19/12/22.

Equilibrium.

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* **Equilibrium of Beam :-**

Beam is horizontal structural membare which transfers a load horizontally.

Beam is said to be in equilibrium if resultant force acting on it is zero.

Hence for equilibrium of beam

$\Sigma f_x = 0$

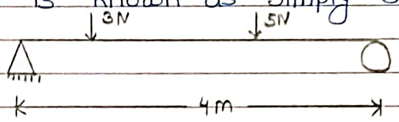
$\Sigma f_y = 0$

$\Sigma M_{@pt.} = 0$

* **Types of beam :-**

① Simply supported beam (SS beam) :-

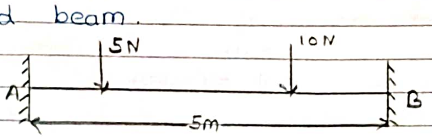
The beam which is supported with hinge at one end & Rollar at other end is known as simply supported beam.



dist. betn two supports is known as span of the beam or length of the beam.

② Fixed beam :-

The beam supported with fixed support at both ends is known as fixed beam.



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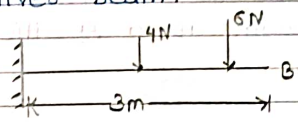
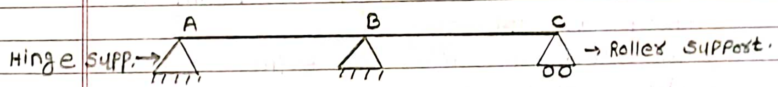
③ Cantiliver beam :-

The beam which one end is fixed & other is hanging is known as cantiliver beam.



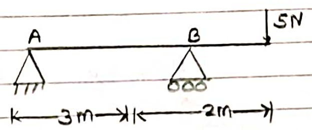
④ Countinous beam :-

The beam (larger span) supported with more than two supports is known as countinuous beam.



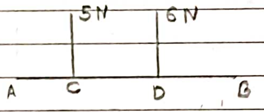
⑤ Overhanging beam :-

If the part of the beam is extended beyond the support then the beam is known as overhanging beam.



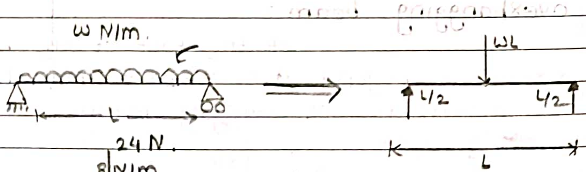
* Types of load :-

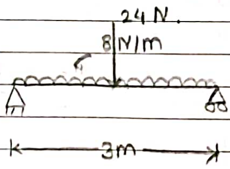
① point load :-
This are the external forces (Load) acting on a beam at a specified points.



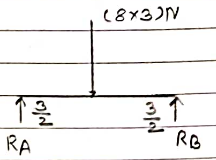
load of 5N acting on Pt. C & 6N on Pt. D

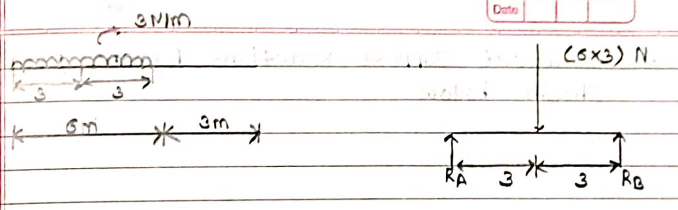
② uniformly distributed load :- (UDL)
The load which is uniformly distributed over length of beam is known as UDL.
expressed in N/m & kN/m.
while solving numericals based on UDL it must be converted in point load

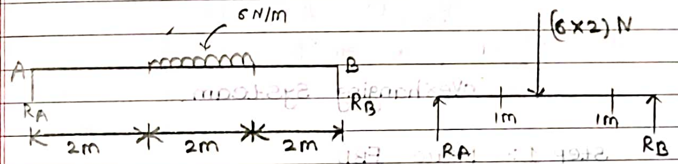




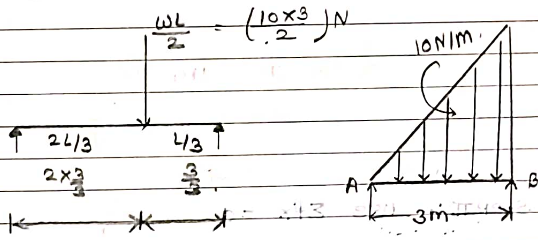
8 N × 3m = 24 N.

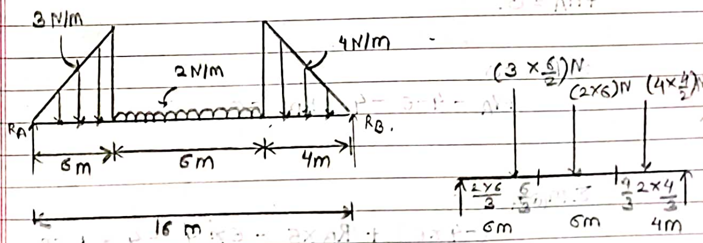


②


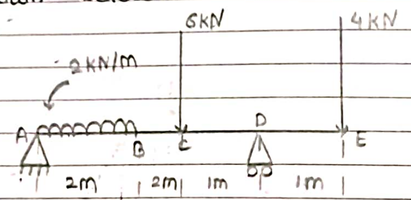
③


③ Triangular load :- UVL (uniformly Varying Load).



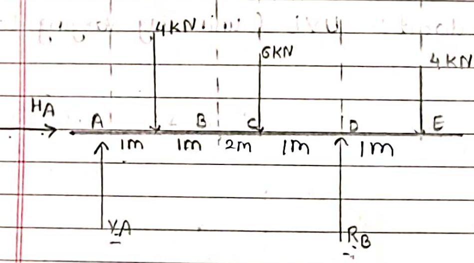


Q Find out support reactions for the Beam shown below.



overhanging systeam.

Step 1 :- Draw FBD.



StepII :- use $\Sigma f_x = 0$

$+H_A = 0.$

$\Sigma f_y = 0$

$+V_A - 4 - 6 - 4 + R_B = 0$

$\Sigma m@A = 0$

$(-4 \times 6) + R_B \times 5 - 6 \times 4 - 4 \times 1 = 0$

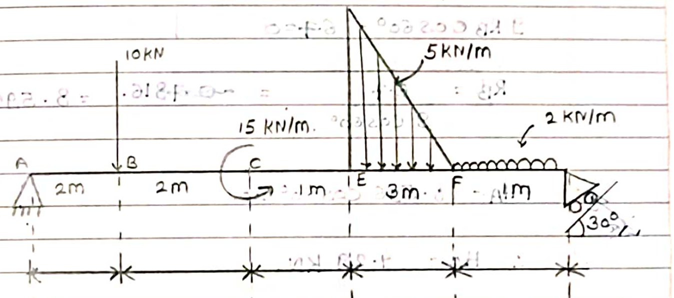
$-24 + 5R_B - 24 - 4 = 0$

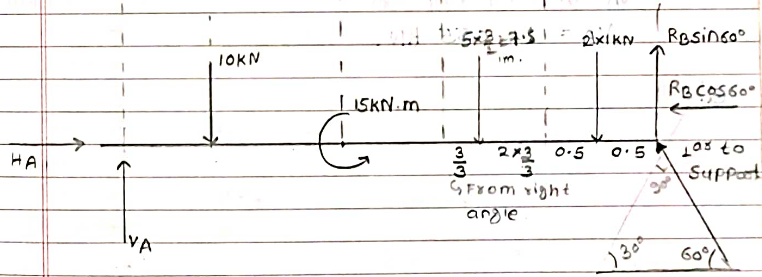
$5R_B = 52 \qquad R_B = \frac{52}{5} = 10.4$

$V_A - 4 - 6 - 4 + 10.4 = 0$

$V_A = 3.6.$

Q.





$\Sigma f_x = 0$

$+H_A - R_B \cos 60° = 0 \quad -①$

$\Sigma f_y = 0$

$+V_A - 10 - 7.5 - 2 + R_B \sin 60° = 0 \quad -②$

$\Sigma M@A = 0$

$9 \times R_B \cos 60^\circ + (-2 \times 8.5) - 7.5 \times 6 + 15 - 10 \times 2 = 0$

$9 R_B \cos 60^\circ - 67 = 0$

$R_B = \dfrac{6.7}{9 \cos 60^\circ} = \cancel{0.7816} = 8.596$

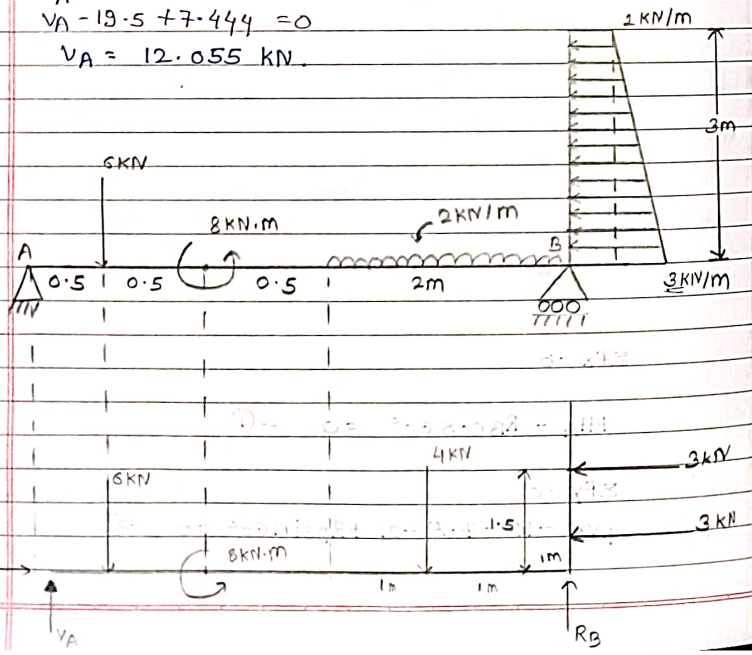
$+H_A - 8.596 \cos 60^\circ = 0$

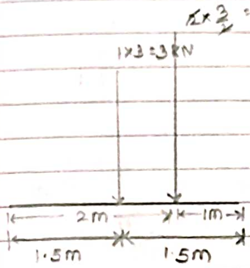
$\therefore H_A = 4.29$ kN.

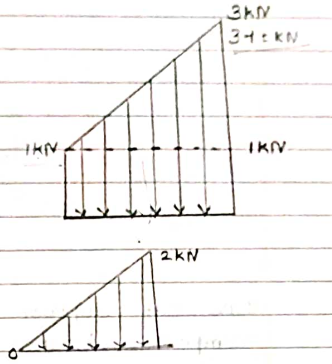
$V_A - 10 - 7.5 - 2 + 8.596 \sin 60^\circ = 0$

$V_A - 19.5 + 7.444 = 0$

$V_A = 12.055$ kN.







$\Sigma fx = 0$

$+H_A - 3 - 3 = 0$ $\quad H_A = 6$

$\Sigma fy = 0$

$V_A - 6 - 4 + R_B = 0$

$\Sigma M@A = 0$

$-6 \times 0.5 + 8 - 4 \times 2.5 + 3 + 3(1.5) + R_B \times 3.5 = 0$

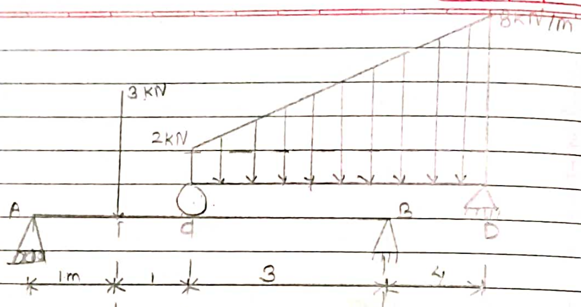
$2.5 + R_B 3.5 = 0$

$R_B = \dfrac{-2.5}{3.5} = -0.714$ kN. our assumption wrong.

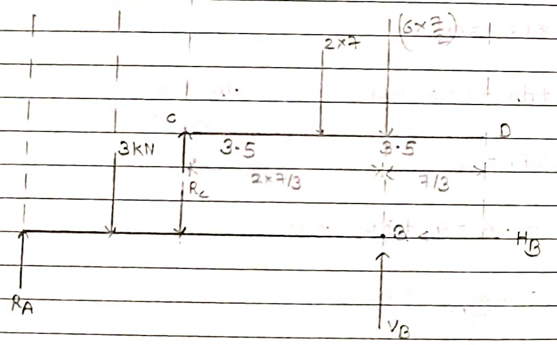
↓ downword. (not upword) (sign or direction).

$V_A - 6 - 4 + (-0.714) = 0$

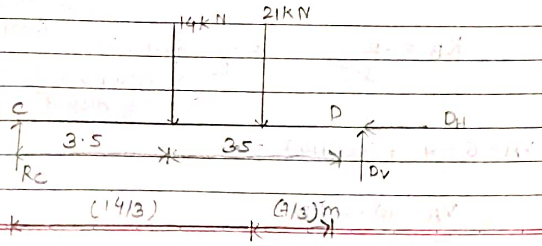
$V_A = 10.71$ kN.



Find out support reactions for pt. ABCD for compound beam shown above.



There are 4 unknowns on beam AB So we consider beam CD.



$\Sigma f x = 0$

$-D_H = 0$

$D_H = 0.$

$\Sigma f y = 0$

$+R_C - 14 - 21 + D_V = 0$

$m@c = 0$

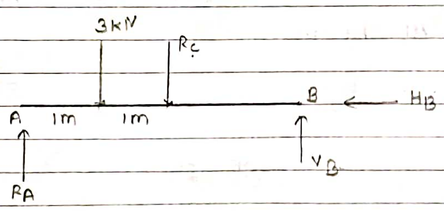
$+D_V \times 7 - 21 \times 4.67 - 14 \times 3.5 = 0$

$7D_V = 147.07$

$\boxed{D_V = 21.01}$

$R_C - 14 - 21 + 21.01 = 0$

$\boxed{R_C = 13.99}$



$\Sigma f x = 0$

$-H_B = 0$ $\quad H_B = 0.$

$\Sigma f y = 0$ $\quad +R_A - 3 - R_C + V_B = 0$

$R_A - 3 - 13.99 + V_B = 0$

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$M@A = 0$

$+V_B \times 5 \ -13.99 \times 2 \ -3 \times 1 \ = 0$

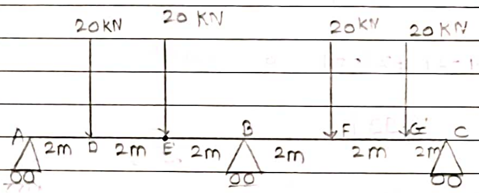
$5V_B = 30.98$

$V_B = 6.196$. KN .

$R_A - 3 - 13.99 + 6.196 = 0$

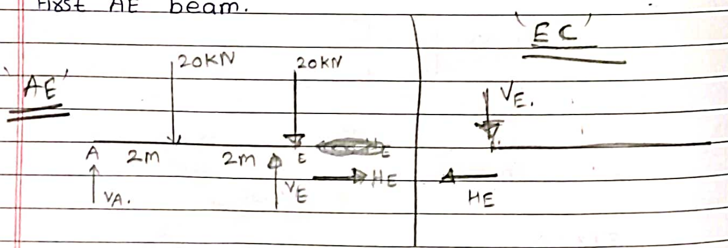
$R_A = 10.794$

Q.



@ E internal hinge.

First AE beam.



$\Sigma f_x = 0 \quad +H_E = 0$

$\Sigma f_y = +V_A - 20 - 20 + V_E = 0$ $\quad$ $V_E =$

$\Sigma M@A = 0 \quad (-20 \times 4) + (V_E \times 4) - (20 \times 2) = 0$

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$H_E = 0$

$\Sigma f_y = \ +V_A - 20 - 20 + V_E \ = 0$

$\Sigma M@A \ = 0$

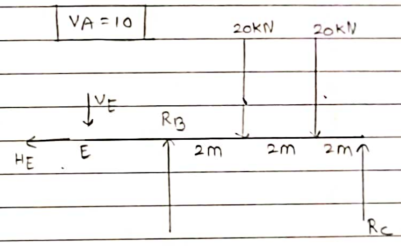
$(-20 \times 4) + (V_E \times 4) - (20 \times 2) \ = 0$

$4V_E = 120$

$V_E = 30$

$V_A - 40 + 30 = 0$ $\quad$ $\boxed{V_A = 10}$

For EC



$\Sigma f_y = 0$

$-V_E + R_B - 20 - 20 + R_c - 20 = 0$

$\Sigma f_x = 0 \qquad -H_E = 0$

$\Sigma M@c = 0$

$+V_E \times 8 - R_B \times 6 + 20 \times 4 + 20 \times 2 + \qquad = 0$

$-R_B 6 + \qquad = 0$

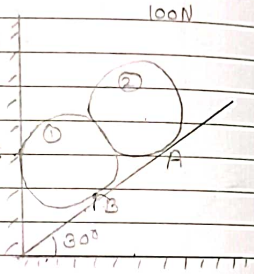
$R_B = \frac{520}{6} = 86.66$ $\qquad R_B =$

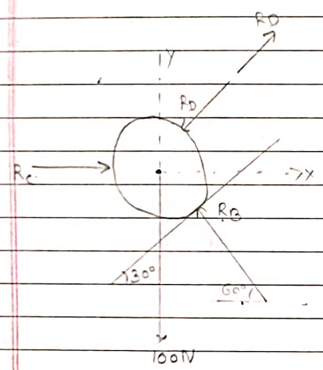
$R_c = 23.34$.

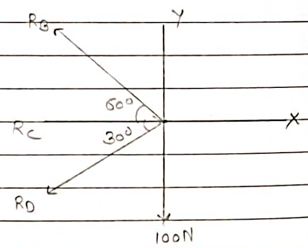
$240 - 6R_B + 80 + 40 = 0$

Q. Two identical roller's each of weight 100N are supported by Incined plane & vertical wall. Find out reactions at A, B, C

Draw FBD.





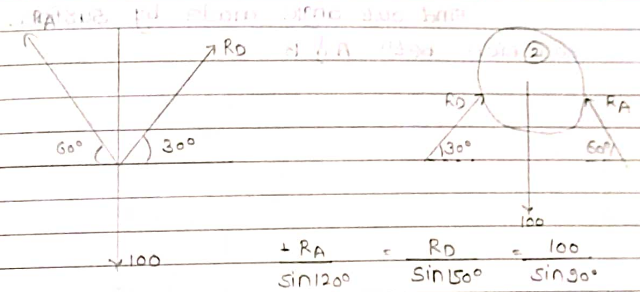


$\Sigma fx = 0$

$+R_C - R_B \cos 60° - R_D \cos 30° = 0$

$\Sigma fy = 0$

$-100 + R_B \sin 60° - R_D \sin 30° = 0$

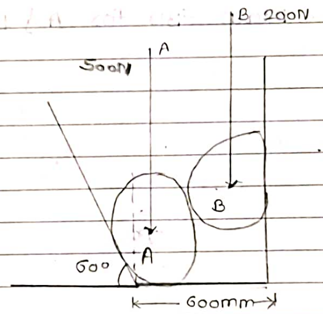


$$\frac{R_A}{\sin 120°} = \frac{R_D}{\sin 150°} = \frac{100}{\sin 90°}$$

$R_D = 50KN.$

$R_A = 86.602\ KN.$

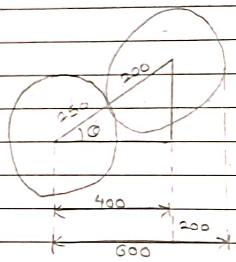
$\boxed{R_C = 115.47\ KN.}$



Two spheres A & B are resting in Smooth trough As shown in fig. below.
Find out reactions at All contact Surfaces
Henece $r_A = 250\text{ mm}$ $r_B = 200\text{ mm}$.

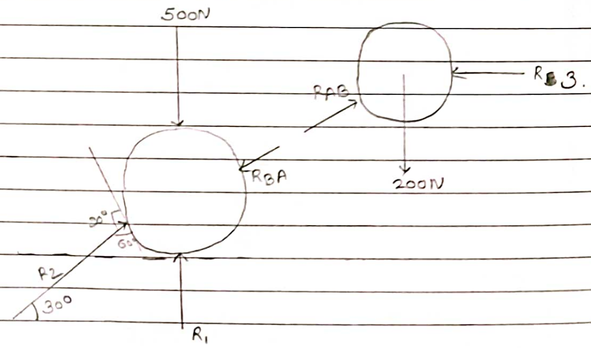
→

Step 1 Find out angle made by surface reaction betn A & B.



$\therefore \cos\theta = \dfrac{400}{450}$ $\qquad \therefore \theta = 27.27^\circ$

Step 2: Draw FBD for A & B.



$\Sigma f_x = 0$

$-R_3 + R_{AB}\cos 27.27^\circ = 0$

$\therefore R_3 = R_{AB}\cos 27.27^\circ$

$\Sigma f_y = 0$

$-200 + R_{AB}\sin 27.67 = 0$

$\therefore R_{AB} = \dfrac{200}{\sin 27.67} = 436.506.$

$R_3 = 436.506 \cos 27.27 = 387.992.$

For Block 'A'

$\Sigma f_x = 0$

$-R_{BA}\cos 27.27 + R_2\cos 30^\circ = 0$

$R_{BA} = \dfrac{R_2 \cos 30^\circ}{\cos 27.27}$ $\qquad \therefore R_{BA} = 0.974\, R_2.$

$\Sigma f_y = 0$

$-500 + R_1 + R_2 \sin 30^\circ - R_{BA}\sin 27.27 = 0$

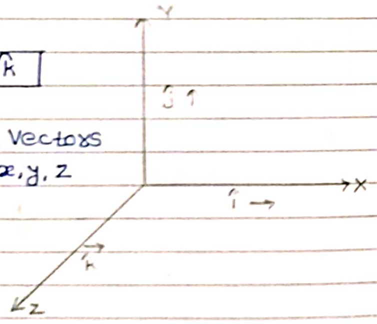
* Space force system :-

To solve Real life engg. problems 3-Dimentional's force system is involved hence this problems cannot be solve only considering 2-D system.

To solve this type of numericals the basic step is to express the forces given in system in vector form. i.e. in the form of dot product of magnitude of the force & unit vector in the direction of force.

The process of expressing a force into the vector form can be catagarised into 4 types depending upon data provided.

$$\boxed{\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}}$$



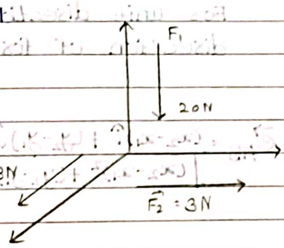
$\vec{i}, \vec{j}, \vec{k}$ are unit vectors in the direction x, y, z respectively.

* Types :-

① Given force is parallel to x, y or z-axis.

$$\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$$

The force f_1 is parallel to Y-axis so

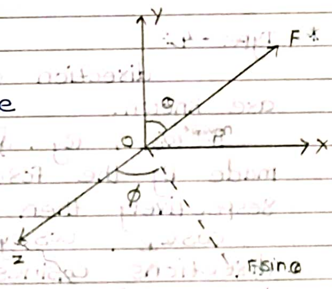


$$F_1 = 0\hat{i} - 20\vec{j} + 0\vec{K}$$

$$F_2 = 3\vec{i} + 0\vec{j} + 0\vec{K}$$

$$F_3 = 0\vec{i} + 0\vec{j} + 3\vec{K}$$

* Type 2 :- Angle made by the force with the axis is given & angel made by it's projection in the plane with other axis is given.



$\therefore$ $F_Y = F\cos\theta$.

Projection of the force $F\sin\theta$.

$\therefore$ $F_X = F\sin\theta \cdot \sin\phi$.

$\therefore$ $F_Z = F\sin\theta \cdot \cos\phi$.

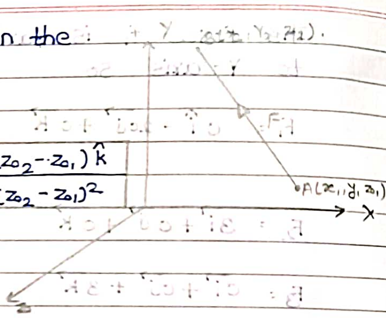
$\therefore$ Force in vector form

$$\hat{F} = F_x\hat{i} + F_Y\hat{j} + F_Z\hat{k}$$

* Type 3 :-

Two points on the line of Action of the force are given.

For unit direction in the direction of force



$$\therefore \boxed{\vec{e}_{AB} = \frac{(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_{o2}-z_{o1})\hat{k}}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_{o2}-z_{o1})^2}}}$$

$$\therefore \quad \vec{F} = \vec{e}_{AB} \cdot F$$

$$\therefore \quad \vec{F} = F\left[\frac{(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_0)^2\hat{k}}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}\right]$$

* Type - 4 :- direction cosines of the force are known.

if θ_x, θ_y, & θ_z are the angles made by the force with x, y, z axis respectively then $\cos\theta_x$, $\cos\theta_y$, $\cos\theta_{zo}$ are known as directions cosines of the force.

The components are Given by

$f_x = F\cos\theta_x$.

$$\therefore \quad \theta_x = \cos^{-1}\left[\frac{f_x}{F}\right]$$

Similarly.

$f_y = F\cos\theta_y$ $\qquad$ $f_{zo} = F\cos(\theta_{zo})$

$\therefore \theta_y = \cos^{-1}\left[\frac{f_y}{F}\right]$ $\qquad$ $\therefore \theta_{zo} = \cos^{-1}\left[\frac{f_{zo}}{F}\right]$.

Also the relation betn direction cosines is $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_{zo} = 1$.

& The magnitude of force

$$F = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$$

Q. determine the magnitude & direction of $\vec{F} = 500\hat{i} + 600\hat{j} - 300\hat{k}$ $\qquad$ Type - 4 example.

→ Given force can be written as

$\vec{F} = F_x\hat{i} + F_y\hat{j} - F_z\hat{k}$.

$$F = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$$

$$F = \sqrt{(500)^2 + (600)^2 + (300)^2}$$

$$F = 836.66N.$$

$$\cos\theta_x = \frac{F_x}{F} \qquad \therefore \theta_x = \cos^{-1}\left(\frac{500}{836.66}\right)$$

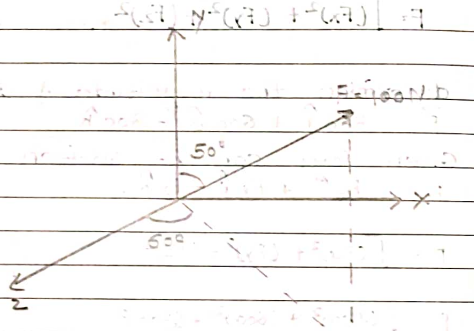
$$\therefore \theta_x = 53.30°$$

$$\theta_y = \cos^{-1}\left(\frac{F_y}{F}\right) = \cos^{-1}\left(\frac{600}{836.66}\right) = 44.181°$$

$$\theta_z = \cos^{-1}\left(\frac{F_z}{F}\right) = 111.012°.$$

Q. determine x, y, z components of the 100N force shown in fig. below . Also find out the angel made by force with x, y, z axis.

Type 2 :- projection.



$F_x = 100 \sin 50° \sin 60° = 66.341$

$F_y = 100 \cos 50° = 64.27$

$F_z = 100 \sin 50° \cos 60° = 38.302$

$\vec{F} = 66.34\,\hat{i} + 64.27\,\hat{j} + 38.30\,\hat{k}$

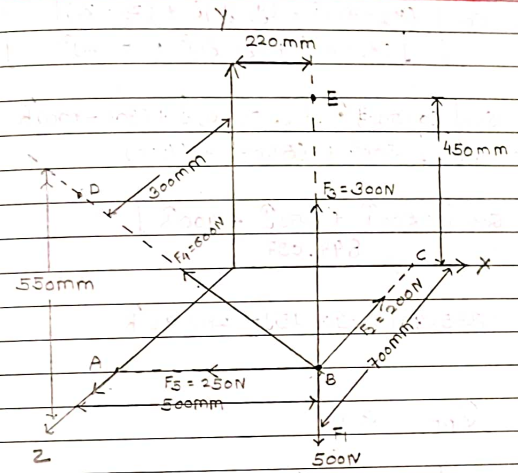
$|F| = \sqrt{(66.34)^2 + (64.27)^2 + (38.30)^2}$

$|F| = 100$

$\theta_x = \cos^{-1}\left(\frac{66.34}{100}\right) = 48.440°$

$\theta_y = \cos^{-1}\left(\frac{64.27}{100}\right) = 50°$

$\theta_z = \cos^{-1}\left(\frac{38.302}{100}\right) = 67.47°$



For $F_3 = \hat{e}\ 300\,N$

$F_4 = \hat{e}\ 600\,N$.

Find out co-ordinates of E, B, D.

Bcoz E is in xy plane.

E (220, 450, 0)

D (0, 550, 300) dist. from Y axis.

For Y-co-ordinate take dist. from z axis.

B (500, 0, 700) → dist from z-axis.

dist. from x-axis

$\therefore$ F_4 & F_3

$\therefore$ $F_4 = \hat{e}_{BD} \cdot 600$ $\qquad F_3 = \hat{e}_{BE} \cdot 300$

where $\hat{e}_{BD}$ is unit vector along force $\vec{F_4}$

& $\hat{e}_{BE}$ is unit vector along force $\vec{F_3}$

BD
↳ starts from B & end with D.
so cordinates of D are $x_2 y_2 z_2$.

$$F_4 = 600\left[\frac{(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}\right]$$

$$F_4 = 600\left[\frac{(0-500)\hat{i} + (550-0)\hat{j} + (300-700)\hat{k}}{\sqrt{500^2 + (550)^2 + (400)^2}}\right]$$

$$F_4 = 600\left[\frac{-500\hat{i} + 550\hat{j} - 400\hat{k}}{844.097}\right]$$

0.7108

$$F_4 = -355.40\hat{i} + 390.95\hat{j} - 284.32\hat{k}$$

$$F_3 = \hat{e}_{BE} \cdot 300$$

$$F_3 = 300\left[\frac{(220-500)\hat{i} + 450\hat{j} - 700\hat{k}}{\sqrt{280^2 + 450^2 + 700^2}}\right]$$

$$F_3 = \frac{300}{878.009}\left[-280\hat{i} + 450\hat{j} - 700\hat{k}\right]$$

$$F_3 = 0.3416\left[-280\hat{i} + 450\hat{j} - 700\hat{k}\right]$$

$$F_3 = -95.67\hat{i} + 153.72\hat{j} - 239.12\hat{k}$$

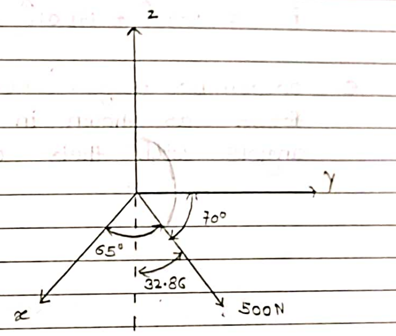
we can use Type -1 for finding $\vec{F_1}, \vec{F_5}, \vec{F_2}$

$\vec{F_1} = -500\hat{j}$ N.

$\vec{F_5} = -250\hat{i}$ N.

$\vec{F_2} = -200\hat{k}$ N.

Q. Determine x, y, & z components of 500 N force shown in fig.



$F_z = 500\cos(180-32.86)$
$= -419.99$

$\theta_x: 65°$
$\theta_y = 70°$
θ_z : with positive z axis
$= 180 - 32.86$
$= 147.14°$

These are the direction cosines.

$F_x = F\cos\theta_x = 500\cos 65°$
$= 211.309$ N.

$F_y = F\cos\theta_y = 500\cos 70°$
$= 171.01$ N

$F_z = 500\cos(147.14)$
$= -419.99$

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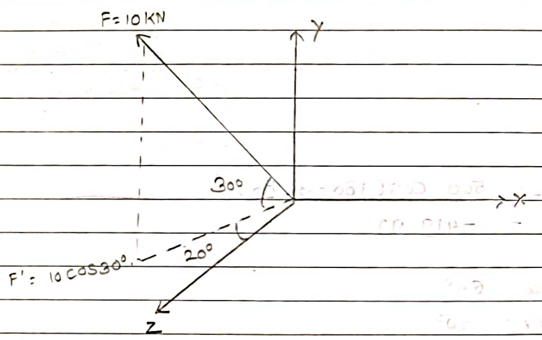
$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{F} : 211.309\,\hat{i} + 171.01\hat{j} - 419.99\,\hat{k}$.

Q. Determine x, y, z components of 10 N force as shown in fig. below & also find angles with their co-ordinate axes.



→

$F' : 10\cos 30° = 8.66$

$F_z : \overset{10\cos30°\cos20°}{8.66\cos 20°} = 8.137$ N

$F_x : \overset{10\cos30°\sin20°}{8.66\sin 20°} = -2.788$ N — due to direction

$F_y : 10\sin 30° = 5$ N

$\theta_x : \cos^{-1}\left[\frac{F_x}{F}\right] = \cos^{-1}\left[\frac{-2.788}{10}\right] = 106.188°$

$\theta_y : \cos^{-1}\left[\frac{F_y}{F}\right] = \cos^{-1}\left[\frac{5}{10}\right] = 60°$

$\theta_z = \cos^{-1}\left[\frac{8.137}{10}\right] : 35.540°$.

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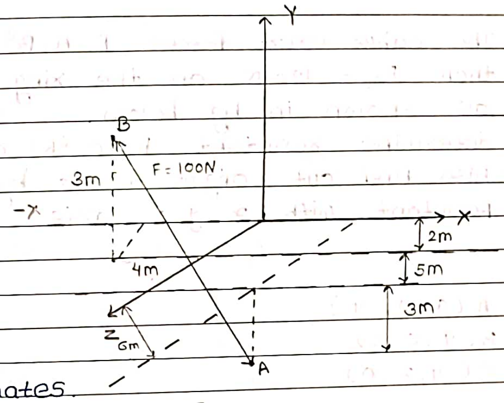
Q. Express the 100 N force F as shown in fig below. Also find out it's angles with x, y, z axis.



Step-1: Co-ordinates.

B (-4, 3, 2) x_2, y_2, z_2

A (6, -3, 7) x_1, y_1, z_1

$\vec{F} = \hat{e}_{AB}\,F$.

$= 100\left[\frac{(x_2-x_1)\hat{i} + (y_2-y_1)\hat{k} + (z_2-z_1)\hat{k}}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}\right]$

$= 100\left[\frac{-10\hat{i} + 6\hat{j} - 5\hat{k}}{\sqrt{10^2 + 6^2 + 5^2}}\right]$

$= 7.8811\left[-10\hat{i} + 6\hat{j} - 5\hat{k}\right]$

$= -78.811\,\hat{i} + 47.286\hat{j} - 39.4055\,\hat{k}$

$\theta_x = \cos^{-1}\left[\frac{-78.811}{1500}\right] = \cancel{99.068°}$ 142°

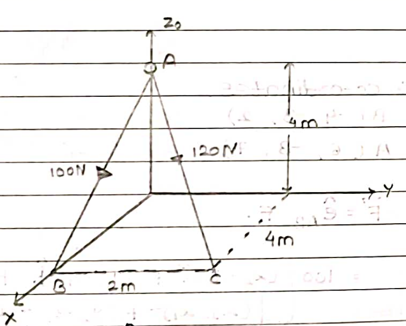
$\theta_y = \cos^{-1}\left[\frac{47.286}{1500}\right] = \cancel{84.573°}$ 61.779°

$\theta_z = \cos^{-1}\left[\frac{-39.405}{1500}\right]$ $\cancel{94.5}$ = 113.20

* Type :- 2.
* Numericals on concurrent force system in space.

Q. The cable exert forces $F_{(A,B)} = 100\,N$. then $F_{AC} = 120\,N$ on the ring at 'A' as shown in fig below. determine resultant force 'R' acting at 'A'. Also find out angles made by 'R' Resultant with x, y, z axis.

A (0, 0, 4)
B (4, 0, 0)
C (4, 2, 0)



$\vec{F}_{AB} = F_{x}\hat{i} + F_{y}\hat{j} + F_{z}\hat{k}$.
$\vec{F}_{AC} = F_{x_2}\hat{i} + F_{y_2}\hat{j} + F_{z_2}\hat{k}$.

$\vec{F}_{AB} = F_{AB} \cdot \hat{e}_{AB}$
$= 100 \times \left[\dfrac{4\hat{i} - 4\hat{k}}{\sqrt{16 + 16}}\right]$
$= 17.677 \times (4\hat{i} - 4\hat{k})$
$= 70.71\,\hat{i} - 70.71\,\hat{k}$

$\vec{F}_{AC} = F_{AC}\;\hat{e}_{AC}$
$= 120 \left[\dfrac{4\hat{i} + 2\hat{j} - 4\hat{k}}{\sqrt{16 + 4 + 16}}\right]$

$= 80\,\hat{i} + 40\,\hat{j} - 80\,\hat{k}$

$\vec{R} = \vec{F}_{AB} + \vec{F}_{AC}$

$\vec{R} = 150.71\,\hat{i} + 40\,\hat{j} - 150.71\,\hat{k}$

$|R| = \sqrt{R_x^2 + R_y^2 + R_z^2} = 216.857\ N.$

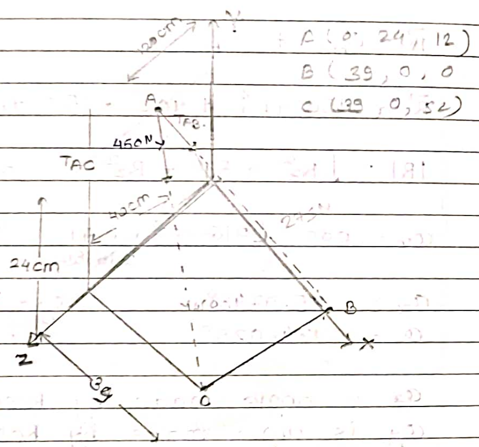
$\theta_x = \cos^{-1}\left(\dfrac{\cancel{216.857}\ 150.71}{216.857}\right)$

$\theta_x = 45.9748°$ $\qquad$ $\theta_y = 79.37°$
$\theta_z = 134.025°$.

θ_x is angle made by Resultant with x-axis.
θ_y is angle made by Resultant with y-axis.
θ_z is angle made by Resultant with z-axis.

Q. Find out resultant force acting at point A for the force system shown below. where Tension in string AC is 450 N & tension in string AB is 275 N.

A (0, 24, 12)
B (39, 0, 0)
C (39, 0, 52)



$T_{AB} = F_{AB} =$

$T_{AC} = F_{AC}$

$$\vec{F}_{AB} = 275\left[\frac{39\hat{i} - 24\hat{j} - 12\hat{k}}{\sqrt{39^2 + 24^2 + 12^2}}\right]$$

$$= 5.809\left[39\hat{i} - 24\hat{j} - 12\hat{k}\right]$$

$$= 226.551\hat{i} - 139.416\hat{j} - 69.708\hat{k}$$

$$\vec{F}_{AC} = 450\left[\frac{39\hat{i} - 24\hat{j} + 40\hat{k}}{\sqrt{39^2 + 24^2 + 40^2}}\right]$$

$$= 7.4\left[39\hat{i} - 24\hat{j} + 40\hat{k}\right]$$

$$= 288.637\hat{i} - \cancel{69}177.6\hat{j} + 296.04\hat{k}.$$

$$\vec{R} = 515.188\hat{i} + (-317.016\hat{j}) + 226.332\hat{k}$$

$$|R| = \sqrt{(515.188)^2 + (317.016)^2 + (226.322)^2} = 645.866$$

$\theta_x = 37.091°$ $\theta_y = 119.395°$ $\theta_z = 69.486°$

Q. Determine resultant of two forces shown in fig. below.