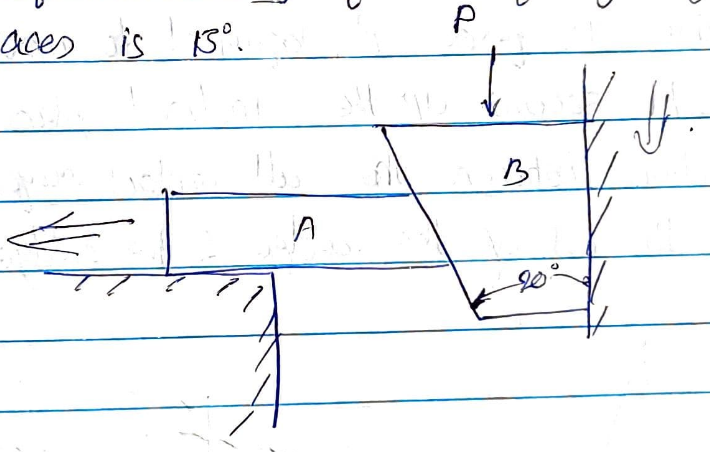
* Wedges:

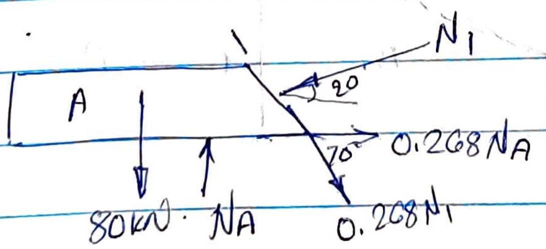
Wedges are small pieces of materials with two of their opposite surfaces not parallel. They are used to slightly lift heavy blocks, machinery, precast beams etc. for final alignments or to make place for inserting heavy devices. The weight of the wedge is very small compared to the weight lifted. Hence in all problems, the self-weight of wedge is neglected.

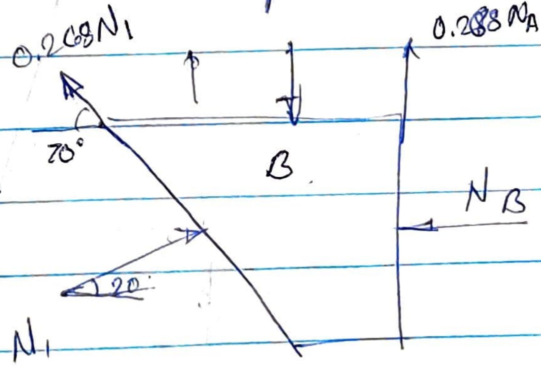
* Numericals

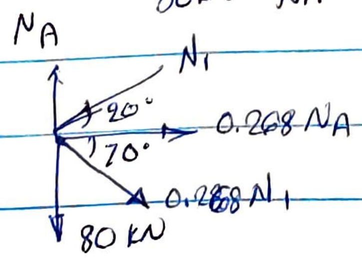
① A block A weighing 80 kN is to be moved towards left by light wedge B. Find necessary force P if angle of friction at all rubbing surfaces is 15°.



$\mu = \tan 15 = 0.268$







$\Sigma F_x = 0$

$-N_1 \cos 20 + 0.268 N_1 \cos 70 + 0.268 N_A = 0$

$-0.848 N_1 + 0.268 N_A = 0$

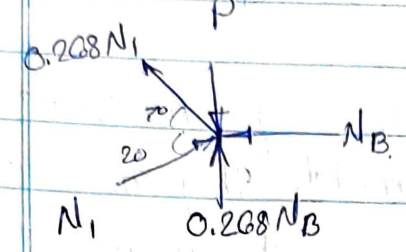
$\Sigma F_y = 0 \quad N_A - 80 - N_1 \sin 20 - 0.268 N_1 \sin 70 = 0$

$N_A = 80 + 0.593 N_1$

$-0.848 N_1 + 0.268 (80 + 0.593 N_1) = 0$

$-0.848 N_1 + 21.44 + 0.158 N_1 = 0$

$N_2 = 31.114$ kN.
$N_1 = 98.45$ kN



$\Sigma f_x = 0$

$N_1 \cos 20 - N_B - 0.268 \times 31.114 \cos 70 = 0$

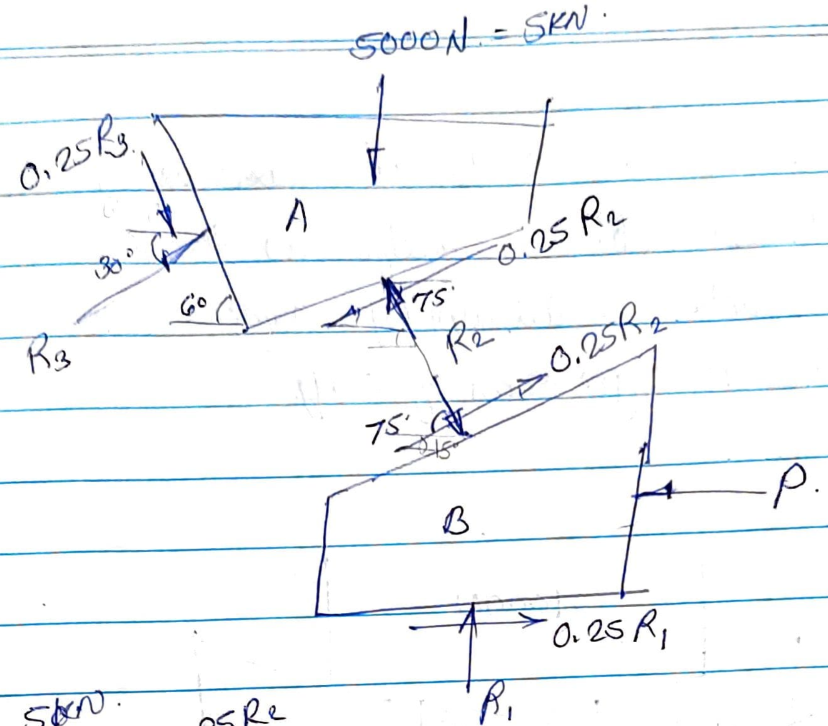
$N_B = 26.385$ kN.

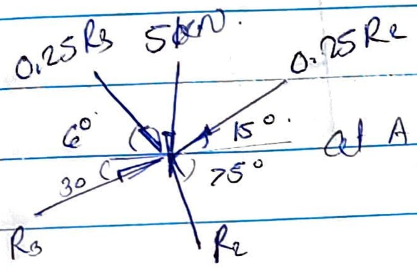
$\Sigma f_y = 0$

$0.268 N_B - P + N_1 \sin 20 + \overset{0.268}{N_1} \sin 70 = 0$

$P = 25.548$ kN.

(2) Determine the force P required to move the block of weight 5000N up the inclined plane. Coefficient of friction between the all contact surfaces is 0.25.
(1) Neglect the wt. of the wedge & the wedge angle is 15°





$\Sigma F_x = 0.$

$R_3 \cos 30 + 0.25 R_3 \cos 60 - 0.25 R_2 \cos 15 - R_2 \cos 75 = 0$

$R_3 - 0.5 R_2 = 0$ — ①

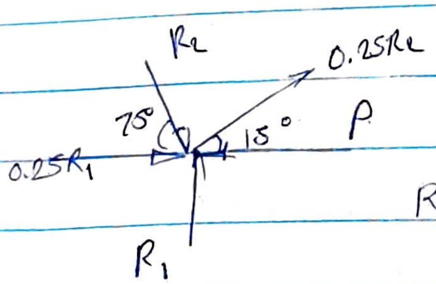
$\Sigma F_y = 0$

$R_3 \sin 30 + R_2 \sin 75 - 0.25 R_3 \sin 60 - 0.25 R_2 \sin 15 = 0.$

$\therefore 0.28 R_3 + 0.9 R_2 = 5$ — ②

$\therefore R_3 = 2.404$ kN

$R_2 = 4.807$ kN.



$\Sigma F_y = 0$

$R_1 - R_2 \sin 75 + 0.25 R_2 \sin 15 = 0.$

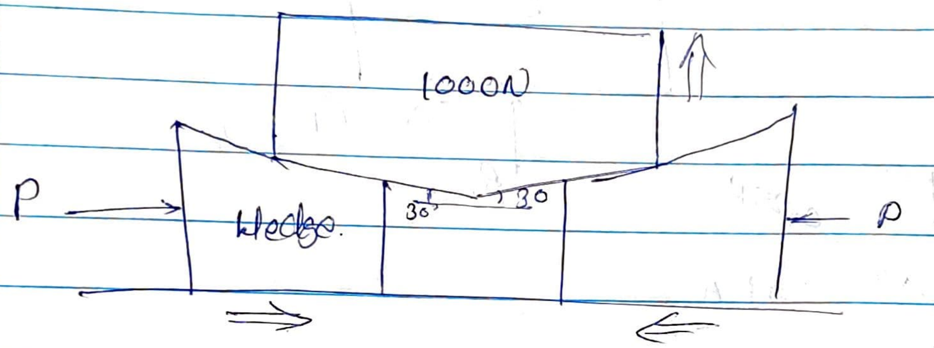
$R_1 = 4.332$ kN

$\Sigma F_x = 0$ $\quad -P + 0.25 R_1 + R_2 \cos 75 + 0.25 R_2 \cos 15 = 0$

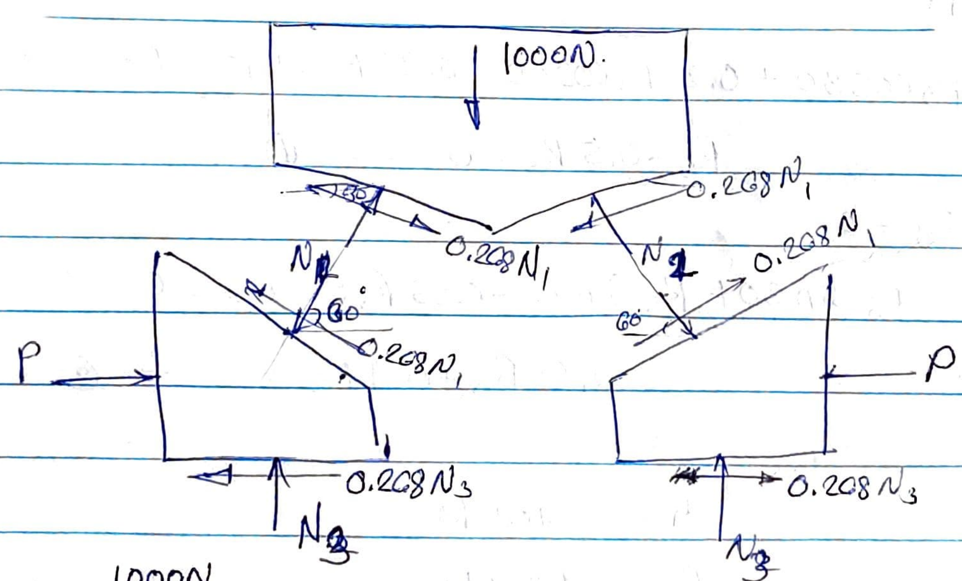
$P = 3.488$ kN.

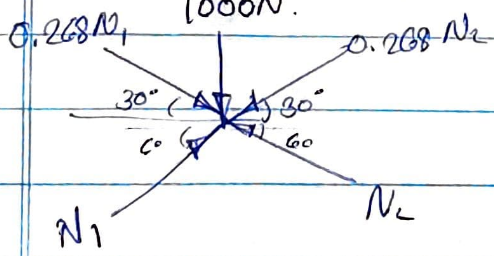
③ A block of 1000N is to be raised up by means of force P each acting on wedges as shown in fig. If angle of friction at all rubbing surfaces is 15°. determine P. Ignore wt. of wedge

②



$\mu_s = \tan 15 = 0.268$





$\Sigma F_x = 0$

$N_1 \cos 60 + 0.268 N_1 \cos 30 - N_2 \cos 60$

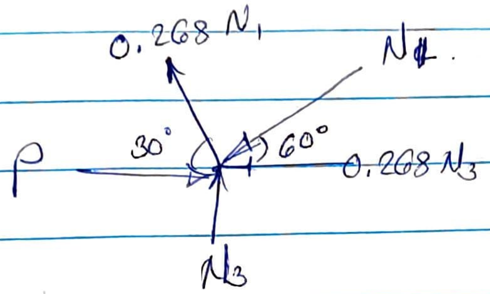
$- 0.268 N_2 \cos 30 = 0$

$\therefore N_1 = N_2$

$\Sigma F_y = 0$

$N_1 \sin 60 + N_2 \sin 60 - 0.268 N_1 \sin 30 - 0.268 N_2 \sin 30 - 1000 = 0.$

$N_1 = N_2 = 683.036\,N.$



$\Sigma F_y = 0$

$N_3 - N_1 \sin 60 + 0.268 N_1 \sin 30 = 0$

$N_3 = 500\,N.$

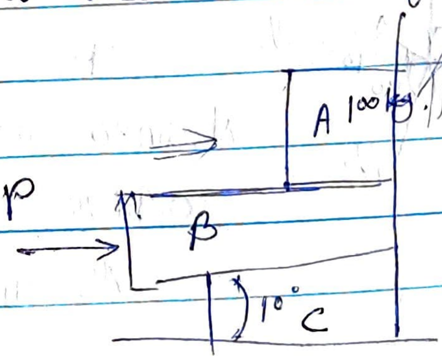
$\Sigma F_x = 0$

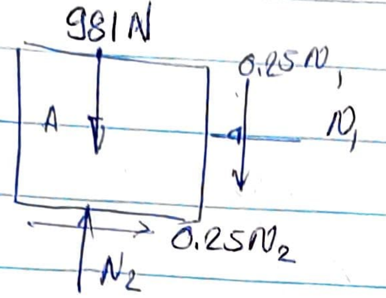
$-N_1 \cos 60 - 0.268 N_1 \cos 30 - 0.268 N_3 + P = 0$

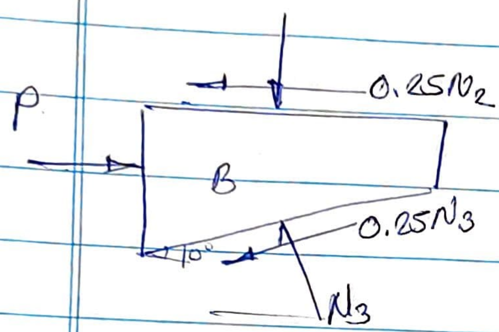
$P = 634.047\,N.$

(4) Two 10° wedges of negligible wt. are used to move a block of mass 100kg. If $\mu = 0.25$ at all surfaces of contact, find the smallest force P that should be applied to one of the wedges.

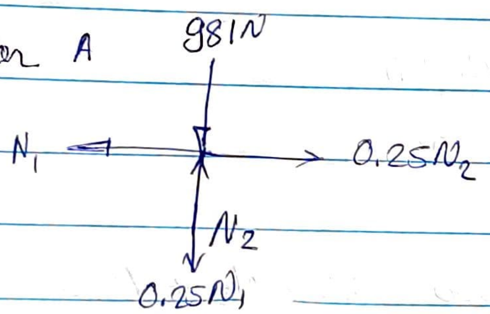
:- Here block A is required to just move. Hence impending motion of A will be upward & that for wedge B towards right. Here the wedge C is at rest it is functioning as a supporting wedge.







For A



$\Sigma F_x = 0$

$0.25N_2 - N_1 = 0 \qquad N_1 = 0.25N_2$

$\Sigma F_y = 0$

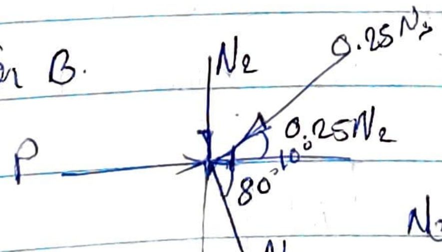
$N_2 - 981 - 0.25N_1 = 0.$

$N_2 = 981 + 0.25(0.25N_2)$

$\therefore N_2 = \cancel{1064.4} \quad 1046.4\,N.$

$N_1 = 261.6\,N$

For B.



$\Sigma F_y = 0$

$N_3 \sin 80 - 0.25 N_3 \sin 10 - N_2 = 0$

$0.94\,N_3 = N_2 = \cancel{1064}\ 1046.4$

$N_3 = 1113.19\,N.$

$\Sigma F_x = 0.$

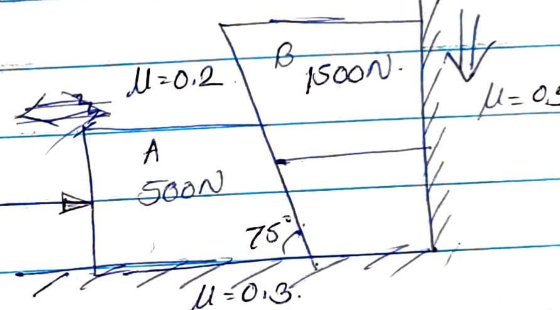
$P - 0.25N_2 - 0.25N_3 \cos 10 - N_3 \cos 80 = 0$

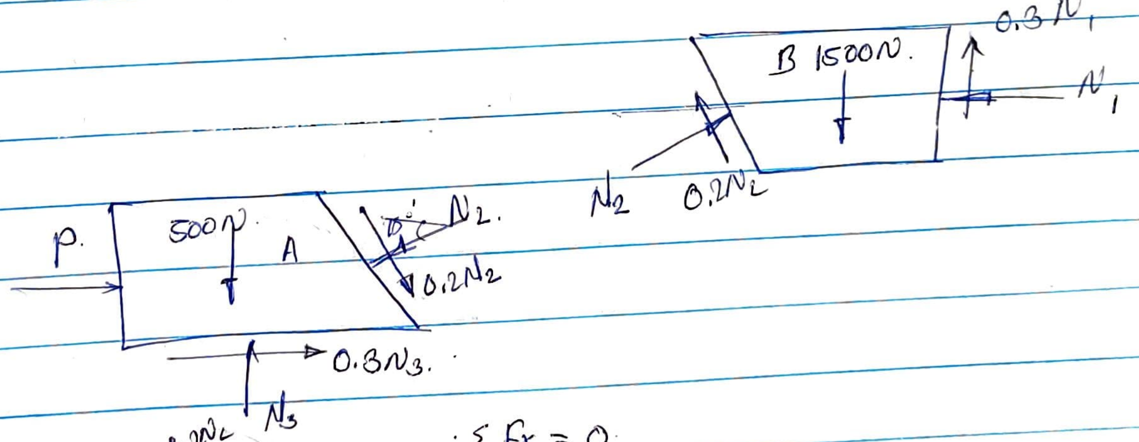
$P = 0.25N_2 + 0.43 N_3$

$= 0.25 \times 1046.4 + 0.42 \times 1113.19$

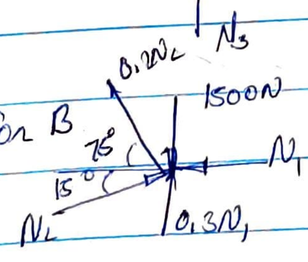
$= 729.14$ kN.

(3) Find the minimum horizontal force P to be applied to block A weighing 500N so as to keep block B, of 1500N in limiting condition of equilibrium.





For B



$\Sigma F_x = 0$

$-N_1 + N_2 \cos 15 - 0.2 N_2 \cos 75 = 0.$

$N_1 = 0.914 N_2$

$\Sigma F_y = 0$

$0.3N_1 - 1500 + 0.2N_2 \sin 75 + N_2 \sin 15 = 0.$

$0.3 \times 0.944 N_2 - 1500$

$0.735 N_2 =$

$N_2 = 2065.4$N $\quad N_1 = 1888.1$ N.

For A.

$$\Sigma F_y = 0$$

$$N_3 - 500 - 0.2N_2 \sin 75 - N_2 \sin 15 = 0$$

$$N_3 = 1433.57\,N.$$

$$\Sigma F_x = 0$$

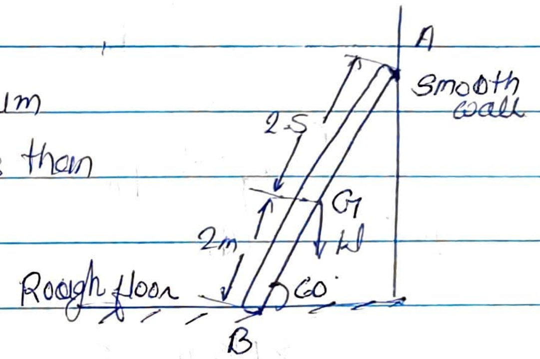
$$P - N_2 \cos 15 + 0.2N_2 \cos 75 + 0.3N_3 = 0$$

$$P = 1458.04\,N.$$

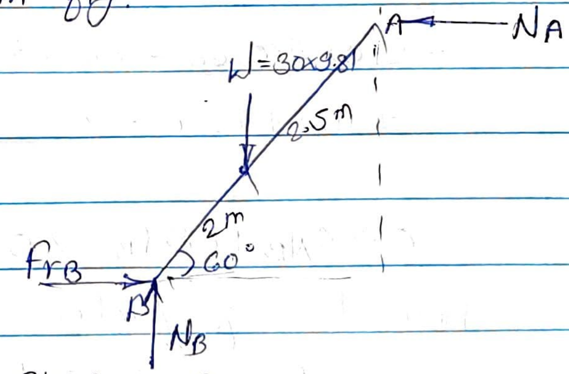
* Ladder Friction

① A non-homogeneous ladder, as shown in fig. rests against a smooth wall at A & rough horizontal floor at B. The mass of the ladder is 30kg & is concentrated at 2m from the bottom. The coefficient of static friction between the ladder & the floor is 0.35. Will the ladder stand in 60° position as shown?

The ladder will be in equilibrium if the frictional force at B is less than the limiting frictional force at B.



Consider the F.B.D. shown in fig.



$$\Sigma F_y = 0$$

$$N_B - 30\times 9.81 = 0$$

$$N_B = 294.3\,N$$

$$\Sigma M @ A = 0$$

$$(30\times 9.81)\times 2.5\cos 60 - N_B \times 4.5\cos 60 + F_{rB} \times 4.5 \sin 60 = 0$$

$$F_{rB} = 75.52\,N$$

Now,

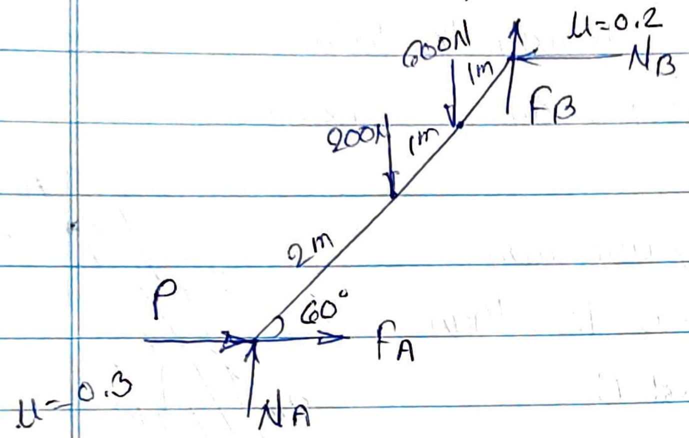
the limiting frictional force at B is

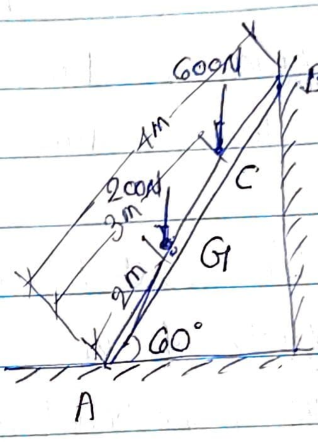
$$(F_{rB})_{max} = \mu N_B = 0.35\times 294.3 = 103\,N$$

$\therefore$ $F_{rB} < (F_{rB})_{max}$

$\therefore$ The ladder will remain in equilibrium.

② A ladder of length 4m, weighing 200N, is placed against a vertical wall as shown in fig. The coefficient of friction between the wall & the ladder is 0.2 & that between the floor & ladder is 0.3. In addition to self wt, the ladder has to support a man weighing 600N at a distance of 3m from A. Calculate the minimum horizontal force to be applied at A to prevent slipping.





$\Sigma M @ A = 0$

$-200 \times 2\cos 60 - 600 \times 3\cos 60 + N_B \times 4 \sin 60$

$+ F_B \times 4\cos 60 = 0$

$0.866 N_B + 0.5 F_B = 275$

From law of friction $F_B = 0.2 N_B$

$0.866 N_B + 0.5(0.2 N_B) = 275$

$\therefore N_B = 284.68\,N$

$F_B = 56.934\,N$

$\Sigma F_y = 0$

$N_A + F_B - 200 - 600 = 0.$

$\therefore N_A = 743.066\,N$

$F_A = 0.3 N_A = 222.92\,N$

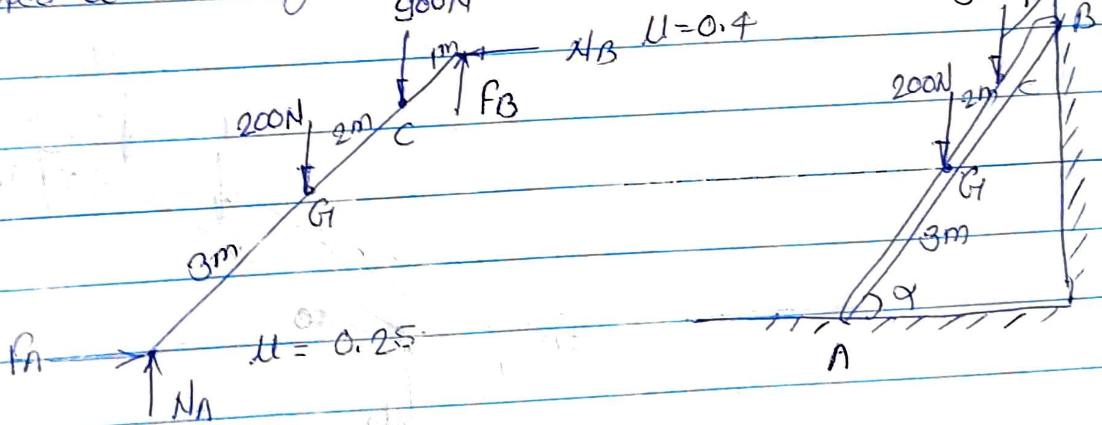
$\Sigma F_x = 0$

$P + f_A - N_B = 0$

$P = N_B - f_A = 284.68 - 222.92$

$P = 61.76N$

③ The ladder shown in fig. is 6m long & is supported by a horizontal floor & vertical wall. The coefficient of friction bet^n floor & ladder is 0.25 & between wall & ladder is 0.4. The self wt of ladder is 200N & may be concentrated at G. The ladder also supports a vertical load of 900N at C which is at a distance of 1m from B. Determine least value of α which the ladder may be placed without slipping. Determine the reactions developed at that stage



$f_B = 0.4 N_B$

$f_A = 0.25 N_A$

$\Sigma F_y = 0$

$N_A + f_B - 200 - 900 = 0$

$N_A + 0.4 N_B = 1100$ —— ①

$\Sigma F_x = 0$

$f_A - N_B = 0$

$0.25 N_A - N_B = 0$ —— ②

Solving ① & ② $\quad N_A = 1000N \Rightarrow f_A = 250N$

$N_B = 250N \Rightarrow f_B = 100N$

$\Sigma M @ A = 0$

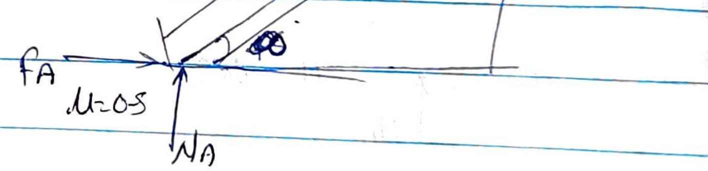
$N_B \times 6 \sin\alpha + f_B\, 6\cos\alpha - 200 \times 3\cos\alpha - 900 \times 5\cos\alpha = 0$

$1500 \sin\alpha = (-600 + 600 + 4500)\cos\alpha$

$\tan\alpha = 3$

$\alpha = 71.563°$

④ A ladder 7m long rests against a vertical wall with which it makes an angle of 40° & resting on a floor. If a man whose wt. is one half of that the ladder, climbs it. At what distance along the ladder will he be when ladder is about to slip & $\mu_s = \frac{1}{3}$ at wall & $\frac{1}{2}$ at floor.



Let x be the distance climbed by man just before ladder slips.

Let W be the wt. of ladder.

$\therefore$ wt. of man $= \frac{W}{2}$

$\Sigma F_x = 0$

$f_A - N_B = 0$

$0.5 N_A - N_B = 0$ —— ①

$\Sigma f_y = 0$

$N_A + f_B - W - \frac{W}{2} = 0$

$N_A + 0.333\, N_B = 1.5W$ —— (2)

Solving (1) & (2)

$N_A = 1.285W$

$N_B = 0.642W$

$\Sigma M @ A = 0$

$N_B \times 7\cos 40 + 0.3 N_B \times 7\sin 40 - W \times 3.5 \sin 40$

$- \frac{W}{2} \times x . \sin 40 = 0$

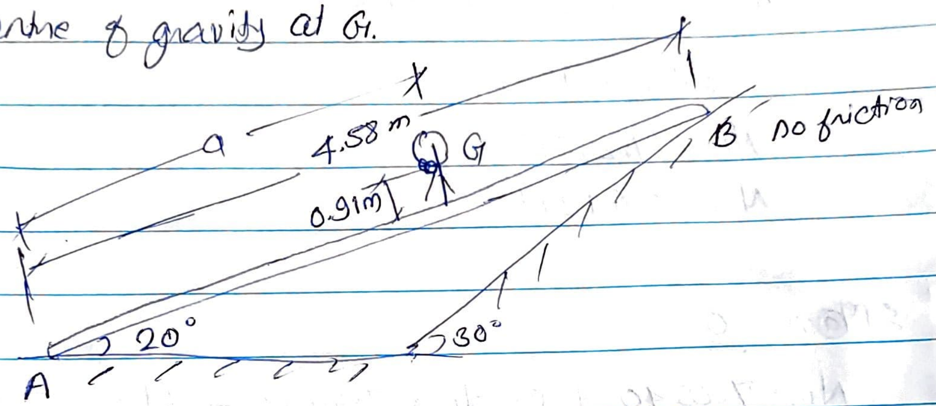
$0.642W \times 7\cos 40 + 0.3 \times 0.642W \times 7\sin 40 - W \times 3.5 \sin 40$

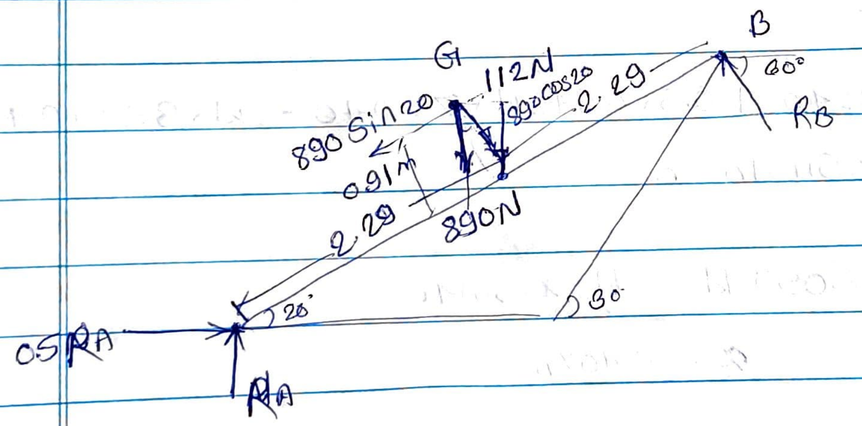
$- \frac{W}{2} \times x . \sin 40 = 0$

$2.059W = \frac{W}{2} . x . \sin 40$

$x = 6.407\,m$

⑤ Determine how far 'a' the man can walk up the 112N plank without causing it to slip. Take $\mu_A = 0.5$ & surface at B as smooth. The wt. of man is 890 N. & centre of gravity at G_1.





G_1 is C.g. man

G_1 is C.G. of ladder

$\Sigma f_x = 0$

$0.5R_A - R_B \cos 60 = 0$

$\therefore R_A = R_B$

$\Sigma f_y = 0$

$R_A + R_B \sin 60 - 890 - 112 = 0$

$1.866\, R_A = 1002$

$\therefore R_A = R_B = 536.98\,N$

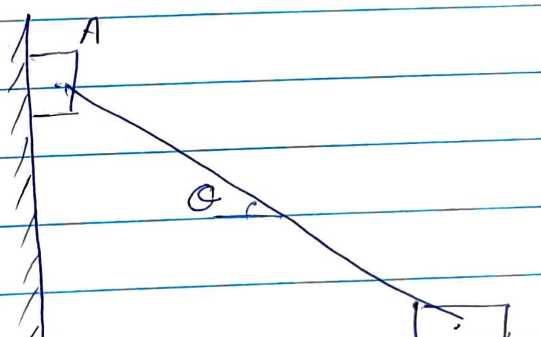
$\Sigma M_{@A} = 0.$

$890 \sin 20 \times 0.91 - 890 \cos 20 \times a - 112 \times 2.29 \cos 20$

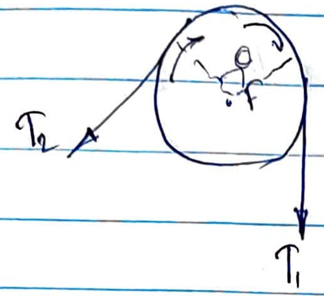
$+ R_B \sin 60 \times 4.58 \cos 20 + R_B \cos 60 \times 4.58 \sin 20 = 0$

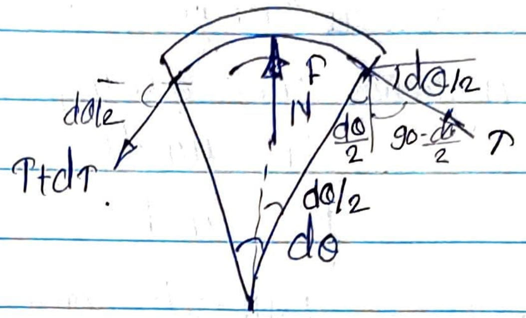
$a = 2.94\,m$

⑥ Two identical blocks A & B are connected by rod & rest against vertical & horizontal planes respectively as shown in fig. If sliding impends when $\theta = 45°$. determine the coefficient of friction μ assuming it to be the same at both floor & wall.



* Belt Friction





$T_2 > T_1$

Σ Forces in radial direction = 0

$$N - T.\sin\frac{d\theta}{2} - (T+dT).\sin\frac{d\theta}{2} = 0$$

Since $d\theta$ is very small angle, $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$

$$N - T.\frac{d\theta}{2} - (T+dT).\frac{d\theta}{2} = 0$$

$$N = \left(T+\frac{dT}{2}\right).d\theta \quad \text{--- (1)} \qquad \left(-T.\frac{d\theta}{2} - T.\frac{d\theta}{2} - dT.\frac{d\theta}{2}\right.$$

From law of friction. $F = \mu N = \mu\left(T+\frac{dT}{2}\right).d\theta$ --- (2)

Σ Forces in tangential direction = 0 gives.

$$T.\cos.\frac{d\theta}{2} - (T+dT)\cos.\frac{d\theta}{2} + F = 0.$$

Since $d\theta$ is very small angle, $\cos\frac{d\theta}{2} = 1$

$$T + dT = F + T$$

$$\therefore dT = F$$

from eqⁿ ② & ③

$$dT = \mu \left(T + \frac{dT}{2}\right) . d\theta .$$

Neglecting small quantity of higher order we get,

$$dT = \mu . T . d\theta$$

$$\frac{dT}{T} = \mu . d\theta$$

Integrating both sides over 0 to θ we get,

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\theta} \mu . d\theta$$

$$(\log T)_{T_1}^{T_2} = \mu (\theta)_0^{\theta}$$

$$\log \frac{T_2}{T_1} = \mu\theta$$

$$\therefore T_2 = T_1 e^{\mu . \theta}$$ (θ should be in radian)

$$\frac{T_2}{T_1} = e^{\mu\beta} \qquad T_2 > T_1$$

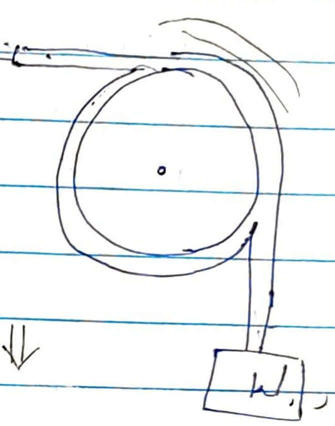
As tensions on two sides of the pulley are different, a net torque acts on the pulley & is given by,

$$\tau = (T_2 - T_1) r .$$

r = radius of the pulley

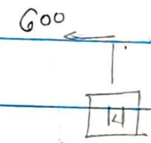
If belt drives the pulley, the driving torque & rotation of pulley will have same sense of rotation for breaking mechanisms, the torque will be opposite to the rotation of pulley. This is useful in identifying larger tension T_2 & smaller tension T_1

(1) A rope making $1\frac{1}{4}$ turns around a stationary horizontal drum is used to support a weight W. If the coefficient of friction is 0.3 what range of weight can be supported by exerting a 600N force at the other end of the rope?



$$\beta = 1.25 \times 2\pi$$
$$= 2.5\pi$$

Case I: Let the impending motion of the weight be downward.

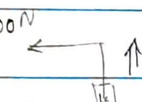


$T_1 = 600\,N \qquad T_2 = W.$

$\therefore$ from law of rope friction,

$W > 600$

$$T_2 = T_1 e^{\mu\beta}$$
$$W = 600\, e^{0.3 \times 2.5\pi}$$
$$= 6330.43\,N.$$



$600 > W$

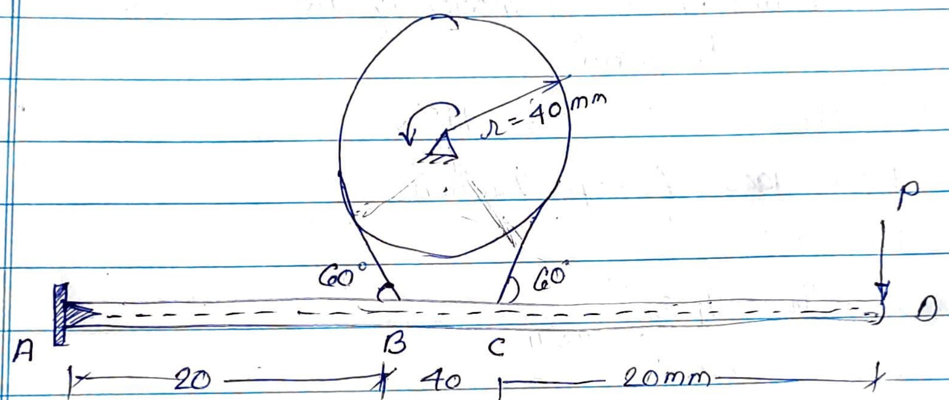
Case II: Let the impending motion of the wt. be upwards

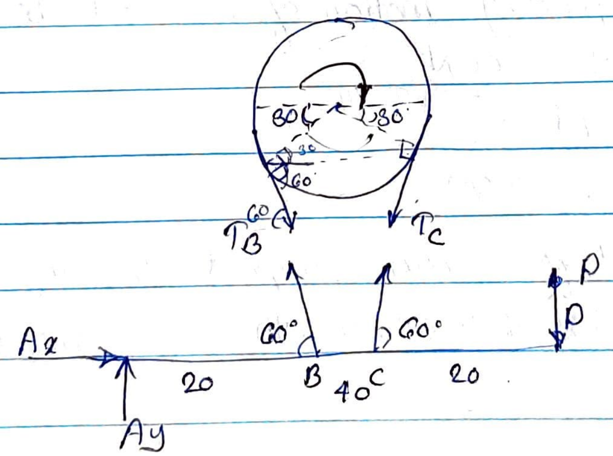
$T_1 = W \qquad T_2 = 600\,N.$

$$600 = W.\, e^{0.3 \times 2.5\pi}$$
$$\therefore W = 56.87\,N.$$

Thus a 600N force can support a range of loads between 56.87 to 6330.43N wt. on other side of the drum.

② As shown in fig. a flexible and inextensible flat belt placed around a rotating drum of 40mm radius, acts as a brake when the arm ABCD is pulled down. Assuming $\mu_k = 0.2$ between drum & belt, find the force P that would result in breaking torque of 4000 N.mm assuming that the drum is rotating counter clockwise.





As the drum is rotating counter clockwise, the breaking ~~the~~ torque will be clockwise. Hence

$T_C > T_B$. for the drum.

$$\frac{T_C}{T_B} = e^{\mu\beta}$$

$$\beta = 240^\circ = \frac{240 \times \pi}{180} = \frac{4}{3}\pi = 4.188 \text{ radians}$$

$$T_C = T_B \cdot e^{0.2 \times 4.188}$$

$$T_C = 2.311\, T_B \quad \text{—— (1)}$$

The breaking torque is $Z = 4000\ N.mm$

$$(T_C - T_B) \times 40 = 4000$$

$$T_C - T_B = 100 \quad \text{—— (2)}$$

$$2.311\, T_B - T_B = 100$$

$$\therefore T_B = 76.266\ N.$$

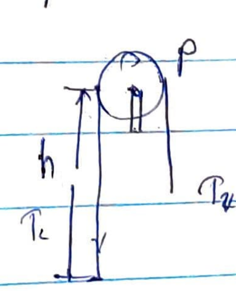
$$T_C = 176.266\ N.$$

For the arm ABCD, $\Sigma M_A = 0$

$$T_B \sin 60 \times 20 + T_C \sin 60 \times 60 - P \times 80 = 0$$

$$P = 131\ N.$$

(3) A cord having weight of 0.5 N/m & total length of 10m is supported over a peg P as shown in fig. If the coefficient of static friction between the peg & cord is $\mu_s = 0.5$, determine the longest length h which one side of the suspended cord can have without causing motion. Neglect size of peg & length of cord draped over it.



$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\beta = 180^\circ = \pi$$

$$T_1 = (10-h) \times 0.5$$

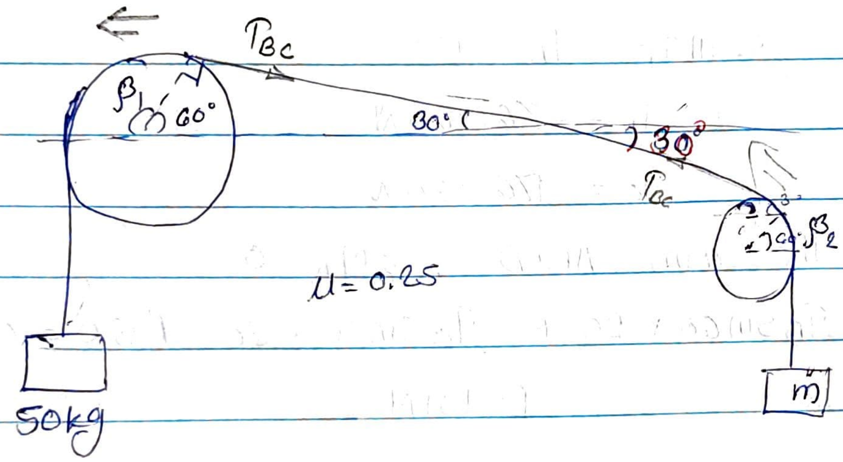
$$T_2 = h \times 0.5$$

$$\frac{0.5h}{(10-h) \times 0.5} = e^{0.5\pi}$$

$$\therefore h = 8.28\, m$$

④ A belt ABCD is placed over two pipes as shown in fig. to support a mass of 50 kg at end A. Determine
ⓐ The smallest value of mass m at end D for which equilibrium is possible.
ⓑ Tension in portion BC of the belt for the above value of m.

Assume coefficient of friction between the belt & the pipes to be 0.25.



As smallest value of m is required

$50 \times 9.81 > T_{BC} > m \times 9.81$

$\beta_1 = 180 - 60 = 120° = \frac{120 \times \pi}{180} = 2.094$

$\beta_2 = 60° = \frac{60 \times \pi}{180} = 1.047 \text{ rad.}$

$\mu = 0.25$

For larger pipe,

$\frac{50 \times 9.81}{T_{BC}} = e^{0.25 \times 2.094}$

$T_{BC} = 290.565 N$

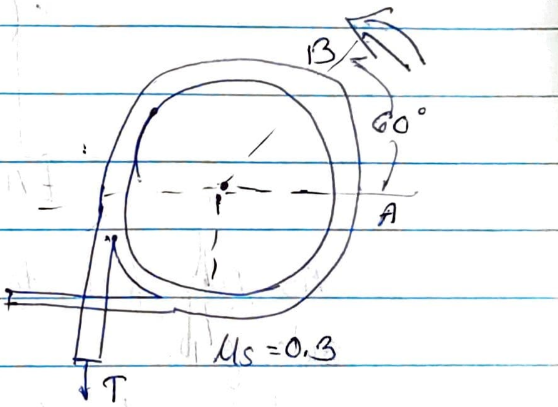
for smaller pipe,

$$\frac{T_{BC}}{m \times 9.81} = e^{0.25 \times 1.047}$$

$$m = \frac{290.505}{9.81 \times e^{0.25 \times 1.047}}$$

$$m = 22.8\ kg$$

⑤ Determine the maximum tension in the rope at points A & B that is necessary to maintain equilibrium. Take $\mu_s = 0.3$ between the rope & the fixed ~~rope~~ post D.



For maximum tension at A & B,

$T_A > 3kN$ & $T_B > T_A$

$T_A > 3kN.$

$$\frac{T_A}{3} = e^{0.3\frac{\pi}{2}}$$

$\beta = 60° = \frac{60 \times \pi}{180} = \frac{\pi}{3} = 1.047\ rad$

$\therefore\ T_A = 4.806\ kN$

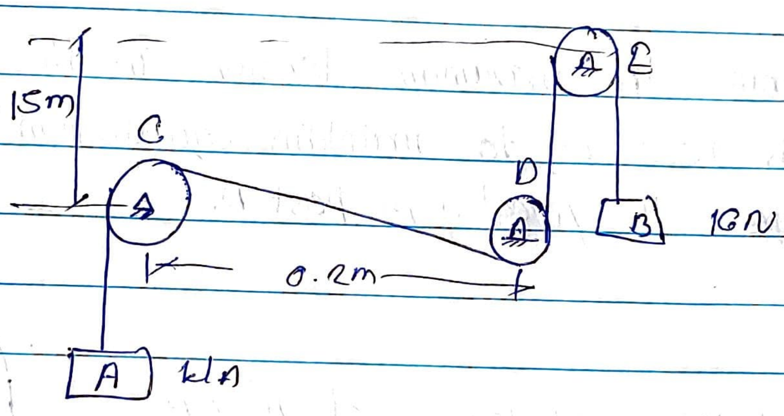
from the point whe, the horizontal rope touches the post to A, the lap angle is $\frac{\pi}{2}$

As $T_B > T_A$

$$\frac{T_B}{T_A} = e^{0.3\frac{\pi}{3}}$$

$$T_B = 6.58\ kN$$

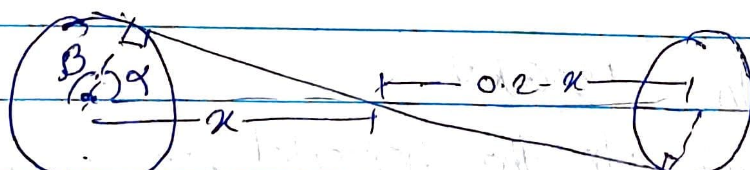
⑥ A cable passes around three 0.05m radius pulleys & supports two blocks as shown in fig. Pulleys C & E are locked to prevent rotation & the coefficient of friction between the cable & pulleys are $\mu_s = 0.2$. Determine the range of values of the wt. of block A for which equilibrium is maintained, if the pulley D is free to rotate.



As D is free to rotate the tension is same on both sides of D.

Lap angle of E is π.

To find lap angle for C, use similarity of Δles



$$\frac{x}{0.2-x} = \frac{0.05}{0.05}$$

$$x = 0.2 - x$$

$$2x = 0.2$$

$$x = 0.1\,m$$

$$\cos\alpha = \frac{0.05}{0.1} \qquad \alpha = 60^\circ$$

$$\beta_c = 180 - 60 = 120^\circ = 120 \times \frac{\pi}{180} = 2.0943 \text{ rad.}$$

As the coefficient of friction is same for both C & E, we can use the total lap angle.

$$\beta = \pi + 2.0943 = 5.235 \text{ rad.}$$

For min^n W_A,

$$\frac{16}{W_A} = e^{0.2 \times 5.235}$$

$$W_A = 5.615 N$$

For max^m W_A

$$\frac{W_A}{16} = e^{0.2 \times 5.235}$$

$$W_A = 45.6 N.$$

$\therefore$ Range of W_A is

$$5.615 \leq W_A \leq 45.6 N$$