UNIT I

FIRST ORDER ORDINARY DIFFERNTIAL EQUATIONS.

* **Defination:**

An equation involving dependant variable, an independant variable and the differntial coifficent of various order is called a differntial Eq^n (D.E)

(i) $\frac{dy}{dx} - y = 0$ (ii) $\frac{dy}{dx} = \cos x$ (iii) $\frac{d^2y}{dx^2} = 0$

(4) $(2x-y)dx + (y-x)dy = 0$

(7) $\frac{dy}{dx} + \frac{2}{x}y = x^2$

(5) $\sqrt{1+\frac{dy}{dx}} = \frac{d^2y}{dx^2}$

(8) $x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = nz$

(6) $\frac{dy}{dx} = \frac{x+y-3}{2x-y+1}$

(9) $\frac{\partial^2 y}{\partial t^2} = c^2 \cdot \frac{\partial^2 y}{\partial x^2}$

(10) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

* **Ordinary differntial Equations:**

Differntiable equation involving only one independant variable and one or more dependant variable and their derivative w.r.to independant variable are called Ordinary Differntial eq^n (O.D.E)

* **Partial differntial Equation:**

Differntial eq^n involving two or more independant variable and one or more dependant variable and theire partial Derivative w.r.to independant variables are called P.D.E

* **Order of differntial Equation.**

The order of diff. eq^n is the order of the highest derivative appears in the eq^n

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* Degree of Differential Eqⁿ:
The degree of differential eqⁿ is the degree of highest order differential coifficent or derivative when the differential coifficent free from radical and fractions.

Ex ① $\frac{dy}{dx} + \frac{2}{x} y = x^2$

order = 1, degree = 1

② $\sqrt{1+\frac{dy}{dx}} = \frac{d^2y}{dx^2}$

squaring on both side

$1+\frac{dy}{dx} = \left(\frac{d^2y}{dx^2}\right)^2$

order = 2, degree = 2

③ $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{d^2y}{dx} + 1 = 0$

order = 2, degree = 3.

④ $x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} = ny$

order = 2, degree = 1

* ⑤ $\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^2 z}{\partial x} = \frac{\partial z}{\partial y}$

order = 2, degree = 1

* solution of differential Eqⁿ

consider eqⁿ

$\frac{dy}{dx} = \cos x$ & $y = \sin x + c$

Here $\frac{d}{dx}(y) = \frac{d}{dx}(\sin x + c)$

$$\frac{dy}{dx} = \cos x + 0$$

$$\frac{dy}{dx} = \cos x$$

$\therefore$ $y = \sin x + c$ is solⁿ of Diff. eqⁿ $\frac{dy}{dx} = \cos x$

where c is arbitory constant

Defⁿ: A solⁿ is relation betⁿ dependant and independant variable which is free from derivative and satisfy differential eqⁿ.

* Genral solution of a Differential Eqⁿ:

Genral solⁿ of a Diff. eqⁿ is solⁿ which contain arbitory constant is equal to the order of the diff. eqⁿ is called genral solⁿ or complete integral.

Ex. $\frac{dy}{dx} = \cos x$ and solⁿ is $y = \sin x + c$

* Perticular solution of Differential Eqⁿ

The solⁿ obtained by assigning perticular value to the arbitory constants in Genral solⁿ (G.S) of diff. eqⁿ is called a perticular solⁿ or perticular Integral.

Note

① An arbitory constant may be written in such a form as to make the answer simple. Thus it may be written as c or $\log c$, $\sin^{-1}c$ or $\tan^{-1}c$ or e^{c} etc

(2) Total number of arbitory constant in genral solⁿ is equal to the order of the eqⁿ

Ex(1) obtain the differential equation whose genral solⁿ is $y = a\cdot e^{-9t}\cdot\cos(3t+b)$ where a and b arbitory constant.

$\Longrightarrow$ Given G.S is $y = a\cdot e^{-9t}\cdot\cos(3t+b)$

where two arbitory constant a and b

Hence we diff w.r.to t, twice

$$\frac{dy}{dt} = -a\cdot e^{-9t}\cdot\sin(3t+b)(3) - 9a\cdot e^{-9t}\cdot\cos(3t+b)$$

$$= -3a\cdot e^{-9t}\cdot\sin(3t+b) - 9y$$

$$\frac{dy}{dt} + 9y = -3a\cdot e^{-9t}\cdot\sin(3t+b) \quad \text{---} (1)$$

diff. w.r to t

$$\frac{d^2y}{dt^2} + 9\cdot\frac{dy}{dt} = -3a\left[3e^{-9t}\cos(3t+b) + 9\cdot e^{-9t}\sin(3t+b)\right]$$

$$\frac{d^2y}{dt^2} + 9\cdot\frac{dy}{dt} = -9a\cdot e^{-3t}\cdot\cos(3t+b) + 9\left[3a\cdot e^{-9t}\cdot\sin(3t+b)\right]$$

$$= -9y - 9\left(\frac{dy}{dt} + 9y\right)$$

$$= -9y - 9\cdot\frac{dy}{dt} - 81y$$

$$= -90y - 9\cdot\frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} + 18\frac{dy}{dt} + 90y = 0.$$

Q.2 Form the differential equation of which a genral solution is $y = \log \cos(x-a) + b$, where a and b are arbitory constant.

$\Rightarrow$ Given G.S $\quad y = \log \cos(x-a) + b$

Here two arbitory constant a and b so diff. twice.

$$\frac{dy}{dx} = \frac{1}{\cos(x-a)}(-\sin(x-a)) + 0$$

$$\frac{dy}{dx} = -\tan(x-a) \quad \text{——— (1)}$$

$$\frac{d^2y}{dx^2} = -\sec^2(x-a) = -[1+\tan^2(x-a)]$$

$$\frac{d^2y}{dx^2} = -1 - \tan^2(x-a)$$

$$\frac{d^2y}{dx^2} + 1 = -\left(\frac{dy}{dx}\right)^2 \qquad \left(\because \frac{dy}{dx} = -\tan(x-a) \text{ from (1)}\right)$$

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0$$ is the required diff. eq^n

order 2 and degree 1.

Q.3 From the differential equation whose genral sol^n is $y = a\cdot e^{-2x} + b\cdot e^{-3x}$

since y contains two arbitory constants, we have to differntiate twice.

$$y = a\cdot e^{-2x} + b\cdot e^{-3x} \quad \text{——— (1)}$$

$$\frac{dy}{dx} = -2a\cdot e^{-2x} - 3b\, e^{-3x} \quad \text{——— (2)}$$

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$$\frac{d^2y}{dx^2} = 4a\cdot e^{-2x} + 9b\cdot e^{-3x} \quad \text{—} ③$$

multiply eq^n ② by 5, eq^n ① by 6 and add to eq^n ③.

$$6y = 6a\cdot e^{-2x} + 6b\, e^{-3x}$$

$$+\ 5\cdot\frac{dy}{dx} = -10a\cdot e^{-2x} - 15b\cdot e^{-3x}$$

$$+\ \frac{d^2y}{dx^2} = 4a\cdot e^{-2x} + 9b\cdot e^{-3x}$$

$$\frac{d^2y}{dx^2} + 5\cdot\frac{dy}{dx} + 6y = 0.$$

* ORDINARY DIFFERENTIAL EQUATIONS OF THE FIRST ORDER AND FIRST DEGREE.

An ordinary diff. eq^n of first order and first degree is of the form $M + N\cdot\frac{dy}{dx} = 0$ or $M\cdot dx + N\cdot dy = 0$ where M and N are fun of x and y or constants.

There are Three Type of diff. eq^n.

(i) Diff. eq^n of variable seprable form
(ii) Exact diff. eq^n or Reducible to Exact form
(iii) Linear diff. eq^n or Reducible to linear form.

I] Differential Eq^n are in variable seprable form. or Reducible to variable seprable form.

A] variable seprable form (V.S)

First order diff. equation can be reduce to the form $\frac{dy}{dx} = \frac{f(x)}{g(y)}$

i.e $g(y)\cdot dy = f(x)\cdot dx$

Integrate on both side.

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$$\int g(y)\,dy = \int f(x)\,dx + c$$

B] Differentiable eq^n Reduciable to V.S form by using substitution.

① **Linear substitution:**
If diff. eq^n is of the form $\frac{dy}{dx} = f(ax+by+c)$
Then we substitution $ax+by+c = u$
~~Given diff eq^n is reduce to variable u, x~~

(ii) **Quotient substitution:**

(i) if $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ Then substitute $\frac{y}{x} = u$

$$\Rightarrow y = ux.$$

$$\Rightarrow \frac{dy}{dx} = u + x\cdot\frac{du}{dx}$$

(ii) if $\frac{dx}{dy} = f\left(\frac{x}{y}\right) \longrightarrow$ put $\frac{x}{y} = u \Rightarrow x = uy$

$$\frac{dx}{dy} = u + y\cdot\frac{du}{dy}.$$

Ex ① solve $y - x\cdot\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

$$\Rightarrow y - x\cdot\frac{dy}{dx} = ay^2 + a\cdot\frac{dy}{dx}$$

$$y - ay^2 = (a+x)\cdot\frac{dy}{dx}$$

$$\frac{dx}{(a+x)} = \frac{dy}{y(1-ay)}.$$

$$\int \frac{dx}{a+x} = \int\left(\frac{1}{y} + \frac{a}{1-ay}\right)dy + \log c$$

$$\log(a+x) = \log y + a\cdot\log(1-ay) + \log c$$

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$$\log(a+x) + \log(1-ay) - \log y = \log c$$

$$\log\left[\frac{(a+x)(1-ay)}{y}\right] = \log c$$

$$\frac{(a+x)(1-ay)}{y} = c$$

$$\boxed{(a+x)(1-ay) = yc}$$ is the G.S.

Q.2 solve $\frac{dy}{dx} + \frac{1+y^2}{1+x^2} = 0$

$$\Longrightarrow \int \frac{1}{(1+y^2)} \cdot dy + \int \frac{1}{1+x^2} \cdot dx$$

$$\tan^{-1} y + \tan^{-1} x = \tan^{-1} c$$

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1} c$$

$$\boxed{c = \frac{x+y}{1-xy}}$$ is the G.S

Q.3 solve $y \cdot \frac{dy}{dx} = \sqrt{1+x^2+y^2+x^2 \cdot y^2}$

$$y \cdot \frac{dy}{dx} = \sqrt{(1+x^2) + y^2(1+x^2)}$$

$$y \cdot \frac{dy}{dx} = \sqrt{(1+x^2)(1+y^2)}$$

$$\int \frac{y}{\sqrt{1+y^2}} \cdot dy = \int \sqrt{1+x^2} \cdot dx$$

$$\therefore \sqrt{1+y^2} = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left(x + \sqrt{1+x^2}\right) + c$$

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Q. solve $\frac{dy}{dx} = 1 - x \cdot \tan(x-y)$

put $x - y = u$

$1 - \frac{dy}{dx} = \frac{du}{dx}$

$\therefore \frac{du}{dx} = x \cdot \tan u$

$\frac{du}{\tan u} = x \cdot dx$

$\int x \cdot dx - \int \cot u \cdot du = 0$

$\frac{x^2}{2} - \log \sin u = C$

$\boxed{\frac{x^2}{2} - \log \sin(x-y) = C}$ is the G.S.

C] Homogeneous Differential Equations:

consider diff eq^n is in the form of $M(x,y)\,dx + N(x,y)\,dy = 0$ or $\frac{dy}{dx} = \frac{M(x,y)}{N(x,y)}$

above diff eq^n is said to be homogeneous if $M(x,y)$ and $N(x,y)$ same degree.

Ex solve $x \cdot \frac{dy}{dx} + \frac{y^2}{x} = y$

Given diff eq^n is homogenous

$\frac{dy}{dx} + \frac{y^2}{x^2} = y/x$

put $y/x = v$

$y = vx$

$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

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$$v + x\cdot\frac{dv}{dx} + v^2 = v$$

$$x\cdot\frac{dv}{dx} + v^2$$

$$\frac{1}{v^2}\cdot dv + \frac{1}{x}\cdot dx = 0$$

$$-\frac{1}{v} + \log x + C$$

$$\log x - \frac{1}{v} = C$$

$$\boxed{\log x - \frac{x}{y} = C}$$

Q. solve $(y^4 - 2x^3y)\,dx + (x^4 - 2xy^3)\cdot dy = 0$

$$y^4 - 2x^3y\cdot dx = (2xy^3 - x^4)\,dy$$

divide $x^4\cdot dx$

$$\left(\frac{y}{x}\right)^4 - 2\left(\frac{y}{x}\right) = \left[2\left(\frac{y}{x}\right)^3 - 1\right]\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^4 - 2\left(y/x\right)}{2\left(y/x\right)^3 - 1}$$

put $y/x = v$

$y = xv$

$$\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$$

$$v + x\cdot\frac{dv}{dx} = \frac{v^4 - 2v}{2v^3 - 1}$$

$$x\cdot\frac{dv}{dx} = \frac{v^4 - 2v}{2v^3 - 1} - v$$

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$$x\cdot\frac{du}{dx} = \frac{u^4-2u-2u^4+u}{2u^3-1} = \frac{-u^4-u}{2u^3-1}$$

$$x\cdot\frac{du}{dx} = \frac{u^4+u}{1-2u^3}$$

$$\frac{1-2u^3}{u^4+u} = \frac{1}{x}\cdot dx.$$

$$\int \frac{1}{u} - \frac{3u^2}{1+u^3}\cdot du = \int \frac{1}{x}\cdot dx$$

$$\log u - \log(1+u^3) = \log x + \log C$$

$$\cancel{\log C + \log x = \log \frac{u}{1+u^3}}$$

$$\cancel{\log xC = \log \frac{u}{1+u^3}}$$

$$x\,C = \cancel{\log}\ \frac{u}{1+u^3}$$

$$x\frac{(1+u^3)}{u} = C'$$

$$x\frac{\left(1+\frac{y^3}{x^3}\right)}{y/x} = C'$$

$$\boxed{x^3+y^3 = Cxy.}$$

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D] Non-Homogeneous Differential Equation Reducible to Homogeneous form

diff. eq^n of the form $\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$

Is called non-homogeneous diff eq.

case (i) If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

In this case $a_1x + b_1y$ & $a_2x + b_2y$ will have common factor of the for $lx + my$ and reduce above eq in V.

put $u = lx + my$

(ii) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

substitute $x = X + h$, $y = Y + k$

where h and k are constant to be determine.

Ex (1) solve $\frac{dy}{dx} = \frac{x+y+1}{2x+2y+1}$

$$\frac{dy}{dx} = \frac{x+y+1}{2(x+y)+1}$$

put $x+y = u$

$$1 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{du}{dx} - 1 = \frac{u+1}{2u+1}$$

$$\frac{du}{dx} = \frac{u+1}{2u+1} + 1$$

$$\frac{du}{dx} = \frac{u+1+2u+1}{2u+1} = \frac{3u+2}{2u+1}$$

$$\frac{du}{dx} = \frac{3u+2}{2u+1}$$

$$\frac{2u+1}{3u+2} \cdot du = dx$$

$$= \frac{\frac{2}{3}\left(3u+2-\frac{1}{2}\right)}{(3u+2)} \cdot du = dx$$

$$\frac{2}{3}\left[1 - \frac{\frac{1}{2}}{3u+2}\right] \cdot du = dx$$

$$\frac{2}{3}\int du - \frac{1}{2}\times\frac{2}{3}\int \frac{1}{3u+2} \cdot du = \int dx + c$$

$$\frac{2}{3}u - \frac{1}{3} \cdot \frac{\log(3u+2)}{3} = x + c$$

$$\frac{2}{3}u - \frac{1}{9}\log(3u+2) = x + c$$

$$6u - \log(3u+2) = 9x + c'$$

$$6(x+y) - \log(3x+3y+2) - 9x = c'$$

$$6y - 3x - \log(3x+3y+2) = c_1$$

* Solve $(2x-y+1) \cdot dy - (x+2y+3) \cdot dx = 0$

Given diff eq^n can be written as

$$\frac{dy}{dx} = \frac{x+2y+3}{2x-y+1}$$

$$\frac{a_1}{a_2} = \frac{1}{2} \qquad \frac{b_1}{b_2} = \frac{2}{-1}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

We use the substitution

$x = X + h, \quad y = Y + k$

$dx = dX \quad dy = dY$

$$\therefore \frac{dy}{dx} = \frac{dY}{dX}$$

$$\frac{dY}{dX} = \frac{X + 2Y + (h + 2k + 3)}{2X - Y + (2h - k + 1)}$$

we use h and k such that

$h + 2k + 3 = 0$

$2h - k + 1 = 0$

Now

$$\begin{array}{r} \cancel{2h} + 4k + 6 = 0 \\ -\ \cancel{2h} - k + 1 = 0 \\ \hline 5k + 5 = 0 \end{array}$$

$\boxed{k = -1}$

$\boxed{h = -1}$

$$\frac{dY}{dX} = \frac{X + 2Y}{2X - Y} = \frac{1 + 2 \cdot \frac{Y}{X}}{2 - \frac{Y}{X}}$$

put. $Y = VX$

$$\frac{dY}{dX} = V + X \cdot \frac{dV}{dX}$$

$$V + X \cdot \frac{dV}{dX} = \frac{1 + 2V}{2 - V}$$

$$X \cdot \frac{dV}{dX} = \frac{1 + 2V}{2 - V} - V$$

$$X \frac{dV}{dX} = \frac{1 + 2V - V(2 - V)}{2 - V}$$

$$x \cdot \frac{dv}{dx} = \frac{1+\cancel{2v}-\cancel{2v}+v^2}{2-v}$$

$$x \cdot \frac{dv}{dx} = \frac{1+v^2}{2-v}$$

$$\frac{2-v}{1+v^2} \cdot dv = \frac{1}{x} \cdot dx$$

$$2\int \frac{1}{1+v^2} dv - \frac{1}{2}\int \frac{2v}{1+v^2} \cdot dv = \int \frac{1}{x} \cdot dx$$

$$2 \cdot \tan^{-1} v - \frac{1}{2} \log(1+v^2) = \log x + C$$

$$2 \tan^{-1} v - \frac{1}{2} \log(1+v^2) - \log x = C$$

$$2 \tan^{-1} \frac{Y}{X} - \frac{1}{2} \log\left(\frac{1+\frac{Y^2}{X^2}}{X}\right) = C$$

$$2 \tan^{-1} \frac{Y}{X} - \frac{1}{2} \log \frac{X^2+Y^2}{X^3} = C$$

put $y = Y + k$ $\quad x = X + h$

$Y = y - k$ $\quad X = x - h$

$Y = y + 1$ $\quad X = x + 1$

$$2 \tan^{-1}\left(\frac{y+1}{x+1}\right) - \frac{1}{2} \log \frac{\cancel{(x+1)^2} (y+1)^2}{(x+1)^{\cancel{3}}} = C$$

$$\boxed{2 \tan^{-1} \frac{y+1}{x+1} - \frac{1}{2} \log \frac{(y+1)^2}{(x+1)} = C}$$

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* Exact Differntial Equations.

Consider the differential equation of the form
$M(x,y)\,dx + N(x,y)\cdot dy = 0$

There exist a fun $u(x,y)$ such that $M\cdot dx + N\cdot dy = du$
Then diff equation is called exact diff eqⁿ

Condition of Exactness:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

When condⁿ of exactness satiesfy, then genral solⁿ can be obtained by following Rule.

Rule 1: $\int_{y=\text{constant}} M\cdot dx + \int [\text{Terms of } N \text{ not containing } x]\, dy = $

Rule 2: If N has no terms which is free from x then

$\int_{y=\text{constant}} M\cdot dx = c$ is the genral solⁿ.

Rule 3:

Some time we write G.S as follow

$\int_{x=\text{constant}} N\cdot dy + \int [\text{Terms of } M \text{ not containing } y]\, dx = $

Remark:

sometimes an equation of the form

$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$ become exact if $b_1 = -a_2$

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$$(a_2x + b_2y + c_2) \cdot dy = (a_1x + b_1y + c_1) \cdot dx$$

$$(a_1x + b_1y + c_1) \cdot dx - (a_2x + b_2y + c_2) \cdot dy = 0$$

$M = a_1x + b_1y + c_1$ $\quad$ $N = -a_2x - b_2y - c_2$

$\frac{\partial M}{\partial y} = b_1$ $\quad$ $\frac{\partial N}{\partial x} = -a_2$

The Given eqⁿ is exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\boxed{b_1 = -a_2}$$

Ex solve $(x+y-2)\,dx + (x-y+4) \cdot dy = 0$

The given eqⁿ is of the form

$$M \cdot dx + N \cdot dy = 0$$

where

$M = x+y-2$ $\quad$ $N = x-y+4$

$\frac{\partial M}{\partial y} = 1$ $\quad$ $\frac{\partial N}{\partial x} = 1$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ Give diff eqⁿ is exact.

It's G.S solⁿ is given by

$$\int M \cdot dx + \int N \cdot dy = C$$

$$\int x+y-2\,dx + \int -y+4 \cdot dy = C$$

$$\frac{x^2}{2} + xy - 2x - \frac{y^2}{2} + 4y = C$$

$$x^2 + 2xy - 4x - y^2 + 8y = C_1$$

$$x^2 + 2xy - 4x - y^2 + 8y = C_1$$

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(Ex) Solve $\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$

Given diff eqⁿ can be written as

$(\tan y - 2xy - y)dx - (x^2 - x\tan^2 y + \sec^2 y)\cdot dy = 0$

$M = \tan y - 2xy - y$ & $N = -x^2 + x\cdot\tan^2 y - \sec^2 y$

$\frac{\partial M}{\partial y} = \sec^2 y - 2x - 1$ $\quad \frac{\partial N}{\partial x} = -2x + \tan^2 y\ 0$

$\frac{\partial M}{\partial y} = \tan^2 y - 2x$ $\quad \frac{\partial N}{\partial x} = \tan^2 y - 2x$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

The given diff. eqⁿ is exact
It's G.S is.

$\int M\cdot dx + \int N\cdot dy = c$

$\int \tan y - 2yx - y\cdot dx + \int -\sec^2 y\cdot dy = c$

$\tan y x - 2y\frac{x^2}{2} - yx - \tan y = C$

$$\boxed{x\cdot\tan y - yx^2 - yx - \tan y = c}$$

(Ex) Solve $(y^2\cdot e^{xy^2} + 4x^3)\cdot dx + (2xy\, e^{xy^2} - 3y^2)dy = 0$

Here $M = y^2\cdot e^{xy^2} + 4x^3$ $\quad \frac{\partial M}{\partial y} = 2y\cdot e^{xy^2} + y^2\cdot e^{xy^2}$

$N = 2xy\cdot e^{xy^2} - 3y^2$ $\quad \frac{\partial N}{\partial x} = 2y\cdot e^{xy^2} + 2xy\, e^{xy^2}(y^2)$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

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Given diff. eq^n is exact
G.S is

$$\int y^2 \cdot e^{xy^2} \cdot dx + 4\int x^3 \cdot dx - 3\int y^2 \cdot dy = c$$

$$y^2 \cdot \frac{e^{xy^2}}{y^2} + x^4 - y^3 = c$$

$$\boxed{e^{xy^2} + x^4 - y^3 = c}$$

* Equation Reducible to Exact form by using I.F

Integrating factor:

A fun^n $k(x,y)$ is said to be an Integrating factor of the equation $M\,dx + N\,dy = 0$ If it is possible to obtain a function $u(x,y)$ such that $k(M\,dx + N\,dy) = du$

In othere word, an I.F is a multiplying factor by which the equation can be made exact.

* Rule to find I.F of the eq^n $M \cdot dx + N \cdot dy = 0$ when it is not exact

Rule 1: If $x \cdot M + y \cdot N \neq 0$ and given diff. eq^n is homogenous then

$$I.F = \frac{1}{M \cdot x + N \cdot y}$$

Rule 2: If diff. eq^n is In the form of

$M \cdot y\,dx + N \cdot x \cdot dy = 0$ & $\frac{1}{Mx - Ny}$ s.t

$Mx - Ny \neq 0$

Then $I.F = \dfrac{1}{Mx - Ny}$

Rule 3

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x) \quad \text{Then } I.F = e^{\int f(x)\,dx}$$

Rule 4: $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)=f(y)$ then

$$I\cdot F = e^{\int f(y)\,dy}$$

Rule 5:

If the equation $M\cdot dx + N\cdot dy = 0$ can be written as

$x^a \cdot y^b (my\,dx + nx\cdot dy) + x^r \cdot y^s (py\cdot dx + qx\cdot dy)$.

where a, b, m, n, r, s, p, q are all constant having any value, then the $I\cdot F = x^h \cdot y^k$

where h and k such that after multiplying the I·F, The condition of exactness is satisfy.

Note: h, k can be determined from the following two equation.

$$nh - mk = (m-n) + (mb - na)$$

$$qh - pk = (p-q) + (ps - qr)$$, provided $mq - np \neq$

Ex. Solve $(xy - 2y^2)dx - (x^2 - 3xy)dy = 0$

$$M = xy - 2y^2 \quad \& \quad N = -x^2 + 3xy$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Given diff eqn is not exact, But It is homogenous. Also

$$~~Mx + Ny \neq 0~~$$

$$Mx + N\cdot y = \cancel{x^2y} - 2y^2x + (\cancel{-x^2y}) + 3xy^2$$

$$= xy^2$$

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By Rule I

$$I.F = \frac{1}{x.M + y.N} = \frac{1}{xy^2}.$$

$$\frac{1}{xy^2}\left([xy - 2y^2]\,dx - (x^2 - 3xy)\cdot dy\right]$$

$$\left[\frac{1}{y} - \frac{2}{x}\right]\cdot dx - \left(\frac{x}{y^2} - \frac{3}{y}\right)\cdot dy$$

$$\int M\cdot dx + \int N\cdot dy = c$$

$$\int \frac{1}{y} - \frac{2}{x}\cdot dx + \int \frac{3}{y}\cdot dy = c$$

$$\boxed{\frac{x}{y} - 2\log x + 3\cdot\log y = c}$$ is G.S.

Q.2 $(x^2y^2+2)y\cdot dx + (2-2x^2y^2)x\cdot dy = 0$

$M = (x^2y^2+2)y = x^2y^3+2y$ & $N = 2x - 2x^3y^2$

$\frac{\partial m}{\partial y} = 3x^2y^2+2$ $\quad \frac{\partial N}{\partial x} = 2 - \cancel{6x^2}$

$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$ Given diff eq^n is not exact.

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{x^3y^3 + \cancel{2yx} - \cancel{2xy} + 2x^3y^3}$$

$$I.F = \frac{1}{3x^3y^3}$$

multiply eq^n by I.F

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$$\frac{1}{3x^2y^3}\left[(x^2y^3+2y)\cdot dx\right]+\frac{1}{3x^2y^3}(2x-2x^2y^2)\,dy=0$$

$$\left(\frac{1}{3\cdot x} + \frac{2}{3x^3y^2}\right)\cdot dx + \left(\frac{2}{3x^2y^3} - \frac{2}{3y}\right)dy=0$$

The eq^n become exact

and It's sol^n is

$$\int \frac{1}{3x} + \frac{2}{3x^3y^2}\cdot dx + \frac{2}{3}\int\frac{1}{y}\,dy = c$$

$$\frac{1}{3}\log x + \frac{2}{3}\frac{1}{y^2}\frac{x^{-2}}{-2} + \frac{2}{3}\log y = c$$

$$\log x - \frac{1}{y^2x^2} + 2\log y = C_1$$

$$\boxed{\log\left(\frac{x}{y^2}\right) - \frac{1}{y^2x^2} = C_1}$$

Q solve $(x^2+y^2+x)\cdot dx + (xy)\cdot dy = 0$ — ①

The given eq^n is of the form of

$M\cdot dx + N\cdot dy = 0$, and eq^n ① is not exact.

$M = (x^2+y^2+x)$ and $N = x\cdot y$

$$= \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$$

$$= \frac{1}{x\cdot y}(2y - y)$$

$$= \frac{1}{x\cdot \cancel{y}}(\cancel{y})$$

$$F = \frac{1}{x} = f(x)$$

$$I\cdot F = e^{\int\frac{1}{x}dx} = e^{\log_e x} = x$$

$$\boxed{I \cdot F = x}$$

Now multiply by I·F to the eqⁿ ①

$$x(x^2+y^2+x) \cdot dx + x(x \cdot y) \cdot dy = 0$$

$$(x^3 + y^2x + x^2)\,dx + x^2y \cdot dy = 0 \quad \text{——} \ ②$$

$$\frac{\partial M}{\partial y} = 2xy \qquad \frac{\partial N}{\partial x} = 2xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{eq}^n \ ② \text{ is exact}$$

So solⁿ is

$$\int x^3 + xy^2 + x^2 \cdot dx + \int 0 \cdot dy = c$$

$$\frac{x^4}{4} + \frac{x^2 \cdot y^2}{2} + \frac{x^3}{3} = c$$

$$\boxed{3x^4 + 6x^2y^2 + 4x^3 = c_1}$$

Q. solve $y(2x^2y + e^x) \cdot dx = (e^x + y^3)\,dy$

$\Rightarrow$ The given eqⁿ $(2x^2y^2 + y \cdot e^x)dx - (e^x + y^3)\,dy = 0$ —— ①

$$\frac{\partial M}{\partial y} = 4x^2y + e^x \qquad \frac{\partial N}{\partial x} = -e^x$$

$$\Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow \text{eq}^n \ ① \text{ is not exact}$$

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{2x^2y^2 + y \cdot e^x}\left[-e^x - 4x^2y - e^x\right]$$

$$= \frac{-2(\cancel{e^x + 2x^2y})}{y(\cancel{2x^2y + e^x})} = \frac{-2}{y}$$

$$\therefore \ I\cdot F = e^{\int -\frac{2}{y}\cdot dy}$$

$$= e^{-2\log y}$$

$$= e^{\log y^{-2}}$$

$$= y^{-2} = \frac{1}{y^2}$$

$\therefore$ eqⁿ ① becomes

$$\left(2x^2 + \frac{e^x}{y}\right)\cdot dx + \left(\frac{-e^x}{y^2} - y\right)\cdot dy = 0 \quad \text{--- ②}$$

eqⁿ ② is exact and it's general solⁿ

$$\int 2x^2 + \frac{e^x}{y}\cdot dx + \int -y\cdot dy = c$$

$$\boxed{2\frac{x^3}{3} + \frac{e^x}{y} - \frac{y^2}{2} = c}$$

* Integrating Factors Found by Inspection.

(i) $x\cdot dy + y\cdot dy = d(xy)$

(ii) $\frac{x\cdot dy + y\cdot dx}{xy} = d(\log(xy))$

(iii) $\frac{x\cdot dy - y\cdot dx}{x^2} = d\left(\frac{y}{x}\right)$

(iv) $\frac{x\cdot dy - y\cdot dx}{x\cdot y} = d\left[\log\left(\frac{y}{x}\right)\right]$

(v) $\frac{x\cdot dy - y\cdot dx}{x^2+y^2} = d\left[\tan^{-1}\left(\frac{y}{x}\right)\right]$

VI] $\frac{x\cdot dy - y\cdot dx}{x^2-y^2} = d\left[\frac{1}{2}\log\frac{x+y}{x-y}\right)$

VII] $\frac{y\cdot dx - x\cdot dy}{y^2} = d\left(\frac{x}{y}\right)$

(8) $\dfrac{y \cdot dx - x \cdot dy}{x^2+y^2} = d\left(\tan^{-1}\left(\frac{x}{y}\right)\right)$

(9) $\dfrac{y \cdot dx - x \cdot dy}{xy} = d\left(\log\left(\frac{x}{y}\right)\right)$

(10) $\dfrac{x \cdot dx + y \cdot dy}{x^2+y^2} = \frac{1}{2}\, d\left(\log(x^2+y^2)\right)$

(11) $\dfrac{x \cdot dx + y \cdot dy}{\sqrt{x^2+y^2}} = d\left(\sqrt{x^2+y^2}\right)$

(12) $x \cdot dx + y \cdot dy = \frac{1}{2} d(x^2+y^2)$

(13) $dx + dy = d(x+y)$

(14) $\dfrac{dx+dy}{x+y} = d\left(\log(x+y)\right)$

(15) $(x+y)^n (dx+dy) = d\left[\dfrac{(x+y)^{n+1}}{n+1}\right]$ if $n \neq -1$

(16) $\dfrac{x \cdot dy + y \cdot dx}{x^2 y^2} = d\left(\dfrac{-1}{xy}\right)$

(17) $\dfrac{y \cdot 2x \cdot dx - x^2 \cdot dy}{y^2} = d\left(\dfrac{x^2}{y}\right)$

(18) $\dfrac{x \cdot 2y \cdot dy - y^2 \cdot dx}{x^2} = d\left(\dfrac{y^2}{x}\right)$

(19) $\dfrac{2x^2 y \cdot dy - 2y^2 x \cdot dx}{x^4} = d\left(\dfrac{y^2}{x^2}\right)$

(20) $\dfrac{2xy^2 dx - 2yx^2 \cdot dx}{y^4} = d\left(\dfrac{x^2}{y^2}\right)$

(21) $\dfrac{y \cdot e^x \cdot dx - e^x \cdot dy}{y^2} = d\left(\dfrac{e^x}{y}\right)$

Ex $(x+y)^2\left(x\cdot\frac{dy}{dx}+y\right)=xy\left(1+\frac{dy}{dx}\right)$

$\Rightarrow$ Given
$$(x+y)^2\left(x\cdot\frac{dy}{dx}+y\right)=xy\left(1+\frac{dy}{dx}\right)$$

$$(x+y)^2\left(\frac{x\cdot dy+y\cdot dx}{dx}\right)=xy\left(\frac{dx+dy}{dx}\right)$$

$$\int\frac{x\cdot dy+y\cdot dx}{x\cdot y}=\int\frac{dx+dy}{(x+y)^2}$$

$$\log x\cdot y=-\frac{1}{x+y}+c$$

$$\boxed{\log xy+\frac{1}{x+y}=c}$$ is the G.S.

Ex $e^{x+y}\left(x\cdot\frac{dy}{dx}+y\right)=e^{xy}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow$ $e^{x+y}(x\cdot dy+y\cdot dx)=e^{xy}(dx+dy)$

$$e^{-xy}\cdot d(xy)=e^{-(x+y)}d(x+y)$$

$$\int e^{-xy}\cdot d(xy)=\int e^{-(x+y)}\cdot d(x+y)$$

$$\boxed{e^{-xy}=e^{-(x+y)}+c}$$

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* Linear Differential Equation of First order

Defination:

A diff. eq^n is said to be linear if the dependant variable and it's derivative appear only in the first degree.

A diff. eq^n is of the form

$$\frac{dy}{dx} + Py = Q \quad , P, Q \text{ are fun of } x \text{ or constant.}$$

Method of sol^n

(i) find $I.F = e^{\int P \cdot dx}$

(ii) GS is given by $y \cdot e^{\int P\, dx} = \int Q \cdot e^{\int P\, dx} \cdot dx + C$

Similarly

diff. eq^n $\frac{dx}{dy} + Px = Q$ P and Q are fun of y or constant.

method of sol^n

(i) find $I.F = e^{\int P \cdot dy}$

(ii) Gs sol^n is

$$x \cdot e^{\int P \cdot dy} = \int Q \cdot e^{\int P \cdot dy} \cdot dy + C$$

Note: The coifficent of $\frac{dy}{dx}$ or $\frac{dx}{dy}$ must be equal to one

Q.1 solve $(1+y^2) + (x - e^{\tan^{-1} y}) \cdot \frac{dy}{dx} = 0$

we can write it as.

$$\frac{dx}{dy} + \left(\frac{1}{1+y^2}\right) x = \frac{e^{\tan^{-1} y}}{1+y^2}$$ which is linear in x

Here $I.F = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

G.S is

$$x \cdot e^{\tan^{-1} y} = \int \frac{e^{-\tan^{-1} y}}{1+y^2} \cdot e^{\tan^{-1} y} \cdot dy + c$$

$$= \int \frac{1}{1+y^2} \cdot dy + c$$

$$\boxed{x \cdot e^{\tan^{-1} y} = \tan^{-1} y + c}$$

Q.2 $x^2(x^2-1)\frac{dy}{dx} + x(x^2+1)y = x^2-1$

$$\frac{dy}{dx} + \left(\frac{x^2+1}{x(x^2-1)}\right) y = \frac{\cancel{x^2-1}}{x^2\cancel{(x^2-1)}}$$

$$\frac{dy}{dx} + \frac{x^2+1}{x(x^2-1)} y = \frac{1}{x^2}$$ which is linear in y

$$P = \frac{x^2+1}{x(x^2-1)} \qquad Q = \frac{1}{x^2}$$

$$\int P \cdot dx = \int \left(\frac{1}{x-1} + \frac{1}{x+1} - \frac{1}{x}\right) dx$$

$$= \log(x-1) + \log(x+1) - \log x$$

$$\int P \cdot dx = \log\left(\frac{x^2-1}{x}\right)$$

$$I.F = e^{\int P \cdot dx} = e^{\log\left(\frac{x^2-1}{x}\right)} = \frac{x^2-1}{x}$$

Solⁿ is $y \cdot I.F = \int Q \cdot I.F \cdot dx + C$

$$y\left(\frac{x^2-1}{x}\right) = \int \frac{1}{x^2} \cdot \frac{x^2-1}{x} \cdot dx + C$$

$$= \int \left(\frac{1}{x} - \frac{1}{x^3}\right) \cdot dx + C$$

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$$\frac{y(x^2-1)}{x} = \int\left(\frac{1}{x} - \frac{1}{x^3}\right).dx + c$$

$$\frac{y(x^2-1)}{x} = \log x + \frac{1}{2x^2} + c$$

$$\frac{y(x^2-1)}{x} - \log x - \frac{1}{2x^2} = c$$

Q. solve $(1+\sin y)\frac{dx}{dy} = 2y.\cos y - x(\sec y + \tan y)$

divide by $1+\sin y$

$$\frac{dx}{dy} = \frac{2y.\cos y}{1+\sin y} - \frac{x(\sec y + \tan y)}{1+\sin y}$$

$$\frac{dx}{dy} + x\frac{\left(\frac{1}{\cos y} + \frac{\sin y}{\cos y}\right)}{1+\sin y} = \frac{2y.\cos y}{1+\sin y}$$

$$\frac{dx}{dy} + x\frac{\left(\frac{\cancel{1+\sin y}}{\cos y}\right)}{\cancel{1+\sin y}} = \frac{2y.\cos y}{1+\sin y}$$

$$\frac{dx}{dy} + x.\sec y = \frac{2y.\cos y}{1+\sin y}$$

$P = \sec y \quad Q = \frac{2y.\cos y}{1+\sin y}$

$$I.F = e^{\int P.dy} = e^{\int \sec y.dy} = e^{\log(\sec y + \tan y)}$$

$I.F = \sec y + \tan y$

$$x.(I.F) = \int Q.I.F.dy + c$$

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$$x(\sec y+\tan y)=\int\frac{2y\cdot\cos y}{1+\sin y}\cdot(\sec y+\tan y)\cdot dy+c$$

$$=\int\frac{2y\cdot\cancel{\cos y}}{\cancel{1+\sin y}}\cdot\frac{\cancel{1+\sin y}}{\cancel{\cos y}}\cdot dy+c$$

$$=\int 2y\cdot dy+c$$

$$x(\sec y+\tan y)=y^2+c$$

$$\boxed{x(\sec y+\tan y)-y^2=c}$$

* Equations Reducible to the linear form.

(1) Bernoulli's Differential Equation

$\frac{dy}{dx}+Py=Q\cdot y^n$ is called Bernoulli's diff eq^n.

method.

Step 1 - divided by y^n

$$y^{-n}\cdot\frac{dy}{dx}+P\cdot y^{1-n}=Q \quad \text{--- (1)}$$

put $y^{1-n}=u$

$$(1-n)y^{-n}\frac{dy}{dx}=\frac{du}{dx}$$

$$y^{-n}\frac{dy}{dx}=\frac{1}{1-n}\cdot\frac{du}{dx}$$

eq^n (1) becomes

$$\frac{1}{1-n}\cdot\frac{du}{dx}+Pu=Q$$

$$\frac{du}{dx}+(1-n)P\cdot u=(1-n)Q$$

This linear eq^n

we solve this by methode of solving linear equations.

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(2) similar for equation $\frac{dx}{dy} + Px = Q \cdot x^n$.

Step 1

dividing by x^n

$$x^{-n} \frac{dx}{dy} + P \cdot x^{1-n} = Q$$

put $x^{1-n} = u$

$$(1-n) x^{-n} \frac{dx}{dy} = \frac{du}{dy}$$

$$x^{-n} \frac{dx}{dy} = \frac{1}{(1-n)} \cdot \frac{du}{dy}$$

$$\frac{1}{(1-n)} \cdot \frac{du}{dy} + Pu = Q$$

$$\frac{du}{dy} + (1-n) P \cdot u = (1-n) Q$$

which is linear therefor we can solved.

(3) Equation of the form

$$f'(y) \frac{dy}{dx} + P(f(y)) = Q$$

$\Rightarrow$ put $f(y) = u$

$$f'(y) \cdot \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{du}{dx} + Pu = Q \quad \text{which is linear.}$$

(4) similarly

$$f'(x) \cdot \frac{dx}{dy} + P \cdot f(x) = Q$$

put $f(x) = u$

$$f'(x) \cdot \frac{dx}{dy} = \frac{du}{dy}$$

$\therefore$ eqⁿ reduce to $\frac{du}{dy} + P\cdot u = Q$ which is linear

Q.1 solve $\sin y \cdot \frac{dy}{dx} = \cos x\,(2\cos y - \sin^2 x)$

put $\cos y = u$

$-\sin y \cdot \frac{dy}{dx} = \frac{du}{dx}$

$-\frac{du}{dx} = \cos x\,(2u - \sin^2 x)$

$\frac{du}{dx} + (2\cdot\cos x)\,u = +\sin^2 x \cdot \cos x$

which is linear in u

$P = 2\cdot\cos x \qquad Q = \sin^2 x \cdot \cos x$

$I.F = e^{\int P\,dx} = e^{\int 2\cdot\cos x\cdot dx} = e^{2\sin x}$

G.S is

$u\cdot e^{2\sin x} = \int \sin^2 x \cdot \cos x \cdot e^{2\sin x}\cdot dx + c$

put $\sin x = t \Rightarrow \cos x\cdot dx = dt$

$= \int t^2\cdot e^{2t}\cdot dt + c$

$= t^2\cdot\frac{e^{2t}}{2} - 2t\cdot\frac{e^{2t}}{4} + 2\,\frac{e^{2t}}{8} + c$

$u\cdot e^{2\sin x} = \frac{t^2\cdot e^{2t}}{2} - \frac{t\cdot e^{2t}}{2} + \frac{1}{4}\cdot e^{2t} + c$

$4u\cdot e^{2\sin x} = 2t^2\cdot e^{2t} - 2t\cdot e^{2t} + e^{2t} + c$

$4\cdot\cos y = 2\sin^2 t\cdot e^{2\sin t} - 2\sin t\cdot e^{2\sin t} + e^{2\sin t} + c$

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$$\boxed{u \cdot \cos y = 2\sin^2 t - 2\sin t + 1 + C_1 \cdot e^{-2\sin t}}$$

Q.2 $\cos y - x \cdot \sin y \cdot \frac{dy}{dx} = \sec^2 x$

put $\cos y = u$

$-\sin y \frac{dy}{dx} = \frac{du}{dx}$

$u + x \cdot \frac{du}{dx} = \sec^2 x$

$\frac{du}{dx} + \left(\frac{1}{x}\right) u = \frac{\sec^2 x}{x}$

$P = \frac{1}{x}$, $Q = \frac{\sec^2 x}{x}$

$I.F = e^{\int \frac{1}{x} \cdot dx} = e^{\log_e x} = x$

$u \cdot x = \int \frac{\sec^2 x}{\cancel{x}} \cancel{x} \cdot dx + c$

$\boxed{\cos y \cdot x = \tan x + c}$ is the G.S.

Q.3 $xy - \frac{dy}{dx} = y^3 \cdot e^{-x^2}$

Given diff eqn can be written as

$\frac{dy}{dx} - xy = -y^3 \cdot e^{-x^2}$

This is Bernoulli's eqn

divided by y^3

$y^{-3} \cdot \frac{dy}{dx} - x \cdot y^{-2} = -e^{-x^2}$

put $y^{-2} = u$

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$$-2\cdot y^{-3}\cdot\frac{dy}{dx} = \frac{du}{dx}$$

$$-\frac{1}{2}\cdot\frac{du}{dx} - x\cdot u = -e^{-x^2}$$

$$\frac{du}{dx} + 2x u = 2e^{-x^2}$$

$$P = 2x \quad , \quad Q = 2\cdot e^{-x^2}$$

$$I\cdot F = e^{\int 2x\cdot dx} = e^{x^2}$$

$$u\cdot e^{x^2} = \int 2\cdot e^{-x^2}\cdot e^{x^2}\cdot dx + C$$

$$= 2x + C$$

$$\boxed{\frac{e^{x^2}}{y^2} = 2x + c}$$ is G.S.

* Transformation to polars.

Some time it is not possible to solve a diff. eq^n in variable x, y by usual methods, but when the eq^n is transformed to polar by using $x = r\cos\theta$ $y = r\sin\theta$

In such case

$$x = r\cdot\cos\theta$$

$$y = r\sin\theta$$

$$x^2 + y^2 = r^2$$

$$\frac{y}{x} = \frac{r\cdot\sin\theta}{r\cdot\cos\theta}$$

$$\frac{y}{x} = \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

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$$2x\cdot dx + 2y\cdot dy = 2r\cdot dr$$

$$\Rightarrow x\cdot dx + y\cdot dy = r\cdot dr$$

Also $x\cdot dy - y\cdot dx$

$$= r\cdot\cos\theta\,(r\cdot\cos\theta\cdot d\theta + \sin\theta\cdot dr) - r\cdot\sin\theta\,(\cos\theta\cdot dr - r\sin\theta d\theta)$$

$$\boxed{x\cdot dy - y\cdot dx = r^2\cdot d\theta}$$

Q.1 solve $x^2(x\cdot dx + y\cdot dy) + y\,(x\cdot dy - y\cdot dx) = 0$

$\Rightarrow$ Transforming to polar co-ordinates by using $x = r\cdot\cos\theta$, $y = r\sin\theta$ and therefore $x^2+y^2=r^2$

$$x\cdot dx + y\cdot dy = r\cdot dr$$

Also $\frac{y}{x} = \tan\theta$

$$\frac{x\cdot dy - y\cdot dy}{x^2} = \sec^2\theta\cdot d\theta$$

we have

$$x\cdot dx + y\cdot dy + \frac{y\,(x\cdot dy - y\cdot dx)}{x^2} = 0$$

$$\therefore r\cdot dr + r\cdot\sin\theta\cdot\sec^2\theta\cdot d\theta = 0$$

$$\Rightarrow dr + \tan\theta\cdot\sec\theta\, d\theta = 0$$

$$\int dr + \int \sec\theta\cdot\tan\theta\cdot d\theta = C$$

$$r + \sec\theta = C$$

Hence $\sqrt{x^2+y^2} + \frac{\sqrt{x^2+y^2}}{x} = C$

i.e $\boxed{\sqrt{x^2+y^2}\,(x+1) = \alpha x.}$

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* Equation which are both homogenous and exact
suppose M and N are homo. fun of x and y of degree n $(n \neq -1)$
suppose $M \cdot dx + N \cdot dy = 0$ is exact
Then G.S is $Mx + Ny = c$

eg (1) $(x^3 + 3y^2x) \cdot dx + (y^3 + 3x^2y) \cdot dy = 0$

Given eqn homo. and exact
$\therefore$ The G.S is
$x(x^3 + 3y^2x) + y(x^3 + 3x^2y) = c$