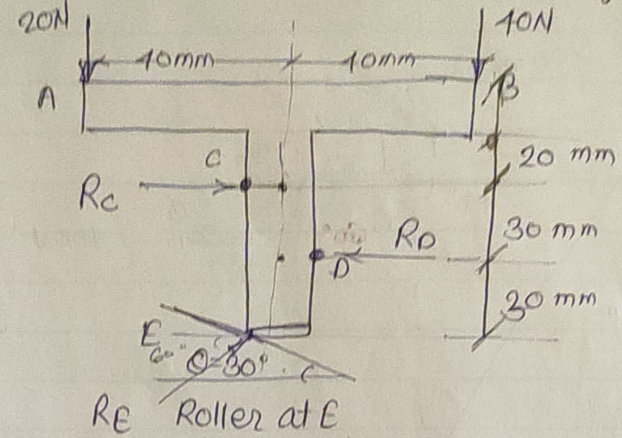
UNIT III

* Equilibrium of Non-concurrent Forces:

① The T shaped bracket shown is supported by a small roller at 'E' & pegs at C & D. Neglecting the effect of friction, determine the reactions at C, D, & E. Refer. fig.



$\Sigma F_y = 0$

$R_E \sin 60 - 20 - 40 = 0$

$R_E = 69.28\ N$

$\Sigma M @ C = 0$

$20 \times \underset{60}{40} - 40 \times \overset{60}{40} - R_D \times 30 + R_E \cos 60 \times 60 = 0$

$\therefore R_D = \overset{29.28\ N}{42.61\ N}$ (22.61N) Thick = 20

$\Sigma F_x = 0$

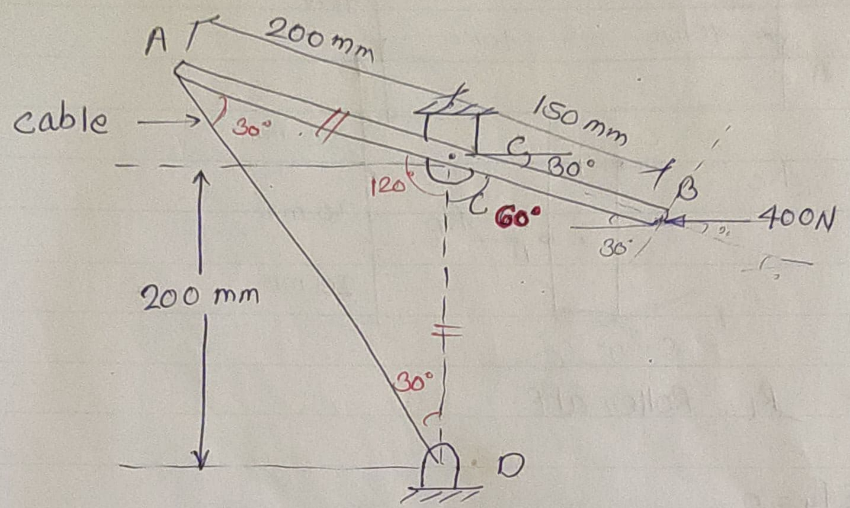
$R_C - R_D + R_E \cos 60 = 0$

$R_C - \overset{29.28}{\cancel{42.61}} + 69.28 \cos 60 = 0$

$R_C = 7.97\ kN.$ (12.02N)

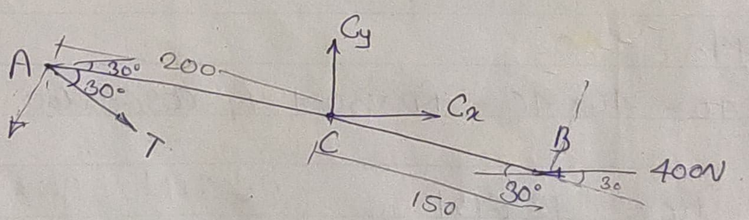
$R_C = 5.30\ N$

② The lever AB is hinged at C & attached to a cable at A. If the lever is subjected at B to a 400N horizontal force, determine
ⓐ The tension in the cable.
ⓑ the reaction at C.



As $CA = CD = 200mm$

$\angle CAD = \angle CDA = \frac{60^\circ}{2} = 30^\circ$



$\Sigma M@C =$

$T\sin 30 \times 200 - 400\sin 30 \times 150 = 0$

$T = 300N$

$\Sigma F_x = 0$

$T\cos 60 + C_x - 400 = 0$

$300\cos 60 + C_x - 400 = 0$

$C_x = 250\ N$

$\Sigma F_y = 0$

$-T\sin 60 + C_y = 0$

$C_y = 259.807\ kN$

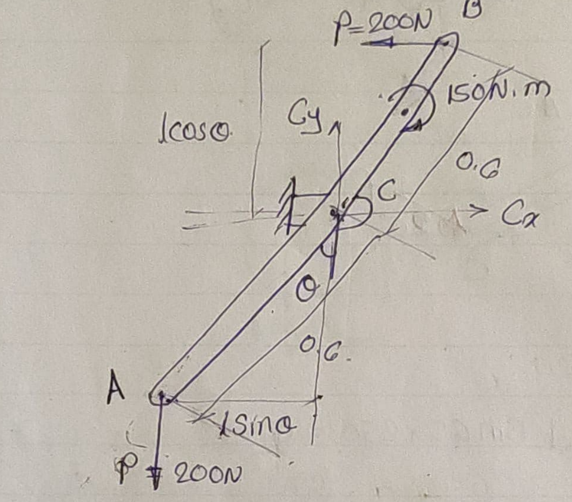
$R_C = \sqrt{250^2 + 259.807^2}$

$= 360.554\ N$

$\theta = \tan^{-1}\frac{259.807}{250}$

$= 46.102^\circ$

③ Rod AB is acted upon by a couple M & two forces, each of magnitude, P. Derive an equation relating θ, P, M & l which must be satisfied when the rod is in equilibrium. Refer fig.. If M = 150 N.m P = 200 N l = 0.6 m, determine θ.



$\Sigma M @ C = 0$

$\cancel{200\times l\theta}$ $200\times0.6\cos\theta + 200\times0.6\sin\theta - 150 = 0$

$$\cos\theta + \sin\theta = \frac{150}{200\times0.6}$$

$$\cos\theta + \sin\theta = 1.25$$

squaring we get,

$$\cos^2\theta + \sin^2\theta + 2\sin\theta.\cos\theta = 1.25^2$$

$$1 + \sin 2\theta = 1.5625$$

$$\sin 2\theta = 0.5625$$

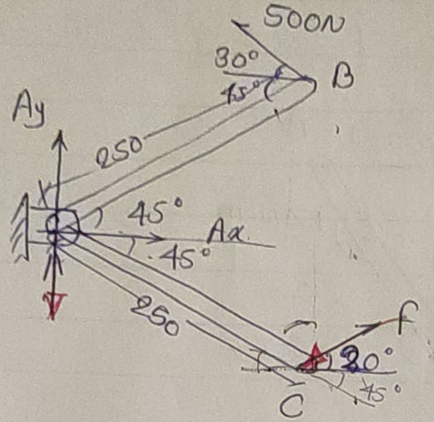
$$2\theta = 34.23^\circ$$

$$\theta = 17.11^\circ$$

$$\theta = 90 - 17.11$$

$$= 72.89^\circ$$

④ Find the force F acting on crank for equilibrium & also find support reaction



$\Sigma M @ A = 0$

$500 \sin 75 \times 250 + F \sin 65 \times 250 = 0$

$F = -532.89 N$

$\Sigma F_x = 0.$

$A_x - 500 \cos 30 - F \cos 20 = 0$

$A_x = 933.77 N$

$\Sigma F_y = 0$

$A_y + 500 \sin 30 - F \sin 20 = 0$

$\therefore A_y = -67.74 N$

$A_y = 67.74 N \downarrow$

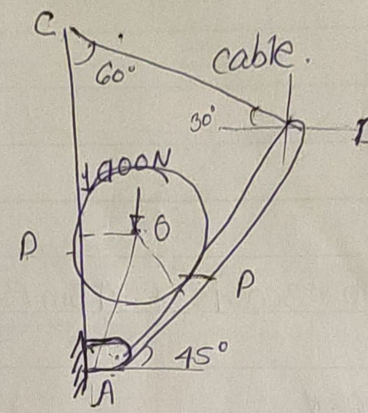
$R_A = \sqrt{933.77^2 + 67.74^2}$

$= 936.22 N$

$\theta = \tan^{-1} \frac{67.74}{933.77}$

$= 4.15°$

⑤ A cylinder weighing 1000N & 1.5m diameter is supported by a beam AB of length 6m & weight 400N as shown in fig. Neglecting friction at the surface of contact of the cylinder, determine ⓐ wall reaction at D. ⓑ Tension in cable BC ⓒ Hinged reaction at support A.



AD & AP are tangents from an external point A. Hence line OA bisect ∠DAP.

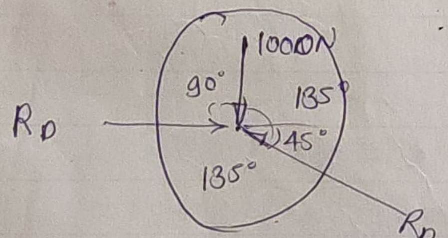
$\angle OAP = \frac{45}{2} = 22.5°$

$\tan 22.5 = \frac{OP}{AP}$

$\therefore AP = \frac{OP}{\tan 22.5}$

$= \frac{0.75}{\tan 22.5}$

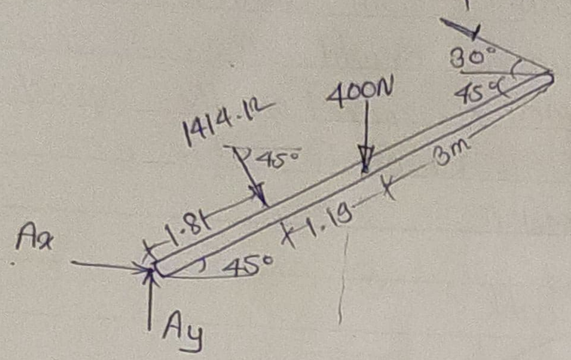
$AP = 1.81\,m$



$\frac{1000}{\sin 135} = \frac{R_P}{\sin 90} = \frac{R_D}{\sin 135}$

$R_D = \frac{1000}{\sin 135} \times \sin 135 = 1000\ KN$

$R_P = \frac{1000}{\sin 135} \times \sin 90 = 1414.21N$



$\Sigma M@A = 0$

$- 1414.12 \times 1.81 - 400 \times 3\cos 45 + T\sin 75 \times 6. = 0$

$\therefore T = 588.08 N$

$\Sigma f_x = 0$

$A_x + R_P \cos 45 - T\cos 30 = 0$

$A_x = -490.64 kN$

$\Sigma f_y = 0$

$A_y - R_P \sin 45 - 400 + T\sin 30 = 0$

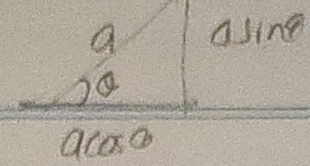
$A_y = 1105.96 N.$

$R_A = \sqrt{490.64^2 + 1105.96^2}$

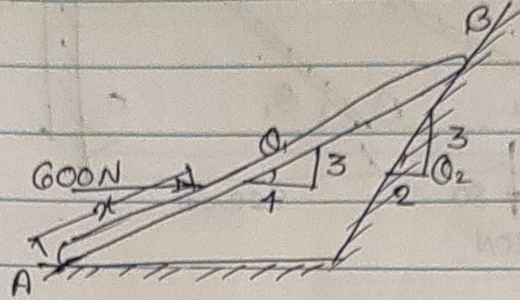
$= 1210 N$

$\theta = \tan^{-1} \frac{1105.96}{490.64}$

$= 66.1°$

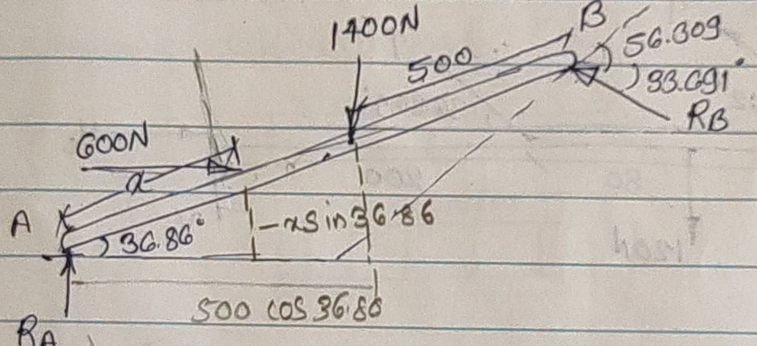


Find the distance a, measured along AB at which a horizontal force of 600N should be applied to hold the uniform bar AB in the position as shown in fig. AB weighs 1400N and measures 1000mm. The incline and the floor are smooth.



$\theta_1 = \tan^{-1}\frac{3}{4} = 36.86°$

$\theta_2 = \tan^{-1}\frac{3}{2} = 56.309°$



$\Sigma F_x = 0$

$600 - R_B \cos 33.691 = 0$

$R_B = 721.118\ N$

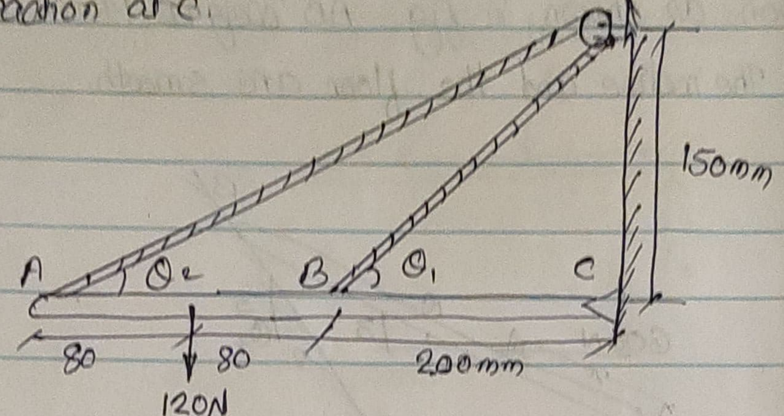
$\Sigma M@A = 0$

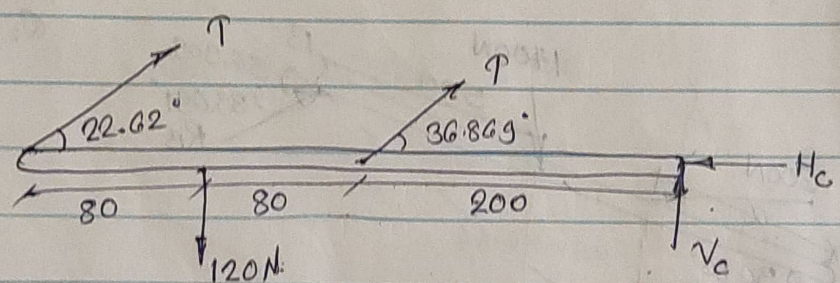
$-600 \times a \sin 36.87 - 1400 \times 500 \times \cos 36.87 + 721.11 \times \cos 33.69 \times 1000 \sin 36.87 + 721.11 \sin 33.69 \times 1000 \cos 36.87 = 0.$

$a = 333.33\ mm$

$R_A = 1000N$

Neglecting the friction & the radius of the pulley shown in fig. determine the tension in the cable ADB and reaction at C.





$\theta_1 = \tan^{-1}\dfrac{150}{200} = 36.869°$

$\theta_2 = \tan^{-1}\dfrac{150}{360} = 22.619°$

$\Sigma M@c = 0$

$120 \times 280 - T\sin 22.62 \times 360 - T\sin 36.869 \times 200 = 0$

$258.459T = 33600$

$T = 130N$

$\Sigma F_x = 0$

$T\cos 22.62 + T.\cos 36.869 - H_c = 0$ $\quad R_c = 224.15$

$H_c = 224 N$

$\Sigma F_y = 0$ $\quad \alpha = 2.03°$

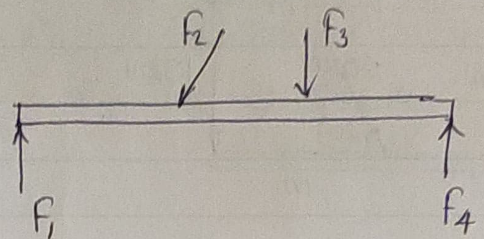
$T\sin 22.62 + T\sin 36.869 + V_c - 120 = 0$

$V_c = -8N \quad V_c = 8N \downarrow$

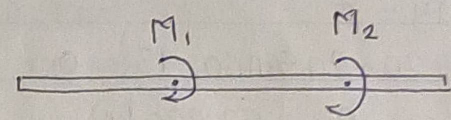
* Equilibrium Of Coplaner Non-Concurrent Force System:

* Types of Loads on Beams:-

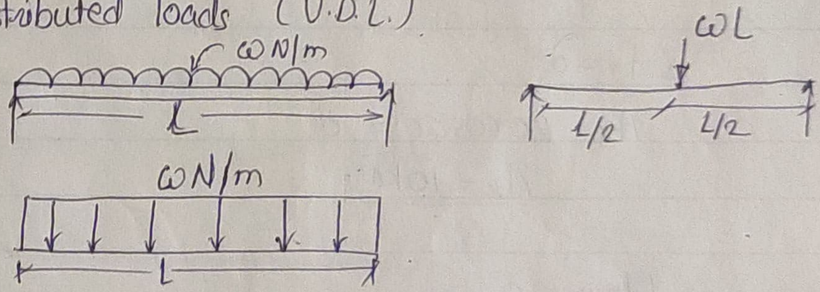
① Point loads or concentrated loads.



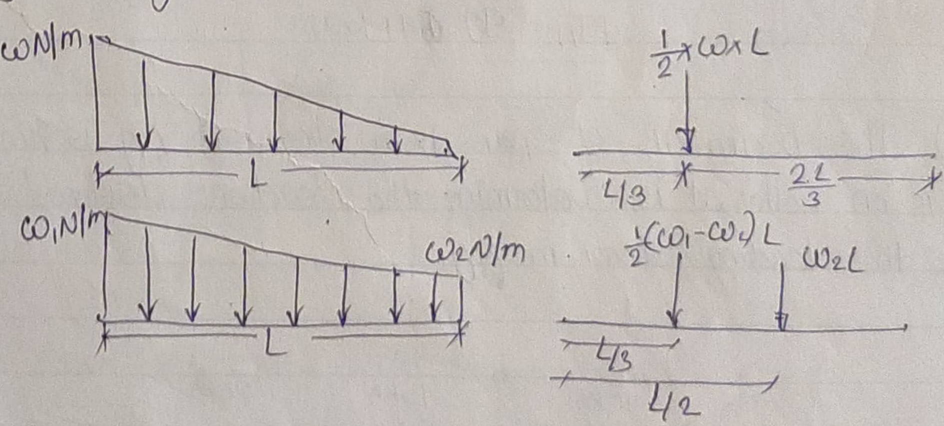
② Couple moments.



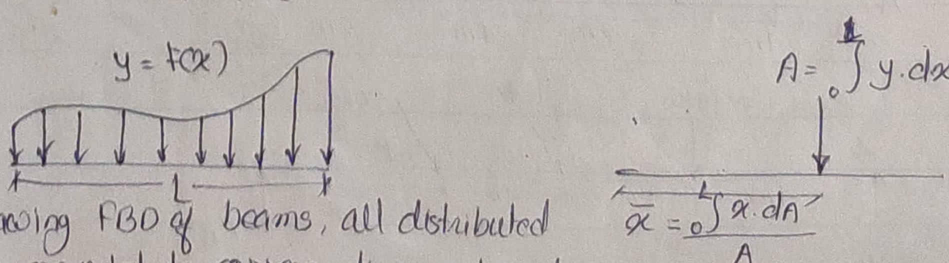
③ Uniformly distributed loads (U.D.L.)



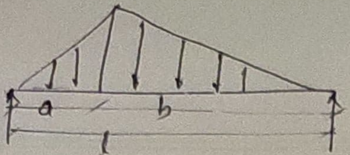
④ Uniformly varying load (u.v.l.)

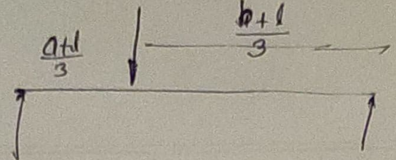


⑤ General variable load



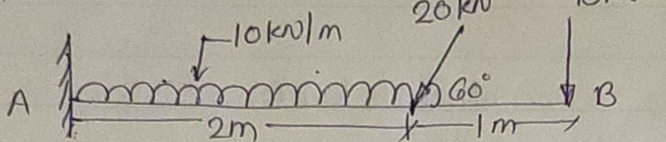
While drawing FBD of beams, all distributed loads must be converted to corresponding point loads.

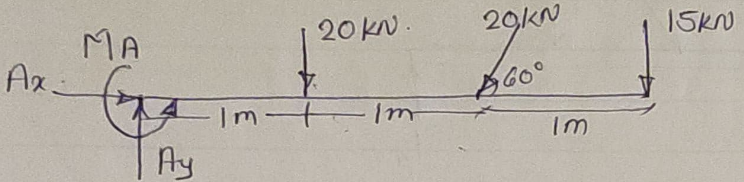




Q① Determine the reactions developed in the cantilever beam shown in fig.

③





$\Sigma F_y = 0$

$A_y - 20 - 20\sin 60 - 15 = 0$

$\underline{A_y = 52.320 \text{ kN}}$

$\Sigma F_x = 0$

$A_x - 20\cos 60 = 0$

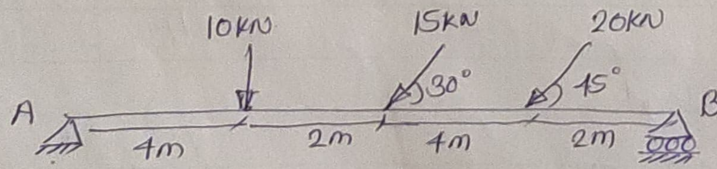
$\underline{A_x = 10 \text{ kN}}$

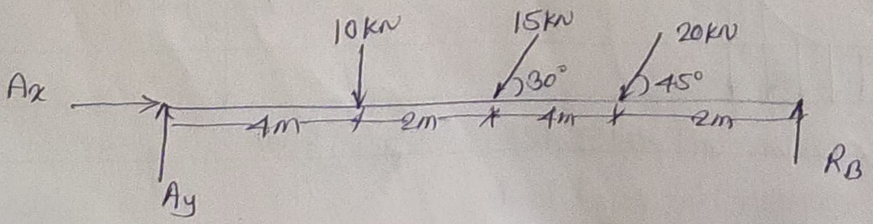
$\Sigma M@A = 0$

$-20\times 1 - 20\sin 60\times 2 - 15\times 3 + M_A = 0$

$\underline{M_A = 99.641 \text{ kN}}$

② The beam AB of span 12m shown in fig. is hinged at A & is on roller at B. Determine the reactions developed at A & B due to the loading shown in fig.





$\Sigma F_x = 0$

$A_x - 15\cos 30 - 20\cos 45 = 0$

$\underline{A_x = 27.132 \text{ kN}}$

$\Sigma F_y = 0$

$A_y + R_B - 10 - 15\sin 30 - 20\sin 45 = 0$

$A_y + R_B = 31.642 \text{ kN.}$

$\Sigma M@A = 0$

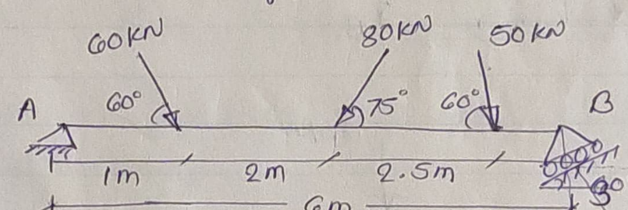
$12R_B - 10\times 4 - 15\sin 30\times 6 - 20\sin 45\times 10 = 0$

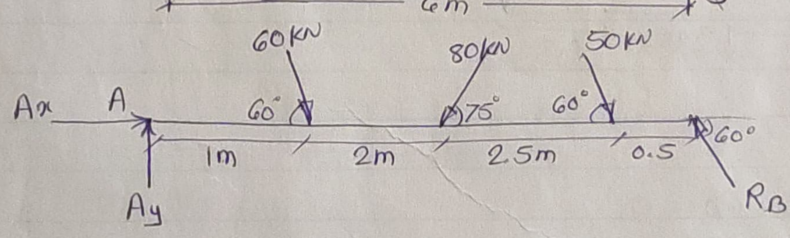
$R_B = 18.868 \text{ kN}$

$A_y = 12.773 \text{ kN.}$

③ find the magnitude & direction of reactions at supports A & B in the beam AB shown in fig.

①





$\Sigma F_x = 0$

$A_x + 60\cos 60 - 80\cos 75 + 50\cos 60 - R_B\cos 60 = 0$

$A_x - R_B\cos 60 = -34.294$ ——— ①

$\Sigma M@A = 0$

$-60\sin 60\times 1 - 80\sin 75\times 3 - 50\sin 60\times 5.5 + 6R_B\sin 60 = 0$

$R_B = 100.447 \text{ kN}$

$\underline{A_x = 15.929 \text{ kN}}$

$\Sigma F_y = 0$

$A_y = 85.547 \text{ kN}$

$A_y - 60\sin 60 - 80\sin 75 - 50\sin 60 + 100.447\sin 60 = 0$

$A_x = 15.929$ kN

$A_y = 85.547$ kN.

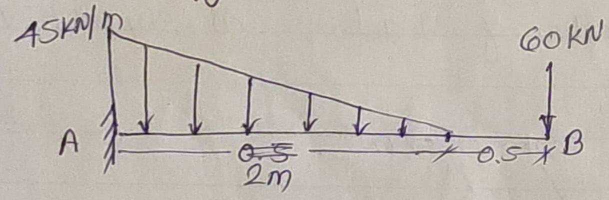
$R_A = \sqrt{15.929^2 + 85.547^2}$

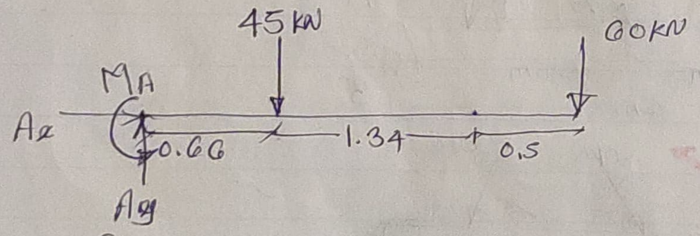
$= 87.017$ kN.

$\alpha_A = \tan^{-1} \dfrac{85.547}{15.929}$

$= 79.442°$

④ Determine the reactions developed in the cantilever beam shown in fig.





$\Sigma F_y = 0$

$A_y - 45 - 60 = 0$

$\therefore A_y = 105$ kN

$\Sigma F_x = 0$

$A_x = 0$

$\Sigma M_A = 0$

$M_A - 45 \times 0.66 - 60 \times 2.5 = 0$

$M_A = 180$ kN.m

5. Determine the reaction developed in the simply supported beam shown in fig.

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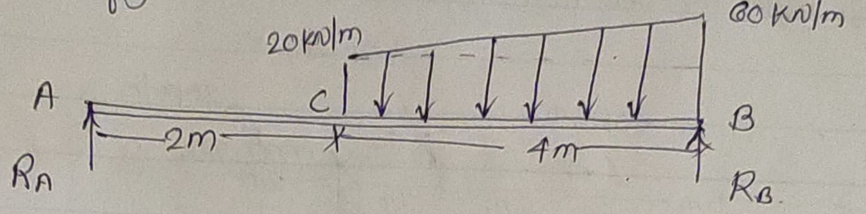
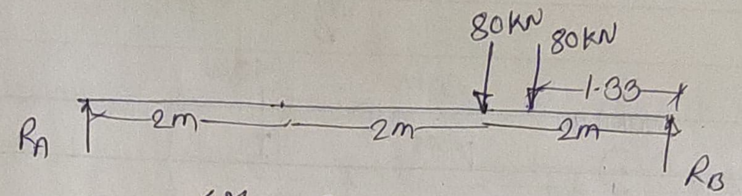




$$\Sigma M_B = 0$$

$$-6R_A + 80\times 2 + 80\times 1.33 = 0$$

$$R_A = 44.44 \text{ kN.}$$

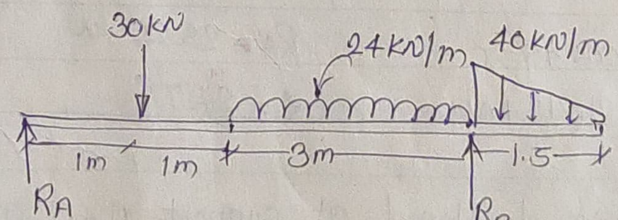
$$\Sigma F_y = 0$$

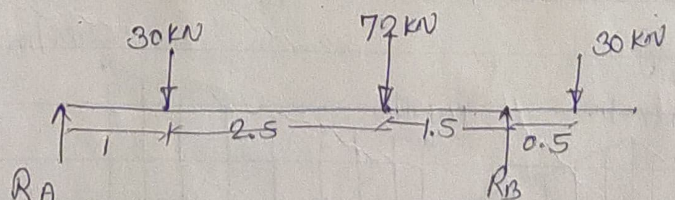
$$R_A + R_B - 80 - 80 = 0$$

$$R_B = 160 - 44.44$$

$$R_B = 115.56 \text{ kN.}$$

6. Determine the reactions at supports A & B of the overhanging beam as shown in fig.





$$\Sigma M_A = 0$$

$$-30\times 1 - 72\times 3.5 - 30\times 5.5 + 5R_B = 0$$

$$R_B = 89.4 \text{ kN.}$$

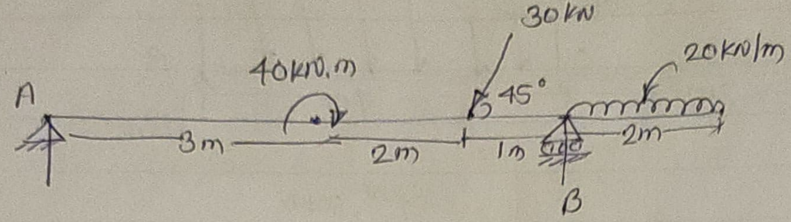
$$\Sigma F_y = 0$$

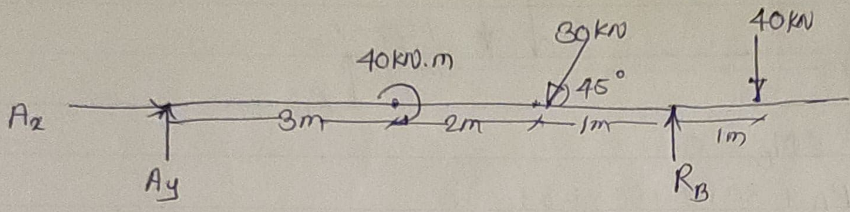
$$R_A + R_B - 30 - 72 - 30 = 0$$

$$R_A = 132 - 89.4 = 42.6 \text{ kN}$$

(7) Determine the reactions developed at supports A & B of overhanging beam as shown in fig.

(2)





$\Sigma M_A = 0$

$-40 - 30\sin 45 \times 5 - 40 \times 7 + 6 R_B = 0$

$\underline{R_B = 71.011\ kN}$

$\Sigma F_y = 0$

$A_y + R_B - 30\sin 45 - 40 = 0$

$A_y = -9.797\ kN = \underline{9.797\ kN \downarrow}$

$\Sigma F_x = 0$

$A_x - 30\cos 45 = 0$

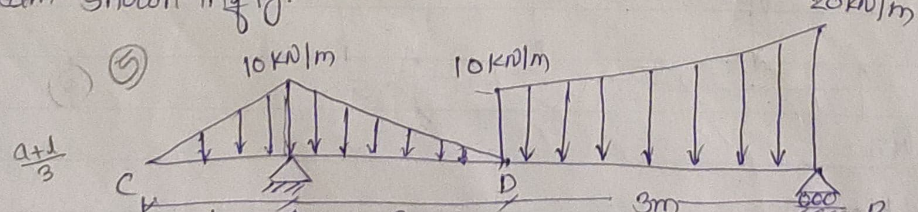
$\underline{A_x = 21.213\ kN}$

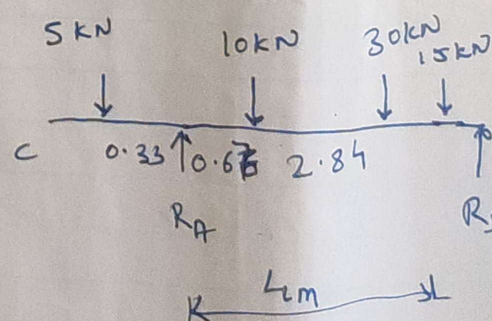
$R_A = 23.26\ kN$

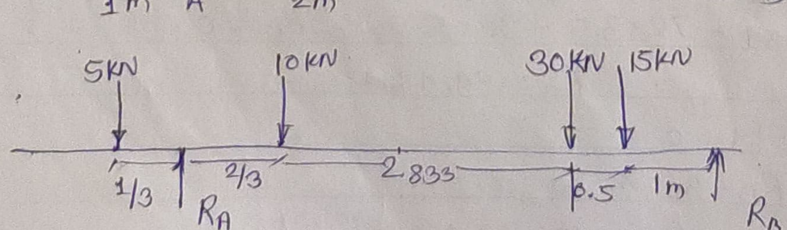
$\angle = 24.77^\circ$

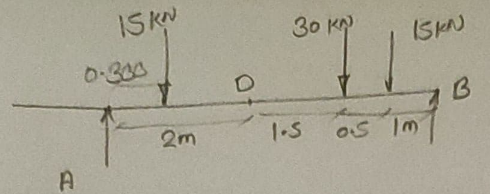
(8) Find the reactions developed at supports A & B of the loaded beam shown in fig.

(5)









$\Sigma M@B = 0$

$-5R_A + 15 \times 4.667 + 30 \times 1.5 + 15 \times 1 = 0$

$\therefore R_A = 26 \text{ kN}$

$\Sigma M_B = 0$

$5 \times 5.333 + 10 \times 4.333 + 30 \times 1.5 + 15 \times 1 - 5R_A = 0$

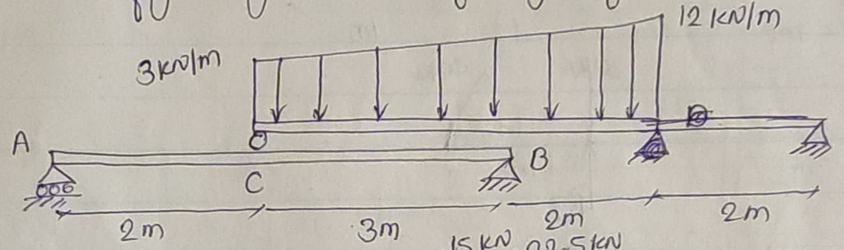
$R_A = 26 \text{ kN}$

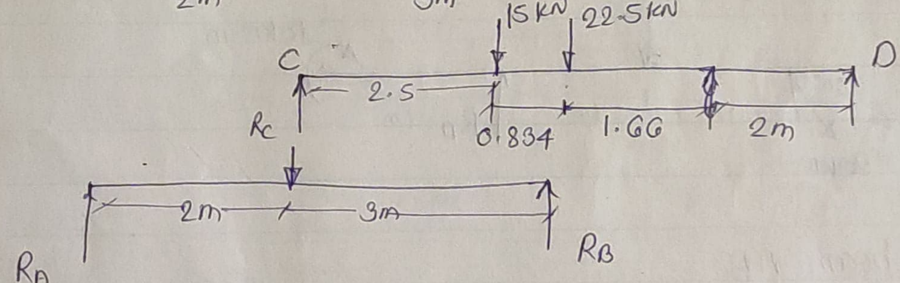
$\Sigma f_y = 0$

$R_A + R_B - 5 - 10 - 30 - 15 = 0$

$\therefore R_B = 34 \text{ kN}$

(9) Determine the reactions at A, B & D of the compound beam shown in fig. Neglect the self weight of the members.





First consider beam CD,

$\Sigma M@D = 0$

$-7R_C - 15 \times 4.494 - 22.5 \times 3.66 = 0$

$R_C = 21.394 \text{ kN} \cong 21.43 \text{ kN}$

$\Sigma f_y = 0$

$R_D + R_C - 15 - 22.5 = 0$

$\therefore R_D = 16.106 \cong 16.07 \text{ kN}$

Consider beam AB

$\Sigma M@A = 0$

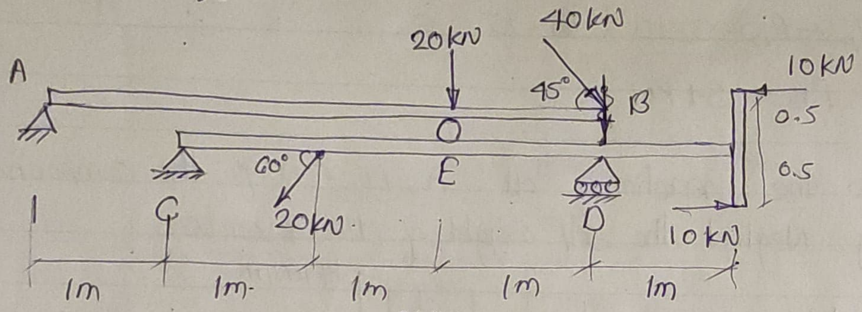
$-2R_C + 5R_B = 0$

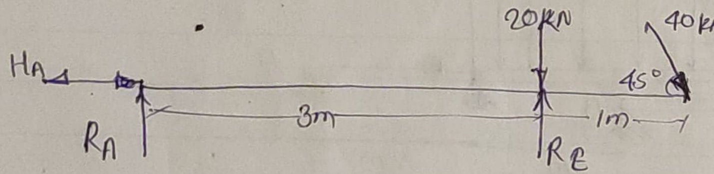
$R_B = \dfrac{2 \times 21.43}{5} = 8.572 \text{ kN}$

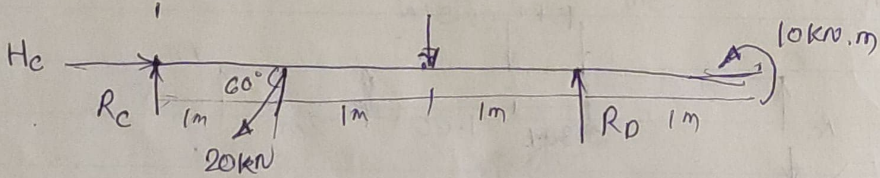
$R_A = 12.858 \text{ kN}$

(10) The beam AB & CF are arranged as shown in fig. Determine the reactions at A, C & D due to the loads acting on the beam as shown in fig.

(6)







Consider beam AB

$\Sigma M @ A = 0$

$-20 \times 3 + 3R_E - 40 \sin 45 \times 4 = 0$

$R_E = 57.712 \text{ kN}$

$\Sigma f_y = 0$

$R_A + R_E - 20 - 40 \sin 45 = 0$

$R_A = 9.427 \text{ kN}$

$\Sigma f_x = 0$

$H_A = 40 \cos 45 = 28.284 \text{ kN}$

$R_A = 29.80$

$\alpha = 18.44$

Consider beam CD

$\Sigma M @ C = 0$

$-2R_E$

$-20 \sin 60 \times 1 - 57.712 \times 2 + 3R_D + 10 = 0$

$R_D = 40.914 \text{ kN}$

$\Sigma F_y = 0$

$R_C + R_D - 20\sin 60 - 57.712 = 0$

$R_C = 34.118\ kN$

$R_C = 35.55\ kN$

$\alpha = 73.66^{\circ}$

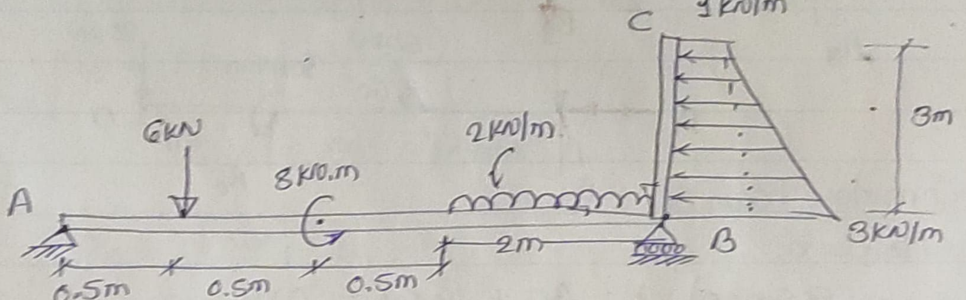
$\Sigma F_x = 0$

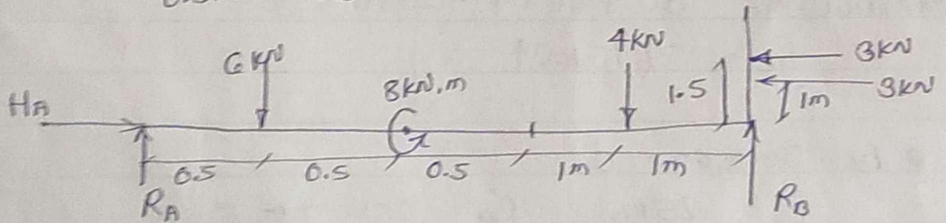
$H_C - 20\cos 60 = 0$

$H_C = 10\ kN$

(11) Find reactions at A & B for a bent beam ABC loaded as shown
3. in fig.

(7)





$\Sigma M@A = 0$

$-6\times 0.5 + 8 - 4\times 2.5 + 3.5 R_B + 3\times 1.5 + 3\times 1 = 0$

$R_B = -0.714\ kN \downarrow$

$\Sigma F_x = 0$

$H_A - 3 - 3 = 0$ $\quad H_A = 6kN \rightarrow$

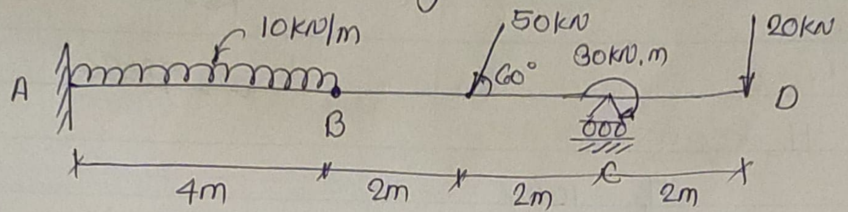
$\Sigma F_y = 0$

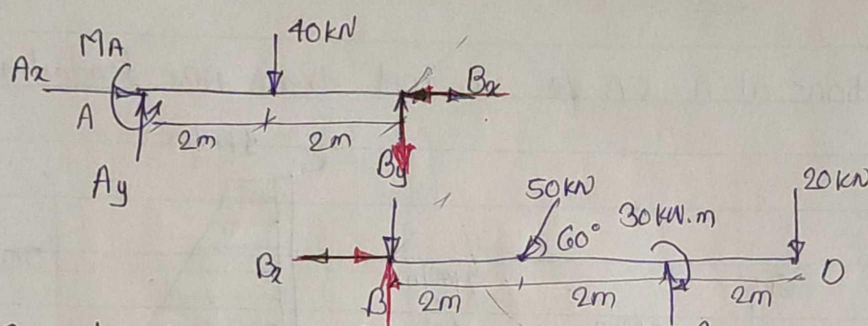
$R_A + R_B - 6 - 4 = 0$

$R_A = 10.714\ kN \uparrow$

(12) Analyse the compound beam shown in fig. to find the reactions at the internal hinge B & at supports A & C.

(8)





Consider beam BD

$\Sigma M_{@B} = 0$

$-(50\sin 60) \times 2 - 30 + R_c \times 4 - 20 \times 6 = 0$

$R_c = 59.15\,kN \uparrow$

$\Sigma F_x = 0$

$-\cancel{\ } B_x - 50\cos 60 = 0$

$B_x = -25\,kN \qquad B_x = 25\,kN \rightarrow$

$\Sigma F_y = 0$

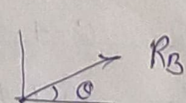
$-B_y - 50\sin 60 - 20 \overset{+59.15}{} = 0$

$B_y = -4.15\,kN \uparrow$

$R_B = \sqrt{B_x^2 + B_y^2} = \sqrt{25^2 + 4.15^2}$

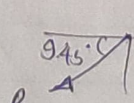
$\boxed{R_B = 25.34\,kN}$

$\theta = \tan^{-1}\dfrac{4.15}{25} = 9.43°$



Consider beam AB.

$R_B = 25.34\,kN \qquad \theta = 9.43°$



$\Sigma F_x = 0$

$A_x + B_x = 0$

$A_x + (-25) = 0$

$A_x = 25\,kN \rightarrow$

$\Sigma F_y = 0$

$A_y - 40 + B_y = 0$

$A_y - 40 + (-4.15) = 0$

$A_y = 44.15\,kN \uparrow$

$\Sigma M_A = 0$

$M_A - 40 \times 2 - B_y \times 4 = 0$

$M_A = 80 - 44.15 \times 4 = 0$

$M_A = 96.6\,kN.m$

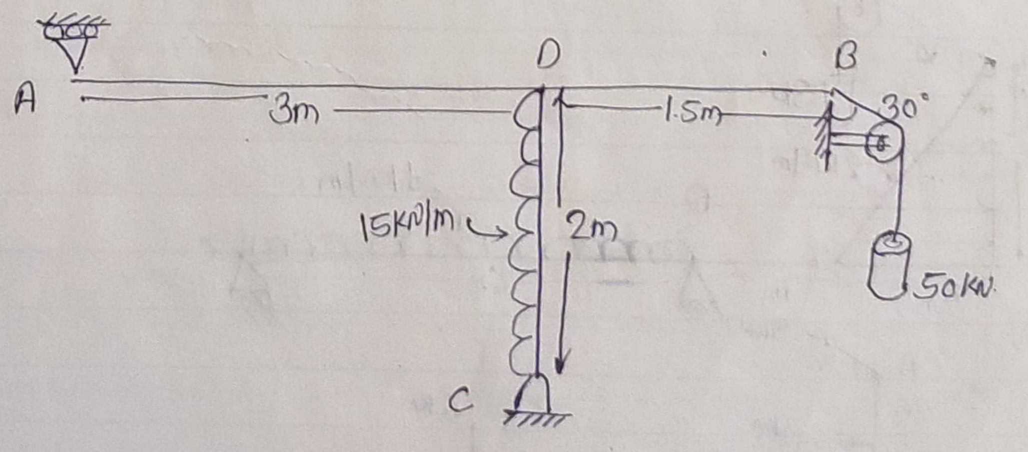
$R_A = \sqrt{25^2 + 44.15^2}$

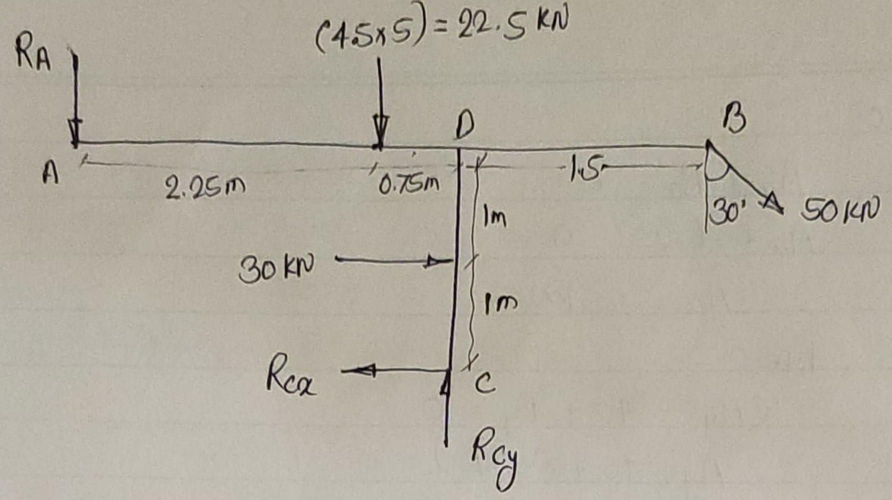
$R_A = 50.74\,kN$

$\theta = \tan^{-1} \frac{44.15}{25}$

$\theta = 60.48°$

(13) Bars AB & CD are rigidly connected by welding at D as shown in fig. Bar AB weighs 5 kN/m whereas weight of bar CD is negligible. Determine the support reactions.





$\Sigma M_{@C} = 0$

$R_A \times 3 + 22.5 \times 0.75 - 50 \cancel{\cos} \sin 30 \times 2 - 50 \cancel{\sin} \cos 30 \times 1.5$
$- 30 \times 1 = 0$

$\therefore R_A = 42.69$ kN ↓

$\Sigma F_x = 0$

$15 \times 2 + 50 \sin 30 - R_{cx} = 0$

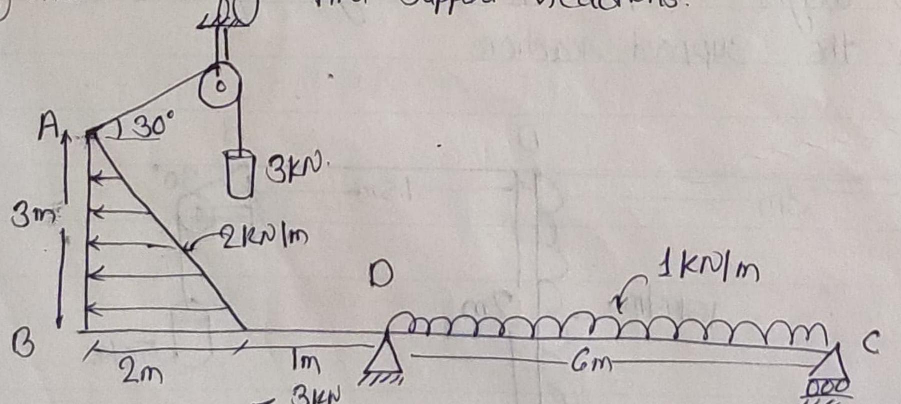
$\therefore R_{cx} = 55$ kN ←

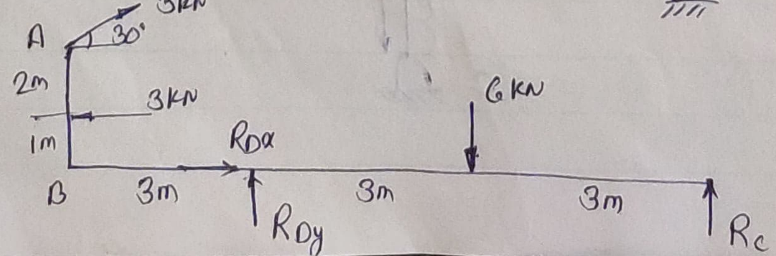
$\Sigma F_y = 0$

$R_{cy} - R_A - 22.5 - 50 \cos 30 = 0$

$\therefore R_{cy} = 108.49$ kN ↑

(14) A single rigid bar ABC of L shape is loaded & supported as shown in fig. Find support reactions.





$\Sigma M_{@D} = 0.$

$6R_c - 3\cos 30 \times 3 - 3\sin 30 \times 3 - 6 \times 3 + 3 \times 1 = 0$

$\therefore R_c = 4.55\,kN \uparrow$

$\Sigma F_x = 0$

$3\cos 30 - \frac{1}{2} \times 3 \times 2 + H_D = 0$

$\therefore H_D = 0.4\,kN \rightarrow$

$\Sigma F_y = 0$

$V_D + 3\sin 30 + R_c - 6 = 0$

$\therefore V_D = 0.05\,kN \downarrow$