EM-1 Resolution and Composition of Forces.

* Fundamental Concepts :-

① Particle :-

It is a material point which cointains definite quantity of matter without dimensions.

② Rigid body :-

- It is a body which doesn't undergo any deformation (change in shape and size) under the action of external force.
- Combination of large no. of particles which remains at fixed distance from each other when external force is applied or removed.

③ Force :-

- An action which tends to change the state of rest of body or motion of body on which it acts.

* Principle of statics :-

① Newton's 1st law :-

- Body remains in the state of rest or in motion untilland unless an external forced is applied.

② Newton's 2nd law :-

- When unbalanced force acts on particle, it will have an accleration which is proportional to the magnitude of force. this acc^n will be in the direction of force along straight line.
- Rate of change of momentum of body is directly proportional to the force acting on it and is in the direction of force.

③ Newton's 3rd law :-

- For Every action there is an equal and opposite reaction.

* Mechanics :-

- The branch of physical science that deals with state of the rest or state of the motion is called as Mechanics.

* Engineering mechanics :-

- The branch of Applied science, which deals with laws and principals of Mechanics along with their applications in real life engineering problems.

Eg :-
planning, designing, construction and machines.

* Classification of Engg. Mechanics :-

- Engg. Mechanics.
  - Mechanics of solids.
    - Mechanics of Rigid bodies.
      - Statics
      - Dynamics.
        - Kinematics
        - Kinetics.
    - Mechanics of Deformable bodies.
  - Mechanics of fluids.

- In our syllabus going to study mechanics of Rigid body. i.e = statics and Dynamics.

* Statics :-

- It is the branch of Engg. Mechanics which deals with the forces and their effects (acting) upon the bodies which are at rest.

* Dynamics :-

- It is the branch of engineering mechanics which deals with forces and their effects (acting) upon the bodies which are in motion.
- Dynamics has two types:-

i) Kinetics ii) Kinematics.

i) Kinetics:-

It is the branch of Dynamics which deals with the bodies in motion by considering the forces which causes the Motion.

ii) Kinematics:-

It is the branch of Dynamics which deals with the bodies in motion without considering the force which causes the Motion.

* Terms :-

i] Mass :- (m).

- The quantity of matter possessed by a body.
- S.I unit :- gram (g)
  kilogram (kg)

1kg = 1000g.

- $f = \frac{p}{t}$.

$f \rightarrow$ force.
$p \rightarrow$ Momentum
$t \rightarrow$ time.

Camlin Page
Date / /

iii) Weight :- (w).

- Force acting on the object due to gravity.
- It is the product of mass and gravitational Accleration.

$\therefore W = m \times g.$

- S.I unit :- Newton (N)

KiloNewton (KN).

- 1 kN = 1000 N.

* Force :- [(F) or (P)].

- It is an external agency which produce's or tend's to produce, destroy the motion.
- It is an external force which tend's to change the state of rest or state of Motion of body.
- It is a vector quantity.
- Force is also known as rate of change of momentum.

$\therefore$ momentum = mass $\times$ velocity.

$\therefore p = m \times v.$

as,

f = mass $\times$ acc^n

P = F = mass $\times$ rate of change of velocity.

$\therefore P = f = m \cdot a.$

- S.I unit :- Newton (N)

Kilo Newton (KN). $= 10^3$ N.

Meganewton (mN) $= 10^6$ N

Giganewton (GN) $= 10^9$ N.

Teranewton (TN) $= 10^{12}$ N.

- Force required to produce unit gravitational acc^n or unit mass.

$\therefore f = m \times a$

$= 1kg \times (9.81)\ m/s^2$. (gravity).

$\therefore f = 9.81\ kg.\frac{m}{s^2} = \underline{\underline{9.81\,N}}.$

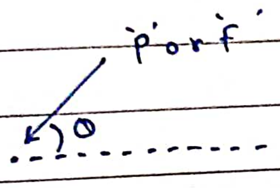
* Characteristics of force :-

i) Magnitude :- the value of force (10N, 20kN), etc.

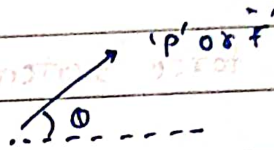
vector ii) Direction :- Line of action and angle formed with fixed axis.

iii) Nature of force or sense :-

It means whether the force is pull or push.



push type force.



Pull type force.

Push :- force acting toward's the point.

Pull :- force acting away from the point.

* Effect's of force:-

Force may produce following effect's on the body :-

i) It may change the state of body.

Rest body in Motion.

In Motion body to rest.

It may produce deformation in non rigid body.

ii) It may produce rotational effect in the body.

iii) It may produce internal stress in the body.

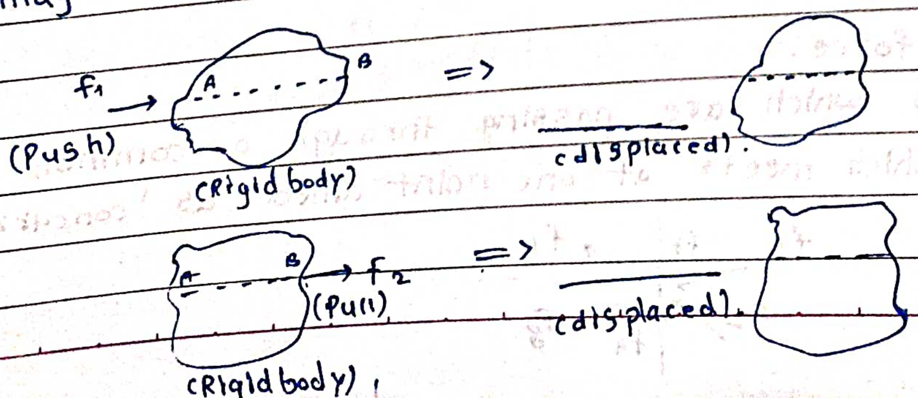
iv) It may produce stable state (equilibrium).

v) It may keep the body in stable state (equilibrium).

* Principle of transmissibility of force.

The state of the rigid body will not change. If force acting on a body is replaced by another force of same magnitude ~~&~~ direction acting along the same line of action.

∴ F_1, F_2 along same line of action (AB) do same ~~task~~.



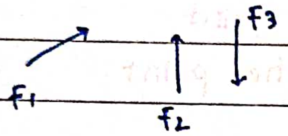
* System of force :-

- When a single force is acting on a body called as 'force'. whereas no. of forces acting simultaneously known as "system of force".

* Types of force system :-

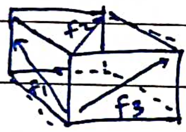
i) co-planer force's :-

- force's lie on same plane. (2D) (x-y axis).



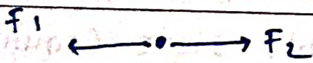
ii) Non-coplaner force's :-

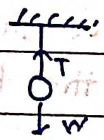
- Force's lie on different plane. (3D) (xyz axis).



iii) Co-linear force :-

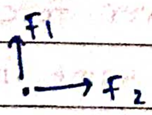
- Force's acting on same line of action.





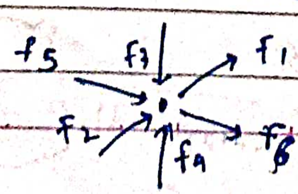
iv) Non co-linear force :-

- Force's not acting along same line of action.



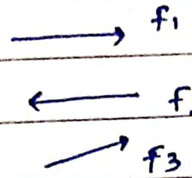
v) Concurrent force :-

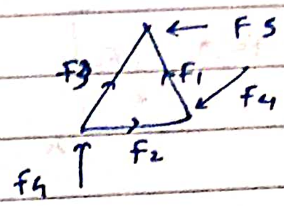
- The force's which are passing through a common point or which meet's at one point called as 'concurren forces'.



vi) Non - concurrent force's :-

- the force's which are not passing through a common point are called as non - concurrent force's.



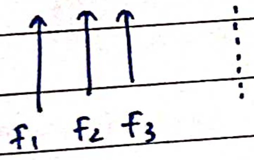


vii) Parallel force's :-

- the force's whose line of action's are parallel to each other are called as " Parallel force's".

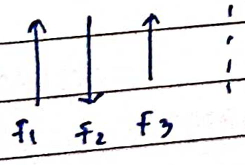
a) Like parallel force's :-

- Are firstly parallel force's and having same direction.



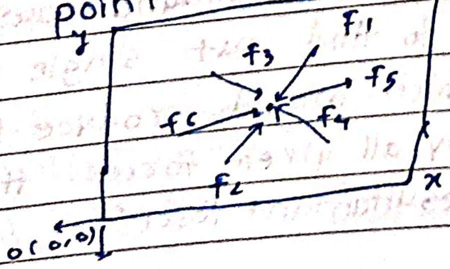
b) Unlike parallel force's:-

- Are firstly parallel force's and having opposite direction.



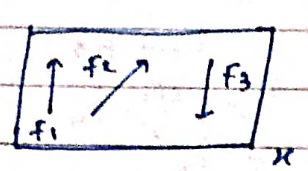
viii) Coplaner concurrent force's:-

- force's lying on same plane (x-y) and passe's or meet's through same point



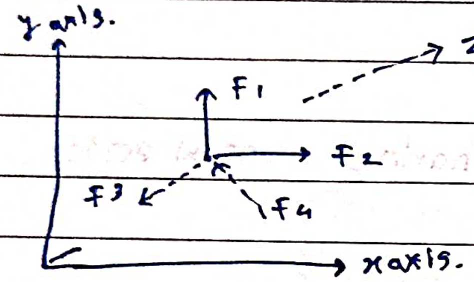
ix) coplaner nonconcurrent force's:-

- force's lying on same plane (xY) and doesn't meet's or pass through same point.

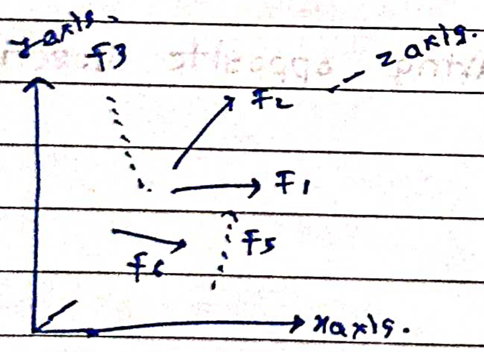


x) Non-coplaner concurrent force's:-

- Force's not lying on same plane (xY) axis but lying on different plane (xYz) axis and passe's through same point.



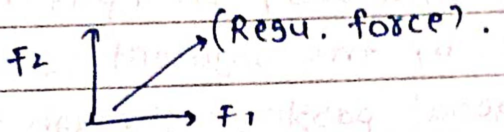
xi) Non-coplaner non-concurrent force's:-



* Resultant force:-

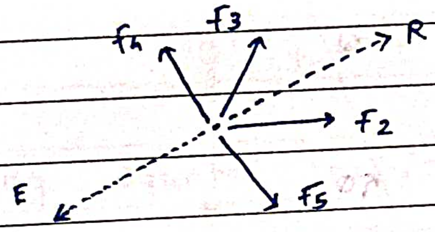
- If number of force's are acting simultaneously on a body then it is possible to find out single force which could replace them. which would produce the same effect as produced by all given force's. that single force is called as 'Resultantant force'.

- A single force which produces the effect that is produced by no. of forces.
- Resultant is denoted by 'R'.



* Equilibrium:-

- It is the single force which when acting with all other forces keeps the body at rest or in Equilibrium.
- Equilibrium is denoted by 'E'.



The Resultant and Equilibrium are equal in magnitude but opposite in the direction.

* Composition and Resolution of force:-

→ Composition:-

- The process of finding out the resultant force of a given force system.

– Resolution:-

- It is a procedure of splitting up a single force into number of components without changing the effect of same on the body.
- forces are generally resolved into two components. mostly forces are mutually ⊥ to each other.

* Methods of composition for Resultant force:-

A) Analytical method

B) Graphical Method.

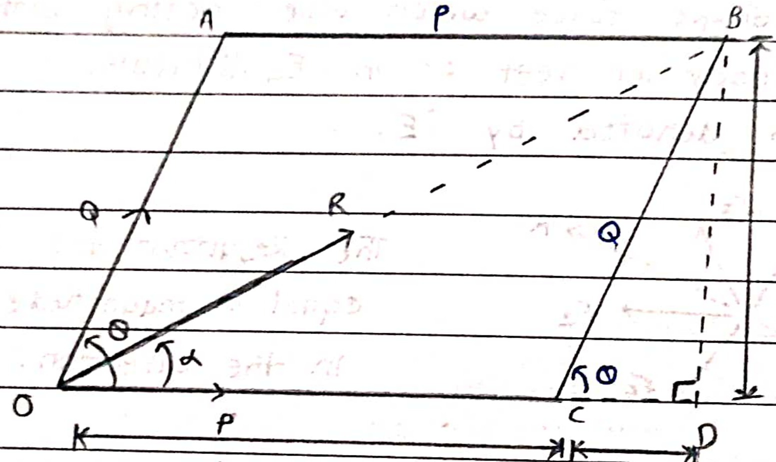
i) Resultant of two concurrent forces.
  ⇒ {Law of parallelogram of forces)

~~ii) Resultant of more than two forces. (con. or nonconcurrent).~~
~~⇒ (method of Resolution).~~

R of 2 concurrent forces.
=> Method is law of parallelogram.

1). Resultant of two concurrent forces:-
[Law of parallelogram of forces].

- If two forces acting simultaneously on a particle be represented in mag. and dir. by two adjacent sides of parallelogram, then the diagonal passing through the point of intersection of two forces will represent the resultant in magnitude and direction.



Consider two forces P and Q acting at a point represented by two sides OA and OC of a ∥gram OABC.

Let, θ is the angle betⁿ two forces (P) and (Q).
α is the angle betⁿ force (P) and resultant (R)

Let draw a line $BD \perp OC$ which intersects at D.
In △ BDC ;

$$\therefore \sin\theta = \frac{BD}{BC} = \frac{BD}{Q}.$$

$$\therefore BD = Q.\sin\theta. \quad -\text{(i)}$$

In same △;

$$\therefore \cos\theta = \frac{CD}{BC} = \frac{CD}{Q}$$

$$\therefore CD = Q.\cos\theta \quad -\text{(ii)}$$

In Δ BDO ;

By pythagarous theorm;

$$\therefore (BD)^2 + (OD)^2 = (OB)^2$$

$$\therefore (Q\sin\theta)^2 + (P + Q\cos\theta)^2 = (R)^2.$$

$$\therefore Q^2.\sin^2\theta + P^2 + 2PQ.\cos\theta + Q^2\cos^2\theta = R^2.$$

$$\therefore Q^2(\sin^2\theta + \cos^2\theta) + P^2 + 2PQ.\cos\theta = R^2.$$

$$\therefore Q^2 + P^2 + 2PQ.\cos\theta = R^2.$$

taking square root:-

$$\boxed{\therefore R = \sqrt{P^2 + Q^2 + 2PQ.\cos\theta}}$$ ---- Magnitude

for direction consider α :-

$$\therefore \tan\alpha = \frac{BD}{OD} = \frac{Q\sin\theta}{P + Q\cos\theta}.$$

$$\boxed{\therefore \tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}.}$$ ----- Direction.

Q. Two forces of 100N and 150N are acting simultanlously at a point. what is the resultant of these two forces if the angle betⁿ two forces is 45°.

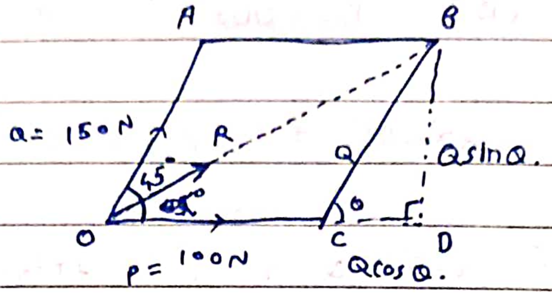
→ Given,

$P = 100\,N.$

$Q = 150\,N.$

$\theta = 45°.$

$R = ?\ ;\ \alpha = ?.$



Accⁿ to law of parallelogram of force :-

$\therefore R = \sqrt{P^2 + Q^2 + 2PQ.\cos\theta}\ .$

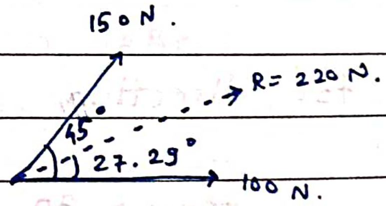
$\therefore R = \sqrt{(100)^2 + (150)^2 + 2.100.150.\cos 45°}\ .$

$\therefore R = \sqrt{(219.680)^2}$

$\therefore R = 219.680813146.$

$\therefore R \simeq 220\,N$ => Magnitude.

$\therefore \tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}.$



$\therefore \tan\alpha = \dfrac{150 \times \sin 45°}{100 + 150 \times \cos 45°}.$

$\therefore \tan\alpha = \cancel{0.5541744368}$
$0.5594713656\,3$

$\therefore \tan\alpha = \tan(27.29°)$

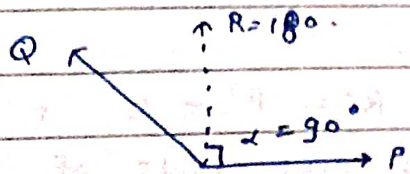
$\therefore \alpha = 27.29°$. => Direction.

Q. The sum of two force's is 270N and their resultant is 180N. if Resultant is perpendicular to P, find the two forces P and Q.

→ $P + Q = 270N.$

$R = 180N.$

$\alpha = 90^\circ.$



According to law of parallelogram;
by direction formulae :-

$$\therefore \tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}.$$

$$\therefore \tan 90^\circ = \frac{Q\sin\theta}{P+Q\cos\theta}.$$

$$\therefore \infty = \frac{Q\sin\theta}{P+Q\cos\theta}. = \frac{1}{0}$$

$$\therefore P + Q\cos\theta = 0$$

$$\therefore -P = Q\cos\theta. \quad \text{------} \quad (i).$$

Now, $\therefore R^2 = P^2 + Q^2 + 2PQ.\cos\theta.$

$$\therefore (180)^2 = P^2 + Q^2 + 2P(-P)$$

$$\therefore (180)^2 = P^2 - 2P^2 + Q^2$$

$$\therefore (180)^2 = -P^2 + Q^2$$

$$\therefore (180)^2 = Q^2 - P^2.$$

$$\therefore (180)^2 = (Q+P)(Q-P).$$

$$\therefore (180)^2 = (270)(Q-P).$$

$$\therefore Q - P = \frac{(180)^2}{270}.$$

$$\therefore Q - P = 120$$

$$\therefore Q - \cancel{P} = 120$$

$$+ \therefore Q + \cancel{P} = 270$$

$$2Q = 390$$

$$\therefore Q = 195$$

$$\therefore P = 75.$$

$\cdot \cos\theta \Rightarrow$

Date / /

③ for two forces P and Q acting at a point. maximum resultant is 2000 N and minimum magnitude of Resultant is 800 N. find values of P and Q.

→ As,

$$R^2 = P^2 + Q^2 + 2PQ.\cos\theta.$$

for max value of R, $\cos\theta \Rightarrow 1$ at $\theta = 0°$.

$$\therefore R_{max} = \sqrt{P^2+Q^2+2PQ.(1)}.$$

$$\therefore R_{max} = \sqrt{(P+Q)^2}$$

$$\therefore R_{max} = P+Q.$$

$$\therefore 2000 = P+Q. \quad -(i)$$

for min value of R, $\cos\theta \Rightarrow -1$ at $\theta = 180°$.

$$\therefore R_{min} = \sqrt{P^2+Q^2+2PQ.(-1)}$$

$$\therefore R_{min} = \cancel{\sqrt{P^2+Q^2}} . \sqrt{(P-Q)^2}.$$

$$\therefore 800 = P-Q \quad -(ii)$$

$$\therefore P + \cancel{Q} = 2000$$

$$+ \therefore P - \cancel{Q} = 800$$

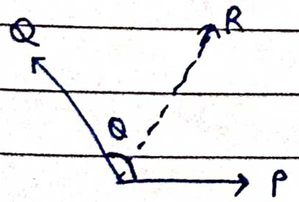
$$\therefore 2P = 2800$$

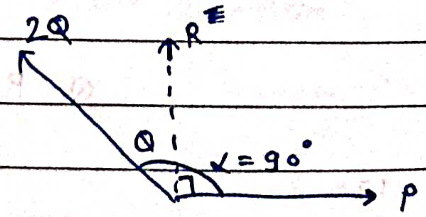
$$\therefore P = 1400\,N.$$

$$\therefore Q = 600\,N.$$

④ The angle between the two forces P and Q is θ. If Q is doubled then new resultant is perpendicular to P. proove that: Q = R.

→





$$\therefore \tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}. \quad \text{(for fig (2))}$$

$$\therefore \frac{1}{0} = \frac{2Q\sin\theta}{P+2Q\cos\theta}.$$

$$\therefore P + 2Q\cos\theta = 0$$

$\Rightarrow \quad -P = 2Q\cos\theta$. — (i)

and,

$\therefore R^2 = P^2 + Q^2 + 2PQ.\cos\theta$. (for fig(1))

$\therefore R^2 = P^2 + Q^2 + 2PQ.\cos\theta$.

$\therefore R^2 = P^2 + Q^2 + \cancel{2}P(-P)$

$\therefore R^2 = Q^2 \cancel{P^2}$

$\therefore R = Q$ Prooved.

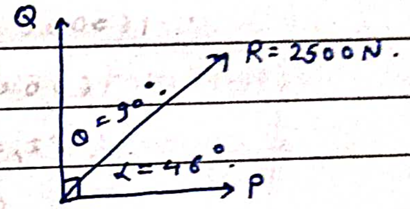
(5) The angle betⁿ the two concurrent forces is 90° and their resultant is 2500 N. the resultant makes an angle of 46° with one force. Determine magnitude of each force.

→ Given;

$\theta = 90°$.

$\alpha = 46°$.

$R = 2500N$.



by using parallelogram law of force:-

$\therefore R^2 = P^2 + Q^2 + 2PQ.\cos\theta$. $\cos 90° = 0$.

$\therefore (2500)^2 = P^2 + Q^2 + 2PQ.\cos 90°$.

$\therefore (2500)^2 = P^2 + Q^2$ — (i)

Now,

$\therefore \tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$.

$= \dfrac{Q\sin 90°}{P + Q\cos 90°}$

$\therefore \tan\alpha = \dfrac{Q}{P}$

$\therefore \tan 46° = \dfrac{Q}{P}$

$\therefore Q = 1.0355P$. — (ii)

Substitute (ii) in (i),

$\therefore (2500)^2 = P^2 + (1.0355P)^2$

$\therefore (2500)^2 = P^2 + (1.0355)^2 (P)^2$.

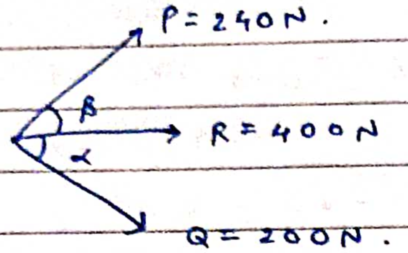
$\therefore (2500)^2 = P^2 (2.07226025)$

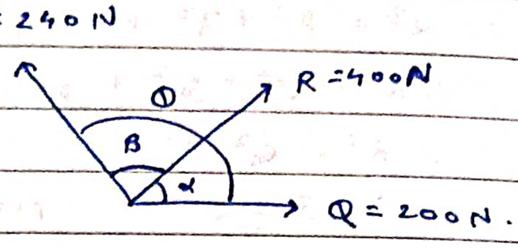
$\therefore P^2 = 3,016,030.44$

$Q = 1798.32$.

$\Rightarrow P = 1736.67$. → ans.

(6) Resultant force R = 400 N has two component forces P = 240 N and Q = 200 N as shown. determine direction components of α & β resultant force.





→ $\theta = (\alpha + \beta)$.

by parallelogram force of law:-

$\therefore R^2 = P^2 + Q^2 + 2PQ\cos\theta$.

$\therefore (400)^2 = (240)^2 + (200)^2 + 2\cdot(240)\cdot(200)\cdot\cos\theta$.

$\therefore 160000 = 57,600 + 40000 + 96,000 \cos\theta$.

$\therefore 160000 = 97,600 + 96,000 \cos\theta$.

$\therefore 62,400 = 96,000 \cos\theta$.

$\therefore 0.65 = \cos\theta$.

$\therefore \theta = \cancel{0.3632}\ 49.45$.

$\therefore \tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$.

$\therefore \tan\alpha = \dfrac{200 \cdot \sin(49.45)}{240 + 200\cdot\cos(49.45)}$

$\therefore \tan\alpha = \dfrac{151.9677}{370.0222}$

$\therefore \tan\alpha = 0.4106988716$.

$\therefore \alpha = 22.3279$.

$\therefore \beta = \theta - \alpha$

$\therefore \beta = 49.45 - 22.32$

$\therefore \beta = 27.13$

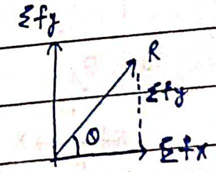
* Resultant of more than two force's :-
[method of Resolution].

- When two or more coplaner concurrent or non-concurrent force's are acting on a body then the resultant can be found out by using resolution of force's. procedure is below:-

i) Resolve all the force's Horizontally and find the algebric sum of all the horizantal component's.
If a force is denoted by f, then
f_x = Horizontal. component of force f (along x direction).
f_y = vertical component of force f (along y direction).
thus, in this step find Σf_x.

ii) Resolve all the force's vertically and find the algebric sum of all vertical component's.
∴ find Σf_y.

iii) The Resultant force will be inclined at an angle θ with horizontal, such that :-



$$\therefore R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$$

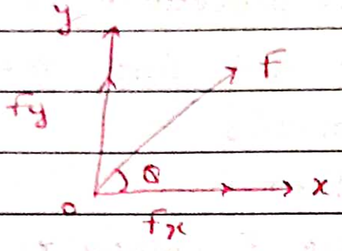
iv) The resultant force will be inclined at an angle θ with horizontal, such that :-
$$\therefore \tan\alpha = \frac{\Sigma f_y}{\Sigma f_x}$$

* Resolution of force :-
- following 3 method's are used to resolve the force.
i) Orthogonal Resolution (perpendicular resolution).
ii) Non-perpendicular / Non-orthogonal resolution.
iii) Resolution into two parallel component's.

Date / /

i) Orthogonal or perpendicular Resolution :-

In this method, force is split up into two components which are perpendicular to each other along x-direction & y-direction.



then;

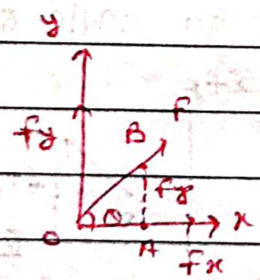
f_x = Horizontal component of force F along x-axis.

f_y = Vertical component of force F along y-axis.

θ = angle made by force F with x-axis.

angle betⁿ x & y axis is 90°.

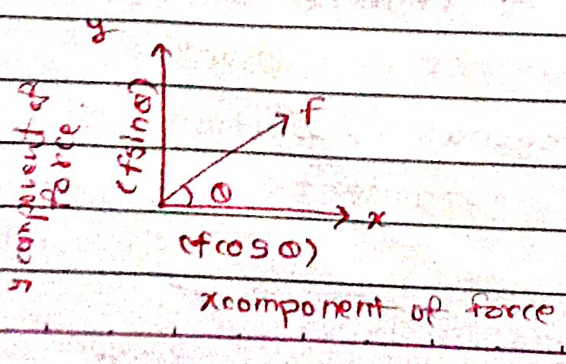
- draw AB ⊥ x axis.



$\therefore \sin\theta = \frac{AB}{OB}$ $\qquad$ $\therefore \cos\theta = \frac{OA}{OB}$

$\sin\theta = \frac{f_y}{F}$ $\qquad$ $\therefore \cos\theta = \frac{f_x}{F}$

$\therefore \underline{f_y = F\sin\theta}$. $\qquad$ $\therefore \underline{f_x = F\cos\theta}$.

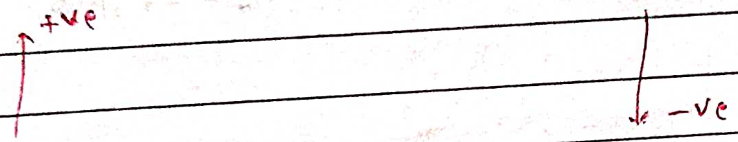


* Following are the different cases of resolution of force into two perpendicular components :-

- Sign conventions :-

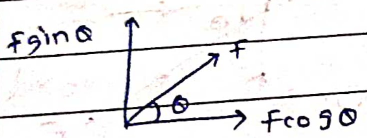
i) Components acting towards Right are positive & vice versa.

ii) Components acting upward are positive & vice versa.



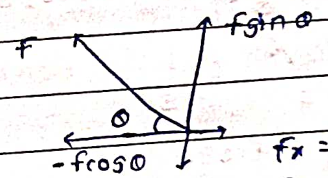
- Cases :- (force acting away from point).

a) force in 1st quadrant.



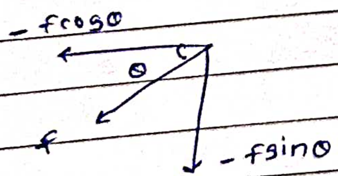
$F_x = F\cos\theta$.
$F_y = F\sin\theta$.

b) force in 2nd quadrant.



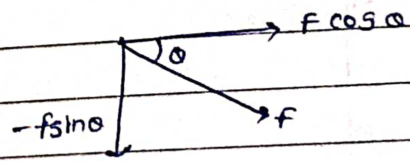
$F_x = -F\cos\theta$.
$F_y = F\sin\theta$.

c) force in 3rd quadrant.



$F_x = -F\cos\theta$.
$F_y = -F\sin\theta$.

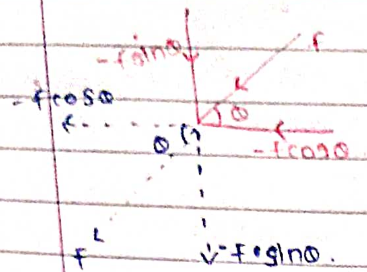
d) Force in 4th quadrant.



$F_x = F\cos\theta$.
$F_y = -F\sin\theta$.

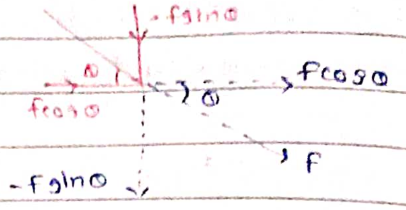
→ Cases of Resolution :- (Force acting towards the point).

a) Force in 1^{st} quadrant.



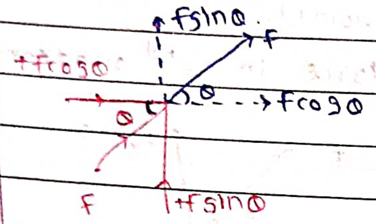
$F_x = -F\cos\theta$.
$F_y = -F\sin\theta$.

b) Force in 2^{nd} quadrant.



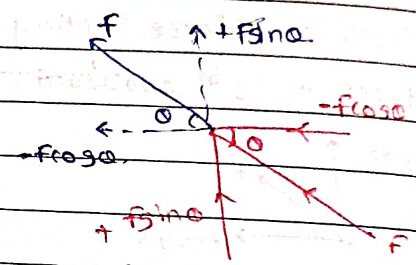
$F_x = F\cos\theta$.
$F_y = -F\sin\theta$.

c) Force in 3^{rd} quadrant.



$F_x = F\cos\theta$.
$F_y = F\sin\theta$.

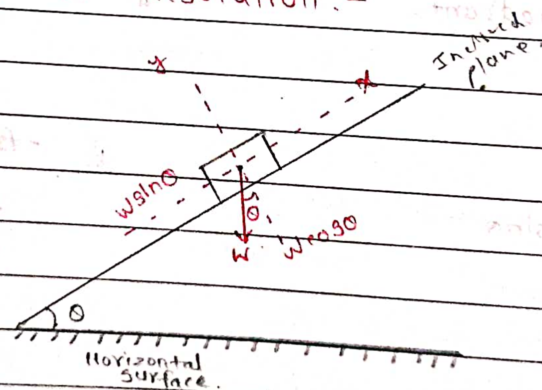
d) Force in 4^{th} quadrant.



$F_x = -F\cos\theta$.
$F_y = +F\sin\theta$.

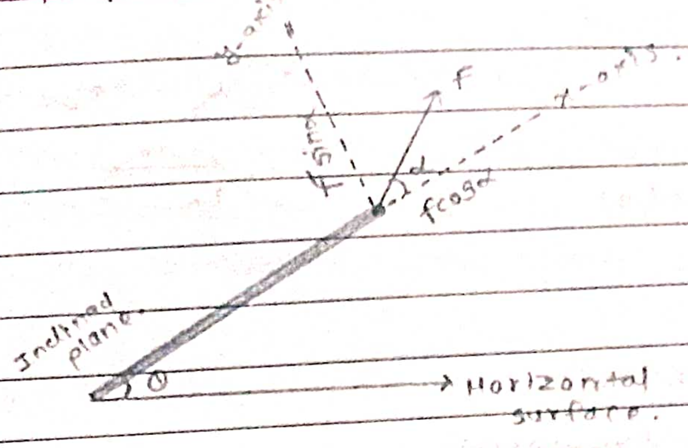
* Special cases of Resolution :-

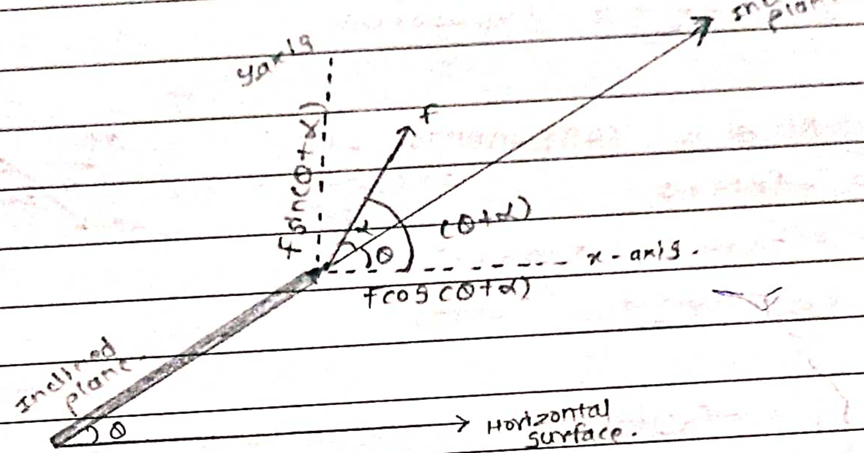
i)



component along the plane = $W\sin\theta$.
component ⊥ to plane = $W\cos\theta$.

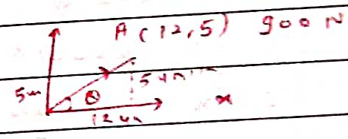
ii)





– Example's based on orthogonal Resolution :- ($\perp$ component's).

1). A force of 900 N act's from origin to a point A(12, 5). find two orthogonal component's.



$\rightarrow$ $\therefore \tan\theta = \frac{5}{12}$.

$\therefore \theta = \tan^{-1}\left(\frac{5}{12}\right)$.

$\therefore \theta = 22.62°$.

$\therefore F_x = F \times \cos\theta$

$= 900 \times \cos(22.62°)$

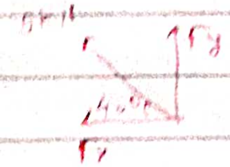
$F_x = 830.77\ N$.

$\therefore F_y = F_y \sin\theta$

$= 900 \times \sin(22.62°)$

$F_y = 346.16\ N$.

② find x & y components of :-



→ $\therefore f_x = -f \times \cos\theta$

$= -(5000) \times \cos(40°)$

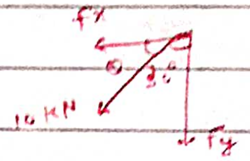
$\therefore f_x = 3830\ N.$ (left)

$\therefore f_y = f_y \sin\theta$

$= (5000) \times \sin(40°)$

$\therefore f_y = 3210\ N.$ (upwards).

③ find x & y component's of :-



→ $\therefore f_x = -f_x \cos\theta.$ $\theta = (90° - 30°)$ $= 60°.$

$= -(10{,}000) \times \cos 60°.$

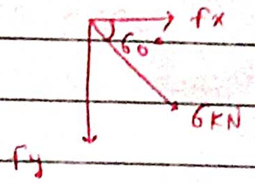
$\therefore f_x = 5\ KN$ (Left).

$\therefore f_y = -f_y \sin\theta.$

$= -(10{,}000) \times \sin(60°).$

$\therefore f_y = 8.66\ KN$ (downwoard).

④ find x & y component's of :-



→ $\therefore f_x = f_x \cos\theta$.

$= (6000) \times \cos 60°.$

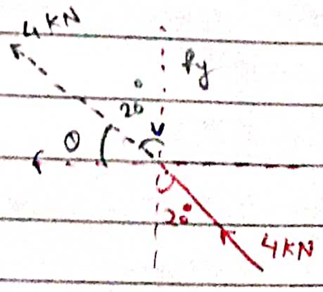
$\therefore f_x = 3000\ N$ (toward's right).

$\therefore f_y = -f_y \sin\theta.$

$= -(6000) \times \sin 60°.$

$\therefore f_y =$ ~~5.2 KN~~ $5200\ N$ (towards downword)

⑤ find x & y component's of :-



→ $\therefore f_x = -f_x \cos\theta$ $\theta = (90° - 20°)$ $= 70°.$

$= -(4000) \times \cos 70°$

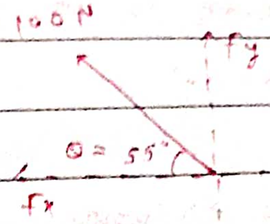
$\therefore f_x = 1370\ N$ (left).

$\therefore f_y = -f_x \sin\theta$

$= -(4000) \times \sin 70°.$

$\therefore f_y = 3.75\ KN$ (upward).

⑥ find x and y components of :-



$\therefore F_x = -F_x \cos 5\theta$.

$= -(100) \cdot \cos 55°$

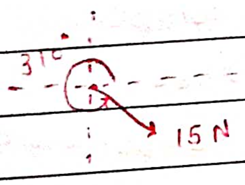
$\therefore F_x = \underline{57.36\,N}$ (left)

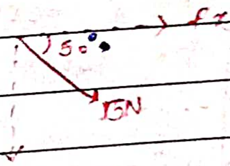
$\therefore F_y = F_y \cdot \sin\theta$.

$= (100) \cdot \sin 55°$

$\therefore F_y = \underline{81.91\,N}$ (upward)

⑦ find x & y components of :-





$F_x = F_x \cos\theta$

$= 15 \cos 50°$

$\therefore F_x = \underline{9.64\,N}$

$\therefore F_y = -F_y \cdot \sin\theta$

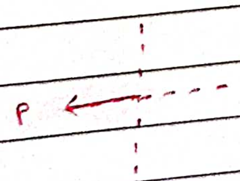
$= -15 \cdot \sin 50°$

$\therefore F_y = \underline{11.49\,N}$ (downwards)

find x and y components for the following :-

Q. Find x and y

a)



Here force is on negative x-axis.

$\therefore \theta = 0°$.

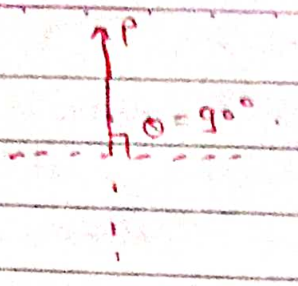
$\therefore F_x = -P\cos 0°$

$\therefore F_x = \underline{-P}$ (left)

$\therefore F_y = \underline{0}$.

$\Sigma f_x = 0$
$\Sigma f_y = 0$.

b)



→ Here force is on y axis

$\therefore \theta = 90°$

$\therefore f_x = f_x \cos\theta$
$= f_x \times \cos 90°$
$\therefore f_x = \underline{0}$

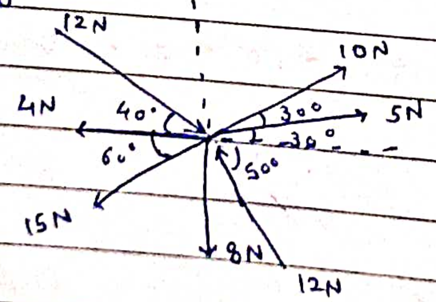
$\therefore f_y = + f_y \sin\theta$
$= P . \sin 90°$
$\therefore f_y = \underline{P}$.

- thus we can say that :-

i) If a force is on x - axis then it's x components is the force itself and y component will be zero.

ii) If a force is on y-axis, then it's y component is the force itself. and x component will be zero.

2] Determine the resultant of following force system as shown in figure.



→ Now Resolving all the forces Horizontally along x-
we have,

$\therefore \Sigma f_x = 10 \cos 60° + 5 \cos 30° . + 12 \cos 40° - 12 \cos 50° - 15 \cos$
$- 4 \cos 0°$.
$= -0.69 N$

$\therefore \Sigma f_x = \underline{0.69 N}$ towards left.

Now resolving all the force's vertically along y-axis we have,

$\therefore \Sigma f_y = -12\sin 40° - 8\sin 90° + 12\sin 50° - 15\sin 60° + 10\sin 80° + 5\sin 30°$

$= -8.35\,N$

$\therefore \Sigma f_y = \underline{8.35\,N}$ (downwards)

$\therefore$ Resultant $= \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$

$= \sqrt{(0.69)^2 + (8.35)^2}$

$\therefore R = \underline{8.38\,N}$ (mag. of Resultant).

$\therefore$ Direction:-

$\tan\alpha = \dfrac{\Sigma f_y}{\Sigma f_x}$

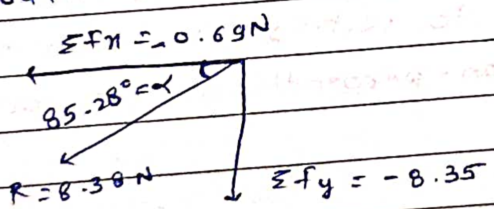
$\therefore \tan\alpha = \dfrac{8.35}{0.69}$

$\therefore \tan\alpha = 12.1014$

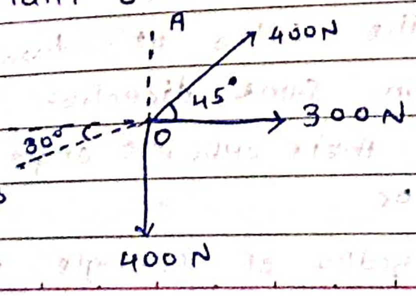
$\therefore \alpha = \tan^{-1}(+12.1014)$

$\therefore \alpha = \underline{85.28°}$

$\therefore$ Layout:-



Q. Three concurrent co-planer force's act's on a body of point O. Determine two additional forces along OA & OB such that Resultant of five forces is zero.



For Resultant to be zero $\Sigma f_x = 0$; $\Sigma f_y = 0$.

$\therefore \Sigma f_x = 0$

$\therefore +400 \cos 45° + 300 \cos 0° \pm 400 \cos 90° \pm A \cos 90° - B \cos 30° = 0.$

$\therefore 400 \cos 45° + 300 - B \cos 30° = 0.$

$\therefore 282.843 + 300 = 0.866 B,$

$\therefore 582.843 = 0.866 B.$

$\therefore \underline{B = 673.03 N}.$

$\therefore \Sigma f_y = 0$

$\therefore +400 \sin 45° - 400 \sin 90° + A \sin 90°. = 0$

$\therefore 282.843 - 400 + A \sin 90° - B \sin 30° = 0$

$\therefore 282.843 - 400 - 673.03 \sin 30° + A = 0,$

$\therefore -453.672 + A = 0.$

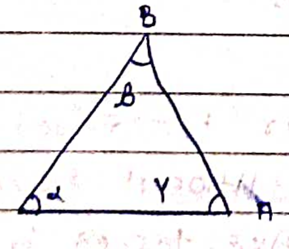
$\therefore \cancel{-}A = \cancel{-}453.672.$

$\therefore \underline{A = 453.672 N}.$

ii) Non - perpendicular / non - orthogonal Resolution:-

When a force is resolved or split into two directions which are not perpendicular to each other, then the resolution is called as Non - perpendicular / non - orthogonal

- Sine Rule



It states that, the ratio of two sides of an oblique triangle is in same direction as that of the ratio of sines of their opposite angle.

or

the ratio of two sides of triangle will be equal to the ratio of sine of their opposite angle.

$\therefore \dfrac{AB}{BC} = \dfrac{\sin\alpha}{\sin\gamma}$ $\qquad$ $\dfrac{BC}{AC} = \dfrac{\sin\gamma}{\sin\beta}$

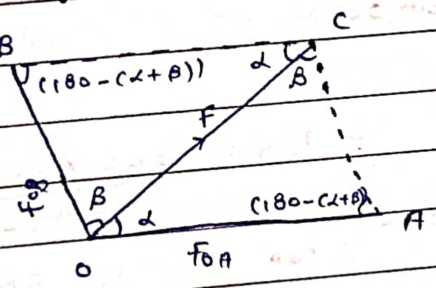
$\Rightarrow$ $\cancel{\sin\gamma}$

$\dfrac{AB}{\sin\alpha} = \dfrac{BC}{\sin\gamma}$ — (i) $\qquad$ $\dfrac{BC}{\sin\gamma} = \dfrac{AC}{\sin\beta}$ — (ii)

from (i) & (ii);

$$\frac{AB}{\sin\alpha} = \frac{BC}{\sin\gamma} = \frac{AC}{\sin\beta}$$ — sine rule.

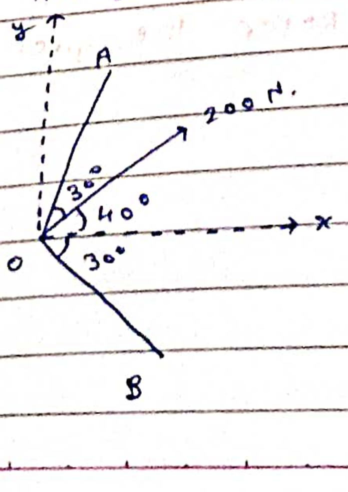
Then to resolve the force into two Non-perpendicular components,
construct the parallelogram by keeping orignal given force along diagonal of two components along two sides of ||gram passing through point of force.

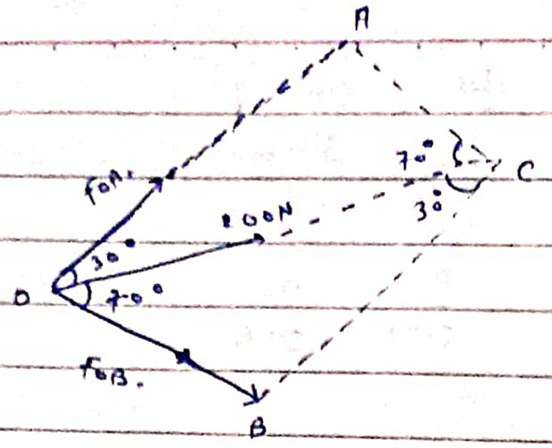


In Δ OAC, by sine rule,

$$\frac{F_{OB}}{\sin\alpha} = \frac{F_{OA}}{\sin\beta} = \frac{F}{\sin[180-(\alpha+\beta)]}.$$

Q: Resolve 200 N force into two components along A and B direction. Refer the given figure.





As angle betⁿ OA and OB is 100° ∴ resolution is non-perpendicular.

∴ by sine rule.

$$\therefore \frac{F_{OA}}{\sin 70^\circ} = \frac{F_{OB}}{\sin 30^\circ} = \frac{200}{\sin[180-(30+70)]}$$

$$\therefore \frac{F_{OA}}{\sin 70^\circ} = \frac{F_{OB}}{\sin 30^\circ} = \frac{200}{\sin 80^\circ}$$

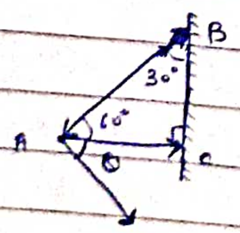
$$\therefore F_{OB} = \frac{200 \times \sin 30^\circ}{\sin 80}$$

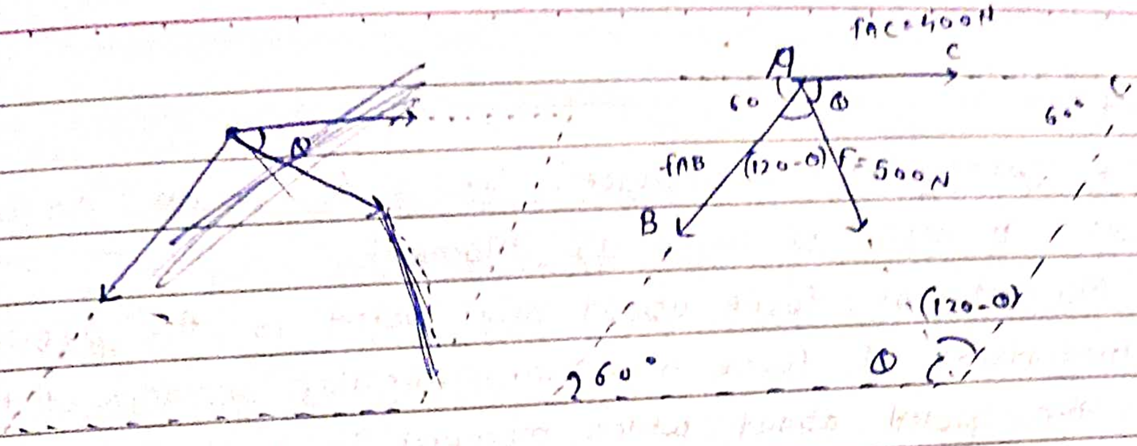
$$\therefore F_{OA} = \frac{200 \times \sin 70^\circ}{\sin 80^\circ}$$

$\therefore F_{OB} = 190.84\,N$.

$\therefore F_{OA} = 101.5\,N$.

Q. The force F acting on the frame has a magnitude of 500 N and it is to be resolved into two components acting along AB & AC. Determine the angle θ so that the component F_{AC} is directed from A towards C & has magnitude of 400N. Refer the given figure.





by sine rule :-

$$\therefore \frac{F_{AC}}{\sin(120-\theta)} = \frac{F_{AB}}{\sin\theta} = \frac{500}{\sin 60}$$

$\therefore$ ~~FAB = 500~~

$$\therefore F_{AC} = 500 \times \frac{\sin(120-\theta)}{\sin 60}$$

$$\therefore \frac{400 \times \sin 60}{500} = \sin(120-\theta)$$

$$\therefore 0.693 = \sin(120-\theta)$$

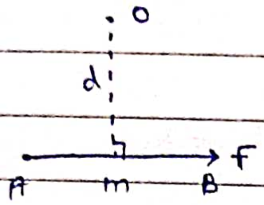
$$\therefore 120-\theta = 43.87$$

$$\therefore \theta = \underline{76.13^\circ}.$$

* **Moment :-**

The turning effect produced by a force on the body on which it acts is called as Moment.

Moment of force about any point is the product of magnitude of force and perpendicular distance of force from the point about which moment is to be taken.



clockwise ↻ => +ve.

Anticlockwise ↺ => -ve.

Let force F is applied at point A as shown. and O is any point about which we want to take moment thus from point O, Draw OM line perpendicular to the line of action of force.

OM = d = perpendicular distance bet^n force & point O.

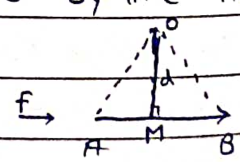
$\therefore$ Moment = M = F x d.

- **Graphical representation of Moment :-**

i) Draw a line parallel to line of action of force.

ii) Length of line is equal to force.

iii) In the figure, force F is represented by line AB. and OM = d = perpendicular distance.



$\therefore$ by def^n :-

M = F x d. - (by def^n).

iv) Now join OA and OB; then, In Δ OAB.

Area (Δ OAB) = $\frac{1}{2}$ x AB x OM.

2 (Area of ΔOAB) = F x d = Mo.

- MOI is equal to product of force and perpendicular dist^n or twice the area of triangle.

* Varignon's theorm [Law of Moment's] :-

- It state's that if number of force's are acting simultaniously on a body, the algebric sum of moment's of all the force's about any point is equal to moment of their resultant about same point.

Mathematically ;

$\Sigma (F \times d) = (R \times x)$

where,

F = All force's acting on a body.

d = $\perp^{lar}$ distance.

R = Resultant of a force.

x = $\perp^{lar}$ distance of Resultant force

- Use :-

The theorm is useful to calculate or find the position or location of Resultant of Non-concurrent force system. (in given figure).

* Couple :-

- A pair of two equal and opposite (unlike) parallel force's (of same magnitude) is known as couple.

— Properties of couple :-

i] Two unlike parallel, non-collinear force's of same magnitude will form couple.

ii] Resultant of couple is always zero.

iii] Moment of couple is product of one of the force of Lever arm of couple. $\therefore M = F \times d$.

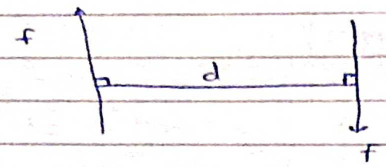
Lever arm of couple = $\perp^{lar}$ distance bet^n couple force's.

Lever arm of couple balanced by single force.

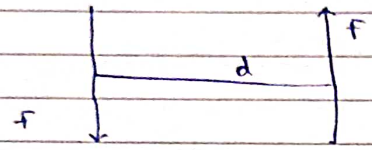
iv] Couple cannot be balanced by single force.

v] Couple can be balanced by another couple of same & opp. nature

vi] Reference point is not required to take moment of couple.

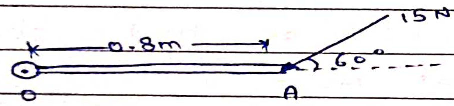
① 

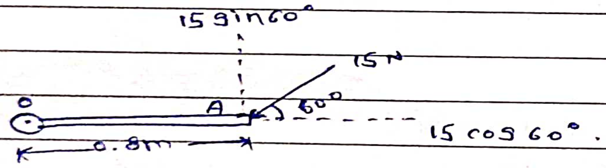
Clockwise couple.

② 

Anticlockwise couple.

Q. A force of 15N is applied at an angle of 60° to the edge of door as shown in figure. Find the moment of this force.



→ 

$\therefore \cancel{M_o = (15 \sin 60° \times 0)} + (15$

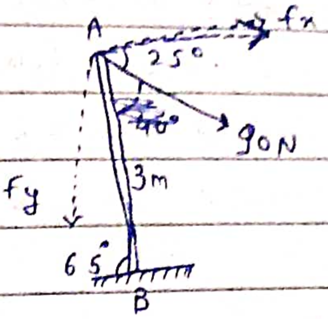
$\therefore M_o = (15 \cos 60° \times 0) + (15 \sin 60° \times 0.8).$

$= 0 + 10.39$.

$\therefore M_o = 10.39 \text{ N.m}$.

Q. A 90 N force is applied to the control rod AB. Determine moment of this force at point B / about point B.

→ At point B rotation is at point B.



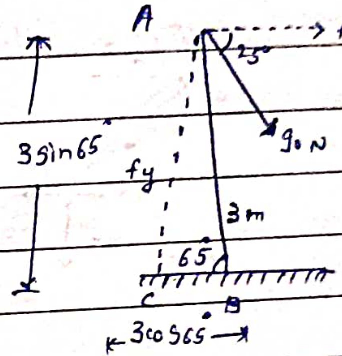
$\therefore f_x = 90 \cos 25° = 81.567$ N.

$\therefore f_y = 90 \sin 25° = 38.036.$

$\therefore M_B = [f_x \times AB] - (f_y \times BC)$

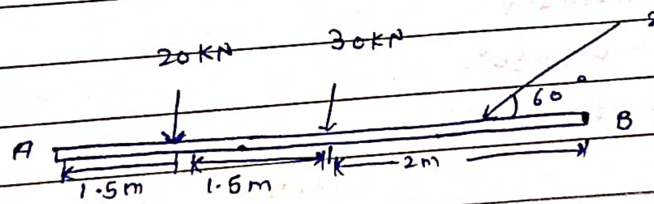
$= [81.567 \times 3 \sin 65°] - [3 \cos 65°]$

$\therefore M_B = \underline{\underline{173.53}}$ N.m.



✱ Example's based on Varignon's theorm:-

Q. Determine the resultant of the system of forces acting on a beam as shown in figure.



→ Above force system is Non concurrent force system.

Resolving forces Horizontally,

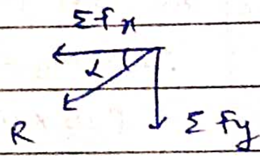
$\Sigma f_x = -20 \cos 60° = -10$ KN $= 10$ KN (toward's left).

Resolving forces vertically,

$\Sigma f_y = -20 \sin 60° - 20 \sin 90° - 30 \sin 90$

$\Sigma f_y = -67.32$ KN $= 67.32$ KN (toward's downward's).

Resultant



$\therefore R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2} = \sqrt{(10)^2 + (67.32)^2}$

$\therefore R = 68.06$ KN.

Direction

$\therefore \tan\alpha = \frac{\Sigma f_y}{\Sigma f_x} = \frac{-67.32}{-10 \text{ KN}} = \boxed{81.55 = \alpha}$.

Camlin Page

Date / /

Now taking moment about point A using Varignon's theorm

$\therefore \Sigma M_A$ = Moment of inertia of Resultant. (ΣF_y)

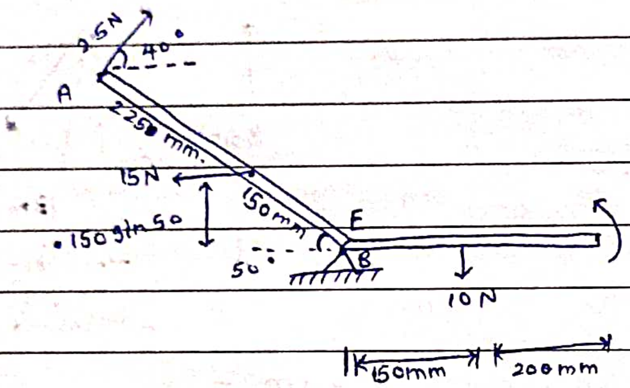
$\therefore (20 \times 1.5) + (30 \times 3) + (20 \sin 60 \times 6)(20 \cos 60 \times 0) = (R \sin\alpha \times x)$

$\therefore 223.92 = (68.06 \times \sin 81.55 \times x)$

$\therefore 223.92 = 67.32 \times x$

$\therefore x = \underline{3.326}$, from pt (A)

Q. For a given force system, find the Resultant in magnitude & direction. also find the location of Resultant.



→ Resolving forces Horizontally.

$\Sigma f_x = -15 \cos 50° + 25 \cos 40$

$= 4.15$ N (forward right).

Resolving forces vertically.

$\Sigma f_y = 25 \sin 40 - 10 \sin 90°$

$= 6.07$ N (upward)

Resultant.

$\therefore R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$

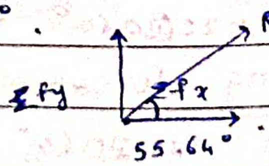
$= \sqrt{(4.15)^2 + (6.07)^2}$

$\therefore R = \underline{7.35 \text{ N}}$

Direction

$\tan\alpha = \frac{\Sigma f_y}{\Sigma f_x} = \frac{6.07}{4.15}$

$\therefore \alpha = \underline{55.64°}$.



to find position of R in given figure.

by Varignon's theorm:-

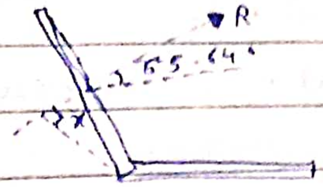
$\therefore \Sigma M_B$ = Moment of inertia at R.

$\therefore (10 \times 150) + (15 \times 150 \sin 50) + (25 \times 375) = (7.35 \times x)$

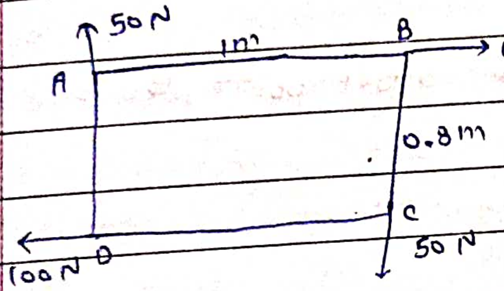
$\therefore x = \frac{2901.4}{7.35}$

$\therefore x = \underline{394.75 \text{ mm}}$

New figure :-



Q. Find resultant moment of two couple's for the loading as shown in fig :-



→ Moment of 50N couple is clockwise,

$\therefore 50 \times 1$

$\therefore$ 50 N.m (clockwise).

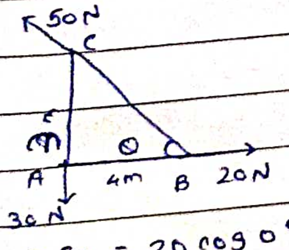
Moment of 100N couple is clockwise,

$\therefore 100 \times 0.8$

$\therefore$ 80 N.m (clockwise).

Resultant moment = $50 + 80 = 130$ N.m (clockwise).

Q. Find the Resultant and it's point of application on y-axis for the force system acting on triangular plate.



$\therefore \tan\theta = \frac{3}{4}$

$\therefore \theta = 36.86°$. (with/AB) along

→ $\Sigma f_x = 20\cos 0° - 50\cos\theta + 0$
$= 20 - 50\cos 36.86°$
$= -20$ N

$\Sigma f_x = 20$N. (towards left)

$\Sigma f_y = 0 + 50 \sin\theta - 30\sin 90°$
$= -30 + 50\sin 36.86°$
$= 0.00007$

$\Sigma f_y = \simeq 0$

$\therefore R = \sqrt{(20)^2 + (0)^2} = \underline{20}$ (left) as $[\Sigma f_y = 0$ & $\Sigma f_x = -20N]$.

Camlin Page

Date / /

Varignon's thm at pt B.

$\therefore \Sigma M_B = R \times X$

$\therefore -(30 \times 4) + (50 \times 0) + (20 \times 0) = -20 \times x$

$\therefore -20 \times x = -120$

$\therefore x = \frac{-120}{-20} = \underline{\underline{6}}$

as $x = 6m$ from pt B and it will be above point B to create anticlockwise moment.

- For couple at any point, find moment at that point and take force shifts there.