Week by week

Week 3

Lecture 5

5.1 Two qubit systems and operations

5.2 Entanglement

Lecture 6

6.1 Dense coding

6.2 Teleportation

Lab 3

Two qubit operations, entanglement, dense coding, teleportation

Lecture overview

In this lecture:

4.1 Dense coding

4.2 Teleportation

- Reiffel: 5.3
- Kaye: Ch 5
- Nielsen and Chuang: 1.3.5, 1.3.7, 2.3

Entanglement and quantum computing

A state which is *not separable* is **entangled**. For example:

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

In this lecture we will see how entangled states can be critical in various quantum computing tasks and apply these in the Lab to gain experience in how entangled states work.

In particular we will discuss

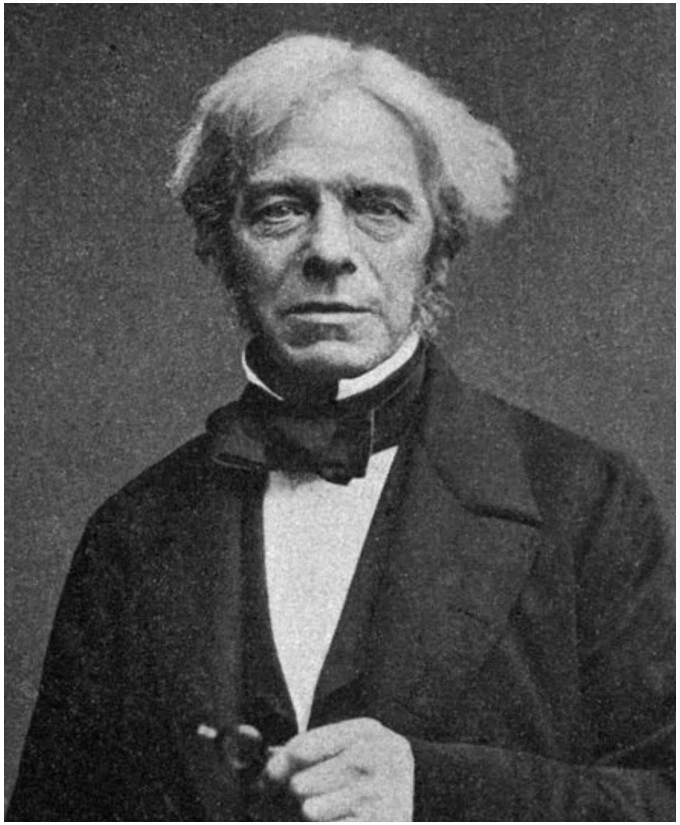
1. Dense Coding
2. No-cloning theorem
3. Quantum teleportation

Entanglement as a resource

When asked what practical use electricity was, Faraday reportedly replied:

"Why sir, there is every probability that you will be able to tax it"

Entanglement is similar, a ***resource*** useful for many quantum information tasks.



Faraday

4.1 Dense coding

PHYC90045

Lecture 4

THE UNIVERSITY OF MELBOURNE

Dense Coding

Alice would like to send **two classical bits** to Bob.

01 → Wants to send → 01

Alice

Bob

Alice and Bob can use a **quantum** NBN, and share some initial entanglement – can they get any advantage?

Dense Coding

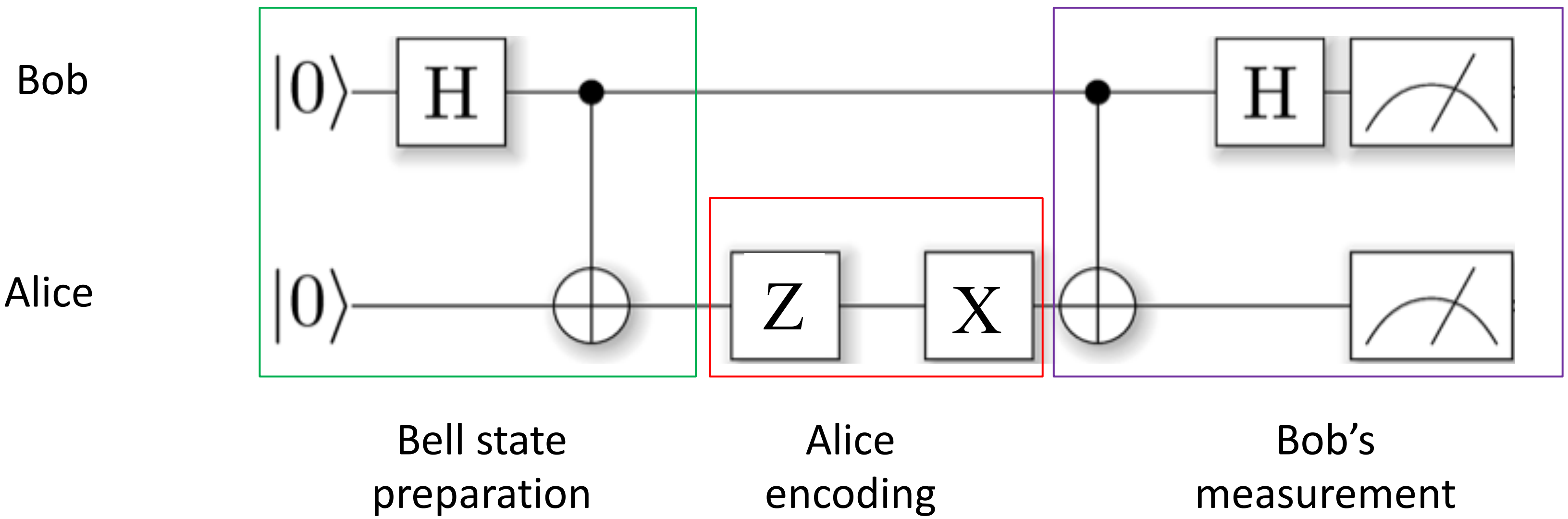
Entanglement makes it possible.

(1) Alice and Bob share an entangled state
(2) Alice flips her qubit one of four ways, based on the state she wants to send
(3) Alice sends her qubit to Bob
(4) Bob measures correlations between the qubits, to reveal which of the four (ie. two bits) operations Alice applied

Alice

Bob

Dense Coding Circuit



Bell state preparation:

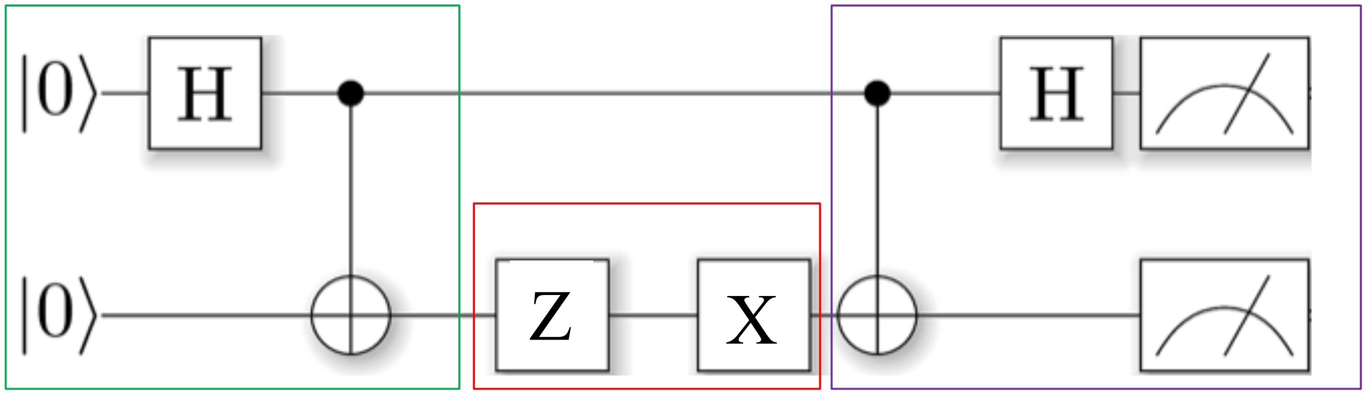
$$|00\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}} |0\rangle$$

$$\text{CNOT} \rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Dense Coding Circuit

Bob

Alice



Bell state preparation

Alice encoding

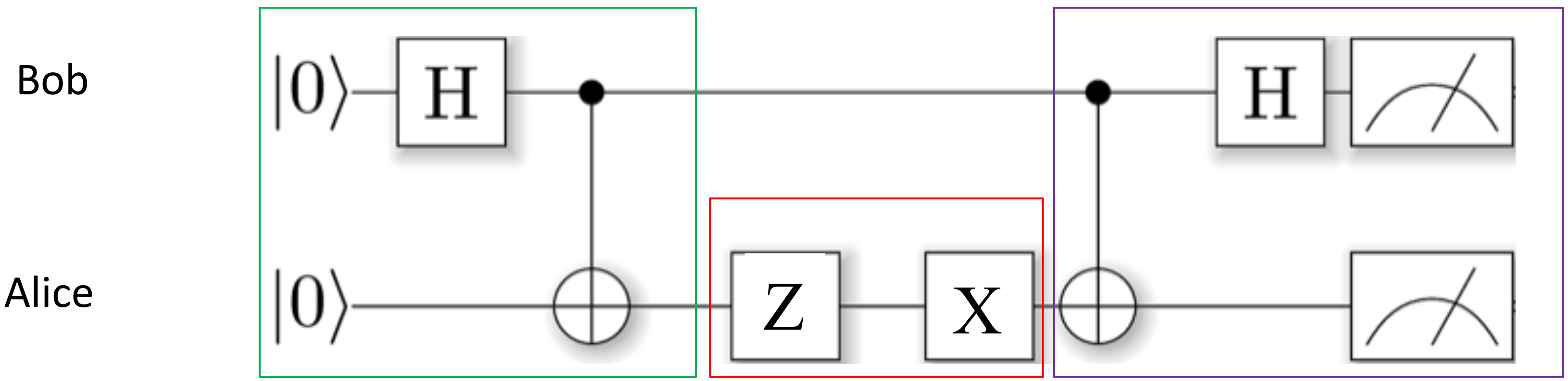
Bob's measurement

$$|00\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}|0\rangle$$
$$\rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$
\begin{array}{lrcl}
0,0 & \frac{|00\rangle+|11\rangle}{\sqrt{2}} & \rightarrow & \frac{|00\rangle+|11\rangle}{\sqrt{2}} \\
0,1 & X_2\frac{|00\rangle+|11\rangle}{\sqrt{2}} & \rightarrow & \frac{|01\rangle+|10\rangle}{\sqrt{2}} \\
1,0 & Z_2\frac{|00\rangle+|11\rangle}{\sqrt{2}} & \rightarrow & \frac{|00\rangle-|11\rangle}{\sqrt{2}} \\
1,1 & X_2Z_2\frac{|00\rangle+|11\rangle}{\sqrt{2}} & \rightarrow & \frac{|01\rangle-|10\rangle}{\sqrt{2}}
\end{array}
$$

Alice applies one of four different operations to her qubit, based on the classical information she would like to send.

Dense Coding Circuit



Bell state preparation

Alice encoding

Bob's measurement

$$0,0 \qquad \frac{|00\rangle+|11\rangle}{\sqrt{2}} \rightarrow \frac{|00\rangle+|11\rangle}{\sqrt{2}}$$

$$0,1 \qquad X_2\frac{|00\rangle+|11\rangle}{\sqrt{2}} \rightarrow \frac{|01\rangle+|10\rangle}{\sqrt{2}}$$

$$1,0 \qquad Z_2\frac{|00\rangle+|11\rangle}{\sqrt{2}} \rightarrow \frac{|00\rangle-|11\rangle}{\sqrt{2}}$$

$$1,1 \qquad X_2 Z_2\frac{|00\rangle+|11\rangle}{\sqrt{2}} \rightarrow \frac{|01\rangle-|10\rangle}{\sqrt{2}}$$

CNOT

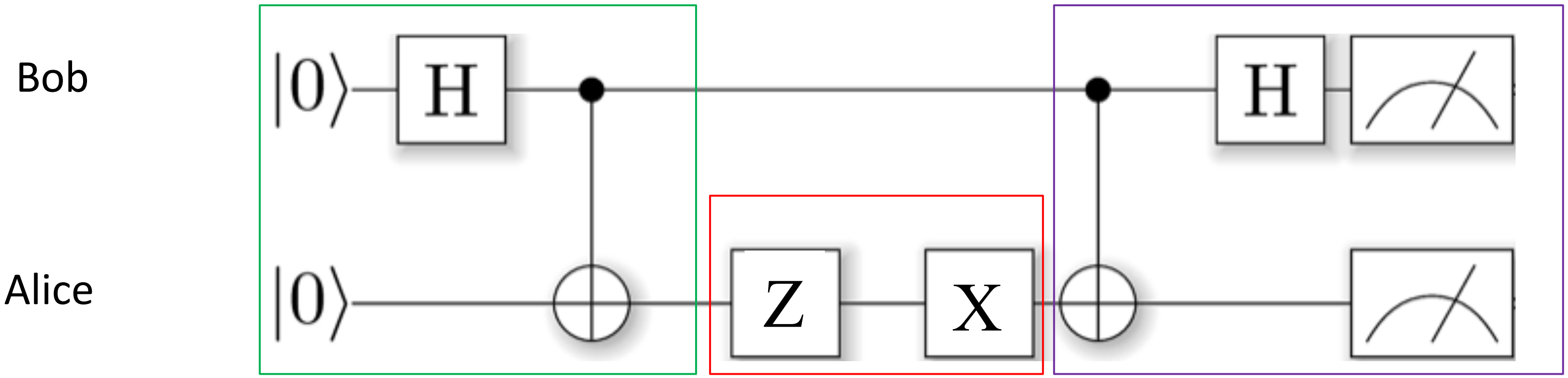
$$\frac{|00\rangle+|10\rangle}{\sqrt{2}} \qquad \frac{|01\rangle+|11\rangle}{\sqrt{2}} \qquad \frac{|00\rangle-|10\rangle}{\sqrt{2}} \qquad \frac{|01\rangle-|11\rangle}{\sqrt{2}}$$

H

$$|00\rangle \qquad |01\rangle \qquad |10\rangle \qquad |11\rangle$$

Dense Coding Circuit

THE UNIVERSITY OF MELBOURNE



Bell state preparation

Alice encoding

Bob's measurement

$$H\left|+\right\rangle = \left|0\right\rangle,\ H\left|-\right\rangle = \left|1\right\rangle$$

$$\begin{array}{lrcl}
0,0 & \frac{|00\rangle+|11\rangle}{\sqrt{2}} & \rightarrow & \frac{|00\rangle+|11\rangle}{\sqrt{2}} \\
0,1 & X_2\frac{|00\rangle+|11\rangle}{\sqrt{2}} & \rightarrow & \frac{|01\rangle+|10\rangle}{\sqrt{2}} \\
1,0 & Z_2\frac{|00\rangle+|11\rangle}{\sqrt{2}} & \rightarrow & \frac{|00\rangle-|11\rangle}{\sqrt{2}} \\
1,1 & X_2Z_2\frac{|00\rangle+|11\rangle}{\sqrt{2}} & \rightarrow & \frac{|01\rangle-|10\rangle}{\sqrt{2}}
\end{array}$$

CNOT $\rightarrow$

$$\begin{aligned}
\frac{|00\rangle+|10\rangle}{\sqrt{2}} &= \frac{|0\rangle+|1\rangle}{\sqrt{2}}\,|0\rangle = |+\rangle\,|0\rangle \\
\frac{|01\rangle+|11\rangle}{\sqrt{2}} &= \frac{|0\rangle+|1\rangle}{\sqrt{2}}\,|1\rangle = |+\rangle\,|1\rangle \\
\frac{|00\rangle-|10\rangle}{\sqrt{2}} &= \frac{|0\rangle-|1\rangle}{\sqrt{2}}\,|0\rangle = |-\rangle\,|0\rangle \\
\frac{|01\rangle-|11\rangle}{\sqrt{2}} &= \frac{|0\rangle-|1\rangle}{\sqrt{2}}\,|1\rangle = |-\rangle\,|1\rangle
\end{aligned}$$

H $\rightarrow$

$$\begin{array}{l}
|00\rangle \\
|01\rangle \\
|10\rangle \\
|11\rangle
\end{array}$$

Two bits communicated but only one qubit "sent".

Makes use of pre-existing entanglement.

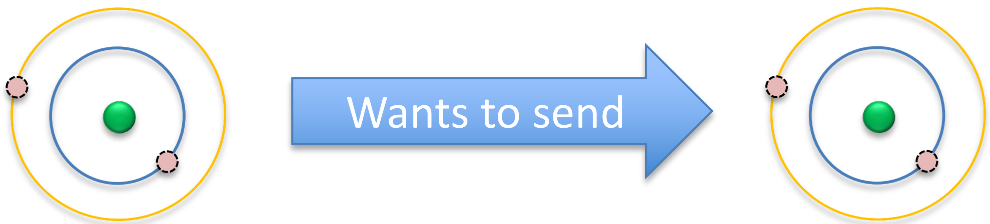
4.2 Teleportation

PHYC90045

Lecture 4

A Quantum Computing Bus?

To understand the role entanglement can play in quantum information processing, we will consider how it can be to transmit quantum information around our quantum computer (and potentially between quantum computers)



$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \qquad\qquad |\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

Alice

Bob

Communication around the quantum computer is an important primitive. We could physically move quantum systems, but there is a (potentially) better way: **teleportation**

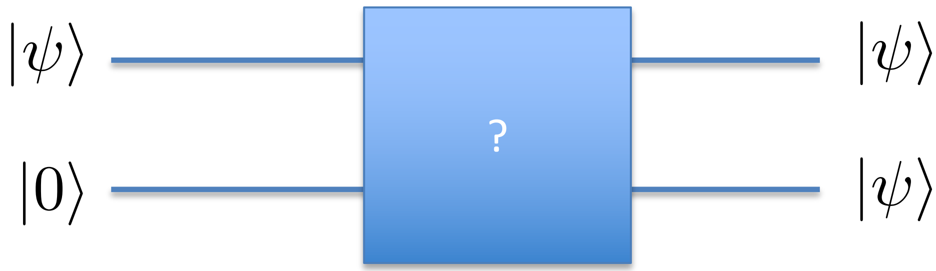
Sending classical information

How would we do this classically? Measure everything about the state, then send that information down (classical) bus and recreate a perfect copy elsewhere.

Problem: we can't do this in quantum mechanics because classical measurement (1) *collapses the system*, and (2) *this clones the system* which we can't do in quantum mechanics.

No-cloning theorem

Can we make a circuit which clones the input state?



That is, we ask if it is possible to make a *unitary* transformation s.t.

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle)$$

$$= \alpha^2|00\rangle + \alpha\beta|01\rangle + \beta\alpha|10\rangle + \beta^2|11\rangle$$

No-cloning theorem: the answer is **no**.

Proof of no-cloning theorem

If we had a cloning circuit, we could use it on two arbitrary states, $|\psi\rangle$ and $|\phi\rangle$

$$U|\phi\rangle|0\rangle = |\phi\rangle|\phi\rangle \qquad U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$$

Inner product on LHS: $\langle 0| \langle \phi| U^\dagger U |\psi\rangle |0\rangle = \langle \phi|\psi\rangle$

Inner product on RHS: $\langle \psi|\langle \psi|\phi\rangle|\phi\rangle = \langle \psi|\phi\rangle^2$

But the only solutions to $x^2=x$ are x=0 or x=1. We can only have a circuit clone states *which are orthogonal (x=0 case), not arbitrary states*.

There can be no unitary transformation which clones two arbitrary states.

Teleportation

Entanglement makes it possible.

(1) Alice has a qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

(2) Alice and Bob share an entangled state

(3) Alice measures correlations between her qubit and half of the entangled state

(4) Alice sends the results of the measurements to Bob

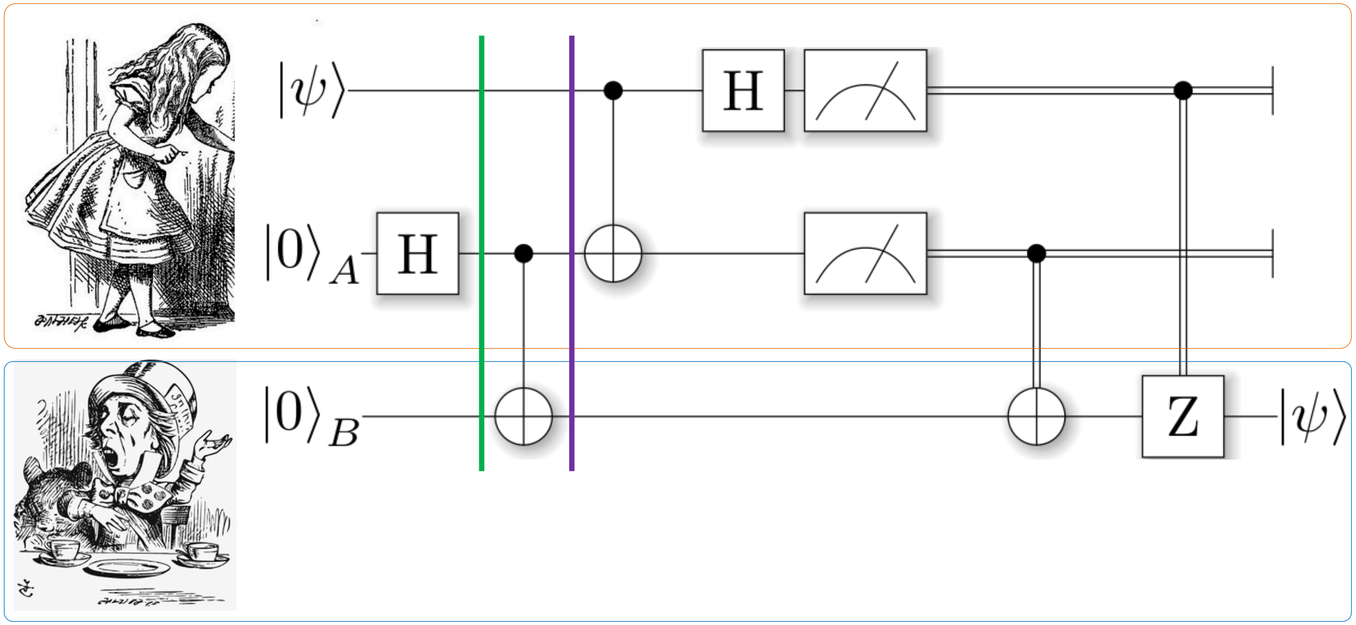
(5) Bob uses them to reconstruct the original state in his qubit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Alice

Bob

Teleportation



Double lines indicate classical operations

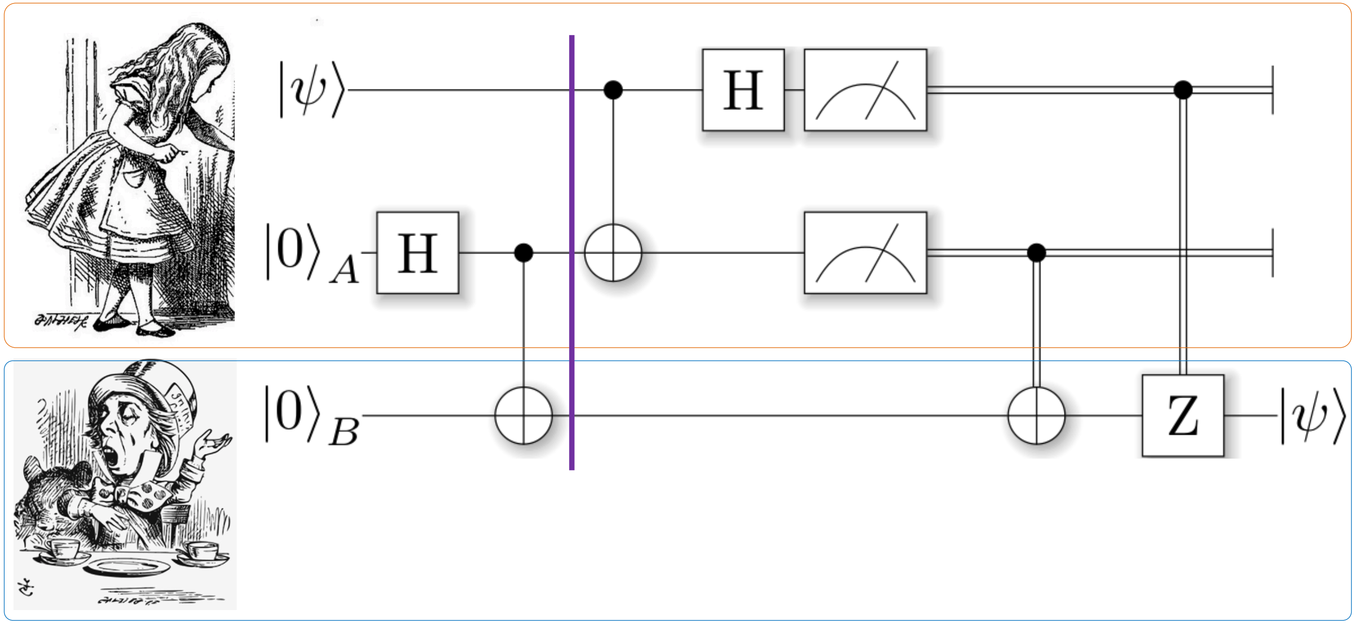
$$(\alpha|0\rangle + \beta|1\rangle) \otimes |00\rangle$$

Hadamard (A) $\longrightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle$

CNOT(A-B) $\longrightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Alice's state Bell state preparation (shared)

Teleportation

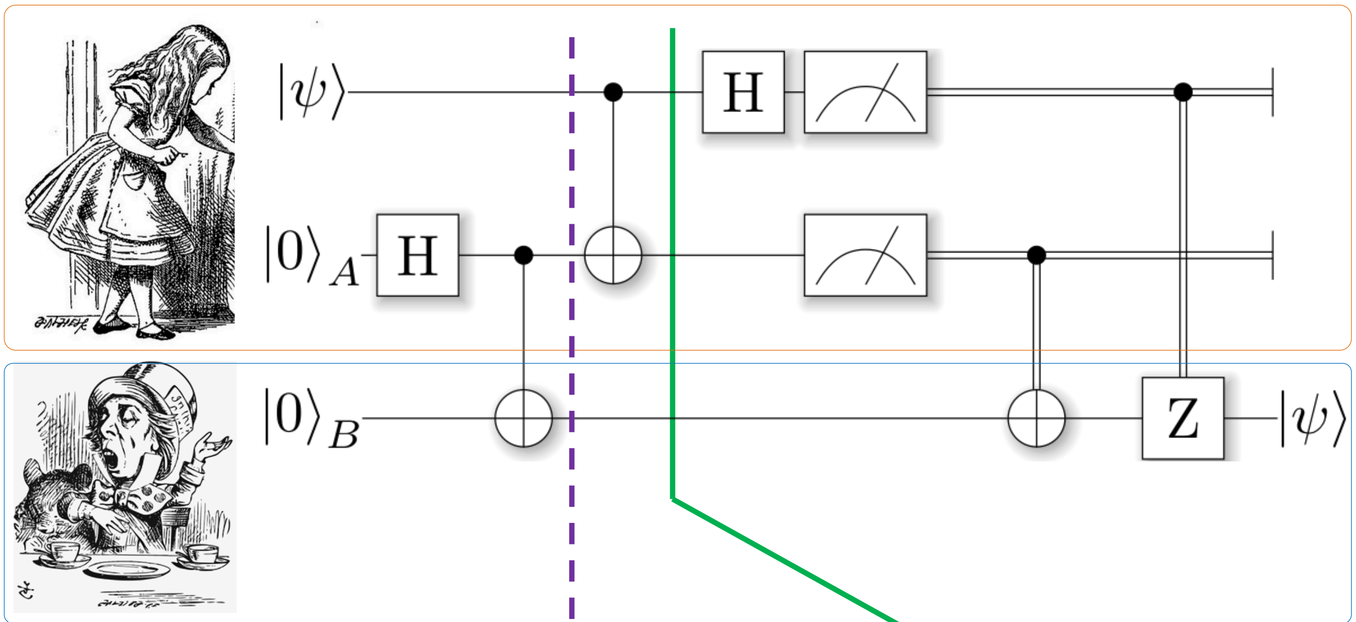


Total system state: $$(\alpha|0\rangle + \beta|1\rangle)\,\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Alice's state $|\psi\rangle$ Shared entangled state A & B

Expand: $\longrightarrow$ $$\frac{1}{\sqrt{2}}\left(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|100\rangle + \beta\,|111\rangle\right)$$

Teleportation



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|100\rangle + \beta\,|111\rangle\right)$$

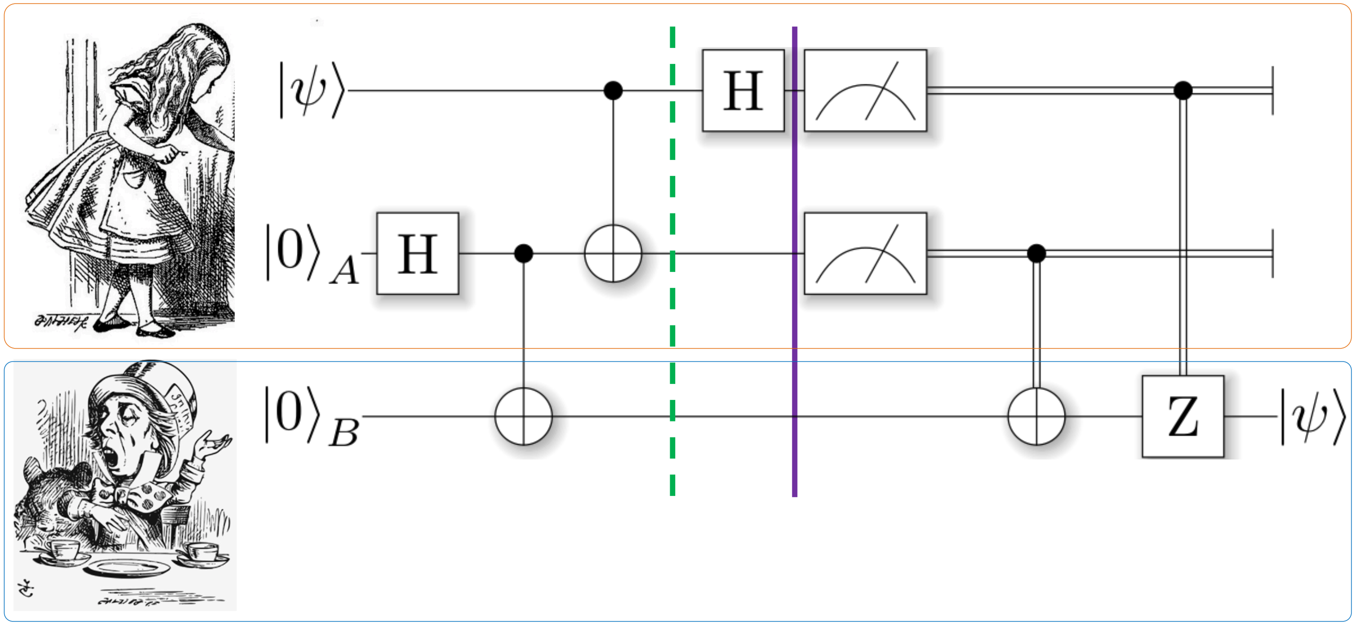
CNOT[1,2]

$$\xrightarrow{} \alpha\frac{|000\rangle + |011\rangle}{\sqrt{2}} + \beta\frac{|110\rangle + |101\rangle}{\sqrt{2}}$$

Rewrite (ex):

$$\frac{|+\rangle|0\rangle}{2}(\alpha|0\rangle + \beta|1\rangle) + \frac{|+\rangle|1\rangle}{2}(\alpha|1\rangle + \beta|0\rangle) + \frac{|-\rangle|0\rangle}{2}(\alpha|0\rangle - \beta|1\rangle) + \frac{|-\rangle|1\rangle}{2}(\alpha|1\rangle - \beta|0\rangle)$$

Teleportation

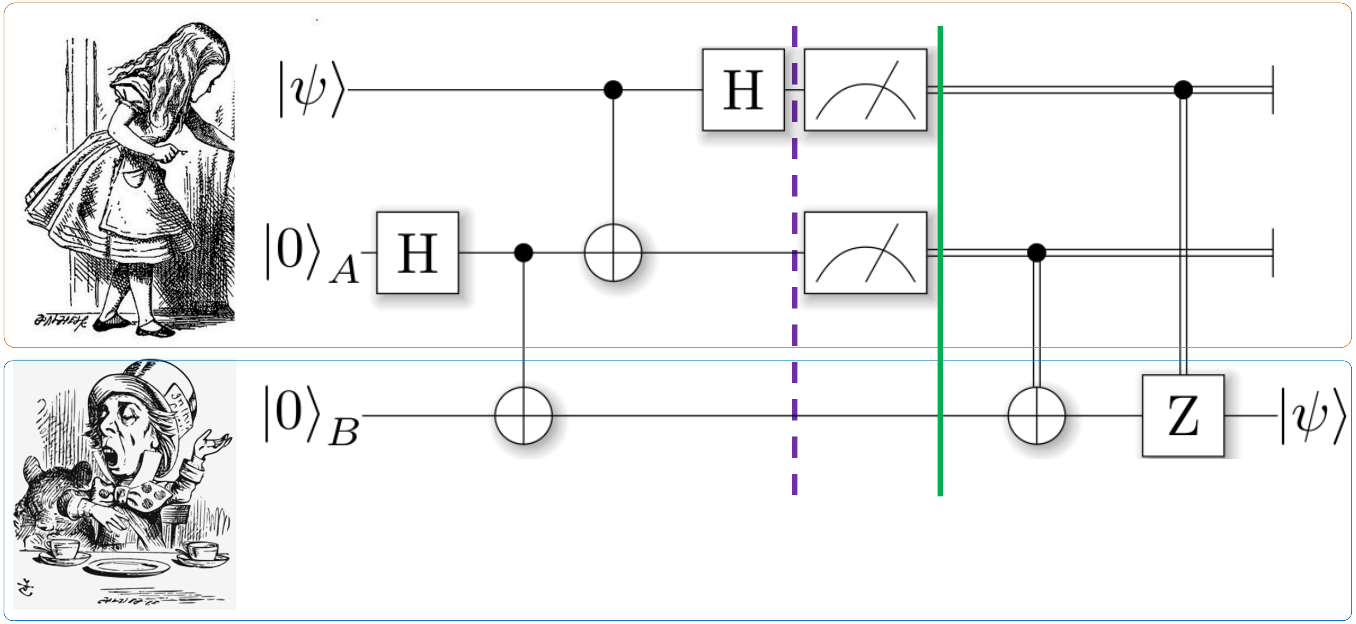


$$\frac{|+\rangle|0\rangle}{2}(\alpha|0\rangle+\beta|1\rangle)$$
$$+\frac{|+\rangle|1\rangle}{2}(\alpha|1\rangle+\beta|0\rangle)$$
$$+\frac{|-\rangle|0\rangle}{2}(\alpha|0\rangle-\beta|1\rangle)$$
$$+\frac{|-\rangle|1\rangle}{2}(\alpha|1\rangle-\beta|0\rangle)$$

Hadamard →

$$\frac{|0\rangle|0\rangle}{2}(\alpha|0\rangle+\beta|1\rangle)$$
$$+\frac{|0\rangle|1\rangle}{2}(\alpha|1\rangle+\beta|0\rangle)$$
$$+\frac{|1\rangle|0\rangle}{2}(\alpha|0\rangle-\beta|1\rangle)$$
$$+\frac{|1\rangle|1\rangle}{2}(\alpha|1\rangle-\beta|0\rangle)$$

Teleportation



$$\frac{|0\rangle|0\rangle}{2}(\alpha|0\rangle+\beta|1\rangle)+\frac{|0\rangle|1\rangle}{2}(\alpha|1\rangle+\beta|0\rangle)+\frac{|1\rangle|0\rangle}{2}(\alpha|0\rangle-\beta|1\rangle)+\frac{|1\rangle|1\rangle}{2}(\alpha|1\rangle-\beta|0\rangle)$$

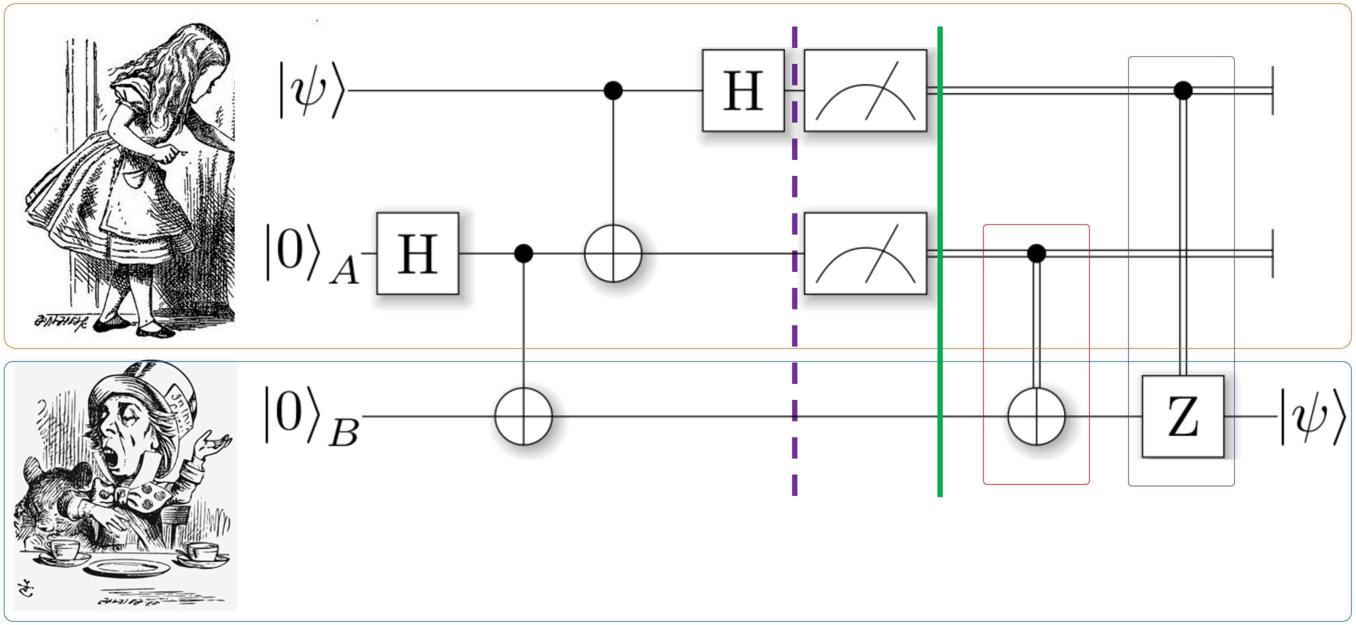
Alice measures her two qubits.

Bob's qubit collapses to one of the four possibilities.

Alice now <u>tells</u> Bob her outcomes (double lines indicate classical communication).

Bob will perform simple corrections shown.

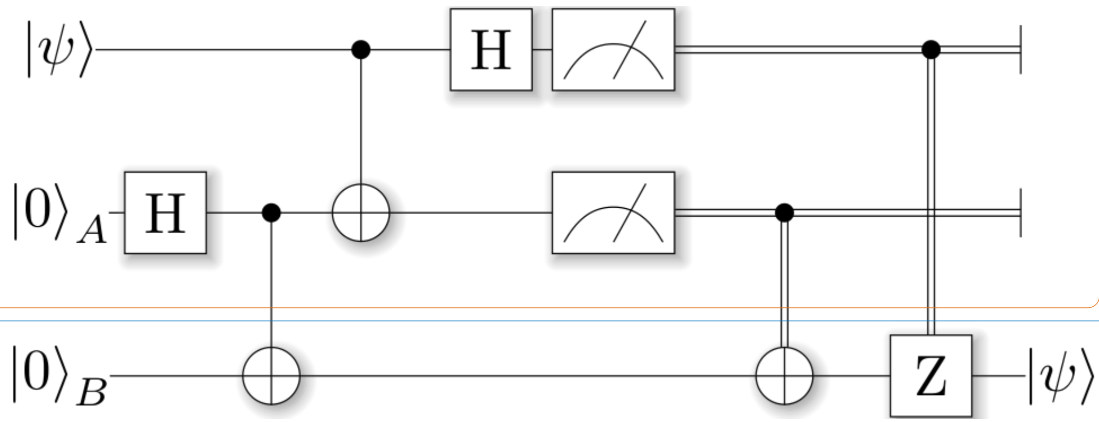
Teleportation



$$
\frac{|0\rangle|0\rangle}{2}(\alpha|0\rangle+\beta|1\rangle)
+\frac{|0\rangle|1\rangle}{2}(\alpha|1\rangle+\beta|0\rangle)
+\frac{|1\rangle|0\rangle}{2}(\alpha|0\rangle-\beta|1\rangle)
+\frac{|1\rangle|1\rangle}{2}(\alpha|1\rangle-\beta|0\rangle)
$$

Alice measures	Bob's qubit
$0,0$	$\alpha|0\rangle+\beta|1\rangle$
$0,1$	$\alpha|1\rangle+\beta|0\rangle \rightarrow \alpha|0\rangle+\beta|1\rangle$
$1,0$	$\alpha|0\rangle-\beta|1\rangle \rightarrow \alpha|0\rangle+\beta|1\rangle$
$1,1$	$\alpha|1\rangle-\beta|0\rangle \rightarrow \alpha|0\rangle-\beta|1\rangle \rightarrow \alpha|0\rangle+\beta|1\rangle$

Teleportation



$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

Alice measures	Bob's qubit
$0, 0$	$\alpha\|0\rangle + \beta\|1\rangle$
$0, 1$	$\alpha\|1\rangle + \beta\|0\rangle$
$1, 0$	$\alpha\|0\rangle - \beta\|1\rangle$
$1, 1$	$\alpha\|1\rangle - \beta\|0\rangle$

i.e. after correction Bob has successfully reconstructed Alice's original state.

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle + \beta|1\rangle$$

$$X(\alpha|1\rangle + \beta|0\rangle) \rightarrow \alpha|0\rangle + \beta|1\rangle$$

$$Z(\alpha|0\rangle - \beta|1\rangle) \rightarrow \alpha|0\rangle + \beta|1\rangle$$

$$ZX(\alpha|1\rangle - \beta|0\rangle) \rightarrow \alpha|0\rangle + \beta|1\rangle$$

Teleportation

Entanglement makes it possible.

(1) Alice has a qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

(2) Alice and Bob share an entangled state

(3) Alice measures correlations between her qubit and half of the entangled state

(4) Alice sends the results of the measurements to Bob

(5) Bob uses them to reconstruct the original state in his qubit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Alice

Bob

Week by week

(1) Introduction to quantum computing
(2) Single qubit representation and operations
(3) Multi-qubit systems
(4) Simple quantum algorithms
(5) Quantum search (Grover's algorithm)
(6) Quantum factorization (Shor's algorithm)
(7) Quantum supremacy and noise
(8) Programming real quantum computers (IBM Q)
(9) Quantum error correction (QEC)
(10) QUBO problems and Adiabatic Quantum Computation (AQC)
(11) Variational/hybrid quantum algorithms (QAOA and VQE)
(12) Solving linear equations, QC computing hardware

Week 3

Lecture 5

5.1 Two qubit systems and operations

5.2 Entanglement

Lecture 6

6.1 Dense coding

6.2 Teleportation

Lab 3

Two qubit operations, entanglement, dense coding, teleportation