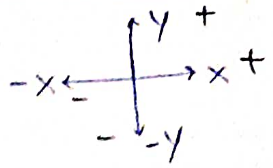
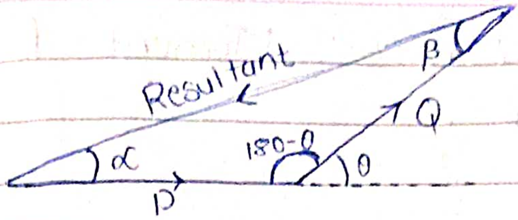
EM



* Law of Triangle :

for Two forces



$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

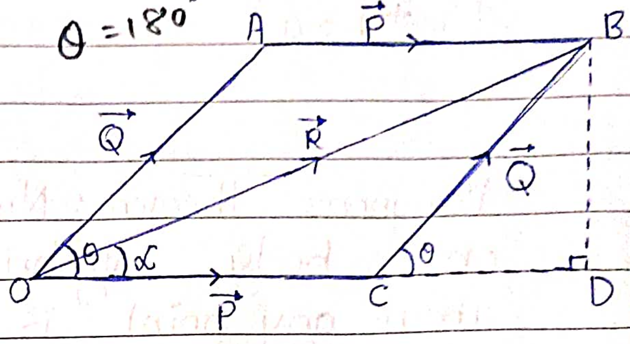
Rmax = θ = 0

Rmin θ = 180°

* Law of Parallelogram :-

for two forces

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$



Cases :-

1] When two forces P and Q are ⊥ with each other

$\therefore \theta = 90°$ then, $\boxed{R = \sqrt{P^2 + Q^2}}$. $\boxed{\alpha = \tan^{-1}\left(\frac{Q}{P}\right)}$

2] When $\theta = 0°$, Maximum (R) $\boxed{R = P + Q}$

3] When $\theta = 180°$ R Minimum $\boxed{R = P - Q}$

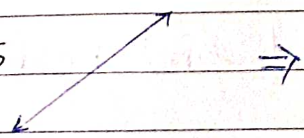
4] P = Q then,

$$\boxed{R = 2P\cos\theta/2 = 2Q\cos\theta/2}$$

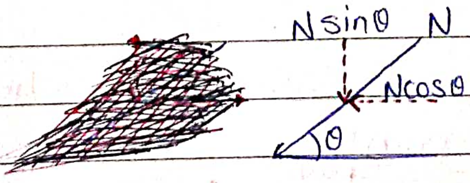
$$\boxed{\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}}$$

* Resolution of Forces :- If the force is inclined (↙, ↘) then

More than two forces



⇒



Force making angle then cos θ

$$\boxed{R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2}}$$

$$\boxed{\tan\alpha = \left|\frac{\Sigma f_y}{\Sigma f_x}\right|}$$

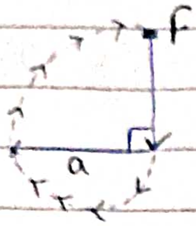
always positive

if R is on X-axis $f_x = 0$

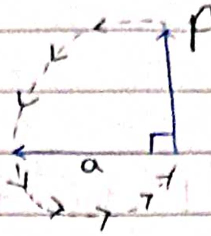
if R is on Y-axis $f_y = 0$

$\boxed{\text{vertical } f_x = 0}$ $\boxed{\text{horizontal } f_y = 0}$

* Moment of force:- Moment = Force × ⊥ distance



Clockwise



Anticlockwise

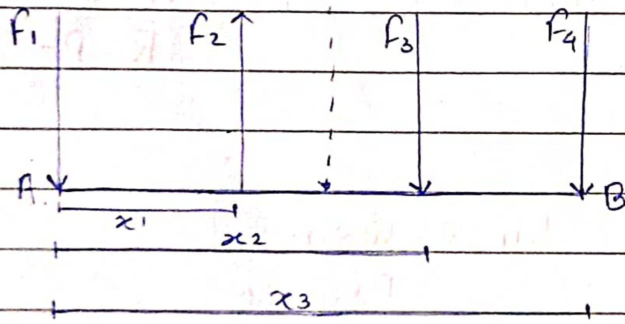
$\boxed{M = +(f \times a)}$

$\boxed{M = -(f \times a)}$

* Varignon's Theorem:- No. of forces acting simultaneously on a body, algebric sum of moments of all forces about any point is equal to moment of their resultant about same point

LHS		RHS
Moment of forces about A	=	Moment of Resultant about A

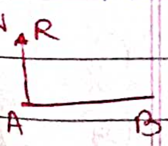
assume A



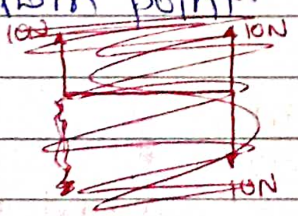
$M_A = Rx$

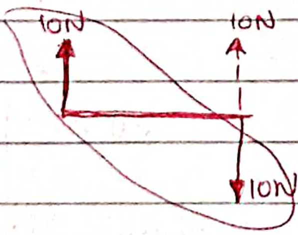
$$-(f_2 x_1) + (f_1 \times 0) + (f_3 \times x_2) + (f_4 \times x_3) = Rx$$

* Coupling:- To find couple you should find Resultant (R) and its distance from point.

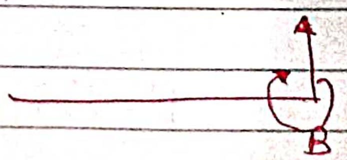


couple at point B

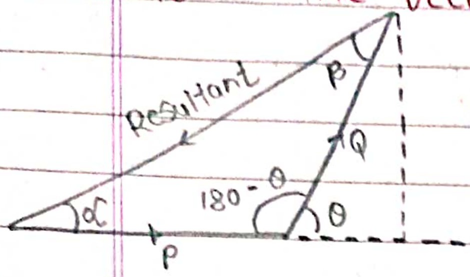




Couple



* **Triangle Law Theorem:-** States that in any system of forces acting on a particle, the resultant force is equal to the vector sum of individual forces



Sine Rule ⇒

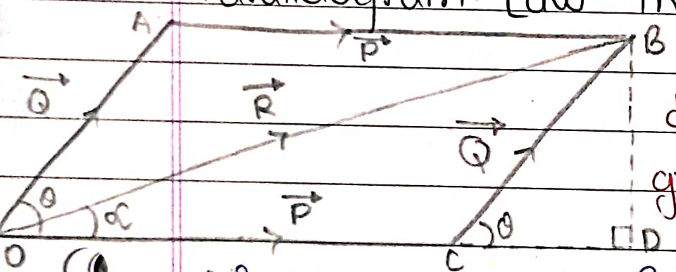
$$\frac{P}{\sin(\beta)} = \frac{Q}{\sin(\alpha)} = \frac{R}{\sin(180-\theta)}$$

Cosine Rule ⇒

$$\cos(180-\theta) = \frac{P^2+Q^2-R^2}{2PQ}$$

$$\boxed{\therefore R = \sqrt{P^2+Q^2+2PQ\cos\theta}}$$

* **Parallelogram Law Theorem:-** If two forces acting at a point are represented both in magnitude and direction by two adjucent sides of parallelogram then their resultant is represented by diagonal of parallelogram from same point.



$\therefore \theta \rightarrow$ angle between forces

$\alpha \rightarrow$ Angle between Resultant force and ~~(Horizontal~~ Horizontal force)

In $\triangle BDC$,

$$\cos\theta = \frac{CD}{CB} = \frac{CD}{Q}$$

$$\therefore CD = Q\cos\theta \text{ ---- (1)}$$

$$\therefore \sin\theta = \frac{BD}{CB} = \frac{BD}{Q}$$

$$\therefore BD = Q\sin\theta \text{ --- (2)}$$

In $\triangle ODB$, (Applying Pythagoras Theorem)

$$OB^2 = OD^2 + DB^2$$

$$R^2 = [OC+CD]^2 + (Q\sin\theta)^2 \text{ ..... from (2)}$$

$$= \cancel{[OC+} [P+Q\cos\theta]^2 + Q^2\sin^2\theta \text{ --- from (1) \& (2)}$$

$$= P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

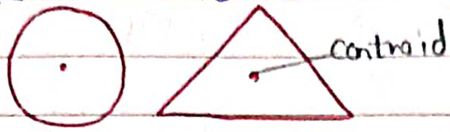
$$= P^2 + 2PQ\cos\theta + Q^2(\sin^2\theta + \cos^2\theta)$$

$$\therefore R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$\boxed{\therefore R = \sqrt{P^2+Q^2+2PQ\cos\theta}}$$

Unit II
EM

* Centroid:- plane figures like triangle circle have area and centre of area of shuch figures is known as centroid



* Procedure to find Centroid:-

i) Select suitable co-ordinate axis, if not given
ii) Divide given areas into different parts having known length, area and distance.
iii) If section is symmetrical about X-axis then we can find $\bar{Y}$ directly.
iv) If section is symmetrical about Y-axis then we can find $\bar{x}$ directly.

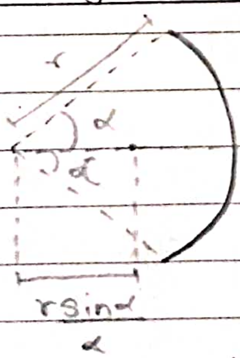
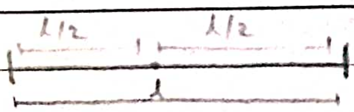
Calculating Centroid of Line:

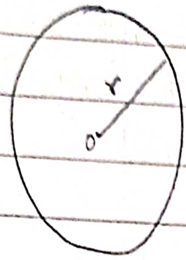
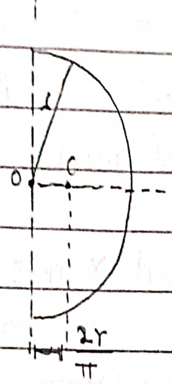
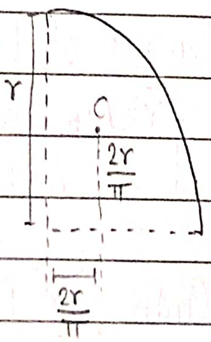
Formulae:

$$\boxed{\bar{X} = \frac{l_1x_1 + l_2x_2 + l_3x_3 + \ldots + l_nx_n}{l_1 + l_2 + l_3 + \ldots + l_n}} \quad \boxed{\bar{Y} = \frac{l_1y_1 + l_2y_2 + l_3y_3 + \ldots + l_ny_n}{l_1 + l_2 + l_3 + \ldots + l_n}}$$

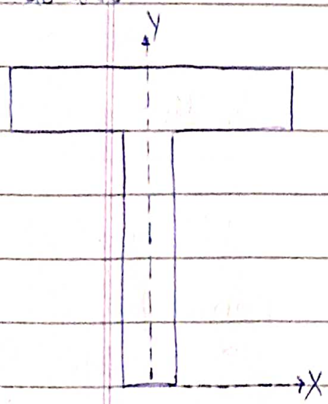
Calculating Centroid of Area:

$$\boxed{\bar{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + \ldots + A_nx_n}{A_1 + A_2 + A_3 + \ldots + A_n}} \quad \boxed{\bar{Y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + \ldots + A_ny_n}{A_1 + A_2 + A_3 + \ldots + A_n}}$$

Description	figure	length	location of Centroid
Arc of circle of radius r and inscribe angle 2α		$2\alpha r$ $\boxed{\bar{x} = \frac{2r}{\pi}}$ $\boxed{\bar{y} = \frac{2r}{\pi}}$	at distance of $\boxed{\frac{r\sin\alpha}{\alpha}}$ from centre
straight line		l	centre of line $\boxed{l/2}$

Description	Figure	length	location of Centroid
Circle of radius r		$2\pi r$	Centre of circle
Semi~~conduct~~circular arc of radius 'r'		πr	according to x,y axis at distance of $\boxed{\frac{2r}{\pi}}$ from centre
Quarter Circle arc of radius 'r'		$\boxed{x=\frac{2r}{\pi}}$ $\boxed{y=\frac{2r}{\pi}}$ $\frac{\pi r}{2}$	at distance $\frac{2r}{\pi}$ from centre along both direction

* T and I section:- If the axis is not given then assume ~~as~~ axis



according to fig, the fig axis is symmetric along Y-axis

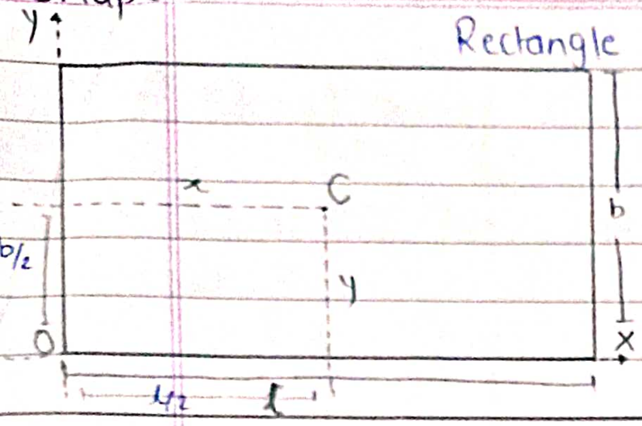
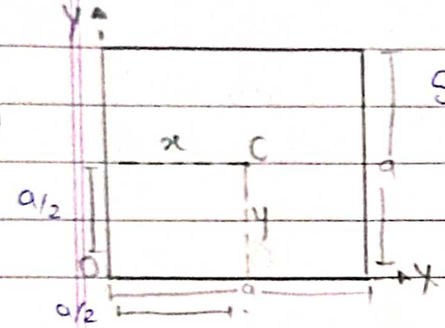
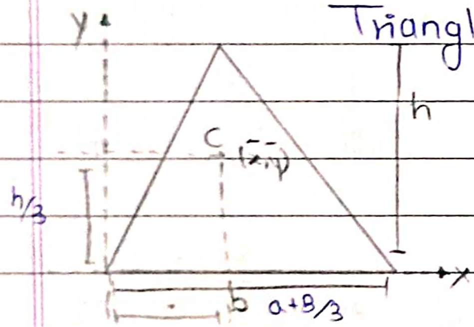
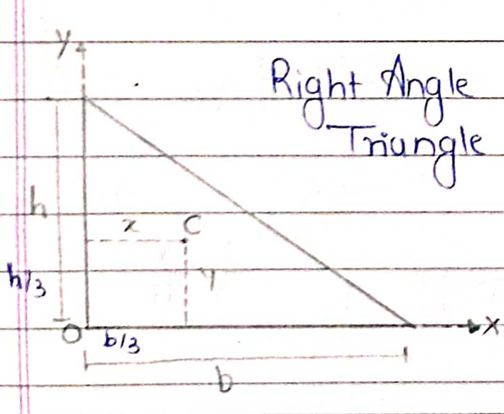
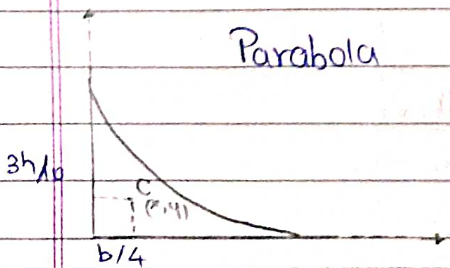
$\therefore$ Symmetric fig along Y-axis

$$\boxed{\therefore \bar{X}=0}$$

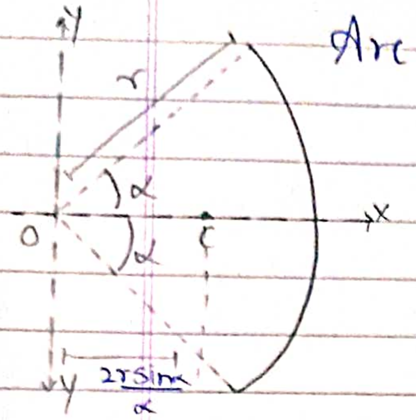
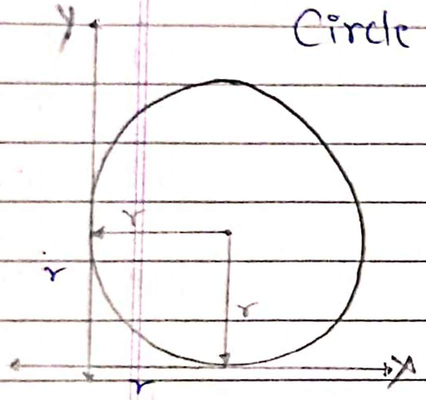
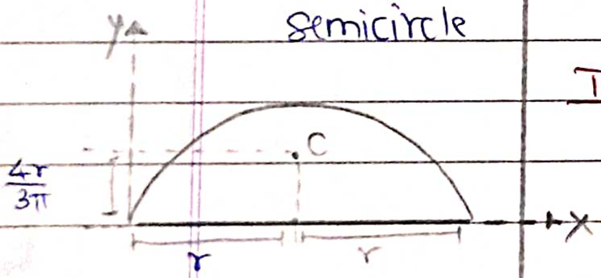
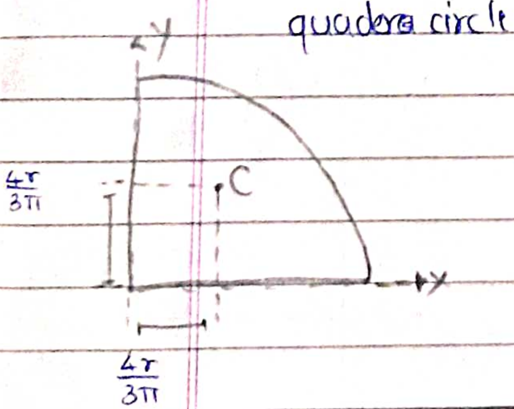
$\therefore$ Symmetric fig along X-axis

$$\boxed{\bar{Y}=0}$$

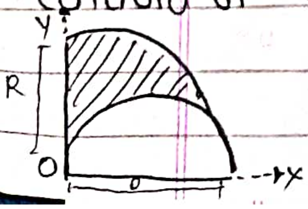
Area of Shapes

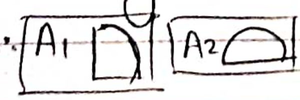
Shapes	Area	x	y
Rectangle 	$l \times b$	$\frac{l}{2}$	$\frac{b}{2}$
Square 	a^2	$a/2$	$a/2$
Triangle 	$\frac{1}{2}bh$	$\frac{a+B}{3}$ can be calculated from fig	$\frac{h}{3}$
Right Angle Triangle 	$\frac{1}{2}bh$	$\frac{1}{3}b$	$\frac{1}{3}h$
Parabola 	$\frac{1}{2}bh$	$\frac{b}{4}$	$\frac{3h}{10}$

EM

Shape	Area	$\bar{x}$	$\bar{y}$
Arc 	αr^2	$\frac{2r\sin\alpha}{\alpha}$	0
Circle 	πr^2	r	r
semicircle 	$\frac{\pi r^2}{2}$	r	$\frac{4}{3}\frac{r}{\pi}$
quadra circle 	$\frac{\pi r^2}{4}$	$\frac{4}{3}\frac{r}{\pi}$	$\frac{4}{3}\frac{r}{\pi}$

* Centroid of Shaded Region



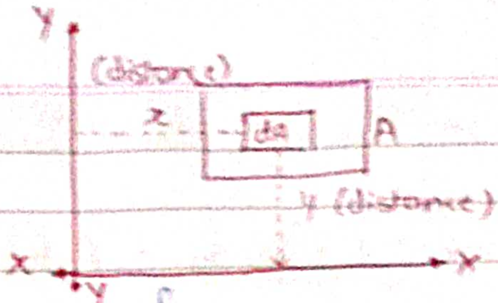
$\therefore$ 

$$\bar{x} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$$\bar{y} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

* Moment of Inertia :-

* Area Moment of Inertia :-



(i) Moment of dA about X-X axis $= y \times dA$

(i) Moment of dA about Y-Y axis $= x\,dA$

dA → small area

(ii) Second Moment of dA about X-X axis

$= y^2 dA$

(ii) Second Moment of dA about Y-Y axis =

$= x^2 dA$

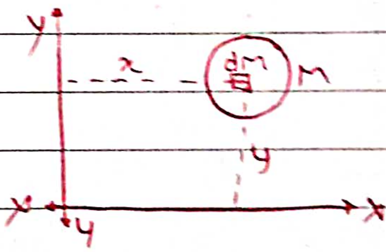
$(I_{dA})_{x-x} = y^2\, dA$

$(I_{dA})_{y-y} = x^2\, dA$

$(I_{dA})_{x-x} = \int y^2\, dA$

$(I_{dA})_{y-y} = \int x^2\, dA$

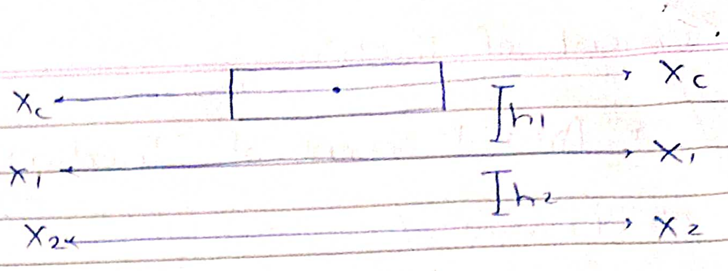
* Mass Moment of Inertia :-



$(I_{dm})_{xx} = \int y^2\, dm$

$(I_{dm})_{yy} = \int x^2\, dm$

* Parallel Axis Theorem :-



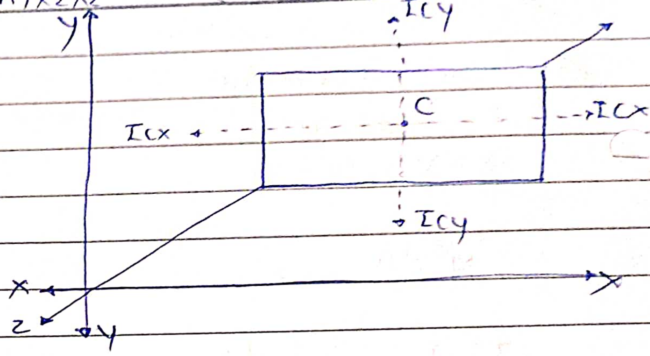
$$(I)_{x_1x_1} = (I)_{x_cx_c} + A \cdot h_1^2$$

$$(I)_{x_2x_2} = (I)_{x_cx_c} + A \cdot (h_1+h_2)^2$$

for two shapes $(I_n)_{x_cx_c} = (I_n)_{x_1x_1} + (I_n)_{x_2x_2}$

$\therefore (I_{A_1})_{x_1x_1} = (I_{A_1})_{x_cx_c} + A_1 h_1^2$

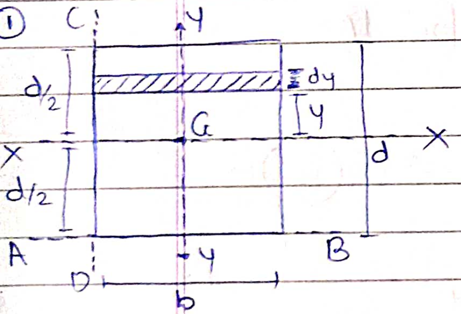
* Perpendicular Axis Theorem :-



$$I_{zz} = I_{xx} + I_{yy}$$

* M.I for Standard Shapes

①



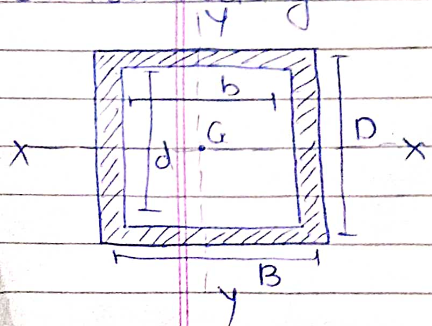
$$I_{xx} = \frac{bd^3}{12} \qquad \text{About Cendroidal Axis}$$

$$I_{xy} = \frac{b^3d}{12}$$

$$I_{AB} = \frac{bd^3}{3} \qquad I_{CD} = \frac{b^3d}{3}$$

$b \rightarrow$ width
$d \rightarrow$ depth

② Hollow Rectangle



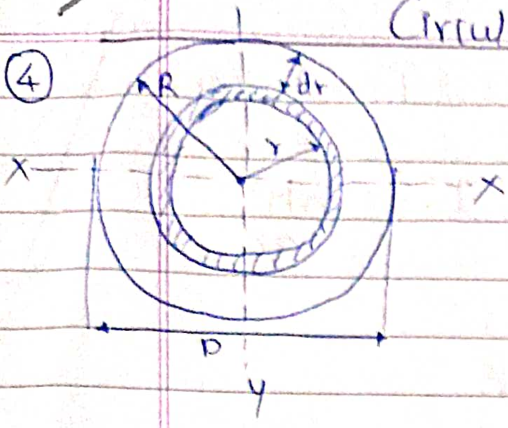
$$I_{xx} = \left(\frac{BD^3}{12} - \frac{bd^3}{12}\right)$$

$$I_{yy} = \left(\frac{B^3D}{12} - \frac{b^3d}{12}\right)$$

③ $I_{xx} = I_{yy} = \frac{a^4}{12}$

Square

(4) Circular Area

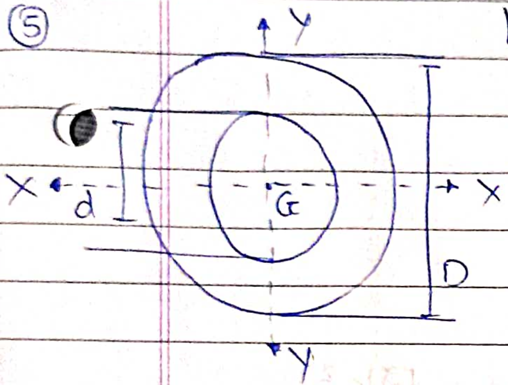


$$I_{zz} = \frac{\pi R^4}{2} \qquad I_{zz} = \frac{\pi D^4}{32}$$

$$\boxed{I_{xx} = I_{yy} = \frac{\pi R^4}{4} \qquad I_{xx} = I_{yy} = \frac{\pi D^4}{64}}$$

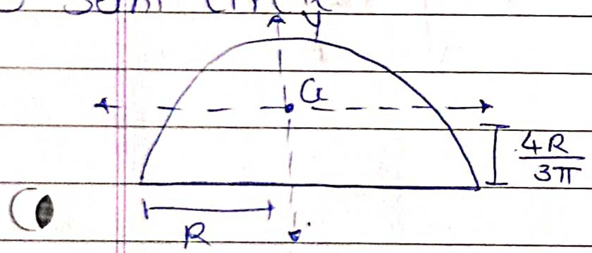
Symmetric (Axis)

(5) Hollow Circular area



$$I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)$$

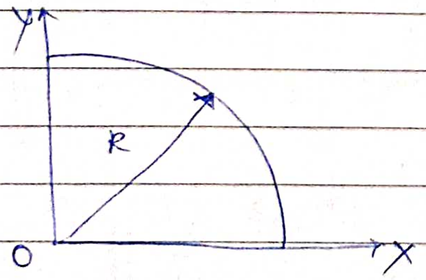
(6) Semi-circle



$$I_{zz} = \frac{\pi R^4}{4} \qquad I_{xx} = I_{yy} = I_{base} = \frac{\pi D^4}{128}$$

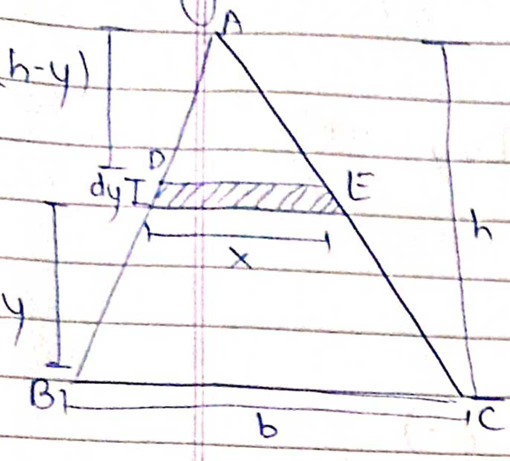
$$I_G = 0.11 R^4$$

(7) Quarter Circle



$$I_{xx} = I_{yy} = \frac{\pi R^4}{16} = \frac{\pi D^4}{256}$$

⑧ Triangle



About BC

$$I_{BC} = \frac{bh^3}{12}$$

About Centroidal

$$I_{xx} = \frac{bh^3}{36}$$

About Vertex 'A'

$$I_A = \frac{bh^3}{4}$$

---

* Radius of Gyration (K) :-

Mass

$$(I_m)_{AA} = MK^2 \qquad (I_A)_{AA} = AK^2$$

$$K = \sqrt{\frac{(I_m)_{AA}}{M}} \qquad K = \sqrt{\frac{(I_A)_{AA}}{A}}$$

$F_x > F_{rmax} \rightarrow$ block slides down

$\phi > \theta \rightarrow$ in equlibrium $\qquad \theta > \phi \rightarrow$ slides

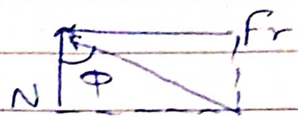
* Friction:-

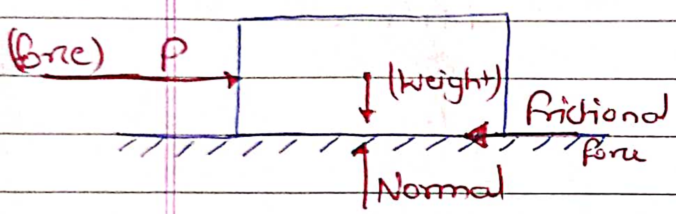
① Static friction:- Friction experienced by the body, when body is in rest condition.

② Kinetic friction:- Friction experienced by a body when it moves.

• Sliding friction:- When one body slides over another then friction experienced by body is sliding friction.

• Rolling friction:- When one body over another, then friction experienced by body is rolling friction.





Resultant force is the reaction between normal reaction and friction force.

$\sum F_y = 0 \qquad \sum F_x = 0$

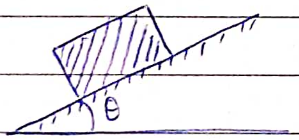
Angle of Friction(ϕ):- $F_r = \mu N \qquad \phi = \tan^{-1}(\mu)$

$\mu_s \rightarrow$ Static friction

$\mu_k \rightarrow$ Kinetic friction

Angle of Repose :- (θ) $\quad \theta = \phi$

$\tan\theta = \mu$



$1kg = 9.81\ N$

$\frac{F_r}{N} = \tan\phi$

$F_{r_{max}} = \mu_s N$

Belt friction :-

$T_2 \rightarrow$ larger tension

$T_1 \rightarrow$ smaller tension

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$\beta \rightarrow$ lap angle in radian

No. of turn of belt

1 Turn = 2π rad

Principle of Transmissibility :- If a force acts at a point on a rigid body, then it is assumed ~~that~~ to act at any other point on the lines of action of force within the body