13/02/2023.

unit 5 :-

kinamatics & Rectilinear motion.

$v = \frac{dx}{dt}$ $a = \frac{dv}{dt}$ Jerck : $\frac{da}{dt}$

where, $x \rightarrow$ displacement.

For Getting displacement from accelration we have integrate accn then we will get velocity then again integrate to get disp.

Engineering mechanics.

Statics

Dynamics
- kinematics
- kinetics

* Kinamatics of Rectilinear motion.

It can be defined as study of effects of the forces (disp, velocity, accn) without considering the 'force' responsible for it.

Kinetics of Rectilinear motion :

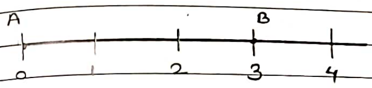
It is branch dynamics which deals with study of force along with it's conseauences OE the effect.

① Position of Particle :-

It defines the location of the particle with reference to a fixed point

② Displacement :- It can be defined as change in position of body.

ex:-



Displacement = Final position - Intial position
= 3 - 0
= 3 m.

Case - II :- A to B & B to A
Displacement = 0 - 0 = 0 m.
Accn is zero if final & Intial position is same.

Q. The velocity of particle moving in a straight line is given by expression $V = t^3 - t^2 - 2t + 2$ the particle is found to be at a dist. 4m from station A. after 2 sec

① Acc^n & disp after 4 sec

② max or min acc^n

→ Acc^n $= a = \frac{dv}{dt} = 3t^2 - 2t - 2$ — ①

$\int ds = \int (t^3 - t^2 - 2t + 2)$

$S = \frac{t^4}{4} - \frac{t^3}{3} - \frac{2t^2}{2} + 2t + C$

For finding const. Put $S = 4m$ $t = 2 sec$

$4 = \frac{16}{4} - \frac{8}{3} - 4 + 4 + C$

$\cancel{4} - \cancel{4} + \frac{8}{3} + \cancel{4} - 4 = C$ $\therefore \boxed{C = 2.666}$

$S = \frac{t^4}{4} - \frac{t^3}{3} - t^2 + 2t + 2.666$ — ②

$= 64 - \overset{21.333}{\cancel{64}} - 16 + 8 + 2.666$

$S = 37.333$ m. ... After $t = 4sec$.

$a_{min}/a_{max} \rightarrow \frac{da}{dt} = 0$

$6t - 2 = 0$ $\boxed{t = 0.333 \text{ sec}}$

$\frac{d^2a}{dt^2} = 6$ .... +ve.

$a_{min} = 3(0.333)^2 - 2(0.333) - 2$

$= -2.333 \ m/s^2$.

Q. A body moves along a straight line & it's accn 'a' which varies with time is given by $a = 2-3t$
5 sec after start of observation it's velocity is found to be 20 m/s.
10 sec after start of observation the body is at 85m from origine determine
① It's accn & velocity & displacement from origine.
② the time in which velocity becomes zero & corresponding disp. is zero.

→

$a = 2-3t$ —①

$\int dv = \int (2-3t)\,dt$

$v = 2t - \frac{3t^2}{2} + c_1$

If $t = 5$ sec $v = 20$ m/s

$20 = 10 - \frac{75}{2} + c_1$

$\boxed{c_1 = 47.5}$

$\therefore v = 2t - \frac{3t^2}{2} + 47.5$

$\therefore \int ds = \int (2t - \frac{3t^2}{2} + 47.5)\,dt$

$s = \frac{2t^2}{2} - \frac{3}{2}\left(\frac{t^3}{3}\right) + 47.5t + c_2$

If $t = 10$ sec $s = 85$ m

$85 = 100 - \frac{1000}{2} + 475 + c_2$

$\therefore \boxed{c_2 = 10}$

$\therefore s = t^2 - \frac{t^3}{2} + 47.5t + 10$.

① at $t = 0$
$a = 2$ m/s²
$v = 47.5$ m/s
$s = 10$ m

② $0 = 2t - \frac{3t^2}{2} + 47.5$

$-\frac{3t^2}{2} + 2t = -47.5$

$\therefore \boxed{t = 6.333}$ → As t cannot be zero

$0 = t^2 - \frac{t^3}{2} + 47.5t + 10$

$\therefore \boxed{t = 10.891}$ sec

Q. A particle moves along a straight line with an accⁿ $a = 4t^3 - 2t$ where $a \to m/s^2$ & $t \to sec$. then $t=0$ the particle is at 2m to the left of origine & when $t = 2sec$ the particle is at 20m to the left of origine.
① determine position of particle at $t = 4 sec$.

→

$\int dv = \int (4t^3 - 2t)\, dt$

$V = \frac{4t^4}{4} - \frac{2t^2}{2} + c_1$

$\therefore \boxed{V = t^4 - t^2 + c_1}$ — ① Here condition not Given to find velocity or const.

$\int ds = \int (t^4 - t^2 + c_1)\, dt$

$S = \frac{t^5}{5} - \frac{t^3}{3} + c_1 t + c_2$

If $t=0$ $x = -2m$

$\boxed{c_2 = -2}$

If $t = 2sec$ $x = -20m$

$-20 = 6.4 - 2.666 + 2c_1 + (-2)$

$\boxed{c_1 = -10.867 \text{ sec}}$

$\therefore S = \frac{t^5}{5} - \frac{t^3}{3} + (-10.867t) - 2$

At $t = 4ses$

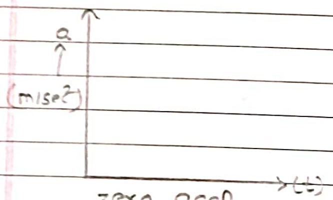
$S = 204.8 - 21.333 - 43.468 - 2$

$\therefore \boxed{S = 137.999 \text{ m}}$

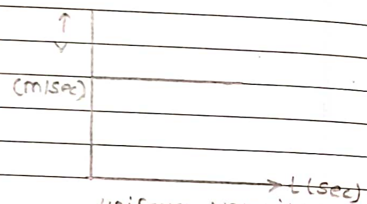
* Motion curves.

Graphical Solution for Kinematics & Rectilinear motion.

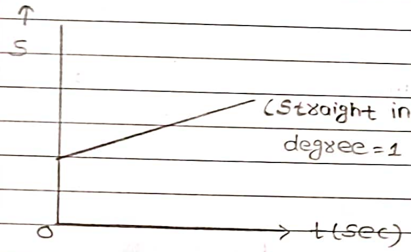
① uniform velocity motion curve:



zero accn.

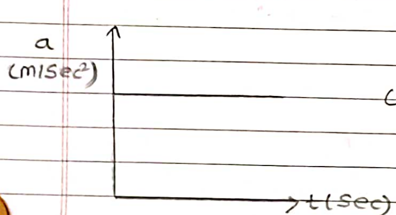


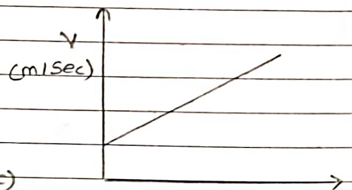
uniform velocity



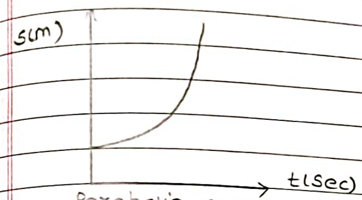
(Straight incined line)
degree = 1

② uniform Accn curve.



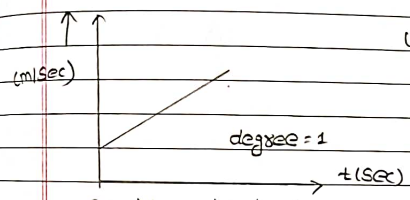


Straight incined line (degree = 1)

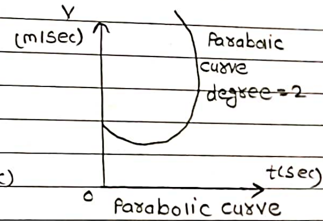


Parabolic current
degree = 2

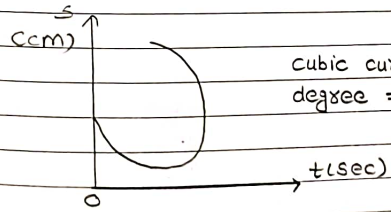
③ variable accn motion curve.



Straight inclined line.



Parabolic curve
degree = 2.



cubic curve
degree = 3

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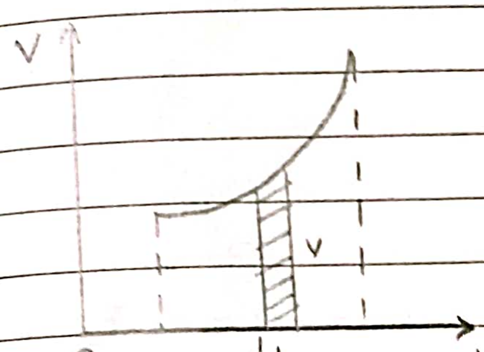
Displacement - time curve.

(S-t curve)



$$\text{Area unde the curve} = \int_{t_1}^{t_2} V \cdot dt \quad \text{— (1)}$$

→ Strip area

but $V = \frac{ds}{dt}$

$\therefore ds = V \cdot dt$

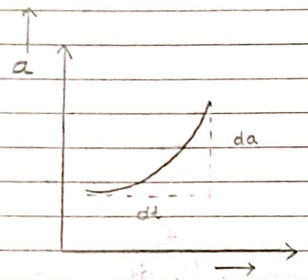
integrating both sides

$$\int_{t_1}^{t_2} V \cdot dt = \int_{S_1}^{S_2} ds$$

$\therefore$ Area of the curve $= S_2 - S_1$

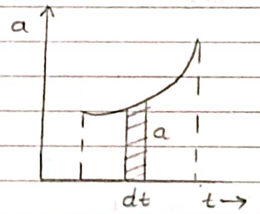
$\therefore$ $\boxed{\text{Area under } V\text{-}t = \text{displacement}}$

③ Graph of Acc^n & time.



Slope to a.t curve = $\frac{da}{dt}$

= Jerk.



Area under a.t curve = $\int_{t_1}^{t_2} a \cdot dt$

$\therefore \; a = \frac{dv}{dt}$ $\quad\therefore\; a \cdot dt = dv$

$\int_{t_1}^{t_2} a \cdot dt = \int_{v_1}^{v_2} dv$

$\therefore$ Area under a.t curve = change in velocity (ΔV) $[V_2 - V_1]$.

Q. Two Sassions P & Q are 5.2 km apart & Automobiles starts from rest from station P & accelrates uniformly to attend sppd of 48 kmph in 30 sec. This P is maintain untill the brakes are applied the automobile comes to rest at station Q. with uniform retardation 1 m/s^2 determine total time required to cover dist bet^n two stations.

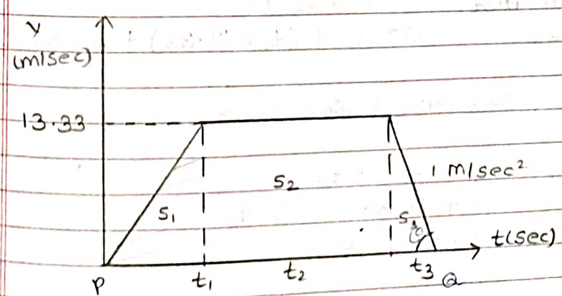
→

kmph ⟶ m/sec $\times \frac{5}{18}$

$\therefore$ 48 kmph $= \frac{48 \times 1000}{60 \times 60}$

$= \frac{48 \times 5}{18}$

$= 13.33$ m/sec.



S = 5.2 km $\quad t_1 = 30$ sec

= 5200 m $\quad T = t_1 + t_2 + t_3$

$S_1 = \frac{1}{2} \times t_1 \times 13.33$

$S_1 = \frac{1}{2} \times 30 \times 13.33$

$\boxed{S_1 = 199.95 \text{ m}}$

$S_2 = t_2 \times 13.33$

$\therefore S_3 = \frac{1}{2} \times t_3 \times 13.33$

$\therefore \tan\theta = \frac{13.33}{t_3}$

$\therefore t_3 = 13.33$

$\therefore \boxed{S_3 = 88.84 \text{ m}}$

Total dist $= S_1 + S_2 + S_3$

$5200 = 199.95 + (t_2 \times 13.33) + 88.84$

$\boxed{t_2 = 368.432}$ Sec

$\therefore T = t_1 + t_2 + t_3$

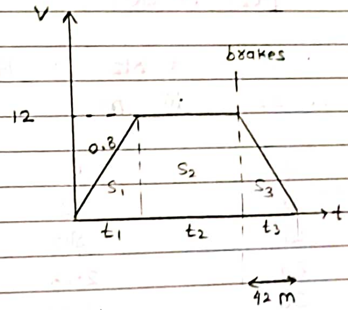
$= 30 + 13.33 + 368.432$

$T = 411.762$ Sec

Q. Bus starts from rest from Pt 'A' & Accelerates at 0.8 m/s² till it reaches maximum Velocity of 12 m/s² after some time brakes are applied so that it comes to rest at 'B' which is 42 m beyond the Pt. where brakes are applied. Knowing that the accⁿ is uniform & total travel time is 36 Sec. Find out dist betⁿ A&B with the help of V-t diagram, Also draw S-t diagram.

→

$T = 36$ Sec.



$\therefore \tan\theta = 0.8$

$0.8 = \frac{12}{t_1}$

As per the data Given slope to V-t curve

$= \frac{12}{t_1} = 0.8$

$\therefore \boxed{t_1 = 15 \text{ Sec}}$

Area under

$S_1 = \frac{1}{2} \times 12 \times 15 = 90$ m we know, $S_3 = 42m$.

$S_3 = \frac{1}{2} \times 12 \times t_3$

$42 = \frac{1}{2} \times 12 \times t_3$

$\boxed{t_3 = 7 \text{ sec}}$

we know, $T = 36$ [Total time taken by motion]

$t_1 + t_2 + t_3 = 36$

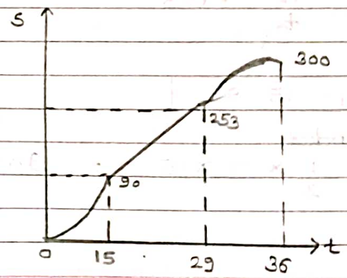
$15 + t_2 + 7 = 36$

$\boxed{t_2 = 14 \text{ sec}}$

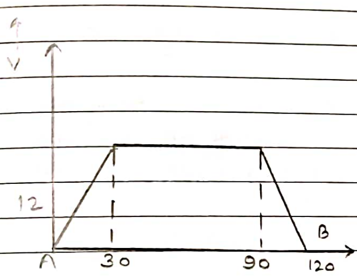
$\therefore S_2 = t_2 \times 12 = 14 \times 12$

$\therefore S_2 = 168 \text{ m}$

t	S
0	0
15	90
29	258
36	300



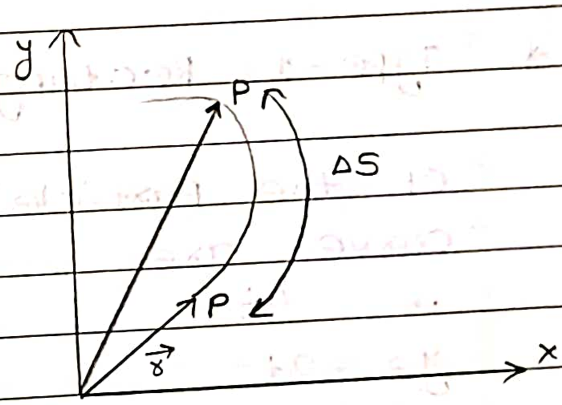
Q. The V.t diagram for the motion of train from Station A to B is as shown in fig below determine Avg. speed for train & dist. betn stations. Also draw A.t curve.



* Kinamatics of curvilinear motion :-
Curvilinear motion of the particle can be defined as the motion along curve or circular path.

This motion can be analysied by using following components

(A) Rectangular components.
(B) Normal & Tangential components.
(C) Radial & Transverse components.



* Type - 1.

Consider motion of particle is defined

$$\bar{r} = x\hat{i} + y\hat{j}$$

$$\therefore \frac{d\bar{r}}{dt} = \vec{V} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$= V_x\hat{i} + V_y\hat{j}$$

$$\therefore |V| = \sqrt{V_x^2 + V_y^2}$$

$$\boxed{\theta_v = \tan^{-1}\left(\left|\frac{V_y}{V_x}\right|\right)}$$

$\vec{a} \cdot \frac{d\vec{v}}{dt}$

$= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$

$\vec{a} = a_x \hat{i} + a_y \hat{j}$

$a = \sqrt{a_x^2 + a_y^2}$

$\theta_a = \tan^{-1}\left(\left|\frac{a_y}{a_x}\right|\right)$

* Type-1 Rectangular components.

the x & y co-ordinates of the particle moveing along a curve are
$x = 4t$
$y = 2t^2$
find out eqn of the path

→

Given : $x = 4t$

$\therefore t = \frac{x}{4}$

$\therefore t^2 = \frac{x^2}{16}$

$\therefore y = \cancel{2} \times \frac{x^2}{\cancel{16}\,8}$

$\therefore y = \frac{x^2}{8}$ $\therefore \boxed{x^2 = 8y}$

Q The motion of the particle moveing in x-y plane is defined by $x = t^2 + 8t - 4$ & $y = t^3 + 3t^2 + 8t + 4$ when x, y are in meters & t is in seconds
find out resultant velocity & accn at $t = 0$ sec & $t = 2$ sec

→

$x = t^2 + 8t - 4$ $y = t^3 + 3t^2 + 8t + 4$

$\frac{dx}{dt} = 2t + 8 = V_x$ $\frac{dy}{dt} = 3t^2 + 6t + 8 = V_y$

$\boxed{V_x = 2t + 8}$ $\boxed{V_y = 3t^2 + 6t + 8}$

$\frac{dv_x}{dt} = a_x = 2$ $\frac{dv_y}{dt} = a_y = 6t + 6$

At $t = 0$
$V_x = 8$ $a_x = 2$
At $t = 2$
$V_x = 12$ $a_x = 2$

At $t = 0$
$V_y = 8$ $a_y = 6$
At $t = 2$
$V_y = 32$ $a_y = 18$

$\therefore$ At $t = 0$

$V = \sqrt{(V_x^2) + (V_y)^2} = \sqrt{8^2 + 8^2} = 11.31$ m/s.

$\alpha = \tan^{-1}\left|\frac{V_y}{V_x}\right| = 45°$

at $t = 2\,sec$

$x = 2^2 + 8(2) + 4 = 24m$

$v_x = 12\ m/sec$

$v_y = 32\ m/sec$

$a_x = 2\ m/sec^2$

$a_y = 18\ m/sec^2$

$v = \sqrt{12^2+32^2} = 34.176$

$\alpha = \tan^{-1}\left|\frac{v_y}{v_x}\right| = 69.443°$

Q. A particle moves along path ~~$x = t^2+8t+4$ & $y = t^3+3t^2+8$~~

$y = x^2-2x+100$ starting with intial velocity $V_0 = (4\hat{i} - 16\hat{j})\ m/sec$

If V_x is constant.

determine velocity & accn at $x = 12m$

→

Given: $V_0 = (4\hat{i} - 16\hat{j})$

$V_{x_0} = 4\ m/sec$ $\quad V_{y_0} = -16\ m/sec$.

$y = x^2 - 2x + 100$

diff. wrt t.

$\therefore \frac{dy}{dt} = 2x \cdot \frac{dx}{dt} - 2\frac{dx}{dt}$

we know $\frac{dy}{dt} = v_y \quad \frac{dx}{dt} = v_x$

$v_y = 2x \cdot v_x - 2v_x$.

$\boxed{v_y = 2v_x(x-1)}$ — ①

diff. again wrt 't'.

$v_y = 2x \cdot v_x - 2v_x$.

$\frac{dv_y}{dt} = 2\frac{dx}{dt} \cdot v_x + 2x\frac{dv_x}{dt} - 2\frac{dv_x}{dt}$

$a_y = v_x \cdot 2v_x + 2x\,a_x - 2a_x$

$= 2v_x^2 + 2x\,a_x - 2a_x$

$\therefore \boxed{a_y = 2v_x^2 + 2x\,a_x - 2a_x}$

At $x = 12\,m$

$v_y = 2\times4\,(12-1)$

$= 88\ m/sec$.

~~$a_y = 32 + 2x\,a_x - 2a_x$~~

~~$a_y = 32 + 24a_x - 2a_x$~~

~~$a_y = 32 + 22a_x$.~~

v_x is constant.

$\boxed{\frac{dv_y}{dt} = 2v_x^2 = a_y}$ $\qquad a_x = \frac{dv_x}{dt} = 0$

$\therefore a_y = 2v_x^2 = 2(4)^2$

$a_y = 32\ m/s^2$.

$v = \sqrt{4^2 + 88^2} = 88.090$ m/sec.

$a = \sqrt{0^2 + 32^2} = 32$ m/sec^2.

* Type - II :

* Normal & Tangential components of v & a

$V_N = 0$ $\qquad a_T = \frac{dv}{dt}$

$V_T = \frac{dx}{dt}$ $\qquad a_N = \frac{v^2}{\rho}$

$\therefore a = \sqrt{(a_T)^2 + (a_N)^2}$

① A particle moves in circular path of radius 0.4 m calculate magnitude of accⁿ 'a' of the particle if it's speed is 0.6 m/sec but increasing at 1.2 m/sec².

→ Given: $\rho = 0.4$m

$v = 0.6$ m/sec

$a = 1.2$ m/sec^2.

$a_T = 1.2$ m/sec^2.

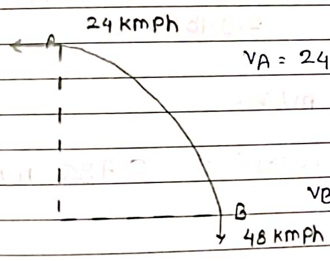
$\therefore a_N = \frac{v^2}{\rho} = \frac{0.6^2}{0.4} = 0.9$ m/s^2

$a = \sqrt{1.2^2 + 0.9^2} = 1.5$ m/s^2

$\therefore$ Resultant accⁿ of particle will be 1.5 m/s².

Q. An automobile enters the curve path which is in the form of quarter circle & of length 360 m at a speed of 24 kmph & then leaves the curve at 48 kmph if car is travelling with constant accⁿ find out resultant accⁿ at ~~0.8~~ pt. A & pt. B.

a_T = constant.



$V_A = 24 \times \frac{5}{18} = 6.66$ m/sec

$V_B = 48 \times \frac{5}{18} = 13.33$ m/sec

For constant accⁿ

$v^2 = u^2 + 2as$

$V_B^2 = u_A^2 + 2 a_T s$

$(13.33)^2 = (6.66)^2 + 2a_T \times 360$

$\boxed{a_T = 0.185 \text{ m/sec}^2}$

Now $\underbrace{\frac{2\pi\rho}{4}}_{\text{length of quarter circle}} = 360$ $\qquad \therefore \boxed{\rho = 229.18 \text{ m}}$

At pt. A.

$a_N = \frac{v^2}{\rho} = \frac{(6.66)^2}{229.18} = 0.193$ m/sec²

$a_T = 0.185$

$a = \sqrt{0.193^2 + 0.185^2} = 0.267$ m/sec²

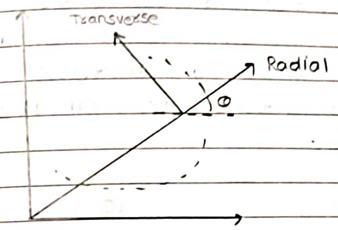
at pt. B.

$a_N = \frac{v_B^2}{\rho} = \frac{13.33^2}{229.18} = 0.775$ m/sec²

$a_T = 0.185$ m/sec²

$a = \sqrt{(0.775)^2 + (0.185)^2} = 0.796$ m/sec².

* Polar coordinates :-
(Radial & Transverse components).



Velocity

v_r = Radial component of velocity.
$= \dot{r}$

v_θ = Transverse component of velocity.
$= r \cdot \dot{\theta}$

$v = \sqrt{(v_r)^2 + (v_\theta)^2}$ $\alpha_v = \tan^{-1}\left|\frac{v_\theta}{v_r}\right|$

• Accleration :-

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = 2\dot{r} \cdot \dot{\theta} + r \cdot \ddot{\theta}$

$a = \sqrt{(a_r)^2 + (a_\theta)^2}$

$a_r = \ddot{r} - r\cdot\dot{\theta}^2$ $a = \sqrt{(a_r)^2+(a_\theta)^2}$

$a_\theta = 2\dot{r}\cdot\dot{\theta} + r\cdot\ddot{\theta}$

$v_r = \dot{r}$ $v_\theta = r\cdot\dot{\theta}$

Q. The polar coordinates of a particle moveing along a plan curve are given by $r = t^3 - 3t + 10$ & $\theta = 0.5r$ where t is in sec, θ is in radians & r is in metere. determine resultant accn of particle at t = 2sec

→

$r = t^3 - 3t + 10$ $\theta = 0.5r$

$\theta = 0.5(t^3 - 3t + 10)$

$\frac{dr}{dt} = \dot{r} = 3t^2 - 3$ $\dot{\theta} = 1.5t^2 - 1.5$

$\ddot{r} = 6t$ $\ddot{\theta} = 3t$

at t = 2sec at t = 2

$\dot{r} = 9$ $\dot{\theta} = 1.5(2)^2 - 1.5$

$\ddot{r} = 6\times2 = 12$ $\dot{\theta} = 4.5$

$\ddot{\theta} = 6$

$V_r = \dot{r} = 9$

$V_\theta = r\cdot\dot{\theta} = (2^3 - 6 + 10)\times 4.5 = 54$

$V = \sqrt{9^2 + 54^2} = 54.744$

$a_r = \ddot{r} - r\cdot\dot{\theta}^2$

$= 12 - (12\cdot(4.5)^2)$

$= -231$

$a_\theta = 2\dot{r}\cdot\dot{\theta} + r\cdot\ddot{\theta}$

$= 2(9)(4.5) + (12)(6) = 153$

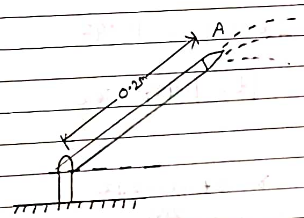
$a = \sqrt{231^2 + 153^2} = 277.073$

$\alpha_v = \tan^{-1}\left|\frac{54}{9}\right| = 80.537°$

$\alpha_a = \tan^{-1}\left|\frac{153}{231}\right| = -33.518°$.

Q. At the instant shown in fig below the water sprikler is rotating with $\dot{\theta} = 2^c/sec$ $\ddot{\theta} = 3^c/sec^2$. If the water is flowing through nozzle at constant rate at 3m/s find out resultant accn of the water particle as it leaves at open end A.

→



$\dot{\theta} = 2\ rad/sec$

$\ddot{\theta} = 3\ rad/sec^2$

$\dot{r} = V_r = 3m/sec$

$\bar{V}_r = \dot{r} = 3m/sec$

$V_\theta = r\dot{\theta} = 0.2\times2 = 0.4$

$V = \sqrt{3^2 + 0.4^2}$

$V = 3.026\ m/sec$

$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

$= 2\times3\times2 + 0.2\times3$

$= 12.6$

$a_r = \ddot{r} - r\dot{\theta}^2$

$= 0 - (0.2)(2)^2$

$= -0.8$

$a = \sqrt{0.8^2 + 12.6^2}$

$a = 12.625\ m/sec^2$

Q. The link OP rotates about hinge 'O' in the vertical plane & in ACW direction starting from horizontal position. A collar can slide on the link as shown in fig. below & the angular position of the link is given by $\theta = 0.5t^2$
$r = 2t - 0.3t^2$ this is position from collar with all the terms having regular meanings compute magnitude of velocity & accn. when $\theta = 60°$

$\rightarrow$

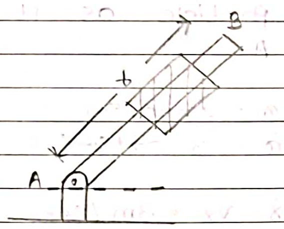
$\theta = 0.5t^2$

$r = 2t - 0.3t^2$

$\theta = 60° = 1.047^c$

$0.5t^2 = 1.047$

$\boxed{t = 1.447}$ sec



$r = 2t - 0.3t^2$ $\qquad$ $\theta = 0.5t^2$

$= 2.265$

$\dot{r} = 2 - 0.6t$ $\qquad$ $\dot{\theta} = \cancel{0.25}t$

$\ddot{r} = -0.6$

$\ddot{\theta} = 1$

$r = 2 - 0.6 \times 1.447$

$\dot{r} = 1.1318$ $\qquad$ $\dot{\theta} = 1.447$

$\ddot{r} = -0.6$ $\qquad$ $\ddot{\theta} = 1$

$a_r = -0.6 - 2.265 \cdot (1.447)^2$

$a_r = -5.342$

$a_\theta = 2 \times 1.131 \times 1.447 + 2.265 \times 1$

$= 5.538$

$a = \sqrt{(5.538)^2 + (5.342)^2} = 7.694 \ m/s^2$

$v_r = \dot{r} = 1.131$

$v_\theta = r \cdot \dot{\theta}$

$= (2.25)(1.44)$

$v_\theta = 3.278$

$v = \sqrt{v_r^2 + v_\theta^2} = \boxed{3.467}$ m/sec

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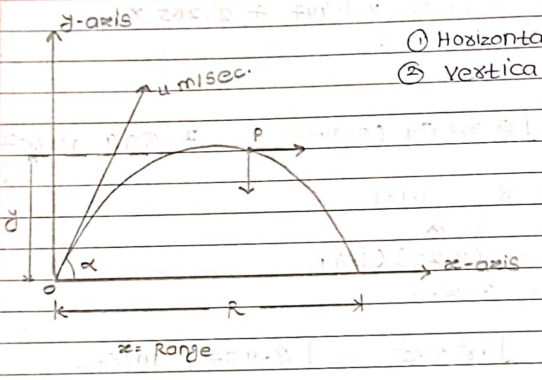
Projectile motion.



① Horizontal motion.
② Vertical motion.

projectile motion can be considered as special case of curvilinear motion when the partical is freely projected in air it follows parabolic path & it is acted upon by two components namely horizontal & vertical component hence the motion haveing combine effect of horizontal component & vertical component is knowns as projectile
hence projectile motion can be analysed by analysing horizontal & vertical motion separetly.

* Important terms in projectile

① Velocity of projection.
It is the velocity with which the projectile is projected upward. it is represented as u m/s.

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② Angle of projection :-
It can be defined as the angle with which partical is projected up. It is represented by 'α'

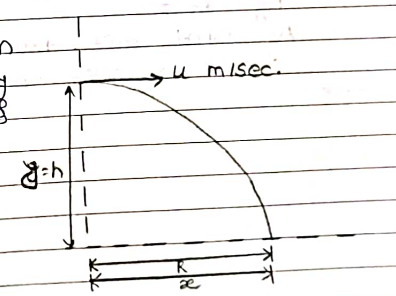
③ Range (x / r) :-
The horizontal dist. covered by projectile during the total flight is known as range.

④ Trajectory :-
The parabolic path traced by the particle during it's flight known as Trajectory

⑤ Time of flight :-
The total time taken by projectile / particle for the flight is known as Time of flight.

* Type-1 :- Particle projected horizontally. ($\alpha = 0°$)

This Type of motion can be analysed by analysing horizontal & vertical motion separatly.



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(A) Vertical motion :- Here acc^n is const. so we can use

$$S = ut + \frac{1}{2} g t^2$$

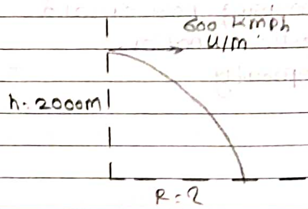
$$\boxed{h = 0 + \frac{1}{2} g t^2}$$ Falling down so g is +ve.

(B) Horizontal motion :- particle travels with const. Velocity (if air resistance is neglected)

$$S = ut + \frac{1}{2} g t^2 \qquad \boxed{g = 0 \text{ const.}}$$

$$\boxed{x = R = ut}$$

Q. A pilot flying with his Bombar at height of 2000m with a horizontal unit velocity 600 kmph wants to strike the target as shown in fig. below.
At what dist. from the target he should release the bomb.



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$$u = 600 \times \frac{5}{18} = 166.67 \text{ m/s.}$$

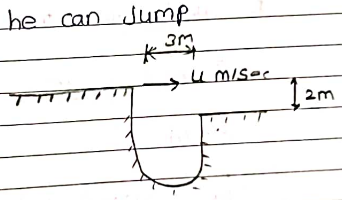
(A) Vertical motion.

$$2000 = \frac{1}{2} \times 9.81 \times t^2$$

$$\boxed{t = 20.192 \text{ Sec}}$$

(B) $x = ut$
$= 166.67 \times 20.192$
$x = 3365.525$ m.

Q. A person wants jump over ditch as shown in fig. below find maximum velocity with which he can jump



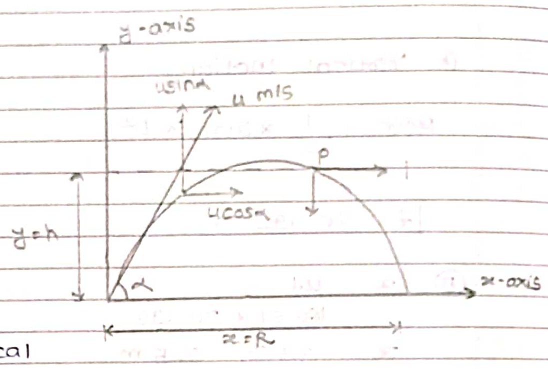
(A) vertical motion

$$2 = \frac{1}{2} \times 9.81 \times t^2$$

$t = 0.6386$ sec

(B) Horizontal
$3 = u \times 0.638$
$\therefore \boxed{u = 4.698}$ m/sec.

* Type - 2 : Inclined projectile on level ground.



For vertical

$S = ut + \frac{1}{2} g t^2$

$\therefore y = u\sin\alpha . t + \frac{1}{2} g t^2$ — ①

For Horizontal motion.

$S = ut + \frac{1}{2} g t^2$

$S = ut$

$\therefore \boxed{x = u\cos\alpha . t}$ — ②

$\therefore t = \frac{x}{u\cos\alpha}$ put in eqn ①

$y = u\sin\alpha + \frac{1}{2} g \left(\frac{x}{u\cos\alpha}\right)^2$

$= \frac{\cancel{u}\sin\alpha . x}{\cancel{u}\cos\alpha} + \frac{1}{2} g \left(\frac{x^2}{u^2\cos^2\alpha}\right)$

$y = x . \tan\alpha - \frac{1}{2} g \frac{x^2}{u^2\cos^2\alpha}$

but $\frac{1}{\cos^2\alpha} = \sec^2\alpha$ & $\sec^2\alpha = 1 + \tan^2\alpha$

$\therefore \boxed{y = x\tan\alpha - \frac{g x^2}{2u^2} (1 + \tan^2\alpha)}$

As this is eqn of parabola hence eqn of trajectory is parabola.

* maximum height reach by the particle.

we have :

$v^2 = u^2 + 2as$.

$u = u\sin\alpha$ . ... As max height

$v = 0$ At max. height

$a = -g$ ... For upward motion

$0 = u^2\sin^2\alpha - 2g h_{max}$

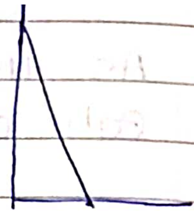
$\therefore \boxed{h_{max} = \frac{u^2\sin^2\alpha}{2g}}$

* Time required to reach max height

$v = u + at$

$0 = u\sin\alpha - gt$

$t = \frac{u\sin\alpha}{g}$ ... Half way flight



$\therefore$ Tota time $= \dfrac{2\,u\sin\alpha}{g}$

* Total horizontal dist. cover in total time.

$S = u\cos\alpha \cdot t$

$x = u\cos\alpha \cdot \dfrac{2u\sin\alpha}{g}$

$\boxed{\therefore x = \dfrac{u^2}{g} \cdot \sin 2\alpha}$ ... As $2\sin\alpha\cos\alpha = \sin 2\alpha$.

* maximum Range covered by particle

For max. Range $\sin 2\alpha = 1$ must.

$\therefore 2\alpha = 90^\circ$

$\therefore \boxed{\alpha = 45^\circ}$

① A ball is projected at an angle s.t. it's horizontal range is 3 times the maximum height covered. Find out angle of projection.

→

Range = 3 max. height

$\frac{u^2 \sin 2\alpha}{g} = 3 \cdot \frac{u^2 \sin^2\alpha}{2g}$

$\sin 2\alpha = 3 \cdot \sin^2\alpha$

$2 \cos\alpha \cancel{\sin\alpha} = \frac{3}{2} \sin\alpha \cdot \cancel{\sin\alpha}$

$\tan\alpha = \frac{4}{3}$

$\therefore \alpha = 53.13°$

② The horizontal component of the velocity of projectile is twice it's vertical component. find out range of horizontal plane, if the projectile passes through a point 18m horizontally & 3m vertically above point of projection. Find the range.

→

$\therefore \cancel{u}\cos\alpha = 2 \cdot \cancel{u}\sin\alpha$

$\tan\alpha = \frac{1}{2}$

$\therefore \boxed{\alpha = 26.565°}$

Now, $x = 18m$.

$y = 3m$

we have

$y = x \cdot \tan\alpha - \frac{g x^2}{2u^2}(1 + \tan^2\alpha)$

$3 = 18 \cdot \tan(26.565) - \frac{9.81 \times 18^2}{2 \times u^2}\left(1 + \frac{1}{4}\right)$

$3 = 9 - \frac{1986.525}{u^2}$

$-6 = \frac{1986.525}{u^2}$ $\therefore \boxed{u = 18.195 \text{ m/s}}$

$\therefore$ Range : $x = u\cos\alpha \cdot t$

$R = \frac{u^2 \cdot \sin 2\alpha}{g}$

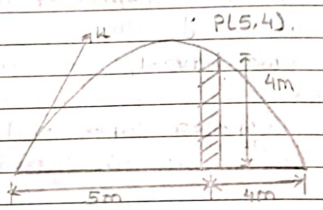
$R = \frac{(18.196)^2 \cdot \sin 2(26.565)}{9.81}$

$\therefore \boxed{R = 27m}$

Q. Find out the ~~dist~~ initial velocity with which projectile is to be projected so that it clears the wall 4m high. located at dist. 5m & strikes a the ground at a dist 4m beyond the wall as shown in fig below. the point of projection is at same level as that of foot of the wall.

R = 9 m.

we have



$R = \frac{u^2 \sin 2\alpha}{g}$

$9 = \frac{u^2 \sin 2\alpha}{g}$

$\therefore u^2 = \frac{9 \times g}{\sin 2\alpha}$ $\qquad$ $\therefore \boxed{u^2 = \frac{88.29}{\sin 2\alpha}}$ —— 1

At any point intial velocity is same. So write Trajectory eqn at pt. P. where $x = 5m$ $y = 4m$

$y = x \cdot \tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$

$4 = 5 \cdot \tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$

$\therefore 4 = 5 \tan\alpha - \frac{\cancel{g}x^2}{2\left(\frac{\overset{9\times g}{\cancel{88.29}}}{2\sin\alpha\cos\alpha}\right)} \times \frac{1}{\cos\alpha \times \cos\alpha}$

$\therefore 4 = 5\tan\alpha - \frac{x^2 \sin\alpha}{\cos\alpha}$

$4 = 5\tan\alpha - \frac{x^2}{9}\tan\alpha$

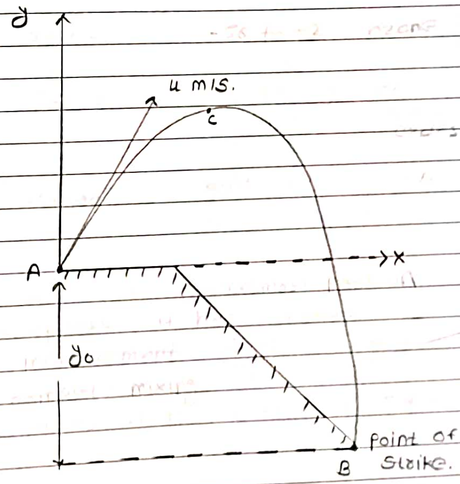
$4 = \cancel{20\tan\alpha}\ 5\tan\alpha - 2.777 \tan\alpha$

$\tan\alpha = \frac{+4}{\cancel{20}\ 2.222}$ $\qquad$ $\therefore \boxed{\alpha = 60.945^\circ}$

$\therefore u^2 = \frac{9 \times 9.81}{\sin 2(60.945)}$

$\therefore u = 10.197 \approx \boxed{10.20}$ m/sec

* Type - III :- Inclined projectile with point of strike & point of projection are at different levels :-



There are two methods to solve this type of projectile

(1) In this method numericals can be solved by putting $y = -y_o$ in the eqn of Trajectory

$\therefore \boxed{-y_o = u\sin\alpha \cdot t - \frac{1}{2}gt^2}$