Unit I

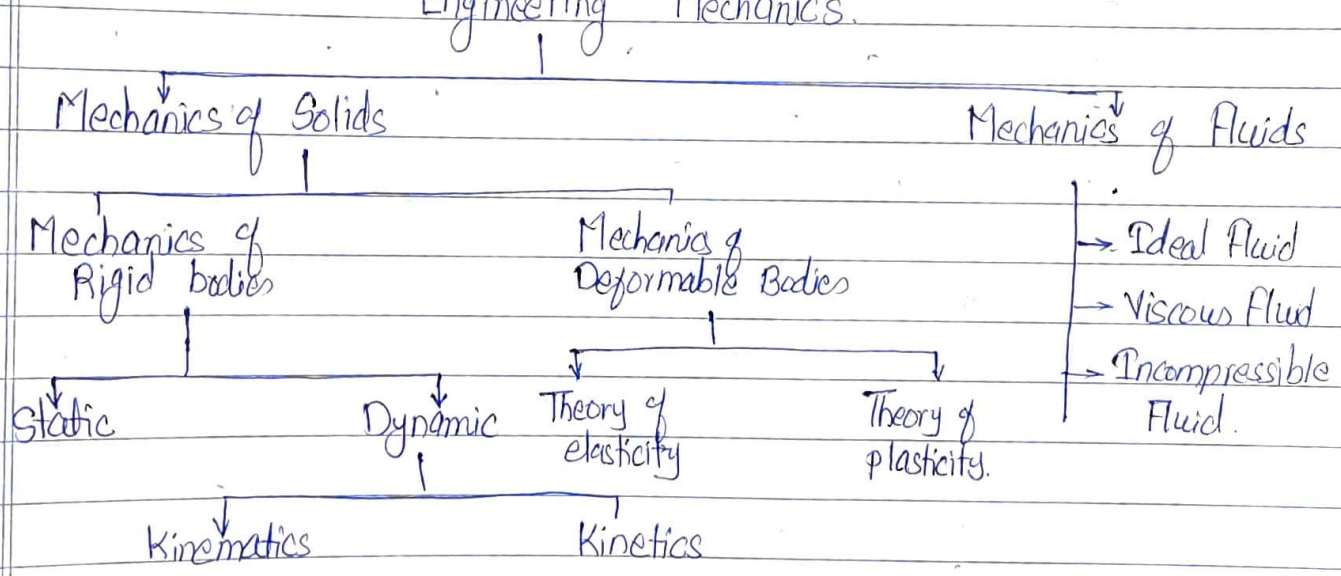
Resolution and Composition of Forces

Principle of statics, Force system, Resolution and composition of forces, Resultant of concurrent forces, Moment of a force, Varignons theorem, resultant of parallel force system, Couple, Equivalent force couple system, Resultant of parallel general force system.

Mechanics :

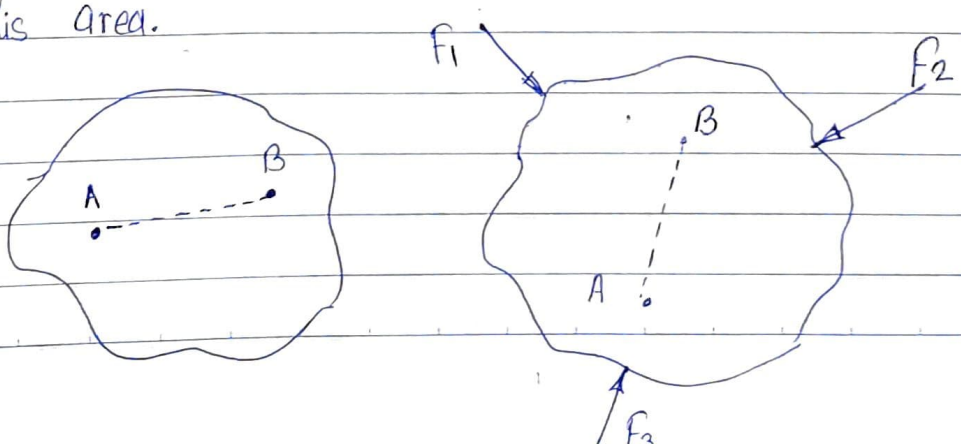
Mechanics can be defined as the physical science, which deals with the state of rest or motion of particles and bodies under the effect of the forces.

- Engineering mechanics is the core subject which deals with the application of laws of mechanics to the real life or field problems
- The most important contribution in the field of mechanics is developed by Sir Issac Newton who has accurately formulated the laws of motion including the gravitational law.
- Further the substantial contributions to the development of mechanics were also made by Da Vinci Varignon, D'Alemberts and others.
- The knowledge of mechanics is very essential for an engineer in planning, designing and contruction of various types of structures. hence the subject of mechanics lies at the core of all engineering and technology.



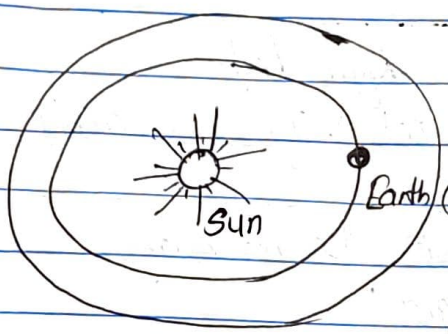
Depending upon the body to which the mechanics is applied, the engineering mechanics is classified as above:

- The solid mechanics is further classified as mechanics of rigid bodies and mechanics of deformable bodies.
- A body which does not deform under the action of forces is called a rigid body. The distance between any two points in rigid body remains fixed.
- The mechanics of rigid bodies dealing with the body at rest is termed as statics and that dealing with the bodies in motion is called as Dynamics.
- The dynamics dealing with the problem without reffering to the forces causing the motion of the body is termed as Kinematics and if it deals with the forces causing the motion of body is called Kinetics.
- If the internal stresses developed in the body are to be studied, the deformation of the body should be considered, that field of mechanics is called Mechanics of deformable bodies / Strength of Materials / Solid Mechanics. This field may be further divided into Theory of Elasticity and Theory of plasticity.
- Liquid and gases deform continuosly with application of very small shear forces. such materials are called fluids. The mechanics dealing with the behaviour of such materials is called Fluid Mechanics.
- Mechanics of ideal fluids, mechanics of viscous fluid and mechanics of incompressible fluids are further classification in this area.



Idealisation In Mechanics

Dr. Chirag N. Patel.



Planetary System

Revolution

Earth

Rotation.

Case I: Earth is moving around the sun

∴ Earth is particle with respect to sun.

Case II: Earth is moving itself i.e. all particle will rotate around the centre.

∴ Earth is rigid body.

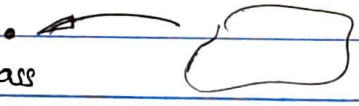
Mathematical description of problem in real engineering can become very complex and difficult

Assumption:

Assumptions are helpful or useful in making practical problems more comprehensible and simple and resulting into a very accurate solution to complex problem

① Particle :-

- Negligible mass
- Size and shape of body - ingnored
- particle is considered as a point mass



- Vehicles

Though it is having some mass, size and shape we are treating it as particle.

It's size & shape doesn't matter if we are considering velocity & acceleration.

- Linear motion takes place as a single unit without rotation about their mass

(2) Rigid Body :-

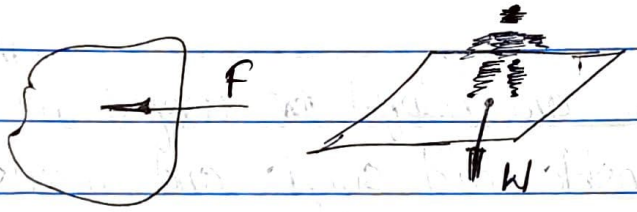
- Body can be assumed as Rigid body.
  - = Dimension of body will not change if we apply load.
  - = Distance between particles does not change

"No body is perfectly rigid."

(3) Continuum :-

- mass of body is dispersed.
  - across its volume
- voids are considered to be zero.

(4) Point (Concentrated) force :-



* Laws of Mechanics.

The following are the fundamental laws of mechanics:

1. Newton's first law
2. Newton's second law
3. Newton's third law
4. Newton's law of gravitation
5. Law of transmissibility of forces
6. Parallelogram law of forces.

1. Newton's First Law of motion:

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion.)

It states that every body continues (keep doing) in its state of rest or of uniform motion in a straight line unless it is compelled (force to do something) by an external agency acting on it. This leads to the defination of force as the external agency which changes or tends to change the change state of rest or uniform linear motion of the body.

It also states the law of inertia.

2. Newton's Second Law of motion:

If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant & in the direction of this resultant force.

It states that the rate of change of momentum of a body is directly proportional to the impressed force & it takes place in the direction of the force acting on it. Thus according to this law.

Thus, Force $\propto$ Rate of change of momentum

But momentum = mass x velocity.

As mass do not change,

Force $\propto$ mass x rate of change of velocity

$F \propto$ mass x acceleration

$F \propto ma$

3. Newton's Third Law of Motion:-

It states that, "for every action there is equal & opposite reaction."

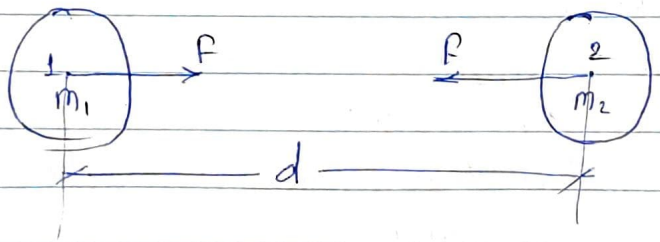
The force of action & reaction between bodies in contact have same magnitude, same line of action & opposite in sense. It must be memorised that reaction coill exerts only when there is action. Hence this law is also called as 'Law of reaction.'

4. Newton's Law of Gravitation:-

Everybody attracts the other body. The force of attraction between any two bodies is directly proportional to their masses & inversely proportional to the square of the distance between them. According to this law the force of attraction between the bodies of mass m_1 & mass m_2 at a distance d as shown in fig.

$$F = G. \frac{m_1 . m_2}{d^2}$$

where G is the constant of proportionality & is known as constant of gravitation.



5. Law of Transmissibility of force.

In a particular case of great importance is that of the attraction of the earth on a particle located on its surface. The force F exerted by the earth on the particle is then defined as the weight W of the particle. Taking M equal to the mass of the earth, m equal to the mass of particle, & r equal to the Radius R of the earth, & introducing the constant,

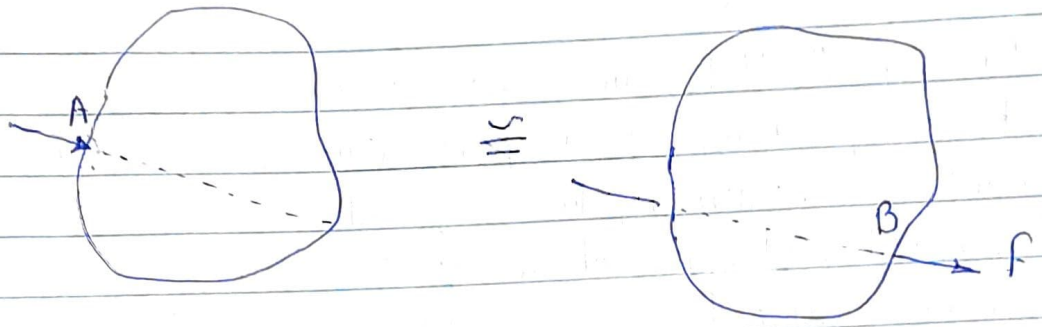
$$g = \frac{GM}{R^2} = 9.81\ m/s^2$$

$$G = 6.673 \times 10^{-11}\ Nm^2/kg^2$$

5. Law of Transmissibility of Force:- (2m)

The principle of transmissibility states, "The points of application of a force can be transmitted anywhere along its line of action. OR

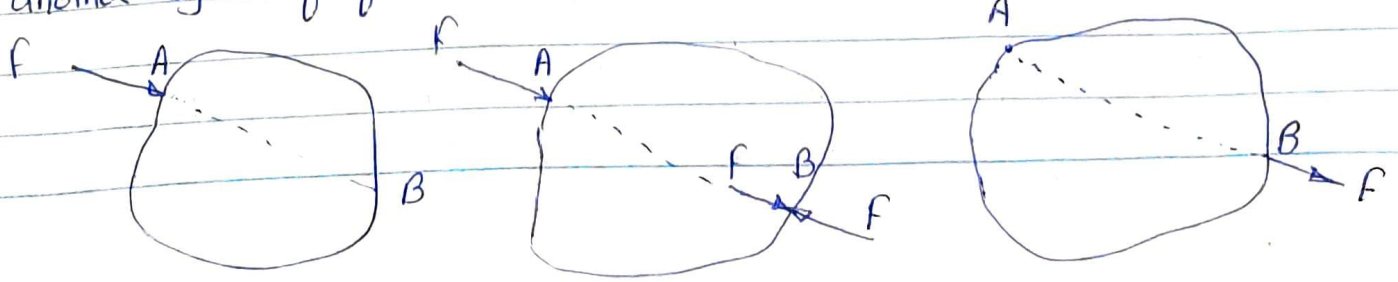
According to this law the state of rest or motion of the rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude & direction but acting anywhere on the body along the line of action of the replaced force.



Let F be the force acting on a rigid body at a point A as shown in figure. According to the law of transmissibility of force, this force has the same effect on the state of body as the force F applied at point B.

[This law is only applicable to rigid bodies it can not be used in the subject of solid mechanics or any deformable body]

The law of transmissibility of forces can be proved using the law of superposition, which can be stated as the action of a given system of forces on a rigid body is not changed by adding or subtracting another system of forces in equilibrium.

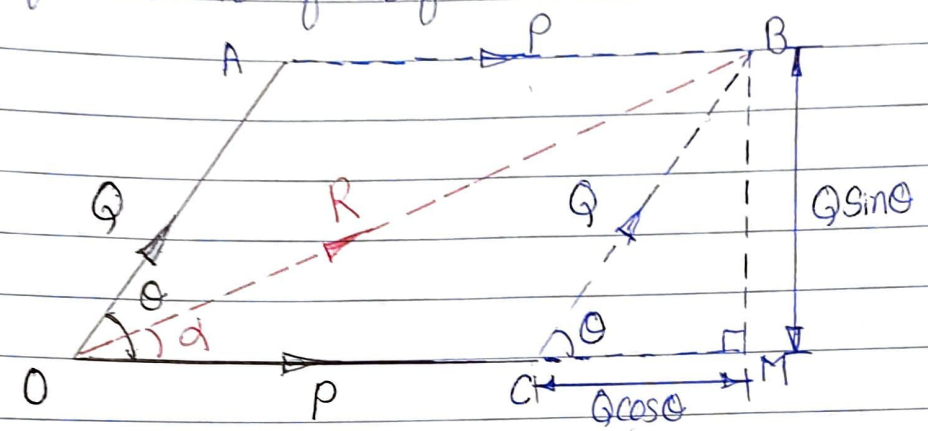


Consider the rigid body shown in fig. It is subjected to a force F at A. Another point B is on the line of action of the force. From the law of superposition it is obvious that if two equal & opposite forces of magnitude f are applied at B along the line of action of given force F, the effect of given force on the body is not altered. Force f at A & opposite force F at B form a system of forces in equilibrium. If these two forces are subtracted from the system, the resulting system is as shown in fig. (c)

Looking at the system of forces in fig. (a) & (c) we can conclude the law of transmissibility of forces is proved.

06. **Law of Parallelogram of forces:**

Law of parallelgram states that "If two forces simultaneously acting at a point can be represented in magnitude and direction by two adjacent sides of parallelogram, the diagonal will represent resultant in magnitude & direction but passing through the point of intersection of two forces."



Consider two forces P and Q acting at a point represented by two sides OA and OC of a parallelogram OABC.

Let θ be the angle between two forces P and Q.

α be the angle between P & R.

Now drop a perpendicular BM & produce CM.

In $\triangle CMB$, we have

$$BM = Q\sin\theta$$

$$CM = Q\cos\theta$$

Magnitude of R:

In $\triangle OMB$,

$$OB^2 = OM^2 + BM^2$$

$$= (OC + CM)^2 + Q^2\sin^2\theta$$

$$= (P + Q\cos\theta)^2 + Q^2\sin^2\theta$$

$$= P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

$$= P^2 + 2PQ\cos\theta + Q^2(\sin^2\theta + \cos^2\theta)$$

$$R^2 = P^2 + 2PQ\cos\theta + Q^2$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad \text{---------- Magnitude}$$

Direction of Resultant:

In $\triangle OMB$

$$\tan\alpha = \frac{BM}{OM} = \frac{BM}{OC + CM}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} \quad \text{---------- Direction}$$

* Particular Case:-

① When two forces P and Q are perpendicular we have $\theta = 90°$

$$\therefore R = \sqrt{P^2 + Q^2}$$

$$\tan\alpha = \frac{Q}{P}$$

② When $\theta = 0$ ---- $R_{max} = P + Q$

③ When $\theta = 180$ $R_{min} = P - Q$

④ If $P = Q = R$ then $\theta = 120°$

⑤ For two equal forces i.e. $P = Q \Rightarrow R = 2P\cos\theta/2$

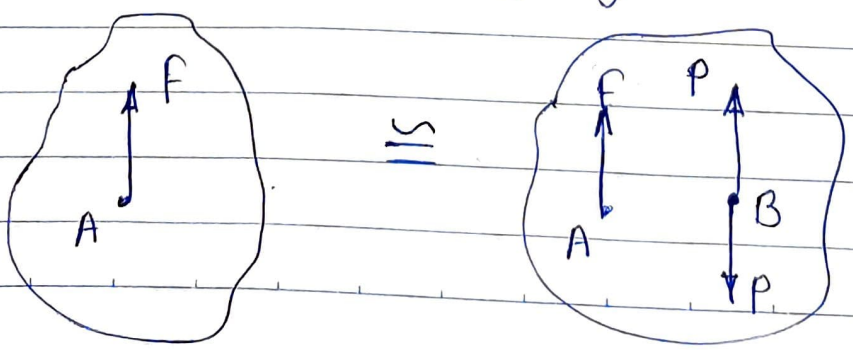
⑥ For two equal forces & $\theta = 90° \Rightarrow R = \sqrt{2}P$

⑦ Law of parallelogram can be proved experimentally not mathematically.

⑧ The mathematical statement of parallelogram law is Cosine Rule.

⑦ Principle of Superposition:

The principle of Superposition states that "The effect of a force on a body remains (unaltered / unchanged / same) if we add or subtract any system which is in equilibrium".



* Defination of force:-

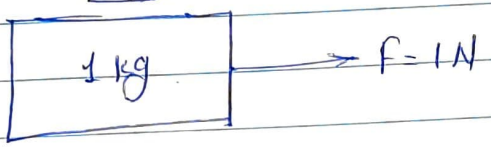
Force is an external agent which changes or tends to change the state of motion or rest of the body acted upon it. Force is a vector quantity & SI unit is Newton. It is also represents in KN, MN, GN, TN etc.

$1\ KN = 1 \times 10^3\ N$, $1\ MN = 1 \times 10^6\ N$

$1\ GN = 1 \times 10^9\ N$, $1\ TN = 1 \times 10^{12}\ N$

* Defination of 1 Newton force:-

When a force acting on a body of mass 1 kg produces an acceleration of $1\ m/s^2$ then the force acting on a body is known as one Newton.



$a = 1\ m/s^2$

Force = 1 kg × 1 m/s²

Force = 1 Newton (N)

In SI system, the unit of force is newton & abbreviated by 'N'.

$1\ kg = 9.81\ N$ $\quad$ $1\ N = 0.101\ kg$

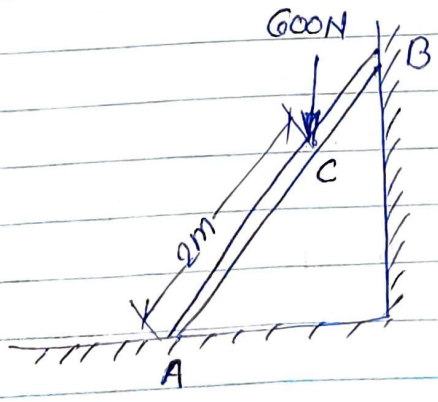
* Characteristics of force:

In order to identify a force completely, we must know the following particulars about it or a force is completely defined only when the following four characteristics are specified.

(a) Magnitude
(b) Direction
(c) Nature or sense
(d) Point of Application.

(Some)

(a)



In figure AB is a ladder kept against a wall. At point C, a person weighing 600N is standing. The force applied by the person on the ladder has following character

(a) Magnitude: This represents the value of force i.e 600N
(b) Direction: Since force is a vector quantity it must have magnitude as well as direction. It is represented by line of action and the angle it forms with some fixed axis.
Here the direction is vertically downward.
(c) Nature or sense: The nature of a force is represented by an arrowhead. Generally it is termed as push or pull. Any force which is acting towards the point is called push & any force which is acting away from the point is called pull.

Push (Comp) Pull (Tension)

Here the nature of the force is push / compression
(d) Point of Application: It is the location of a point on a body where force is acting.

* Effects Of A Force:-

A force when acting on a particle or a body may produce internal or external effects. Following are some effects of force or force system on particle or body.

1. It may change the state of particle or body.
2. It may produce deformation in a non-rigid body.
3. It may produce rotational effect in a body.
4. It may produce internal stresses in a body
5. It may keep the system in stable state (equilibrium).

* System Of Forces:-

When a single agency is acting on a particle it is called as force but when number of forces simultaneously acting the system so formed is called as 'System of forces.'

There are mainly seven types of system of forces

(1) Co-planer forces (2) Non-coplaner forces
(3) Collinear forces (4) Non-collinear forces
(5) Concurrent forces (6) Non-concurrent forces

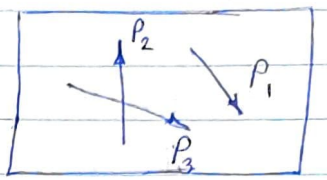
⑦ Parallel forces:

ⓐ Like parallel forces

ⓑ Unlike parallel forces.

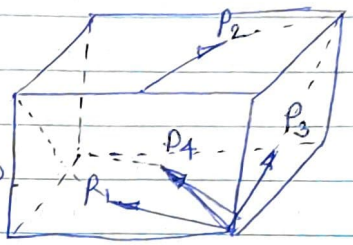
① Co planer forces:

The forces which are acting in the same plane are known as co-planer forces or co-planer force system.



② Non-coplaner forces.

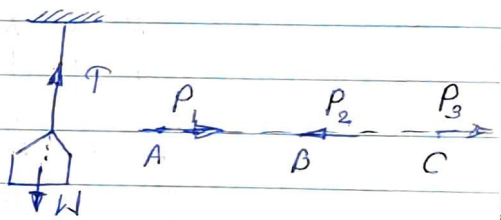
The force system in which the forces acting in different plane is called as Non-coplaner force system. When a force changes its plane it has to pass through space so non-coplaner forces are also called as space forces or spatial force system.



③ Collinear forces

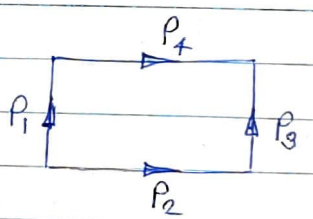
The forces which are acting along the same straight line are called as 'Collinear forces.'

Ex: Tug of war, Forces on rope.

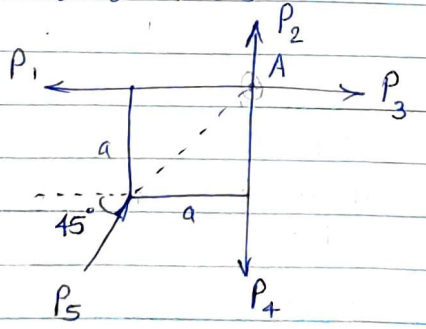


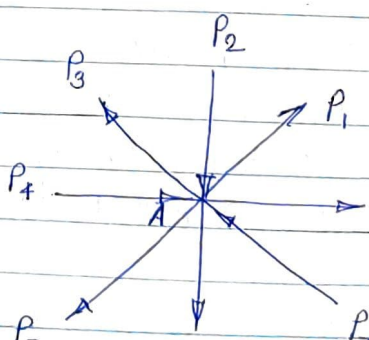
④ Non-Collinear forces:

The forces which are not acting along a straight line are called as Non-Collinear forces.



⑤ Concurrent forces

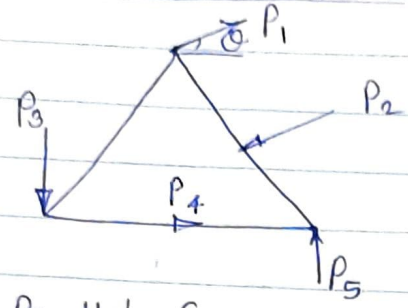




- Tripod for dumpy
- Clock
- Forces on a rod resting against a wall

All the forces which are passing through a common points are called concurrent forces

⑥ Non-concurrent forces:-



The forces which are not passing through a common point are called as non-concurrent forces
forces on moving bus.

⑦ Parallel forces:

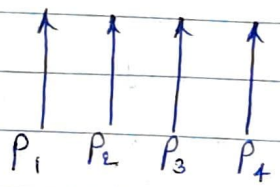
The forces whose lines of action are parallel to each other are known as parallel forces.
There are two types of parallel force system.

ⓐ Like parallel forces:

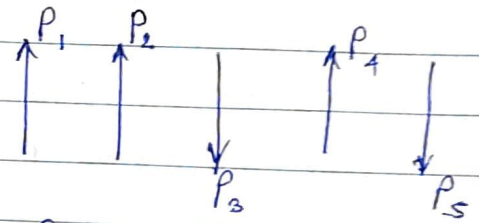
The forces which are parallel to each other & having same direction are called as Like Parallel forces.

ⓑ Unlike parallel forces:

The forces which are parallel to each other & having different directions are called as Unlike parallel forces.



ⓐ Like Parallel forces



ⓑ Unlike Parallel forces.

* Graphical Representation of force:-

Force may be represented in the following two ways
① Vector representation
② Bow's notation.

① Vector Representation of force

A force can be represented graphically by drawing a straight line parallel to the line of action of force. The length of a line to a suitable scale gives the magnitude. direction or sense of a line represents the arrow on a line

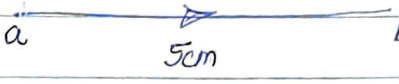
ex. Let the force AB of magnitude 500N acting in the direction AB as shown

Magnitude of force = length of ab vector x scale

scale 1cm = 100N

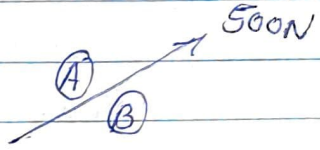
= 5x100

= 500N



This method is limited in use but another, Bow's notation method is commonly used in practice

(2) Bow's Notation.



This method is used for designing a force by writing two capital letters or any numerical numbers, which are marked on each side of a force as shown in fig. where force 500N is represented by AB hence the 500N force may called as force AB.

* Composition & Resolution of Forces:

(a) Composition of Forces:

Composition or compounding is the procedure to find out single resultant force of a force system.

(b) Resolution of Forces:

Resolution is the procedure of spliting up a single force into number of components without changing the effect of the same.

Generally a force to be resolved in to two perpendicular or non-perpendicular components but a force can be resolved into number of components

* Resultant.

→ Graphical method
- Triangle law
- Parallelogram law.
- Polygon law

→ Numerical Method
- Triangle law
- Parallelogram law
- ~~Non-ler~~ Resolution along non-ler directions
- ~~Ler Resolution~~ Resultant of more than three forces.

* Define Resultant & Equilibrant :-

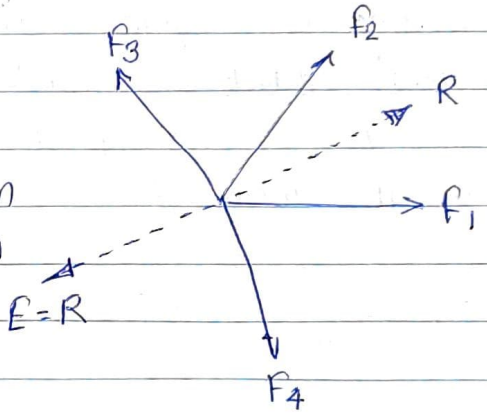
- **Resultant:**

It is a single force which produces the same effect that as produced by number of forces when acting together. The combined effect of all forces is represented by a single force which is known as 'Resultant.' It is denoted by 'R'

- **Equilibrant:**

It is a single force which when acting with all other forces keeps the body at rest or in equilibrium. It is denoted by E.

* The resultant & equilibrant are equal in magnitude but opposite in direction.

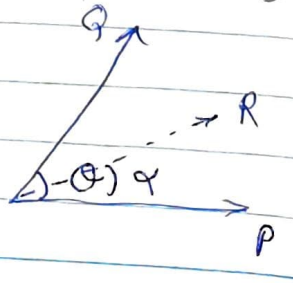


① * Methods of Composition to find R :-

There are different methods to find out resultant of different force system.

① Resultant of two Concurrent Forces :-

When two forces simultaneously acting at a point e in a plane then their resultant can be found out by using Law of parallelogram of forces.



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

Fixed vector - Specified line of action & specified pt. of application ex. moment
Sliding - Specific line of action but no specified pt. of application ex. Principle of transmissibility
Free vector - no specific line of action & no specified pt. of application ex. couple.

② Resultant of Two or More Forces:-

When two or more coplaner concurrent or non-concurrent forces acting on a body the resultant can be found out by using resolution procedure.

Magnitude of resultant $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

& direction $\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$

where ΣF_x = Algebric sum of all x-components (or x component of resultant)

ΣF_y = Algebric sum of all y-component (or y component of resultant)

θ = Angle of 'R' with x-axis.

② Resolution of a force:

There are two methods of resolving a force

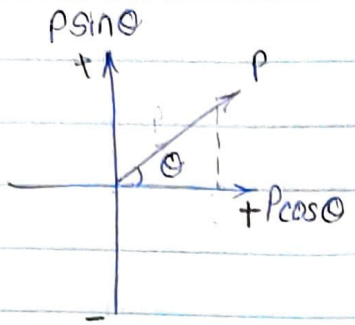
① Orthogonal (Perpendicular) resolution.

② Non-perpendicular resolution.

① Orthogonal or Perpendicular Resolution:

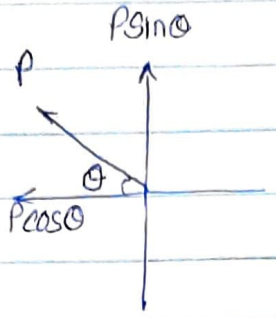
This is the general method of splitting up a single force into two perpendicular components acting along x-axis and y-axis on any two perpendicular axis.

Following examples will help to simplify the resolution of a force into orthogonal components:



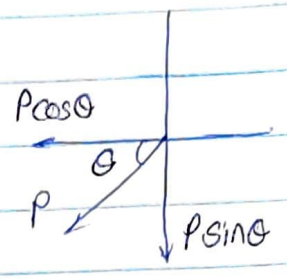
$P_x = +P\cos\theta$

$P_y = P\sin\theta$



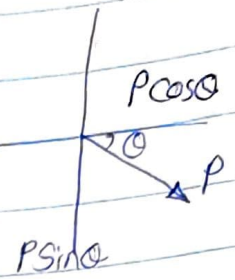
$P_x = -P\cos\theta$

$P_y = P\sin\theta$



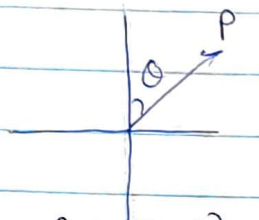
$P_x = -P\cos\theta$

$P_y = -P\sin\theta$



$P_x = P\cos\theta$

$P_y = -P\sin\theta$

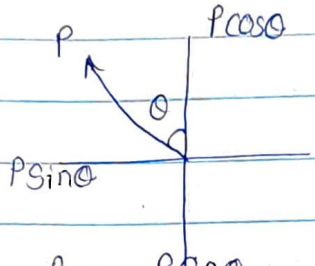


$P_x = P.\cos(90-\theta)$

$= P\sin\theta$

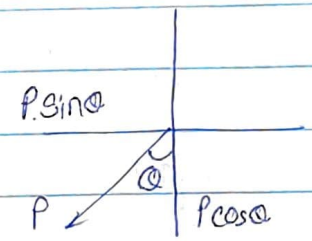
$P_y = P.\sin(90-\theta)$

$= P.\cos\theta$



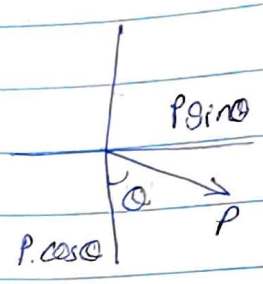
$P_x = -P\sin\theta$

$P_y = -P\cos\theta$



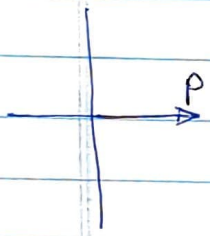
$P_x = -P.\sin\theta$

$P_y = -P.\cos\theta$



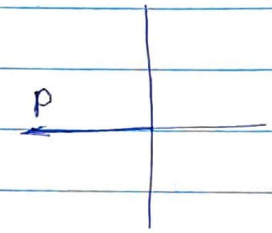
$P_x = P\sin\theta$

$P_y = -P.\cos\theta$



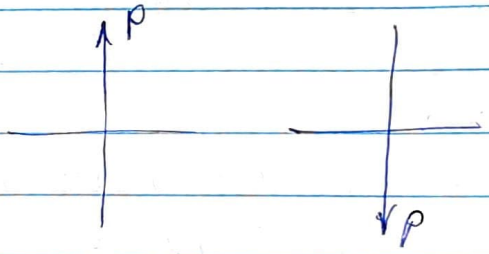
$P_x = P$

$P_y = 0$



$P_x = -P$

$P_y = 0$

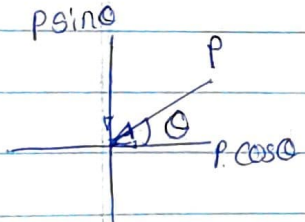


$P_x = 0$

$P_y = +P$

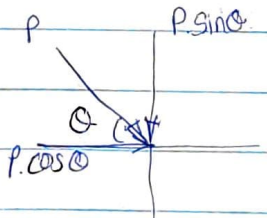
$P_x = 0$

$P_y = -P$



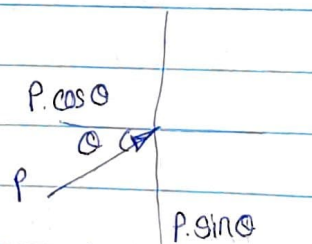
$P_x = -P.\cos\theta$

$P_y = -P.\sin\theta$



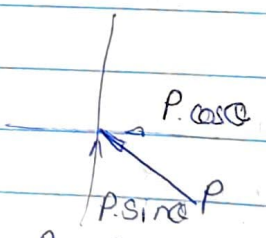
$P_x = P\cos\theta$

$P_y = -P.\sin\theta$



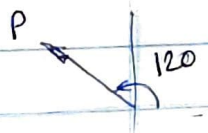
$P_x = P.\cos\theta$

$P_y = P.\sin\theta$



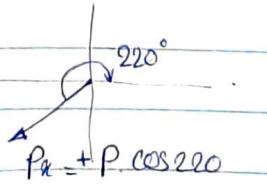
$P_x = -P.\cos\theta$

$P_y = +P.\sin\theta$



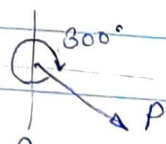
$P_x = P.\cos 120$

$P_y = P\sin 120$



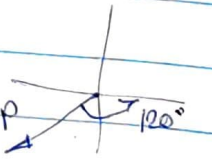
$P_x = +P.\cos 220$

$P_y = +P.\sin 220$



$P_x = P.\cos 300°$

$P_y = P.\sin 300°$

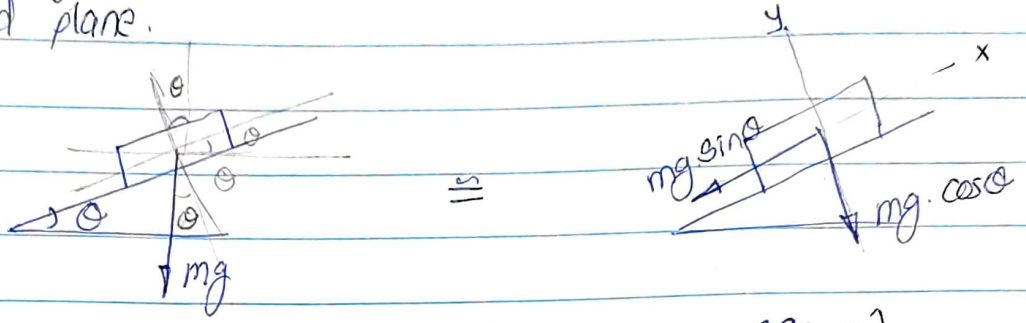


$P_x = P.\cos(-120°)$

$P_y = P.\sin(-120°)$

Special Cases of Resolution:-

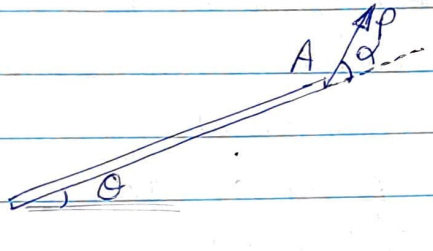
a] Selecting x-axes along the plane & y-axis ⊥er to plane we can resolve weight 'mg' into two components when a body is placed on inclined plane.



① Component along the plane = $mg.\cos(90-\theta)$
$= mg.\sin\theta$

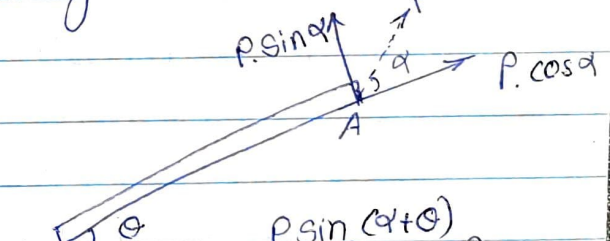
② Component ⊥er to the plane = $mg.\sin(90-\theta)$
$= mg.\cos\theta$

b]

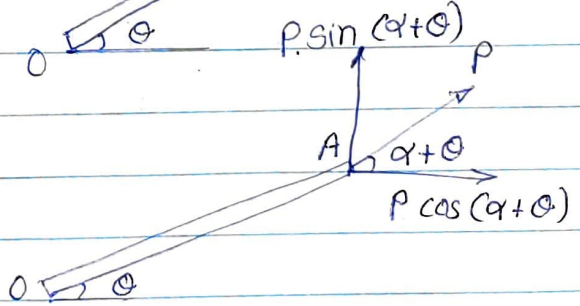


Here force 'P' can be resolved in following two manners.

① Along & perpendicular to bar OA.



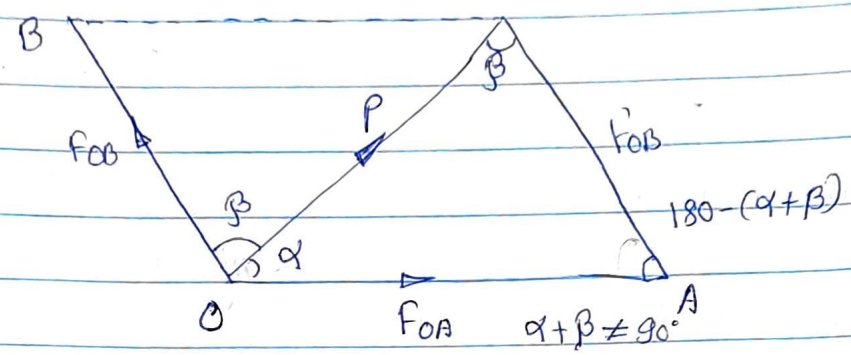
② Horizontal & vertical components.



② Non Perpendicular Components:-

When a force is required to be resolved in to two directions which are not perpendicular to each other the resolution is called Non-perpendicular resolution.

Following is the simple path to resolve a force into two non-perpendicular directions.



Step 1: Construct a parallelogram by keeping origional given force (R) along the diagonal & two components along two adjacent sides of parallelogram (passing through same point.)

Step 2: Find out 3 angles of any one triangle

Step 3: Apply sine rule in that triangle

For example, a force is to be required to resolve along directions OA & OB as shown in figure.

By Sine rule in shaded triangle,

$$\frac{F_{OB}}{\sin\alpha} = \frac{F_{OA}}{\sin\beta} = \frac{P}{\sin[180-(\alpha+\beta)]}$$

$$\therefore \frac{F_{OB}}{\sin\alpha} = \frac{F_{OA}}{\sin\beta} = \frac{P}{\sin(\alpha+\beta)}$$

$$\therefore F_{OA} = \frac{P.\sin\beta}{\sin(\alpha+\beta)} \quad \text{.... component along OA}$$

$$F_{OB} = \frac{P.\sin\alpha}{\sin(\alpha+\beta)} \quad \text{.... component along OB}$$

* Graphical Method Of Finding the Resultant :-

Graphical method is the process of finding out the resultant force by composition of forces.

The composition or compounding of forces by graphical method is possible by the following methods:

① Parallelogram law of forces:

When the two forces acting simultaneously on a rigid body at a point in a plane to be represented in magnitude & direction by the two adjacent sides of the parallelogram, then their represented in magnitude & direction by the diagonal of the parallelogram passing through a point of intersection of the sides of the parallelogram representing its given forces.

Limitation of Law of parallelogram.

① The magnitude & direction of resultant force can be calculated only when two forces are acting in a system of forces.

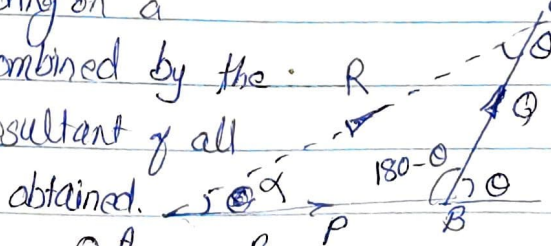
② These two forces must be concurrent & coplaner.

② Triangle Law of forces:

If two forces acting simultaneously are acting at a point on a rigid body are represented in magnitude & direction by the two adjacent side of triangle taken in order, then their resultant is represented by the closing side of the triangle taken in the opposite order.

The line AC represents magnitude of resultant & its inclination gives the direction of resultant as shown in fig. $\cos\alpha = \frac{R^2+P^2-Q^2}{2PR}$

If more than two forces are acting on a body. two forces at a time can be combined by the triangle law of forces & finally the resultant of all the forces acting on the body can be obtained.

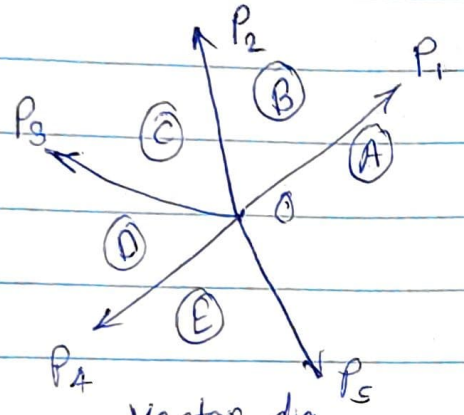


$\cos\alpha = \frac{(\text{Add of adj side})^2 - (\text{Opposite Side})^2}{2 \cdot \text{Adj. side}}$

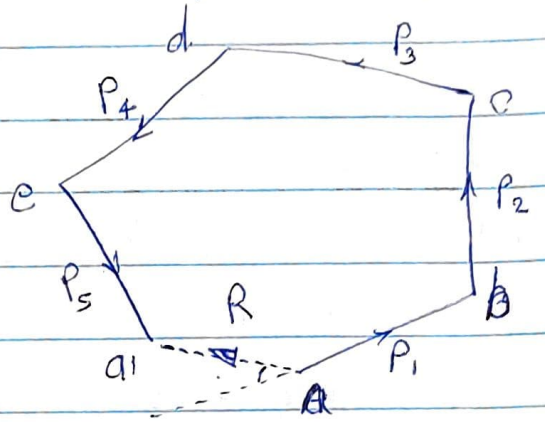
$\frac{P}{\sin(\theta-\alpha)} = \frac{Q}{\sin\alpha} = \frac{R}{\sin(180-\theta)}$

③ Polygon Law of Forces:

Polygon law of forces state that "If a number of concurrent coplaner forces acting simultaneously on a rigid body, are represented in magnitude & direction by the sides of the polygon, taken in order, then the resultant is represented in magnitude & direction by the closing side of the polygon, taken in opposite order."



Vector dia.



Space dia.

Consider the forces having magnitudes P_1, P_2, P_3, P_4 & P_5 are acting at a point as shown in fig.

Let Origin of polygon of force diagram be O' then give the Bow's Notation to the vector diagram. Draw straight line AB parallel & proportional in magnitude to the force P_1. From point B, draw straight line BC parallel & proportional in magnitude to the force P_2. In the same manner, draw straight lines CD, DE & EA, which are parallel & proportional to the forces P_3, P_4 & P_5 respectively. Then the resultant is given by the straight line aa' & its direction is the inclination made from a towards a'.