Stochastic Processes - Homework
Brownian Motion

Yuri F. Saporito

In what follows, $(B_t)_{t\geq 0}$ is a Brownian motion.

1. Show that $X_t = \frac{1}{c}B_{c^2t}$ is also a Brownian motion, for any constant c.
2. Let $(W_t)_{t\geq 0}$ be another Brownian motion independent of B. Prove that for any $\rho \in [-1, 1]$, the process $\rho B_t + \sqrt{1-\rho^2}W_t$ is also a Brownian motion.
3. Let $X_t = B_t - tB_1$, for $0 \leq t \leq 1$. This process is called *Brownian Bridge*. Compute the mean and covariance functions of X. What is the distribution of X_t?
4. Find $\mathbb{P}(B_r \leq B_s \leq B_t)$, for $0 < r < s < t$.
5. Compute $\mathbb{E}[B_r B_s B_t]$, for $0 < r < s < t$.
6. Let τ be an exponential r.v. with parameter λ and assume it is independent of B. What is the characteristic function of B_τ?