$$

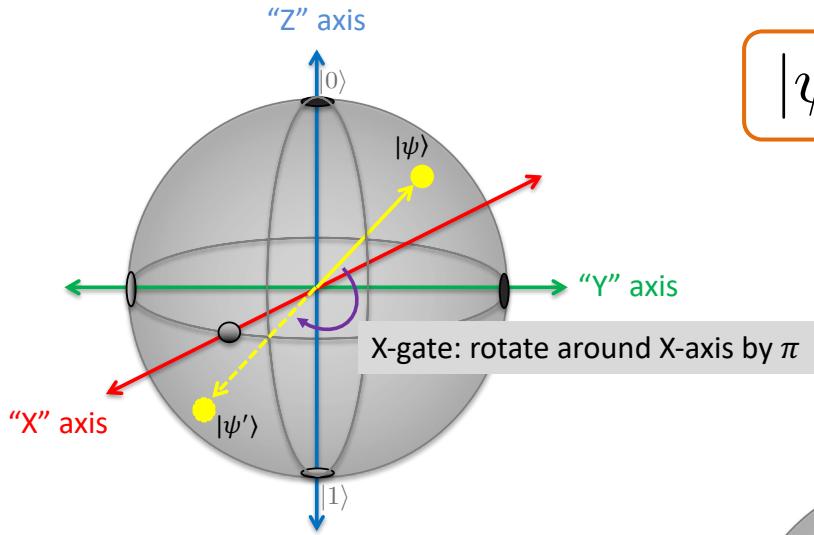
$$|\psi'\rangle = \frac{\sqrt{3}}{2} |0\rangle + \frac{-i}{2} |1\rangle$$



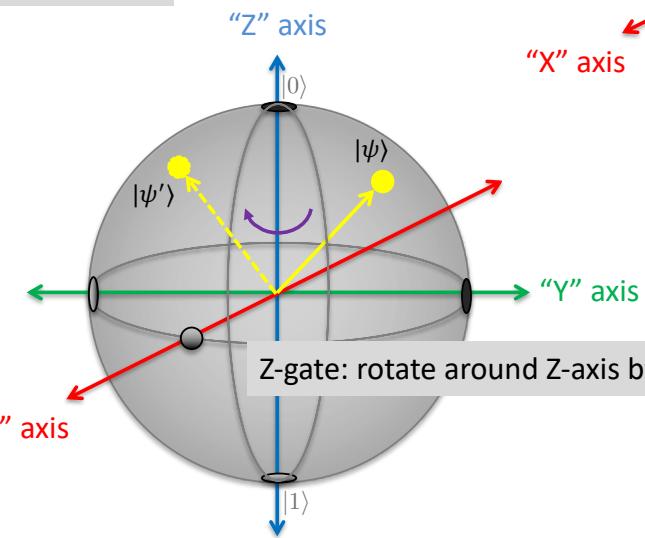
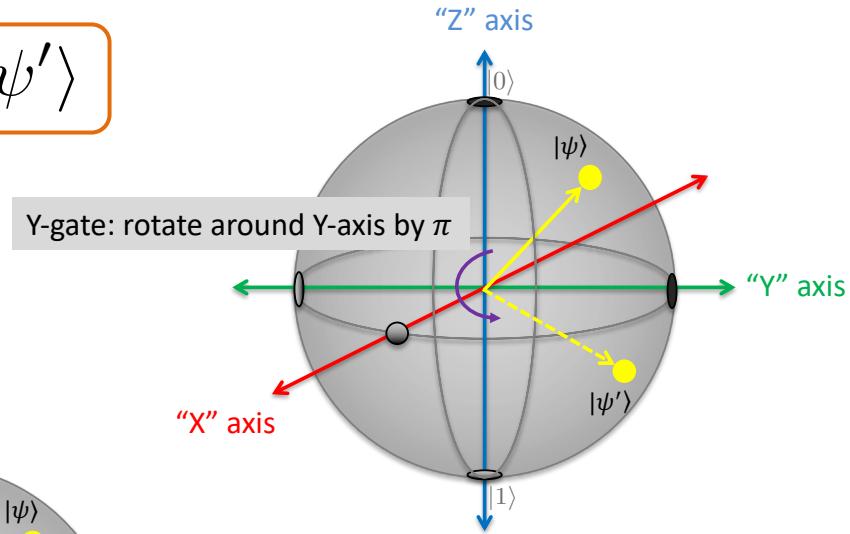
Qubit gates in matrix form and the Pauli matrices

The “Cartesian” quantum operations: X, Y, Z

We can specify the state moving across the Bloch sphere in many ways, but the “Cartesian” operations are very simple – a rotation of π (180°) about any of X, Y, or Z axes:



$$|\psi\rangle \rightarrow |\psi'\rangle$$

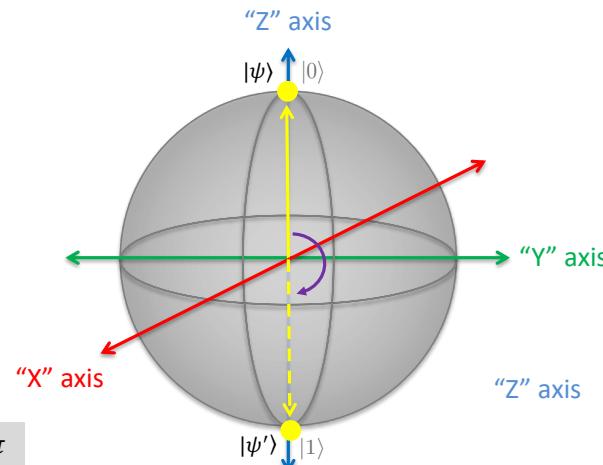
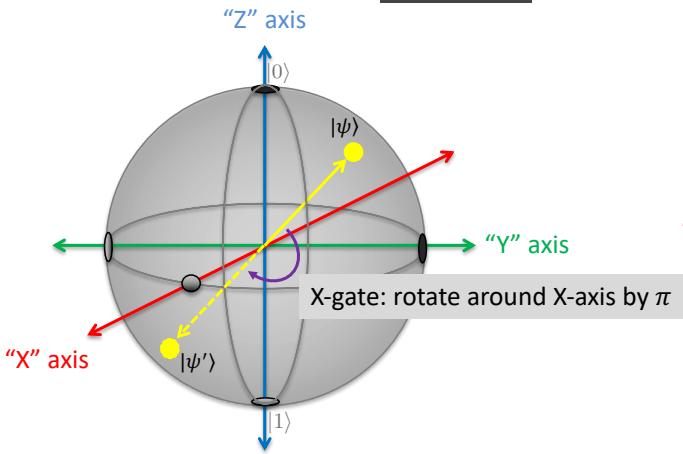
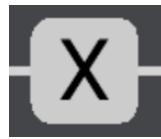


NB. Perspectives not 100% accurate!

The cartesian rotations are usually referred to as the “Pauli” operators X, Y, Z

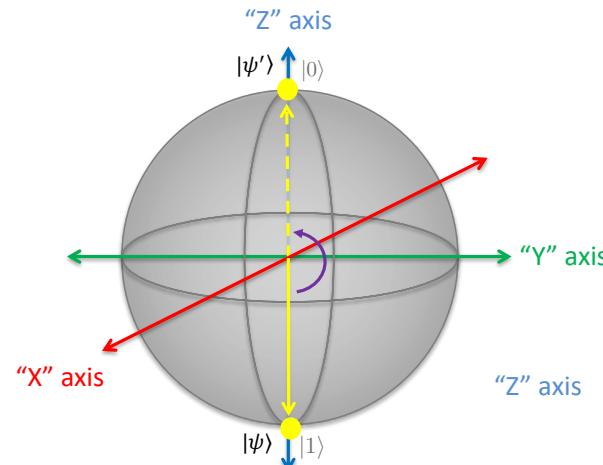
The X gate – acting on the basis states $|0\rangle$ and $|1\rangle$

Circuit symbol:



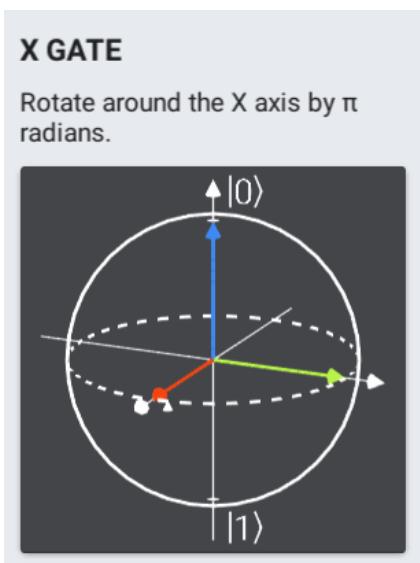
e.g. system starts in: $|\psi\rangle = |0\rangle$
 (north pole)

X gate: rotation about X-axis by π (180°) sends to $|\psi'\rangle = |1\rangle$
 (south pole)



e.g. system starts in: $|\psi\rangle = |1\rangle$
 (south pole)

X gate: rotation about X-axis by π (180°) sends to $|\psi'\rangle = |0\rangle$
 (north pole)



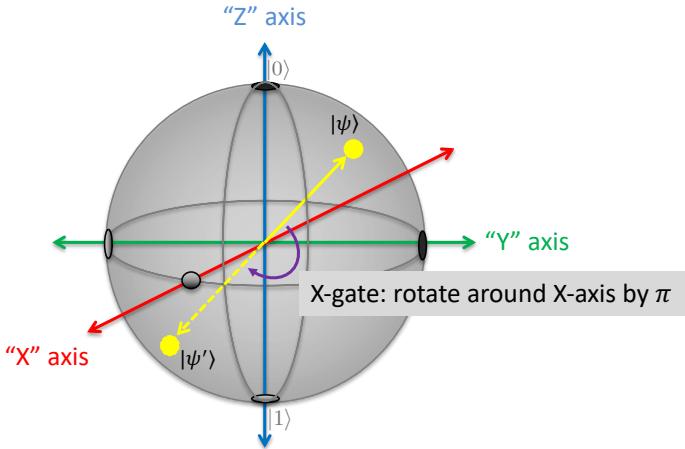
We write $X|\psi\rangle = |\psi'\rangle$

$$\begin{cases} X|0\rangle = |1\rangle \\ X|1\rangle = |0\rangle \end{cases}$$

hence:
 "flip gate"

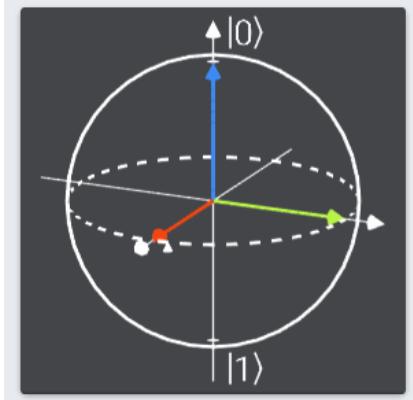
The X gate – acting on general states $a_0|0\rangle + a_1|1\rangle$

Circuit symbol:



X GATE

Rotate around the X axis by π radians.



On the computational states the X-gate operation is a “bit flip”:

$$\begin{cases} X|0\rangle = |1\rangle \\ X|1\rangle = |0\rangle \end{cases}$$

Quantum mechanics is inherently linear, so the X-gate acting on a linear superposition is:

$$X(a_0|0\rangle + a_1|1\rangle) = a_1|0\rangle + a_0|1\rangle$$

Or we can write in “ket” notation:

$$a_0|0\rangle + a_1|1\rangle \xrightarrow{X} a_1|0\rangle + a_0|1\rangle$$

The X gate in matrix form

Action of X-gate in “ket” form: $a_0 |0\rangle + a_1 |1\rangle \xrightarrow{X} a_1 |0\rangle + a_0 |1\rangle$

What is the X-gate in “matrix” form?

Recall “matrix” notation: $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $a_0 |0\rangle + a_1 |1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$ $|\psi\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$

Operations in matrix representation:

$$|\psi'\rangle = U |\psi\rangle$$

$$\begin{bmatrix} a'_0 \\ a'_1 \end{bmatrix} = \begin{bmatrix} 2 \times 2 \\ \text{matrix} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

Action of X-gate in matrix form:

$$a_0 |0\rangle + a_1 |1\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \rightarrow a_1 |0\rangle + a_0 |1\rangle = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

In matrix notation, in general:

i.e. $\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \xrightarrow{X} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

The X gate in matrix form – the Pauli matrices

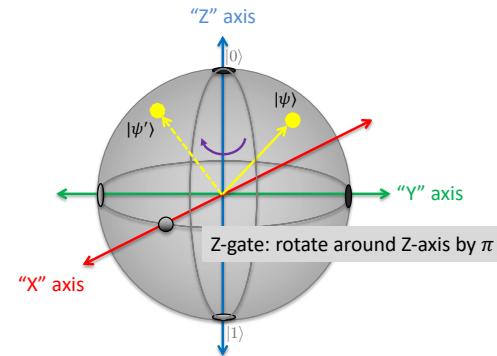
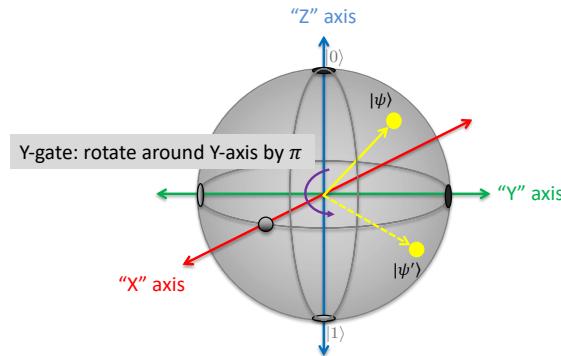
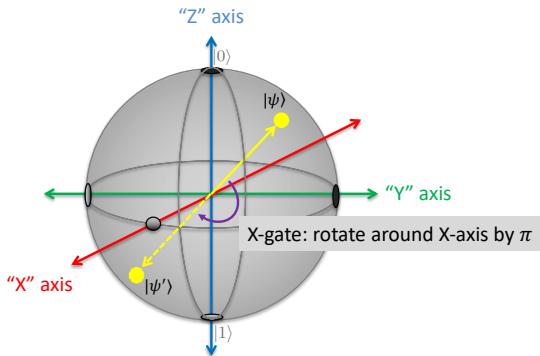
$$|\psi'\rangle = X |\psi\rangle$$

$$\begin{bmatrix} a'_0 \\ a'_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

This is the so-called Pauli X matrix...one of three Pauli matrices representing X, Y and Z operations...

All cartesian axes – the Pauli matrices for X, Y and Z:



$$|\psi'\rangle = X |\psi\rangle$$

$$\begin{bmatrix} a'_0 \\ a'_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$|\psi'\rangle = Y |\psi\rangle$$

$$\begin{bmatrix} a'_0 \\ a'_1 \end{bmatrix} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$|\psi'\rangle = Z |\psi\rangle$$

$$\begin{bmatrix} a'_0 \\ a'_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Week 2

Lecture 3

- 3.1 The Bloch Sphere representation for qubits
- 3.2 Quantum operations on qubits
- 3.3 Qubit gates in matrix form and the Pauli matrices

Lecture 4

- 4.1 The Pauli gates X, Y and Z and the QUI
- 4.2 Qubit operations around non-cartesian axes – H and R gates
- 4.3 Programming sequences over the qubit logic gate library
- 4.4 Note on the context and use of angles

Practice class 2

Bloch sphere and single qubit logic operations on the QUI