Unit Commitment Problem under Uncertainty: Integrating Renewable Energy with Dynamic Scheduling and Storage

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April 17 2025

Note: Most visuals are clearer with coloured printing, especially Figures 2 and 3, however it is not required.
Word count: 4890

Abstract

This study addresses the Unit Commitment Problem, minimising production costs while satisfying demand under renewable integration. Using integer linear programming, we compare formulations derived from Takriti et al. (2000) and Rajan & Takriti (2005), incorporating generators' prior on/off states instead of assuming full initial flexibility. The Rajan-based model proves computationally superior thanks to fewer constraints. We then examine how renewable penetration, excess-production penalties and weather-driven volatility shape total cost. Low penalties favour complete renewable integration, whereas high penalties link optimal penetration to volatility; increasing renewables shifts dispatch toward low-cost generators and away . To manage forecast uncertainty, we propose dynamic rescheduling that updates plans as weather data improve. Two policies are tested: one forbids shortages, the other allows minor shortfalls subject to penalty. The strict policy raises costs proportionally with volatility; the tolerant policy, using a $\pm 10\%$ renewable band and storage, curbs waste and highlights storage's significantly escalating economic value.

1 Introduction

This report explores the Unit Commitment Problem (UCP), an optimisation problem focused on minimising total energy production costs while satisfying demand in each period. The core task is managing the on/off status and output levels of traditional, controllable power generators within their technical and operational constraints. Key considerations include minimum and maximum production limits, cost types (start-up, no-load, variable), and technical requirements such as minimum up- and down-time.

In an increasingly energy-dependent world, optimising electricity production is vital. Developed regions rely on stable electricity for homes, businesses, and infrastructure. This requires planning to ensure that energy production aligns with changing demand. This becomes even more significant considering the financial and environmental impact, since improving production efficiency by just a few percentage points can lead to major savings and reductions in emissions.

Two models are explored, inspired by prior research. Takriti et al. (2000) presents a stochastic model incorporating uncertain fuel and spot-market prices, solved via Lagrangian relaxation and Bender's decomposition. They achieved up to 4% cost savings over 168 hours in a midwestern U.S. system, improving computational performance in real-world scenarios. Rajan & Takriti (2005) suggests a more efficient formulation for the UCP by introducing "turn on/off inequalities", which significantly improve computational performance in real-world instances. This is particularly relevant as we want to make decision-making as efficient as possible. Building on this, we introduce a refinement: our constraints account for the possibility that machines may already have been up or down for some time at the start of the time horizon. This ensures the model better reflects real-world conditions, ensuring improved policy decisions.

The report unfolds as follows: Section 2 provides a verbal description of the problem, outlining assumptions and constraints. Section 3 defines variables and parameters. Next, we present integer linear programming formulations for both models and the solution method. Section 4.1 compares the two formulations' handling of up- and down-time constraints to identify the most efficient one for our use. In section 4.2, we incorporate uncontrollable renewable energy sources, first assuming known production levels, and later in section 4.4, addressing uncertainty in renewable output. Overall, the report aims to deepen understanding of the UCP, evaluate different modelling strategies, and explore renewable integration in energy production planning.

2 Problem Description

In this paper, we study two formulations of the UCP. The UCP seeks to minimise total production costs over a fixed time horizon for a given set of generators, while meeting demand in every period. Each generator is characterised by its own variable costs (per unit produced), no-load costs (when switched on), start-up costs. Operational generators must produce within their minimum and maximum capacity limits, and must also respect minimum up- and down-time requirements—once turned on or off, a generator must stay in that state for a set duration. The model also considers initial states, which may constrain decisions at the start. Furthermore, a penalty is incurred for curtailing excess energy. A production schedule is feasible if it satisfies all these operational constraints:

- Demand is met in each time period.
- Production levels adhere to each generator's minimum and maximum capacity limits.
- Minimum up- and down-time requirements are fulfilled.

To address this problem, we rely on the following assumptions:

- Demand for all periods is known at the start.

- Excess energy production must be curtailed.
- The production schedule for all periods is set before the first period.

Later, we will add to this with renewable energy production.

3 Methodology

3.1 Model 1

The first model used in this paper is based on Takriti et al. (2000) and uses similar notation. However, taking into account the initial conditions requires additional notation. Consequently, the resulting model is as follows.

Sets:

- Generators: $\mathcal{N} = \{1, ..., n\}$
- Time periods: $\mathcal{M} = \{1, ..., T\}$

Parameters:

- q_i := Minimum production of generator i
- Q_i := Maximum production of generator i
- L_i := Minimum up time of generator i
- l_i := Minimum down time of generator i
- c_i^{SU} := Cost of turning on generator i
- c_i^{NL} := Fixed cost of operating generator i
- c_i^{var} := Variable cost of production for generator i
- U_1^i := Periods generator i has been on at $t = 1$
- U_0^i := Periods generator i has been off at $t = 1$
- d_t := Demand at time t
- u_0^i is 1 if generator i is on at $t = 0$ and 0 otherwise.
- λ := Penalty per unit of excess production

Decision variables:

$$u_t^i = \begin{cases} 1 & \text{if generator } i \text{ is on at time } t \\ 0 & \text{if generator } i \text{ is off at time } t \end{cases}$$

$$\mathrm{o}_t^i = \begin{cases} 1 & \text{if generator } i \text{ is turned on at time } t \\ 0 & \text{if generator } i \text{ is not turned on at time } t \end{cases}$$

- x_t^i := The amount of production of generator i at time t
- s_t := Excess production in period t

Model:

$$
\begin{aligned}
& \min \sum_{t\in\mathcal{M}} \left(\sum_{i\in\mathcal{N}} \left(x_t^i c_i^{var} + u_t^i c_i^{NL} + o_t^i c_i^{SU} \right) + \lambda s_t \right) && && (1)\\
\text{s.t.} \quad & d_t \leq \sum_{i\in\mathcal{N}} x_t^i && \forall t \in \mathcal{M}, && (2)\\
& q_i u_t^i \leq x_t^i && \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, && (3)\\
& x_t^i \leq Q_i u_t^i && \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, && (4)\\
& u_0^i \leq u_\tau^i && \forall \tau = 1, ..., L_i - U_1^i, \forall i \in \mathcal{N}, \text{ if } L_i > U_1^i, && (5)\\
& u_t^i - u_{t-1}^i \leq u_\tau^i && \forall \tau = t+1, ..., \min\{t-1+L_i, T\}, && (6)\\
& && \forall t = \max\{l_i - U_0^i, 0\} + 1, ..., T, \forall i \in \mathcal{N}, && \\
& 1 - u_0^i \leq 1 - u_\tau^i && \forall \tau = 1, ..., l_i - U_0^i, \forall i \in \mathcal{N}, \text{ if } l_i > U_0^i && (7)\\
& u_{t-1}^i - u_t^i \leq 1 - u_\tau^i && \forall \tau = t+1, ..., \min\{t-1+l_i, T\}, && (8)\\
& && \forall t = \max\{L_i - U_1^i, 0\} + 1, ..., T, \forall i \in \mathcal{N}, && \\
& s_t = \sum_{i\in\mathcal{N}} x_t^i - d_t && \forall t \in \mathcal{M}, && (9)\\
& -o_t^i \leq u_{t-1}^i - u_t^i && \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, && (10)\\
& o_t^i \leq u_t^i && \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, && (11)\\
& o_t^i \leq 1 - u_{t-1}^i && \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, && (12)\\
& x_t^i, s_t \in \mathbb{R}^+ && \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, && (13)\\
& u_t^i, o_t^i \in \mathbb{B} && \forall t \in \mathcal{M}, \forall i \in \mathcal{N}. && (14)
\end{aligned}
$$

The goal is to minimise the total system cost (1) over all time periods and generators. Constraint (2) ensures that total production in each time period satisfies the demand. Constraints (3) and (4) enforce the minimum and maximum capacity limits in each period for each generator when it is on. Furthermore, constraint (4) guarantees that a generator cannot produce any output when it is off. Constraints (5) and (7) ensure that, given that the generator is found on (off), if the minimum up(down)-time has not passed yet, the generator should stay on (off) until it has passed. Constraints (6) and (8) represent the minimum up- and down-time requirements based on the formulation from Takriti et al. (2000) once the initial up- and down-time constraints are satisfied. However, we modify the up-time constraint by splitting it at $t = U_1^i$ to account for situations where a generator may have already been running for some time and can therefore be switched off earlier. The same logic applies to the minimum down-time constraint. Note that we assume that u_0^i is determined by the state we find it in. The maximum functions ensure that we only invoke the constraint if there is still mandatory state time remaining. Constraint (9) introduces excess production, which must be curtailed to avoid overproduction. Constraints (10)-(12) determine whether generator i is turned on in period t, equalling 1 if switched on and 0 otherwise. Finally, constraints (13) and (14) define the positivity and binary restrictions on the decision variables.

3.2 Model 2

As mentioned in the introduction, the model of Rajan & Takriti (2005) is more efficient than Takriti et al. (2000). For this reason, we formulate a second model which follows the notation from Rajan & Takriti (2005), aiming to reduce computation time once we dive into our deeper analysis. Model 2 also incorporates similar logic as Ostrowski et al. (2012). We use similar notation as model 1, excluding o_t^i and including the following decision variables:

$$
v_t^i = \begin{cases} 1 & \text{if generator } i \text{ is turned on at time } t \\ 0 & \text{otherwise} \end{cases}
$$

$$w_t^i = \begin{cases} 1 & \text{if generator } i \text{ is turned off at time } t \\ 0 & \text{otherwise} \end{cases}$$

Model:

$$\min \sum_{t\in\mathcal{M}} \left(\sum_{i\in\mathcal{N}} \left(x_t^i c_i^{var} + u_t^i c_i^{NL} + v_t^i c_i^{SU} \right) + \lambda s_t \right) \tag{15}$$

$$\text{s.t.} \quad d_t \leq \sum_{i\in\mathcal{N}} x_t^i \qquad \forall t \in \mathcal{M}, \tag{16}$$

$$q_i u_t^i \leq x_t^i \qquad \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, \tag{17}$$

$$x_t^i \leq Q_i u_t^i \qquad \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, \tag{18}$$

$$u_t^i - u_{t-1}^i = v_t^i - w_t^i \qquad \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, \tag{19}$$

$$u_t^i = u_0^i \qquad \forall t = 1, ..., \max\{u_0^i(L_i - U_1^i), (1-u_0^i)(l_i - U_0^i)\}, \tag{20}$$

$$\forall i \in \mathcal{N}, \text{ if } \max\{u_0^i(L_i - U_1^i), (1-u_0^i)(l_i - U_0^i)\} > 0$$

$$\sum_{j=t-L_i+1, j\geq 1}^{t} v_j^i \leq u_t^i \qquad \forall t = \min\{T, \max\{(L_i - U_1^i)u_0^i, 0\} + 1\}, ..., T, \tag{21}$$

$$\forall i \in \mathcal{N},$$

$$\sum_{j=t-l_i+1, j\geq 1}^{t} w_j^i \leq 1 - u_t^i \qquad \forall t = \min\{T, \max\{(l_i - U_0^i)(1-u_0^i), 0\} + 1\}, ..., T,$$

$$\forall i \in \mathcal{N}, \tag{22}$$

$$s_t = \sum_{i\in\mathcal{N}} x_t^i - d_t \qquad \forall t \in \mathcal{M}, \tag{23}$$

$$x_t^i, s_t \in \mathbb{R}^+ \qquad \forall t \in \mathcal{M}, \forall i \in \mathcal{N}, \tag{24}$$

$$u_t^i, v_t^i, w_t^i \in \mathbb{B} \qquad \forall t \in \mathcal{M}, \forall i \in \mathcal{N}. \tag{25}$$

Constraints (21) and (22) are based on Rajan & Takriti (2005). The propositions in their paper show that these inequalities are sufficient to represent the up- and down-time constraints. Constraint (21) implies that if generator i is on at time t, then it could have been turned on at most once within the last L_i periods. If it is off, it cannot have been turned on at all in that same time frame. Equation (22) tells us that, if the generator is on at $t - l_i$, it cannot turn on the in the l_i periods after, as you would first have to turn it off and switch it back on too quickly. If it is off at $t - l_i$, it can only be turned on once in the l_i periods after. Equation (20) fixes the periods until the remaining up- or down-time has passed. Equation (19) is added to ensure a proper relation between u_t^i, v_t^i, and w_t^i. The other constraints are similar to formulation 1.

3.3 Solution Method

To solve the optimisation problem, we use a Branch-and-Bound algorithm. The algorithm divides the feasible region into smaller subproblems and computes bounds to prune suboptimal regions. A more elaborate explanation of the method can be found in Wolsey (2020).

For efficiency, we incorporate cutting planes to tighten the linear programming relaxations. Cutting planes are additional linear constraints that eliminate fractional solutions from the LP relaxation without excluding any feasible integer solutions. This approach improves convergence speed by providing tighter bounds during the pruning process. Our implementation of cutting planes is inspired by Ostrowski et al. (2012).

4 Numerical Results

4.1 Investigating Model Efficiency

To conduct our research, we use three different datasets. The first includes two small examples with two generators and the demand for five time periods. The second contains information about eight different generators. The third includes information about energy demand and five different production scenarios for renewable energy over 168 periods. These were provided by Huisman (2025). No alterations are made to the data. The research was conducted in a Python environment, using Gurobi Optimizer. The optimisation gap was set to 0.0 for both models to ensure equal objective values. The hardware used for the experiments is equipped with an Intel® Core™ i7-8565U CPU and a RAM of 16GB.

We use running time as a measurement to compare which model is more efficient for the three examples, Examples 1 and 2 from the first dataset and Example 3 using eight generator data and demand from the second and third dataset, respectively. We also look at objective values to test whether the two formulations describe the same problem. Table 1 presents the results for both formulations. Each formulation is evaluated on identical problem instances to ensure a fair comparison. The simulation was run for $\lambda = 10$.

Table 1: Comparison of running times and objective values for the two formulations

		Example 1	Example 2	Example 3
Formulation 1	Running time (seconds)	0.03	0.03	1.58
	Objective value	7,110	6,050	18,109,828.92
Formulation 2	Running time (seconds)	0.03	0.03	0.37
	Objective value	7,110	6,050	18,109,828.92

All objective values are equal, confirming correct model implementation. Formulation 2 is more efficient, outperforming Formulation 1 in runtime, primarily due to significantly fewer constraints related to up- and down-time requirements. In formulation 1, the range over which we iterate is notably increased by the introduction of the variable τ, adding an approximate $t * i$ number of constraints per unit increase of τ. Furthermore, the formulation of the turning-on variable o_t^i in formulation 1 is defined by inequalities, compared to the equality constraints for v_t^i in formulation 2. Equality constraints are more easily solved by a branch-and-bound method. For efficiency reasons, we will base our results solely on formulation 2. Furthermore, unless specified otherwise, all further results will be based on the large dataset used for Example 3 (Table 1).

4.2 Incorporating Renewable Production

In an extended version of the UCP, we explore the challenges of integrating renewable energy. Renewable output depends heavily on weather, making it difficult to predict. While renewables have zero marginal costs, their variability adds significant uncertainty to scheduling, complicating the task of matching supply and demand without under- and overproduction.

We first assume deterministic renewable production and examine how different penetration levels affect non-renewable generation and total cost. To address uncertainty, we then introduce a dynamic approach that updates schedules as forecasts improve. Two policies are proposed: the first assumes no supply shortages are allowed, as this may have large consequences for the power grid. The second policy relaxes this, allowing limited shortages while still aiming to minimise them.

4.3 Modelling Certainty in Renewable Production

Thanks to renewable energy production, less energy demand needs to be satisfied by the non-renewable generators. This is represented in the modified demand satisfaction constraint (26). However, renewable energy may also exceed total demand, which is penalised. This is accounted for by the modified excess production equality (constraint (27)). To incorporate renewable energy production we introduce the following parameters:

- α := Penetration rate of renewable energy production
- p_t^{RE} := Renewable energy production in period t

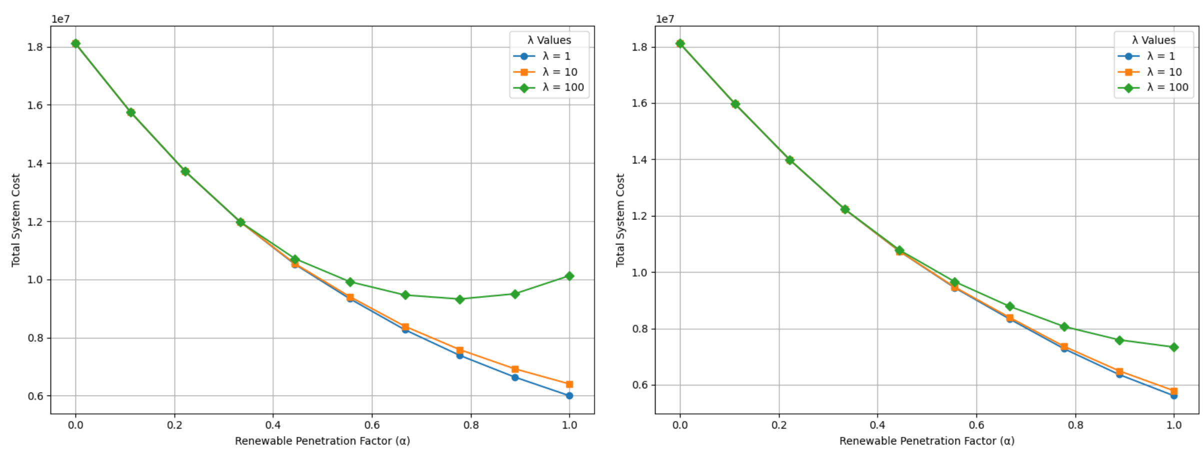
The new constraints are as follows:

$$d_t - \alpha p_t^{RE} \leq \sum_{i \in \mathcal{N}} x_t^i \qquad \forall t \in \mathcal{M}, \tag{26}$$

$$s_t = \sum_{i \in \mathcal{N}} x_t^i - d_t + \alpha p_t^{RE} \qquad \forall t \in \mathcal{M}. \tag{27}$$

To assess the profitability of renewable energy production, we analyse total production costs across different renewable energy penetration levels (α), penalties for excess production, and weather conditions. This analysis helps determine when and whether maximising renewable production is profitable.

We begin by examining how α affects the total cost under different scenarios, covering ten equidistant $\alpha \in [0, 1]$ and three penalty multipliers $\lambda \in \{1, 10, 100\}$. Figures 1a and 1b depict total cost as a function of α for each λ. These figures represent the most and least volatile scenarios, resulting in the most extreme optimal α values (Table 2). The other scenarios show similar trends within the range of these two cases.



(a) Scenario 1

(b) Scenario 5

Figure 1: Total production costs for ten equidistant $\alpha \in [0, 1]$, $\lambda \in \{1, 10, 100\}$, in scenarios 1 and 5

For all scenarios, when the excess production penalty is relatively low ($\lambda = 1$ and $\lambda = 10$), the total system cost shows a monotonically decreasing trend as α increases. This shows that full renewable integration ($\alpha = 1$) is advantageous when excess production penalties are low, as savings from free renewable energy outweigh potential penalty costs.

However, at the highest penalty level ($\lambda = 100$), the relationship between renewable penetration and total cost becomes more complex, with each scenario showing a different near-optimal α. Hence, a heuristic analysis is conducted to estimate a near-optimal α per scenario. Table 2 summarises the optimal α values for each scenario, minimising total costs, for $\lambda = 100$.

Table 2: Descriptive statistics and optimal α per scenario for $\lambda = 100$

Scenario	**Volatility**	**Optimal obj. value**	**Cumulative excess renew. prod.**	**Optimal α**
Scenario 1	1,106.99	9,317,397.12	26,968.89	0.76
Scenario 2	890.56	7,694,021.42	16,695.31	0.82
Scenario 3	876.01	7,396,914.43	15,849.94	0.87
Scenario 4	954.61	8,097,460.06	9,459.47	0.93
Scenario 5	619.80	7,336,296.64	7,672.05	1.0

Note: The optimal αs and optimal objective values are obtained by running the model for 100 equidistant αs ranging between 0 and 1. Volatility refers to the variability of renewable production, which is largely driven by weather conditions.

A clear trend emerges between optimal objective values for $\lambda = 100$ and renewable output volatility (p_t^{RE}). High volatility is challenging, as the up and down-time constraints mean we cannot quickly adapt to sudden changes in renewable supply. As a result, matching supply to demand becomes harder under volatility, increasing the risk of overproduction and suboptimal generator usage, ultimately raising total production costs.

We observe that the order of volatility coincides with the order of total cost (objective value). Scenarios with lower volatility, such as scenario 5, have a lower total cost for the optimal α. The opposite holds for high volatility scenarios, such as scenario 1.

On the other hand, there is a visible negative relation between the total excess production caused by the absolute value of the sum of excess renewable production relative to demand and the optimal α. In other words, when renewable production alone exceeds demand in a period, and when this occurs in large amounts throughout the entire time space, we want to limit the penalty costs of excess production which are certainly incurred by reducing the penetration level.

To sum up the above, we generally observe a perfectly positive relationship in the ranking between scenarios' volatility and their corresponding minimal total cost, and there is a one-to-one correspondence between the rank of scenarios' excess supply caused solely by renewable production and the optimal penetration level α. This is particularly relevant when it comes to budgeting costs. When forecasting models anticipate volatile weather conditions, one can anticipate higher production costs, and vice versa.

With the rise of renewables, a key question is which power plants are most replaced. To explore this, we visualise both absolute production (Figure 2) and each plant's share of total non-renewable output (Figure 3) across scenarios and varying αs. These figures, based on scenario 4 and $\lambda = 10$, are representative of all scenarios. Major deviations are discussed below and shown in the appendix. As trends are largely consistent, we aggregate results into a heatmap (Figure 4), showing how increasing renewable penetration affects each plant's share of non-renewable production.

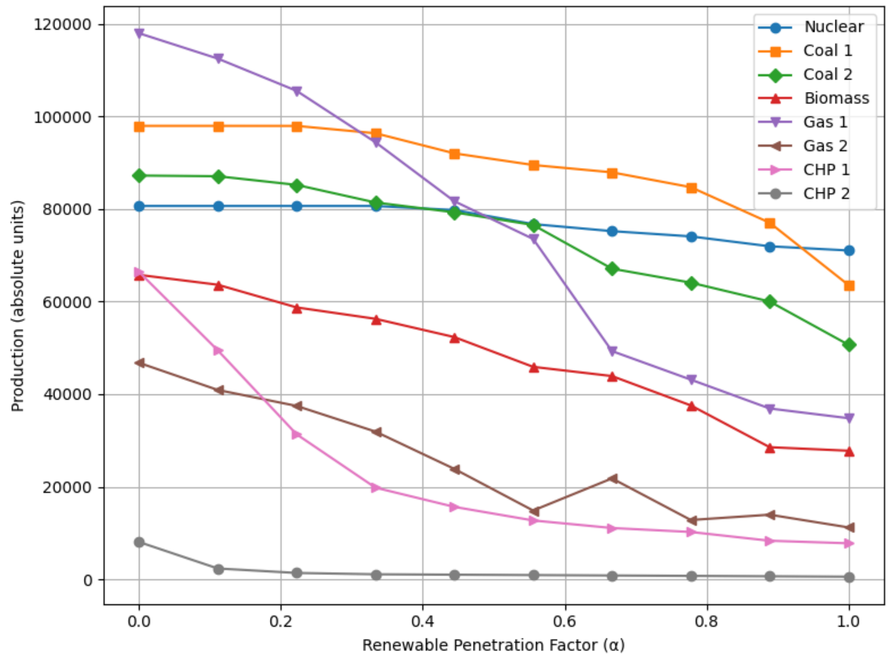


Figure 2: Total production per plant for ten equidistant $\alpha \in [0, 1]$, $\lambda = 10$ in scenario 4

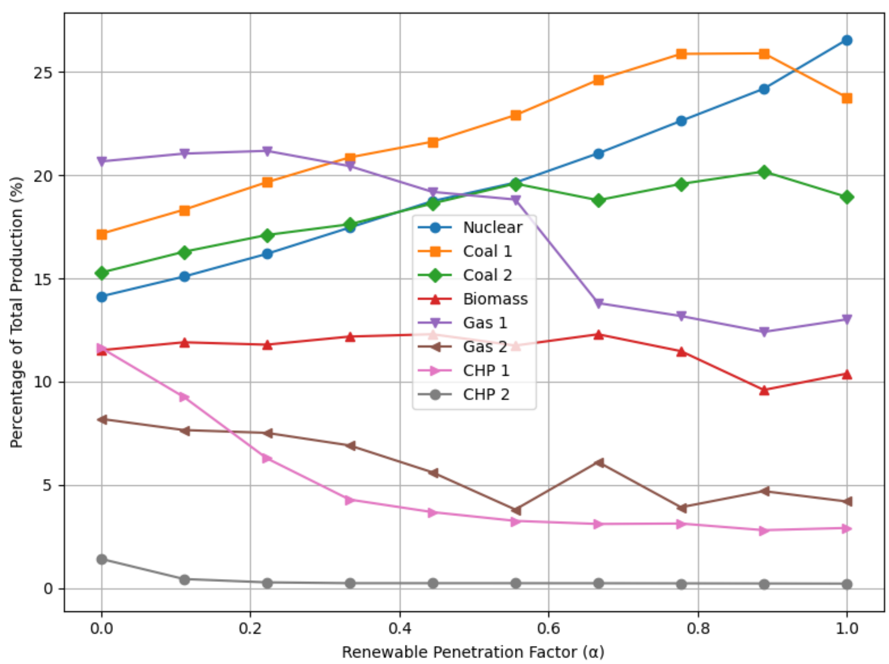


Figure 3: Proportion of total non-renewable production per plant for ten equidistant $\alpha \in [0, 1]$, $\lambda = 10$ in scenario 4

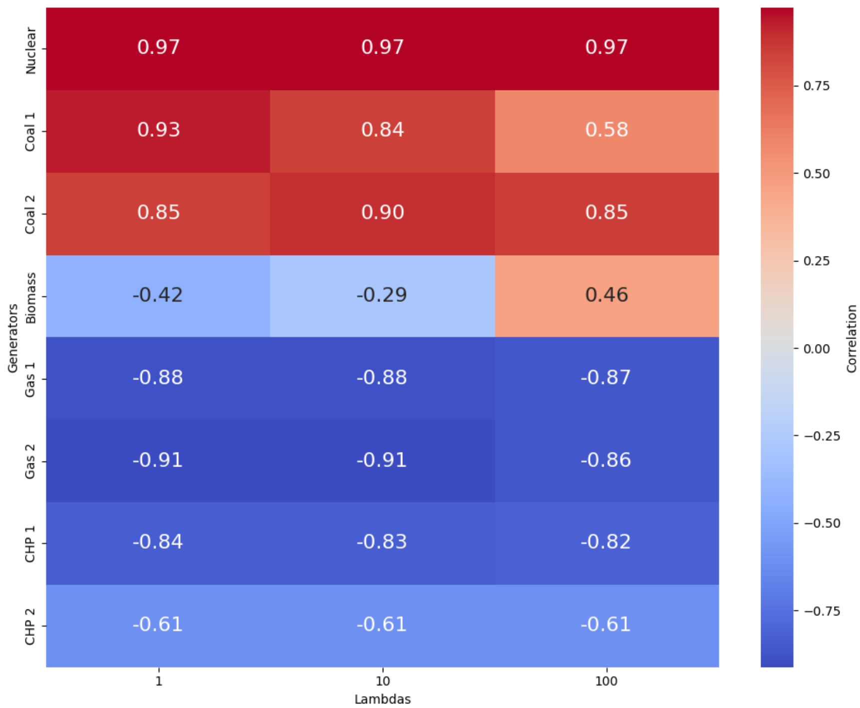


Figure 4: Correlation between α and relative production of a plant for $\lambda \in \{1, 10, 100\}$ across scenarios

Figure 4 reveals a clear pattern: as α increases, the use of high variable cost generators (Gas 1 to CHP 2) consistently declines. This suggests that as renewable energy meets more demand, the optimiser favours cheaper generation, reducing reliance on costlier units. While overall production falls across all generator types (Figure 2), nuclear and both coal generators are used relatively more. Their output also declines, but at a slower rate, due to their low variable costs, indicating that as non-renewables are phased out, cheaper options are prioritised. However, expensive generators are not fully discarded. Figure 3 shows they still play a role, likely due to their flexibility, including short up- and down-times and no start-up costs.

A notable point in Figure 4 is Coal 1, which shows a decreasing positive correlation as λ increases. This is a consistent trend across multiple Figure 3 versions. Biomass, by contrast, maintains stable usage, with correlations below 0.5 and varying by λ setting. Further discussion provides deeper insights into these behaviours.

When interpreting α as a proxy for renewable investment or technological efficiency, the correlations become especially relevant for developing or capital-constrained nations. These countries can use this analysis to phase out depreciating generators and reallocate investments toward renewables, potentially yielding economic benefits.

Coal 1 exhibits a steady production increase for $\lambda = 1$ and $\lambda = 10$, as reasoned above. However, for $\lambda = 100$, the increase flattens or even declines in scenarios 2 and 4 as shown in Figures A1 and A2, explained by long periods of sustained high renewable production. Given that we now have a significant penalty on excess production, combined with the extremely high start-up costs, we conclude that it is not economically beneficial to remain on and keep producing minimum capacity, compared to switching off and avoiding the costs of excess production. In contrast, Coal 2 shows stable, slightly rising relative production, most likely due to its flexibility, given by no start-up costs and similar variable costs to Coal 1. Nuclear production consistently increases across all scenarios, caused by its extremely low variable costs and significant minimum capacity which, paired with its high down-time, means we are never shutting it off.

Gas 1 production declines once renewable penetration reaches a certain threshold and then stabilises. The latter is likely because at its high minimum capacity it becomes economically inefficient due to extremely penalised excess production. For the same reason, we see production shifting from Gas 1 to Gas 2, a very similar generator with much lower minimum and maximum capacities. Despite this drop, Gas 1's relative output later remains stable because its zero start-up costs allow it to efficiently meet sudden high-demand spikes that Gas 2 cannot match. These results hold across all scenarios and lambdas.

Meanwhile, CHP 1 declines rapidly due to high variable costs, and CHP 2 falls to near zero, given even higher variable costs which can be prevented as a higher proportion of demand is covered by renewables. Biomass remains remarkably stable across scenarios and lambdas, most likely explained by its median variable costs, and high start-up costs, discouraging frequent alternation between on- and off-cycles. Its low no-load cost and minimal capacity requirements create flexibility, allowing it to be used as more expensive options disappear and cheaper ones rise with increasing α. Specifically, biomass' low minimum capacity of 32 prevents the sharp drop-off seen at $\lambda = 100$ unlike Coal 1, as it allows us to keep the generator on whilst only paying a penalty of 3200 rather than its very high start-up cost. This therefore explains the slight upward trend as seen in our heatmap analysis in Figure 4.

4.4 Modelling Uncertainty in Renewable Production

We now proceed with the incorporation of uncertainty in renewable energy supply in order to incorporate a more realistic approach. We will use a dynamic approach in which production schedules are gradually updated over time as the uncertainty around renewable production decreases.

We assume a finite set of k possible renewable production scenarios where many sequences are initially the same yet branch off at different moments, simulating uncertainty of as the hours progress. Additionally, the actual scenario corresponds to one of these k options. The other assumption made, is that no shortages are allowed. At time $t = 1$, we have no information about which scenario will occur. To avoid energy shortages at $t = 1$, we schedule the non-renewable generators to produce the difference between demand and the minimum renewable production across all k scenarios. This ensures that demand is always met, regardless of which scenario eventually occurs. We solve the UCP for all time steps $t = 1, \ldots, T$, but at this stage, we only compute the objective value for $t = 1$.

At time $t = 2$, we observe the actual renewable production at $t = 1$. Based on this observation, we eliminate any scenarios that do not match the realised production. This reduces the scenario set to only plausible scenarios. Once again, we conservatively take the minimum renewable production at $t = 2$ across the remaining scenarios, and schedule traditional generation accordingly. We then solve the UCP again for time steps $t = 2, \ldots, T$, and compute the objective value for $t = 2$.

This process is repeated for each subsequent t. Over time, uncertainty decreases as more scenarios are ruled out. Eventually, only one scenario remains — the true one — at which point the uncertainty is resolved. We then solve the UCP one final time for the remaining time periods using the fully known renewable production trajectory and compute the remaining part of the objective value. During this process we also update U_0^i and U_1^i to ensure the minimum up- and down-time requirements are satisfied every time the model is run.

One change is made to the previous model, namely the demand satisfaction constraints (28). For each iteration of the model, we define a new parameter $\hat{p}_t^{RE}$:= Minimum renewable production across the remaining scenarios at time t.

The new constraints are as follows:

$$d_t - \alpha \hat{p}_t^{RE} \leq \sum_{i \in \mathcal{N}} x_t^i \qquad \forall t \in \mathcal{M}, \tag{28}$$

Note that in our first iteration, $\mathcal{M} = \{1, ..., T\}$, in the second, however, it becomes $\mathcal{M} = \{2, ..., T\}$, etc., until our heuristic procedure either reaches time T or rules out the incorrect scenarios. The latter allows the model to optimise for the remaining periods according to the true renewable production levels. Note that this formulation allows for many scenarios. The more extensive the dataset, the more uncertain and realistic the model becomes.

For demonstration purposes, we use a synthetically created dataset, provided by OpenAI (2025), that contains multiple scenarios resembling the five given scenarios. To compare results with the previous model without uncertainty, we run this model at each scenario's corresponding optimal α given in Table 2. The results are presented below:

Table 3: Comparison of objective values for different scenarios for $\lambda = 10$ and optimal αs (Table 2)

Scenario	**Obj. Val. No Uncertainty**	**Obj. Val. Uncertainty**
Scenario 1	9,317,397.12	9,511,173.02
Scenario 2	7,694,021.42	7,741,510.48
Scenario 3	7,396,914.43	7,570,610.22
Scenario 4	8,097,460.06	8,284,129.07
Scenario 5	7,336,296.84	7,336,406.84

As Table 3 shows, the objective values for the model with uncertainty, are higher for every scenario. This was to be expected, as uncertainty leads to the need for estimations, which are never perfect. Note that the exact same trend is observed between optimal objective values and the volatility of renewable energy production. As we generated the dataset and degree of 'branching' arbitrairly, we cannot draw conclusion on absolute differences of the two models. Yet, we expect the objective value to increase as the size of our dataset, and hence uncertainty, increases.

4.5 Modelling Uncertainty in Renewable Production - Moving Towards Reality

In the initial version of our uncertainty model, we assumed that the actual renewable energy production matched a scenario in our dataset exactly, down to two decimal places. However, real-world data is rarely so precise. Noise, variation, and measurement errors are common, and expecting an exact match is too strict and unrealistic. To address this, we drop that assumption and introduce a tolerated range. Now, we consider a scenario in our dataset "close enough" to the actual outcome if its value falls within an acceptable difference at a given time t.

Removing this assumption allows for potential shortages when production fails to meet demand, as the minimum across scenarios may still overestimate actual output. However, with a sufficiently large scenario set, we expect to approximate the real scenario closely, keeping shortages minimal. To account for these, we assign a high penalty (θ) in the objective function, ensuring they are not overlooked.

We also introduce energy storage to handle excess production. Instead of curtailing surplus energy, we store it at a cost (λ).

To model this we add the following decision variables:

- sh_t := Shortage at time t
- sl_t := Energy in storage at the end of time period t
- es_t := Energy flowing into our storage during period t

- os_t := Energy flowing out of our storage during period t

The new objective function becomes:

$$\min \sum_{t \in \mathcal{M}} \left(\sum_{i \in \mathcal{N}} \left(x_t^i c_i^{var} + u_t^i c_i^{NL} + v_t^i c_i^{SU} \right) + \lambda sl_t + \theta sh_t \right) \tag{29}$$

We also add the following constraints to our previously stated uncertainty model:

$$sl_t = es_t \qquad t = 1, \tag{30}$$
$$sl_t = sl_{t-1} + es_t - os_t \qquad \forall t \in \{2, ..., T\}, \tag{31}$$
$$os_t \leq 0 \qquad t = 1, \tag{32}$$
$$os_t \leq sl_{t-1} \qquad \forall t \in \{2, ..., T\}, \tag{33}$$

We delete equation (23) from our model and adjust (28) into the following:

$$\sum_{i \in \mathcal{N}} x_t^i + \alpha \hat{p}_t^{RE} + os_t - es_t + sh_t = d_t \qquad \forall t \in \mathcal{M}, \tag{34}$$

Table 4: Total energy production costs for different $\theta - \lambda$ combinations

θ \ λ	**10**	**50**	**100**	**150**
10	3,732,854.07	9,019,799.54	15,628,481.37	22,237,163.21
50	7,294,397.22	13,216,578.24	19181747.45	26,565,719.92
250	7,348,767.34	13,764,563.31	20452197.32	27,098,866.79
500	7,349,517.34	13,762,426.65	20,451,947.32	27,098,866.79

Table 4 presents total costs across various θ and λ settings, showing how shortage penalties and storage costs impact the system. Notably, a θ of 50 is often sufficient to avoid shortages, as higher values like 250 or 500 yield little additional cost reduction. This suggests the system overproduces and stores energy to avoid penalties, underscoring the value of storage for reliability. However, higher λ increases costs, highlighting the expense of energy storage and the need for balance.

θ can also reflect political priorities. Some governments may tolerate occasional shortages to lower costs, while others favour reliability despite higher expenses. Thus, both storage technology and political decisions shape a cost-effective energy system.

We apply a 20% tolerance, interpretable as level of uncertainty, around observed renewable output, allowing 10% over- or under-estimation, as a proxy for uncertainty. Optimising this interval is important, given volatility varies over time. Time-sensitive weighting functions could improve accuracy. We recommend further research, from basic exponential functions to more advanced, creative implementations of kernel functions.

5 Conclusion

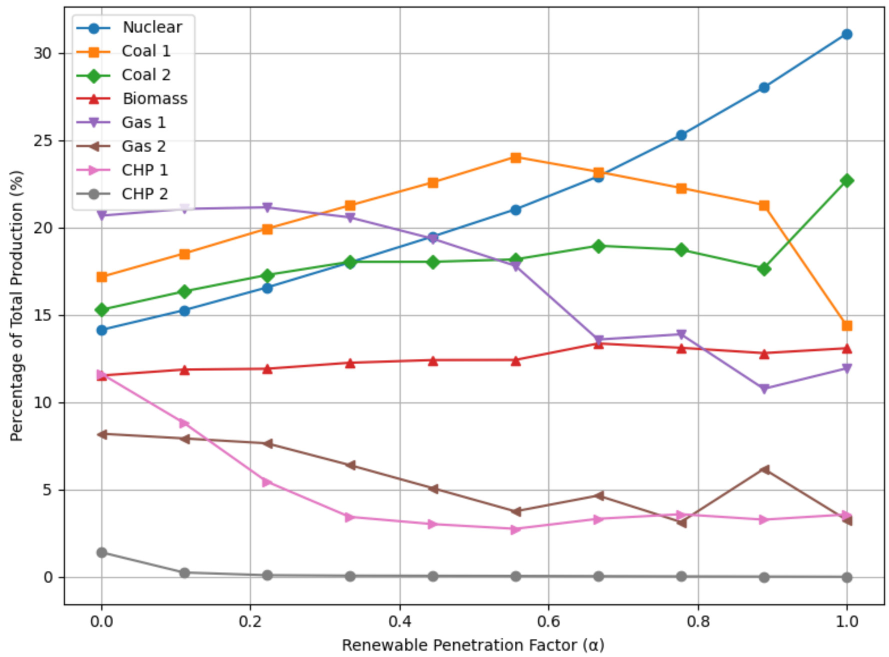
This paper examined the UCP aiming to minimise total energy production costs while meeting demand, focusing on integrating renewable energy. We began by formulating the UCP using integer linear programming, comparing two models based on Takriti et al. (2000) and Rajan & Takriti (2005). Both achieved the same objective values, but the latter was computationally more efficient due to fewer constraints and equality based turn-on variables, making it the preferred choice for further analysis.

We extended the model to incorporate renewable energy and assessed how penetration levels, excess production penalties, and weather-driven volatility impact production costs. With low penalties ($\lambda = 1$ or $\lambda = 10$), full renewable integration was optimal, as zero marginal costs outweighed excess production costs. However, high penalties ($\lambda = 100$) discouraged integration, with optimal α inversely related to cumulative excess renewable output, highlighting a trade-off between renewable usage and system reliability. Volatile weather conditions significantly raised production costs. Generator usage analysis revealed that renewables reduced reliance on expensive units like Gas and CHP, while flexible, low-cost generators such as Nuclear and Coal remained essential. These are insights that could guide future investment.

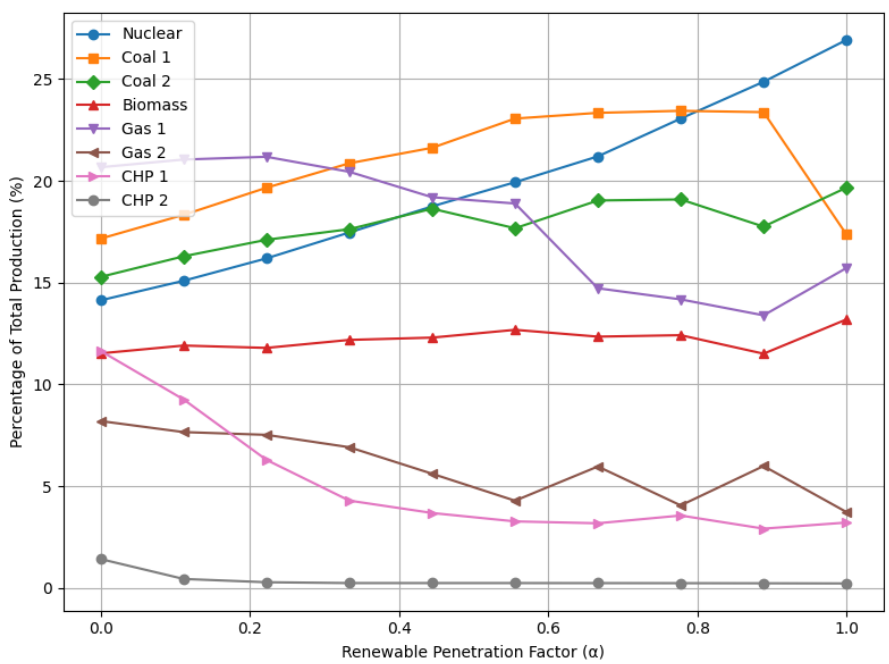
To manage renewable uncertainty, we proposed a dynamic scheduling approach that updates plans as weather forecasts improve. We discussed two policies: one prohibiting shortages and another allowing minimal shortfalls. As expected, modelling uncertainty led to higher objective values due to imperfect predictions. Even in deterministic settings, greater volatility increased costs. To move beyond the unrealistic assumption of perfect forecast accuracy, we adopted a $\pm 10\%$ tolerance for renewable production, allowing shortages but penalising them. The addition of energy storage reduced waste and helped limit shortages, even with modest penalties, reinforcing its role in cost minimisation.

However, our findings have limitations. The use of synthetic data may limit generalisability. Using real-world weather data over longer periods would improve accuracy. The 20% tolerance range lacks empirical justification and may cause overproduction or shortages if poorly set. Future research should explore more precise tolerance thresholds, potentially through cross-validation or weighting functions. Lastly, our assumption of perfect demand knowledge is unrealistic. Incorporating stochastic demand models based on probabilistic forecasts could offer a more robust framework for future analysis.

Appendix A



Appendix A.1: Proportion of total non-renewable production per plant for ten equidistant $\alpha \in [0, 1]$, $\lambda = 100$ in scenario 2



Appendix A.2: Proportion of total non-renewable production per plant for ten equidistant $\alpha \in [0, 1]$, $\lambda = 100$ in scenario 4

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