Unit:

3. Single Phase AC Circuits

* Basic Elements used in AC circuits.

1) Resistor (R)
2) Inductor (L)
3) Capacitor (C)

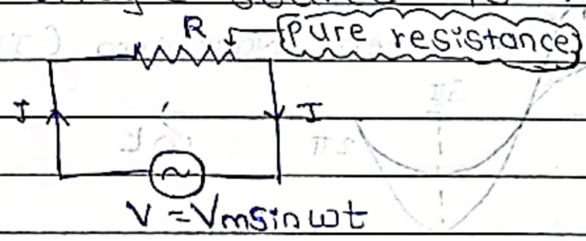
* Single Phase AC circuit

1. Purely resistive AC circuits
2. Purely inductive AC circuits
3. Purely capacitive AC circuits.

* Purely Resistive AC Circuits :-

> Consider a simple circuit consisting of a pure resistance 'R' ohm connected across a AC ~~source~~ voltage source

> Let AC voltage source is $V_m \sin \omega t$ (V_m = max)



* Voltage and current Waveform

> The applied voltage is $V = V_m \sin \omega t$

> It is sinusoidal waveform having peak value $= V_m$

> The basic equation of current is,

$$I = \frac{V}{R} = \frac{V_m \sin \omega t}{R}$$

$\therefore I = \left(\frac{V_m}{R}\right) \sin \omega t$

This equation gives instantaneous value of the current.

The standard equation of current is,

$$I = I_m \sin(\omega t + \phi)$$

Here,

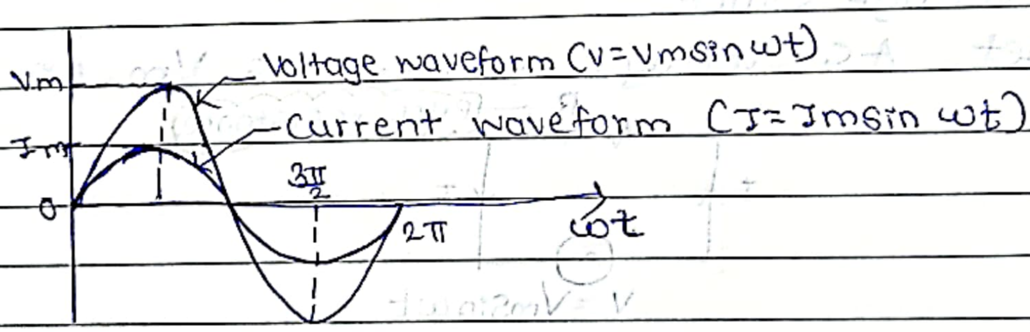
I_m = Peak (Max) value of current

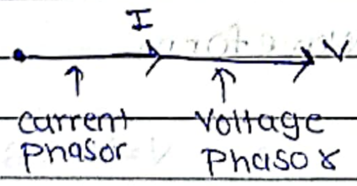
ϕ = Phase difference

$$I_m = \frac{V_m}{R} \quad \text{and} \quad \phi = 0$$

Thus,

$$I = I_m \sin \omega t$$





$\phi = 0$

Hence I and V waveform in Phase

> The waveform of voltage and current with corresponding phasor diagram is show in fig above.

> In the phasor diagram, the phasor are drawn in phase and no phase difference betn them phasors represent the r.m.s value of the alternating quantees.

* Impedence of Purely Resistive circuit

> Impedence is the opposition provided by AC circuit to the flow of current.

> It is denoted by 'Z' and it is ratio of alternating voltage to the alternating current

$$\therefore Z = \frac{\text{Alternating vtg}}{\text{Alternating current}}$$

Rectangular form $Z = \underbrace{R}_{\text{resistance}} + j\underbrace{X}_{\text{Reactance}}$ } equation of impedance

Polar form $Z = |Z| \angle \phi$

For purly resistive ~~AC~~ circuit, only R is present reactance $(X) = 0$

> Impedance in rectangular form

$Z = R + j(0)$

$\therefore Z = R \ \Omega$

> In Polar form

$Z = R \angle 0° \ \Omega$

Page No.:
Date:

* Expression for Power (Purely Resistive AC circuit)

1. Instantaneous power (P)
The instantaneous power in a.c circuit can be obtained by taking product of the instantaneous voltage and instantaneous current.

Thus $P = V \cdot I$
For purely resistive AC circuit,

$$V = V_m \sin\omega t \quad \text{and} \quad I = I_m \sin\omega t$$

$$\therefore P = V_m \sin\omega t \cdot I_m \sin\omega t$$
$$\therefore P = V_m I_m \sin^2\omega t \quad \ldots (1)$$

Using trignometric identity, $\sin^2\omega t = \frac{1-\cos(2\omega t)}{2}$

Equation (1) becomes

$$P = V_m I_m \left[\frac{1-\cos(2\omega t)}{2}\right]$$

$$\therefore P = \frac{V_m I_m}{2}\left[1-\cos(2\omega t)\right]$$

$$\therefore P = \underbrace{\frac{V_m I_m}{2}}_{\text{const term}} - \underbrace{\frac{V_m I_m}{2}\cos(2\omega t)}_{\text{Fluctuating term}} \quad \ldots (2)$$

The equation of intantaneous power consist of 2 terms
i) First term does not contain any sine or cosine so it is const.

ii) Second term contains $\cos 2(\omega t)$, it will change with respect to time so it is called f...

2. Average Power (P_{av})

> The average power of sine wave or cosine wave for one complete cycle is zero.

> The fluctuating term is cosine wave of double frequency ($2\omega t$). So, its avg power is zero

> From eqⁿ (2) avg power is

$$P_{av} = \frac{V_m I_m}{2}$$

2 can be written as $2 = \sqrt{2}\sqrt{2}$

$$\therefore P_{av} = \frac{V_m I_m}{\sqrt{2}\sqrt{2}} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$$

But $\frac{V_m}{\sqrt{2}} = V_{RMS}$; $\frac{I_m}{\sqrt{2}} = I_{RMS}$

$$\therefore P_{av} = V_{RMS} I_{RMS} \text{ Watts.}$$

Page No.:
Date:

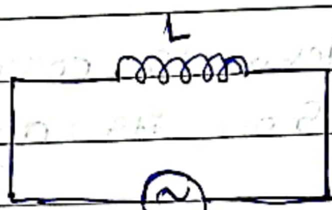
✯ Purely Inductive AC Circuit.

* Circuit Diagram

> This circuit consists of inductor (L) connected across AC voltage source.

> The pure inductor has zero internal resistance.

> It is denoted by 'L' & it is measured in Henries (H)



$V = V_m \sin\omega t$

* Equation of current.

> The alternating current produces alternating (changing) flux and due to this self induced emf is created.

> This self induced emf of inductor is denoted by 'e' & given by,

$$e = -L\frac{di}{dt}$$

> The supply voltage (V) is equal and opposite to self induced emf.

$$\therefore V = -e$$

$$= -\left(-L\frac{di}{dt}\right) = L\frac{di}{dt}$$

Now $V = V_m \sin\omega t$

$$\therefore V_m \sin\omega t = L\frac{di}{dt}$$

Page No.:
Date:

$$\frac{di}{dt} = \frac{V_m \sin \omega t}{L}$$

$$di = \frac{V_m}{L} \sin \omega t \, dt$$

Equation of current(i) can be obtain by integrating di

$$\therefore i = \int di = \int \frac{V_m}{L} \sin \omega t \, dt$$

$$\therefore i = \frac{V_m}{L} \int \sin \omega t \, dt =$$

$$= \frac{V_m}{L} \left[-\frac{\cos \omega t}{\omega} \right]$$

$$= -\frac{V_m}{\omega L} [\cos \omega t]$$

using idendity, $\cos \omega t = \sin \left(\frac{\pi}{2} - \omega t \right)$

$$\therefore i = -\frac{V_m}{\omega L} \left[\sin \left(\frac{\pi}{2} - \omega t \right) \right]$$

But $\sin \left(\frac{\pi}{2} - \omega t \right) = \sin \left[-\left(\omega t - \frac{\pi}{2} \right) \right]$

$$= -\sin \left(\omega t - \frac{\pi}{2} \right)$$

$$\therefore i = -\frac{V_m}{\omega L} \left[-\sin \left(\omega t - \frac{\pi}{2} \right) \right]$$

$$\therefore i = \frac{V_m}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right)$$

The term $\frac{V_m}{\omega L}$ is called as peak value, denoted by I_m

$I_m = \frac{V_m}{\omega L}$ and $\omega L = X_L = 2\pi f L$ is called as reactance of inductance.

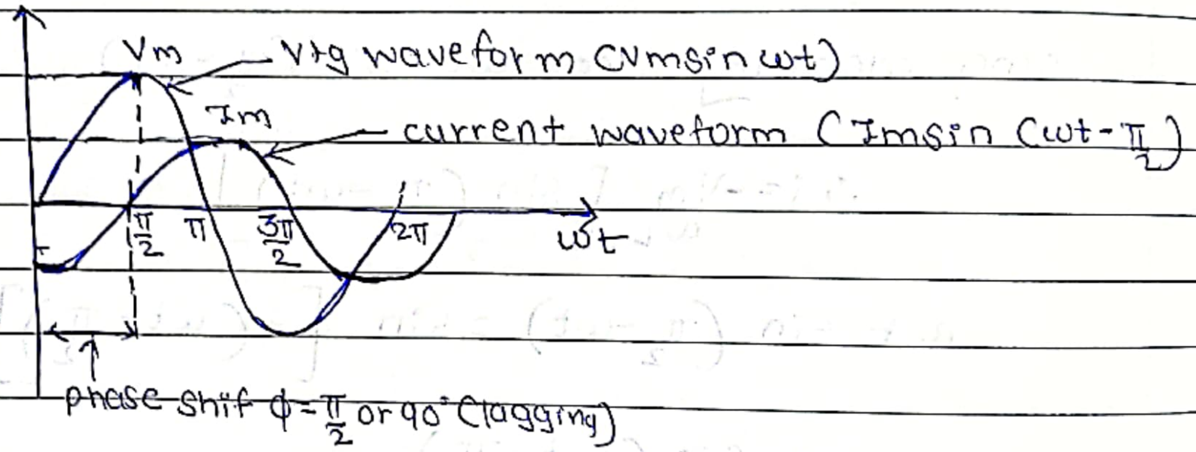
* Voltage Current waveform:

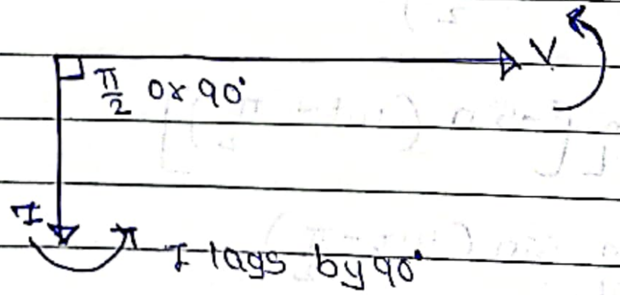
- The voltage applied to purely inductive circuit is
$$V = V_m \sin \omega t$$

- In case of purely inductive circuit, the current lags the voltage by an angle $\frac{\pi}{2}$ radians, i.e. 90°

The equation of current.
$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Here,
$$I_m = \frac{V_m}{X_L} \qquad X_L = \text{Reactance of inductor} = \omega L = 2\pi f L$$





Expression for impedance of Purely Inductive Circuit.

In rectangular form
$$Z = R + jX$$

- For purely inductive circuit, Resistance (R) is zero and reactance = X_L

$$\therefore Z_L = jX_L \; \Omega$$

Polar form,

$$\therefore Z = X_L \angle \frac{\pi}{2} \; \Omega$$

$$\text{Impedance}(Z) = \frac{\text{Vtg}}{\text{Curr}}$$

$$= \frac{V\angle 0}{I\angle -90^\circ} = X_L \angle 90^\circ \; \Omega$$

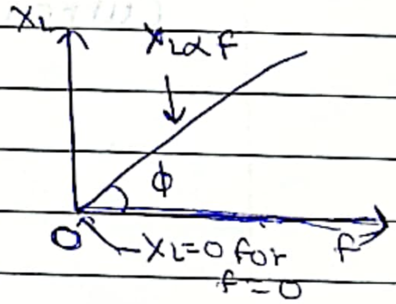
★ Inductive reactance :-

"Inductive rectance is the opposition provided by inductor to the flow of current."

> Denoted by X_L

$$X_L = \omega L \quad \text{and} \quad \omega = 2\pi f$$

$$\boxed{X_L = 2\pi f L}$$

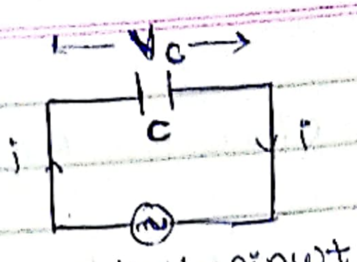


★ Purely Capacitive AC circuit

> The circuit consist of capacitor (C) connected across AC voltage source

> Capacitor has property to store electrical energy when the voltage is applied across it.

> The capacitance is denoted by 'C' and it is measured in Farads (F).



$V = V_m \sin\omega t$

* Equation of Current:

The vtg applied to purely capacitive circuit is

$$V = V_m \sin\omega t$$

The current i charge the capacitor C. The intanteneous charge 'q' on the plate of capacitor is given by

$$q = CV = CV_m \sin\omega t$$

Current is the rate of change of charge

$$i = \frac{dq}{dt} = \frac{d}{dt}[CV_m \sin\omega t] = CV_m \frac{d}{dt}[\sin\omega t]$$

$\therefore$ ~~$i = CV_m \cos$~~

$$i = CV_m \cos\omega t$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \cos\omega t$$

But $\cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$

$$\therefore i = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

Where, $\frac{V_m}{\frac{1}{\omega C}}$ is denoted by I_m

Here $\frac{1}{\omega C}$ is rectance of capacitance denoted by X_C

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

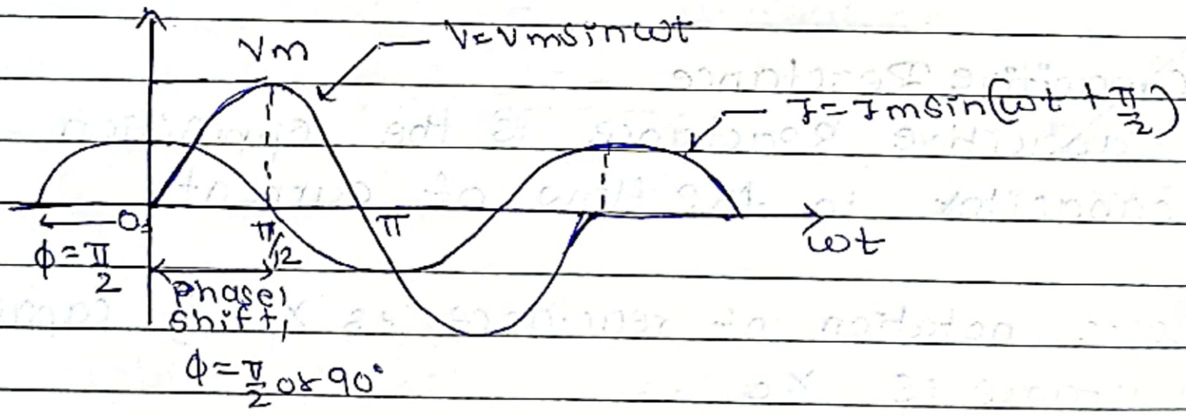
* Voltage and Current Waveforms

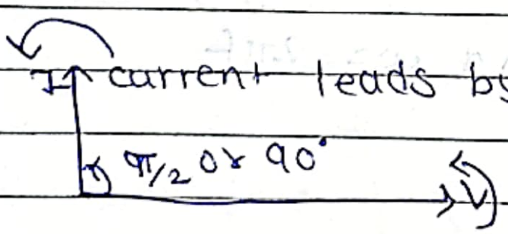
> The voltage applied to purely capacitive cicuit is
$V = V_m \sin \omega t$.

> For purely capacitive circuit, the current leads voltage by an angle $\frac{\pi}{2}$ radian, i.e $90°$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I_m = \frac{V_m}{X_C} \qquad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$





* Impedance of Purely Capacitive Circuit
Rectangular form

$$Z = R + jX$$

> For purely capacitive circuit, resistance (R) is zero reactance is denoted by X_C

> Current is leading & Vtg is lagging the reactance is $-X_C$

$$\therefore Z = 0 - jX_C \ \Omega$$

In polar form, for '-j' term corresponding angle is $-\frac{\pi}{2}$ $-\frac{\pi}{2}$

$$\therefore Z = X_C \angle -\frac{\pi}{2} \ \Omega$$

$$Z = \frac{V}{I} = \frac{V\angle 0^\circ}{I\angle 90^\circ}$$

$$= X_C \angle -90^\circ$$

* Capacitive Reactance

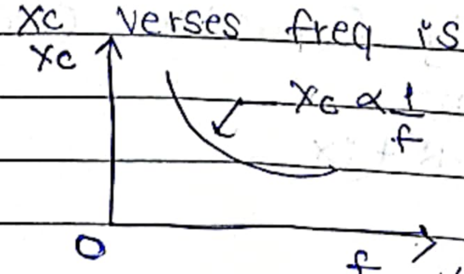
> Capacitive Reactance is the opposition provided capacitor to the flow of current.

> Basic notation of reactance is X & capacitive reactance is X_C

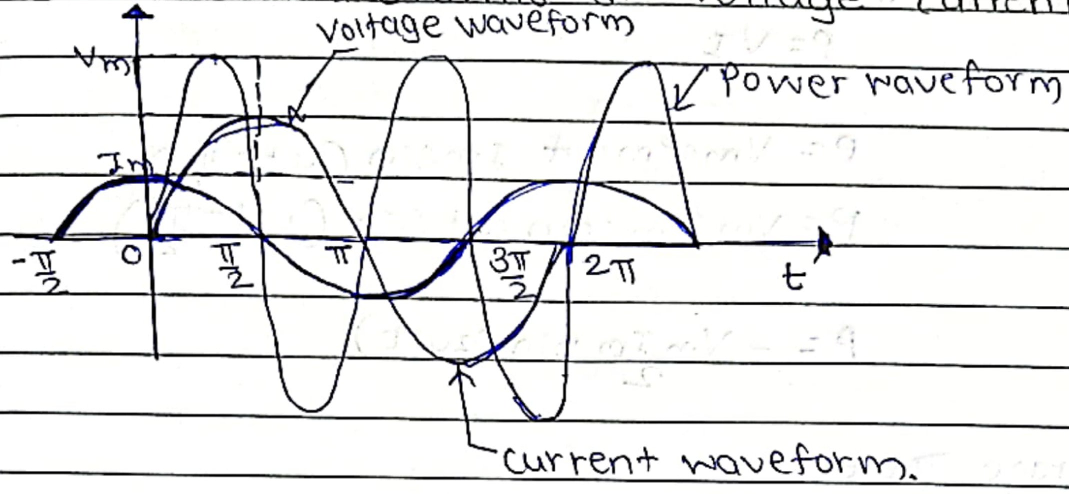
$$X_C = \frac{1}{\omega C} \quad \text{and} \quad \omega = 2\pi f$$

$$\therefore X_C = \frac{1}{2\pi f C}$$

> Graph X_C verses freq is exponential curve.



* Combined waveforms for voltage current and power.



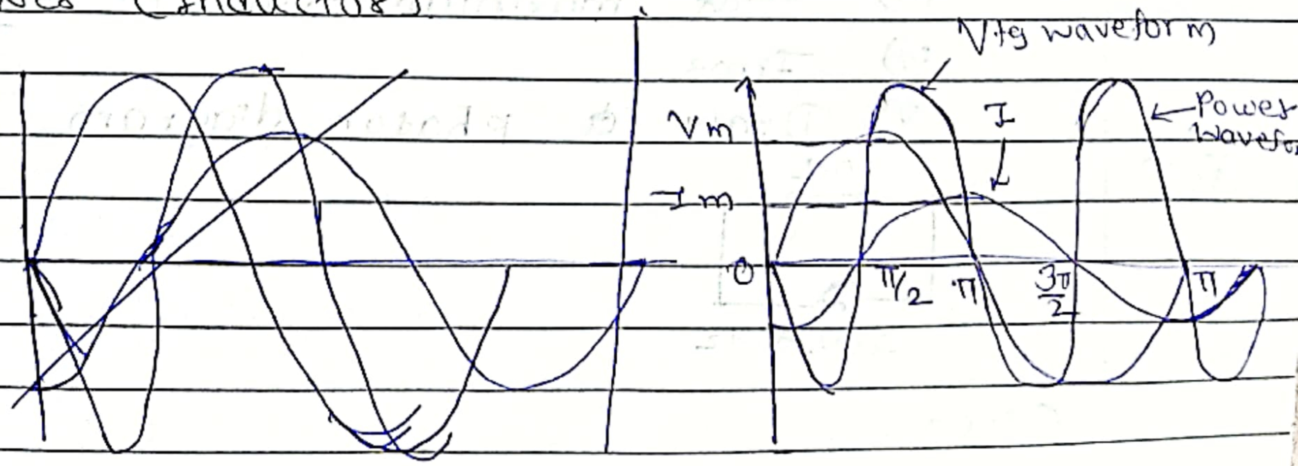
Power is Given as,

$$P = VI$$
$$= V_m \sin\omega t \; I_m \sin(\omega t + \tfrac{\pi}{2})$$
$$= V_m I_m \sin\omega t \cdot \sin(\omega t + \tfrac{\pi}{2})$$

$$P_{avg} = 0$$

* In case of capacitor average power (P_{av}) for a complete cycle is always zero but the energy stored by capacitor is $E = \frac{1}{2} CV^2$ Joules

* Combined waveform for voltage current & power (Inductor).



Page No.:
Date:

* Intantaneous power.

$$P = VI$$

$$P = V_m \sin\omega t \; I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Solving

$$P = V_m I_m \sin\omega t \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$P = -\frac{V_m I_m}{2} \sin(2\omega t)$$

* Average Power

The average Power in case of an inductor complete cycle is always 0, because ⊕ and ⊖ half cycle of power waveform canctes each other but the energy Stored is always

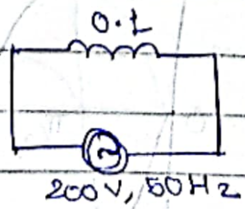
$$E = \frac{1}{2} L I^2 \quad \text{and} \quad E = \frac{1}{2} C V^2$$

$$\boxed{P_{av} = 0}$$

⑤ Numerical:-

Q] An inductive coil having 0.1 Henry Inductance is connected across 200V, 50Hz supply find
i) Inductive reactance (X_L)
ii) rms value of vtg (V_{rms})
iii) V_{max} maximum value of vtg
iv) I_{rms}
v) Draw a phasor diagram.

Soln



Given,

$L = 0.1\,H$; $f = 50\,Hz$
$V_{rms} = 200V$

Page No.:
Date:

i) Inductive reactance (X_L)

$X_L = 2\pi f L$

$= \cancel{2\times 3.14}\ 2\times\pi\times 50\times 0.1$

$= 31.4159\ \Omega$

ii) $V_{rms} = \frac{V_m}{\sqrt{2}}$

$200 = \frac{V_m}{\sqrt{2}}$

$200\times\sqrt{2} = V_m$

$\boxed{V_m = 282.84\,V}$

iii) $I_m = \frac{V_m}{X_L}$

$I_m = \frac{282.84}{31.41}$

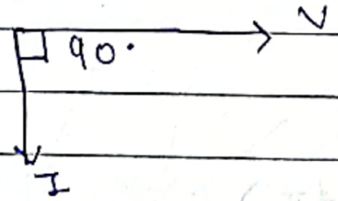
$\boxed{I_m = 9.004\,A}$

iv) $I_{rms} = \frac{I_m}{\sqrt{2}}$

$= \frac{9.004}{\sqrt{2}}$

$I_{rms} = 6.37\,A$

v)



Equation of vtg

$V = V_m \sin \omega t$

$= 282.84 \sin 2\pi\times 50 t$

$V = 282.84 \sin 100\pi t$

$i = I_m \sin(\omega t - \frac{\pi}{2})$

$= 9 \sin(2\pi\times 50 t - \frac{\pi}{2})$

$i = 9 \sin(100\pi t - \frac{\pi}{2})$

eqn of current//

Q2) Find the expression for current it will flow through pure inductor 0.2 H connected across 230V 50 Hz AC supply. Draw the phasor diagram.

Soln Given;

$L = 0.2 H$

$V_{rms} = 230V$

$f = 50 Hz$

$X_L = 2\pi f L$

$= 2 \times \pi \times 50 \times 0.2$

$X_L = 62.83$

$V_{rms} = \frac{V_m}{\sqrt{2}}$

$230 \times \sqrt{2} = V_m$

$V_m = 325.269V$

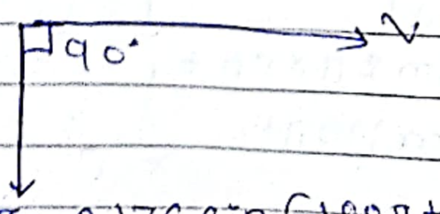
$I_m = \frac{V_m}{X_L}$

$I_m = \frac{325.269}{62.83}$

$\boxed{I_m = 5.176}$

$I = I_m \sin(\omega t - \frac{\pi}{2})$

$I = 5.176 \sin(100\pi t - \frac{\pi}{2})$



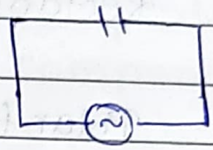
$I = 5.176 \sin(100\pi t - \frac{\pi}{2})$

Capa

Q.3) 50 µF capacitor is connected across 230V, 50 Hz supply calculate
1) Capacitive reactance (X_C) or reactance offered
2) Max value of current
3) Max value of Vtg
4) Write eqⁿ of Vtg and current
5) Phasor diagram.

Soln Given,

$50\mu F = 50\times10^{-6} F$



$C = 50\mu F = 50\times10^{-6} F$

$V_{rms} = 230V$

$F = 50Hz$

i) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\times50\times10^{-6}\times50}$

$X_C = 63.661\,\Omega$

iii) $V_{rms} = \frac{V_m}{\sqrt{2}}$

$230\times\sqrt{2} = V_m$

$\boxed{V_m = 325.269}$

ii) $I_m = \frac{V_m}{X_C} = \frac{325.269}{63.661}$

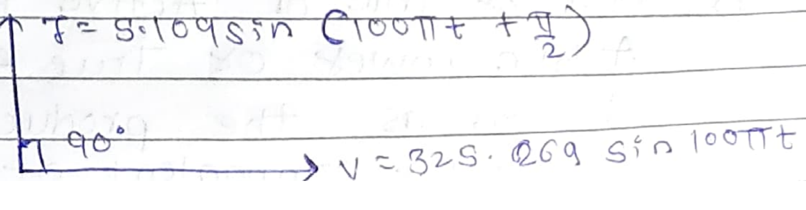
$\boxed{I_m = 5.109 A}$

iv) $I = I_m \sin(\omega t + \frac{\pi}{2})$

$I = \cancel{I_m}\ 5.109 \sin(100\pi t + \frac{\pi}{2})$

$V = 325.269 \sin(100\pi t)$

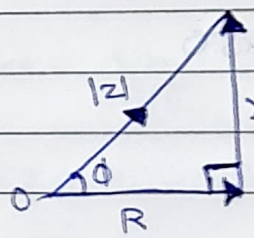
v) Phasor Di :-



Page No.:
Date:

Impedance:-
"The opposition provided by AC circuit to the flow of current is called as impedance

3 basic elements of AC circuit
Resistor ii) Inductor iii) Capacitor
Denoted by Z
Rectangular form:-
$$Z = R + jX$$
Polar form:-
$$Z = |Z| \angle \phi$$
Impedance Triangle:-



Types of Power

Apparent Power (S)
"Apparent power is defined as the product of RMS value of voltage and RMS value of current."
Denoted by 'S'
Unit:- Volt-Ampere (VA)

Formula: $\boxed{S = V_{RMS} \times I_{RMS}}$

Real or True or Active Power (P)
Active power or True p or Real p is defined as the product of supply voltage and active component of current (I cos ϕ)

> Denoted by 'P'
> Unit : Watt (W)

Formula: $\boxed{P = V I \cos\phi}$ // (I: rms current; V: rms vtg; ϕ: phase betn vtg & I)

2) Imaginary or Reactive Power (Q)
Reactive Power is defined as the product of supply voltage and reactive component of current (I sin ϕ)
> Denoted by 'Q'
> Unit :- Volt - Ampere Reactive (VAR)

Formula: $\boxed{Q = V I \sin\phi}$ //

* Power Factor ($\cos\phi$)
It is defined as ratio of real power (P) to apparent power (S).

$$\therefore P.F = \frac{\text{Real power}}{\text{Apparent power}} = \frac{P}{S}$$

$P = VI\cos\phi$ $\quad$ $S = VI$

$$P.F = \frac{VI\cos\phi}{VI} = \cos\phi$$

> Ratio of Resistance (R) to the impedance (Z)

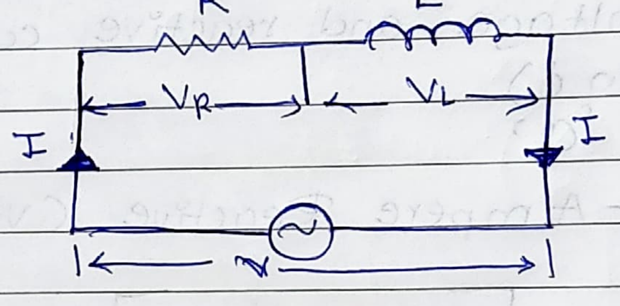
$$\therefore PF = \cos\phi = \frac{R}{Z}$$

$$P.F = \cos\phi = \frac{\text{Fundamental component of current}}{\text{Rms value of total current}}$$

* Significance (Imp) of Power factor

i) Ideally P.F must be equal to 1.
ii) High P.F increases current carrying capacity.
iii) For improved P.F, power losses are reduced.

* A.C through series combination of R-L circuit



Consider a simple circuit consist of series combination of Resistance (R) and inductance (L) connected across V.

The supply vtg is, $V = V_m \sin\omega t$

Let V = is equal to rms value of applied voltage rms value of resultant current Hence voltage drop across R

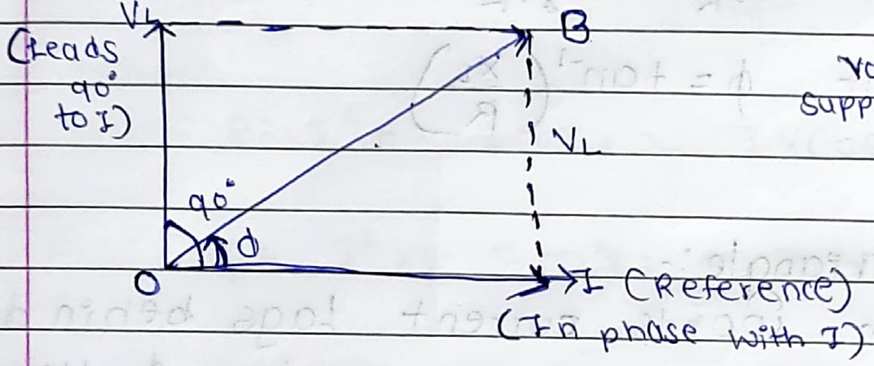
$V_R = I \times R$ (V_R in phase with current i)

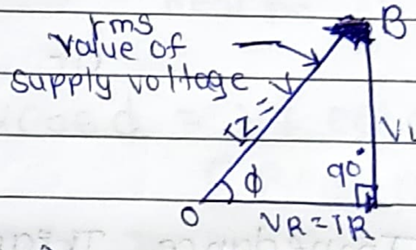
Voltage drop across L

$V_L = I \times X_L$ (V_L leading the current i by magnitude)

* Phasor Diagram

As resistance R and inductance L are in series, their individual voltage drops V_R and V_L also come in series therefore total voltage is found by phasor addition. Current is taken as reference in series circuit as it is common in both the element it is drawn on positive x-axis as shown.





Voltage triangle

* Impedance of Series AC R-L

From vtg triangle vector OB represents total applied voltage which is vector sum of V_R and V_L

$$\bar{V} = \bar{V}_R + \bar{V}_L$$

$$V = \sqrt{V_R^2 + V_L^2}$$

But $V_R = IR$ $\qquad V_R^2 = I^2R^2$

$V_L = IX_L$ $\qquad V_L^2 = I^2X_L^2$

$$V = \sqrt{I^2R^2 + I^2X_L^2}$$

$$= \sqrt{I^2(R^2 + X_L^2)}$$

$$V = I\sqrt{R^2 + X_L^2}$$

$Z = \frac{V}{I}$ i.e $V = IZ$

$\therefore Z = \sqrt{R^2 + X_L^2}$

> Z is known as impedance of the circuit

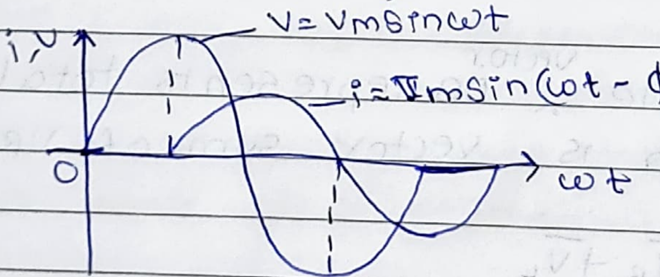
> As shown in phasor diagram applied voltage leads the current I by an angle ϕ such that

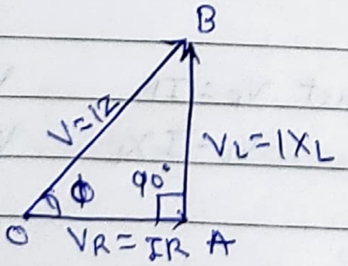
$$\tan\phi = \frac{V_L}{V_R} = \frac{\cancel{I} \cdot X_L}{\cancel{I} \cdot R} = \frac{X_L}{R}$$

$$\text{or} \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

* Impedance Triangle :-

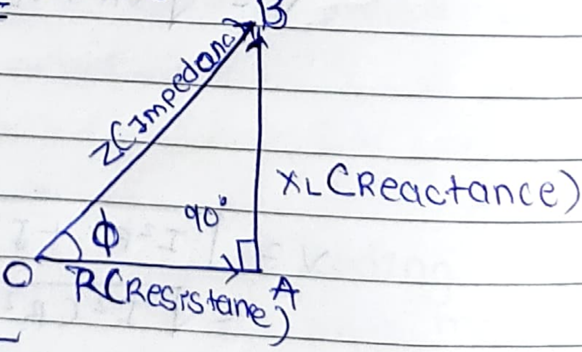
In other words current Lags behind the applied voltage by an angle ϕ, Hence $V = V_m \sin\omega t$ & current is $i = I_m \sin(\omega t - \phi)$.





Divide each by I

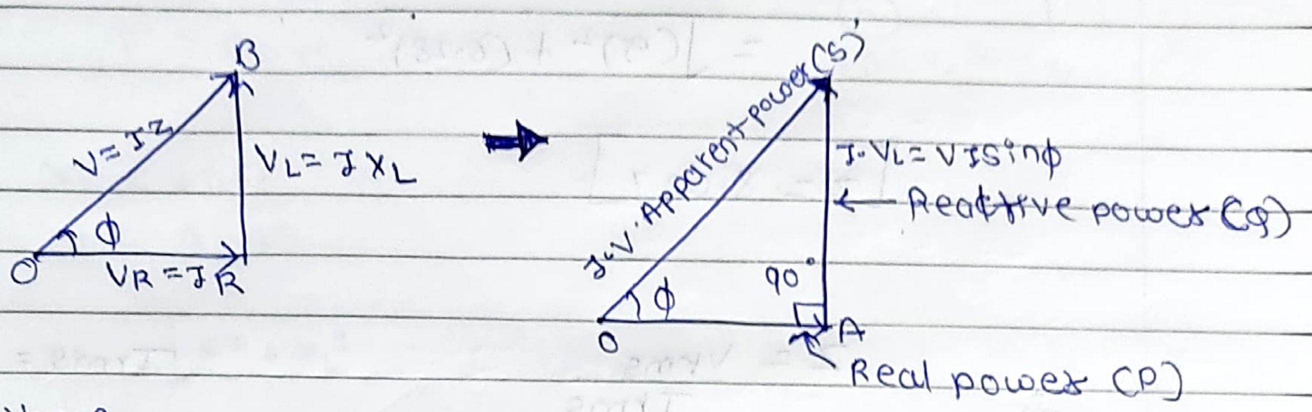
$\Rightarrow$



[Impedance Triangle]

★ Power triangle

Power triangle is obtained by multiplying each side of voltage triangle by current(I).



Where,

$$P = I^2 R = I.V_R = IV\cos\phi = VI\cos\phi \text{ watt}$$

(Real Power)

$$Q = I^2 X_L = I.\cancel{X}_L = I.V_L = I.V\sin\phi \text{ VAR}$$

(Reactive power)

$$S = I^2 Z = I.V = VI \text{ VA}$$

(Apparent power)

Q) A coil has inductance 20 mH and resistance of 5 Ω connected in series across the voltage of $V = 48\sin(314t)$ obtain the expression for current drawn by.

Sol^n Given,

$L = 20\,mH = 20\times10^{-3}\,H$

$R = 5\,\Omega$

$V = V_m \sin\omega t$

$V = 48\sin(314t)$

$\boxed{V_m = 48}$

$X_L = 2\pi f L$

$= 2\pi \times 49.97 \times 20\times10^{-3}$

$X_L = 6.28$

$Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{(5)^2 + (6.28)^2}$

$\boxed{Z = 8.027}$

$Z = \frac{V_{rms}}{I_{rms}}$

$I_{rms} = \frac{I_m}{\sqrt{2}}$

$I_m = \frac{V_m}{Z}$

$= \frac{48}{8.027}$

$\cos\phi = \frac{R}{Z}$

$\phi = \cos^{-1}\left(\frac{R}{Z}\right)$

$\phi = 51.47$

$I = I_m \sin(\omega t - \phi)$

$= 314t - 51.47$

$= 5.979 \sin(314t - 51.47)$

9 A series R-L circuit $R = 25\,\Omega$ and $L = 0.1H$ are connected in series across 250V, 50Hz supply. Calculate:

1) Inductive Reactance (X_L)
2) Impedance (Z)
3) Current (I)
4) Power factor
5) Real, Reactive, apperent power
6) Draw the phasor diagram

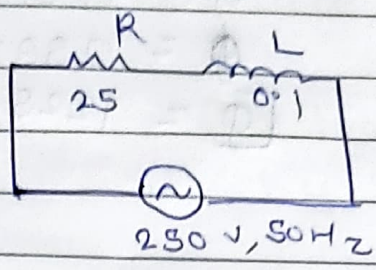
Soln Given,

$R = 25\Omega$

$L = 0.1\ H$

$V_{rms} = 250V$

$F = 50\ Hz$



1)

$X_L = 2\pi f L$

$X_L = 31.45$

2) $Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{(25)^2 + (31.415)^2}$

$Z = 40.148$

3) $V = IZ$

$I = \frac{V}{Z} = \frac{230}{40.148}$

$\boxed{I = 6.266\ A}$

4) $P = VI\cos\phi$

$\phi = \cos^{-1}(R/Z)$

$= \cos^{-1}\left(\frac{25}{40.148}\right)$

$P = VI\cos(51.48)$

$P = 250 \times 6.266\ \cos(51.48)$

$\boxed{P = 975.597\ \text{watt}}$

$Q = VI\sin\phi$

$Q = 250 \times 6.266 \times \sin(51.48)$

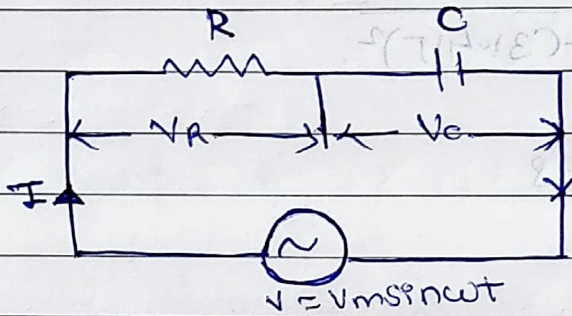
$\boxed{Q = 1225.615 \text{ VAR}}$

$S = VI$

$= 250 \times 6.266$

$\boxed{S = 1566.5 \text{ VA}}$

★ A-C through series ~~R~~-C circuit



Consider series combination of resistance and capacitance connected across voltag(V)

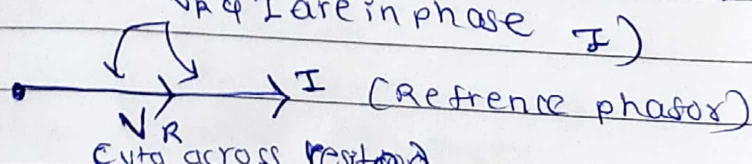
The supply voltage ~~drop across~~ is $V = V_m \sin\omega t$

If instanteneous current "i" then RMS current is denoted by I_{RMS} or I.

Due to flow of current "I", the voltage drop take place across resistor (R) and capacitor(C)

The voltage drop across Resistance $R = V_R$

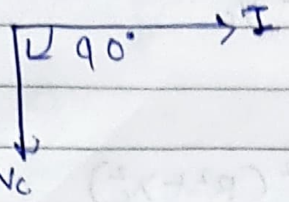
$V_R = I \times R$ ...① (V_R in phase with I)



The voltage drop across capacitance $C = V_c$

$V_C = IX_C$ $\quad X_C = \frac{1}{2\pi f C}$ (Vc lags the I by 90°)

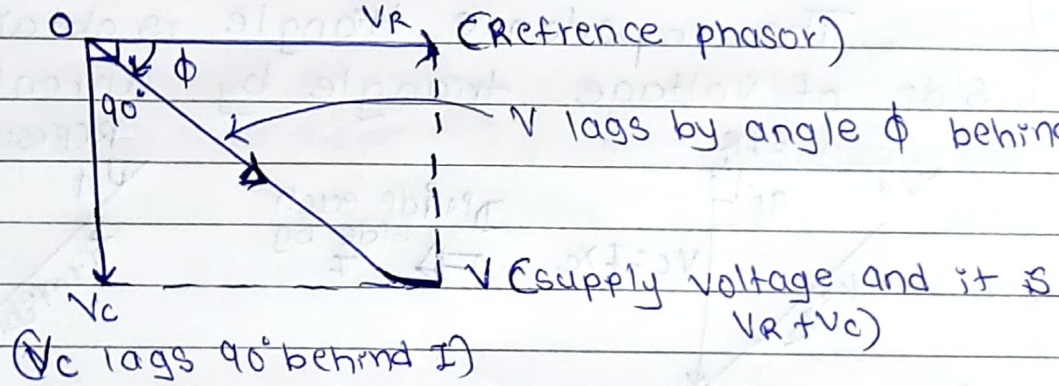
↓
capacitive Reactance



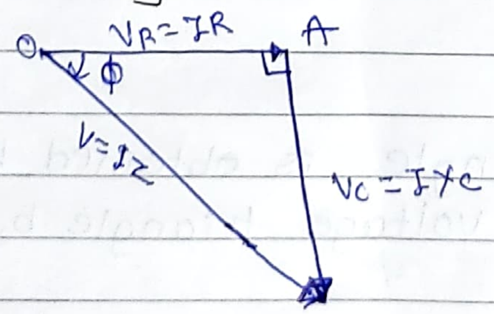
As V_R and V_C are voltage drop across R and C therefore total voltage V is phasor sum of V_R and V_C

$$V = V_R + V_C$$

* Complete phasor diagram.



* Voltage triangle:



- Impedance

From vtg triangle.

$$V = \sqrt{V_R^2 + V_C^2}$$

But $V_R = IR$ $\therefore V_R^2 = I^2 R^2$

$V_C = I X_C$ $\therefore V_C^2 = I^2 X_C^2$

$$V = \sqrt{I^2 R^2 + I^2 X_C^2} = \sqrt{I^2 (R^2 + X_C^2)}$$

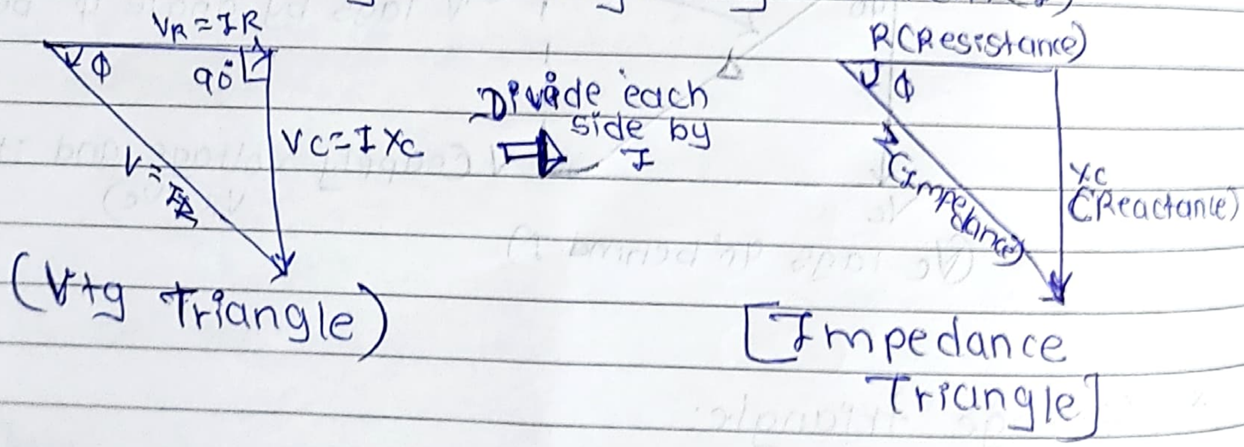
$$V = I\sqrt{R^2 + X_C^2}$$

$$Z = \frac{V}{I} \qquad V = IZ$$

$$\therefore \boxed{Z = \sqrt{R^2 + X_C^2}}$$

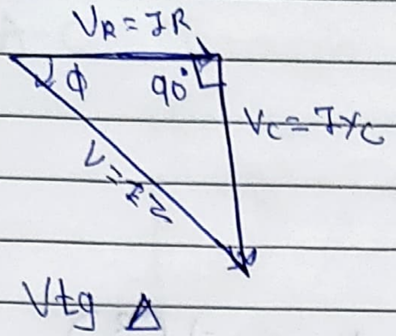
* Impedance triangle

The impedance triangle is obtain by diving side of voltage triangle by current(I)

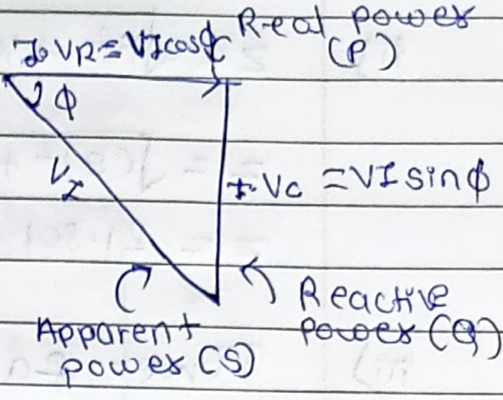


* Power Triangle:

Power Triangle is obtained by multiplying each side of voltage triangle by current(I)



Multiply each side by I



[Power Triangle]

$$\tan\phi = \frac{-V_C}{V_R}$$

$$\phi = \tan^{-1}\left(\frac{-V_C}{V_R}\right)$$

$$\cos\phi = \frac{R}{Z}$$

$$\phi = \cos^{-1}(R/Z)$$

Q) A series circuit consisting of Resistance 5Ω and capacitance of $150\mu F$ is connected to single phase 200V, 50Hz supply Calculate.

i) Capacitive Reactance
ii) Impedence
iii) Power factor.
iv) Current drawn by circuit
v) Actual Power OR Real power.

Soln Given,

i)
$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\times\pi\times50\times150\times10^{-6}}$$

$$\boxed{X_C = 21.22\Omega}$$

ii) $Z = \sqrt{R^2 + X_C^2}$

$Z = \sqrt{(5)^2 + (21.220)^2}$

$Z = 21.801$

iii) Power factor $= \cos\phi$ $\Rightarrow \phi = \cos^{-1}\left(\frac{R}{Z}\right)$

P.F $= 0.229$ $\phi = 76.741$

iv) $V = I \times Z$

$I = \frac{V}{Z}$

$I = \frac{200}{21.801}$

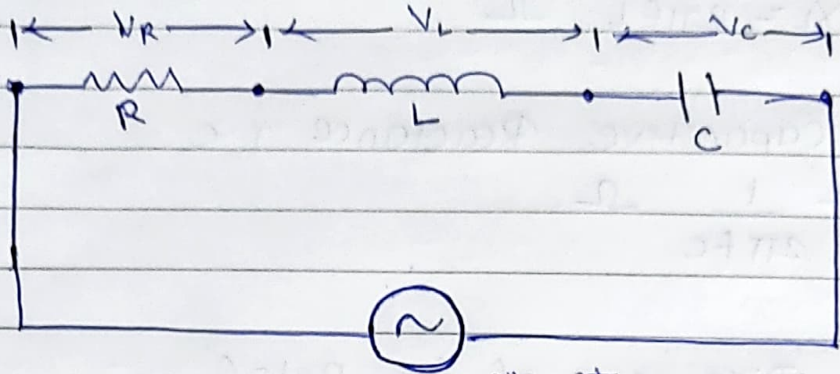
$I = 9.173$

v) $P = VI \cos\phi$

$= 200 \times 9.173 \cos(76.741)$

$P = 420.771$

★ A-C Through R-L-C circuit.



$v = V_m \sin\omega t$
(supply vtg)

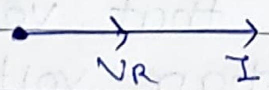
The supply voltage is $V = V_m \sin\omega t$

Consider a series combination of resistor (R), inductor (L) & capacitor (C) connected across a Voltage (V)

Due to flow of current 'I' the voltage drop take place aross R, L, C.

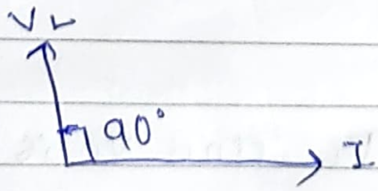
voltage drop across Resistance R

$V = IR$ (V_R in phase with I)



Voltage drop across Inductor L

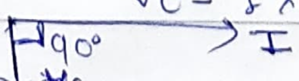
$V_L = IX_L$



(V_L leads the current I by 90°)

Voltage drop across capacitor C

$V_C = IX_C$ (V_C lag the I by 90°)



X_L = Inductive Reactance

i.e $X_L = 2\pi f L \ \Omega$

X_C = Capacitive Reactance i.e

$X_C = \frac{1}{2\pi f C} \ \Omega$

★ Phasor Diagram ~~of~~ for R-L-C

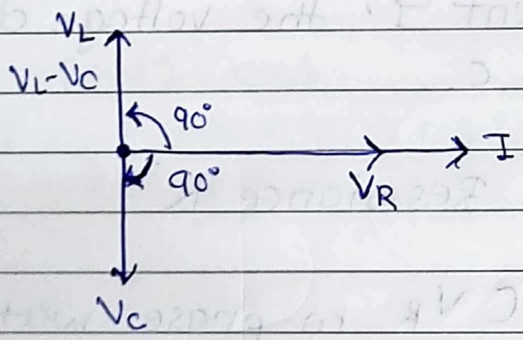
Q Explain the series R-L-C circuit with three cases

i) $X_L > X_C$

ii) $X_L < X_C$

iii) $X_L = X_C$

i) Phasor Diagram for $X_L > X_C$

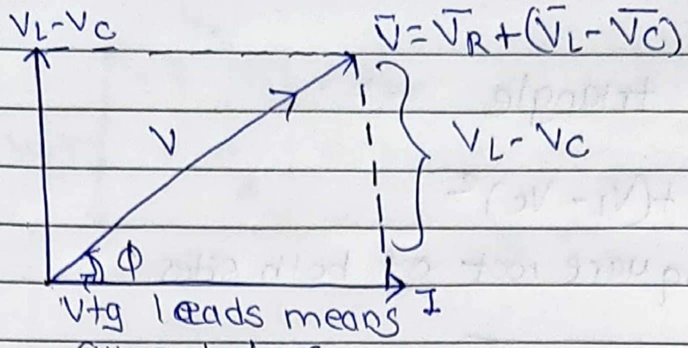


As $X_L > X_C$ it means that voltage drop across L is greater than voltage drop across C

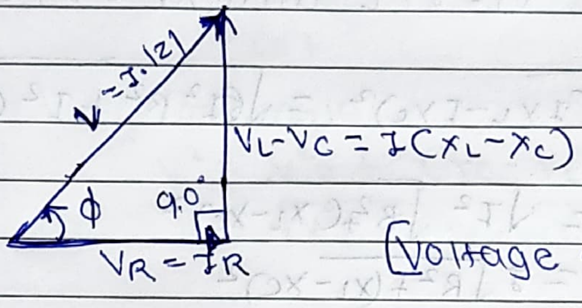
$\therefore V_L > V_C$

Thus the resultant of V_L and V_C will be towards V_L

The resultant of V_L and V_C is $V_L - V_C$



Vtg leads means current lags.



$(V = IZ)$

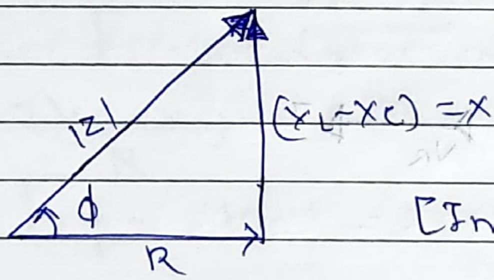
$(V_L - V_C) = I(X_L - X_C)$

$(V_R = IR)$

[Voltage triangle]

* Impedance triangle is obtain by dividing each side of Vtg triangle by current I.

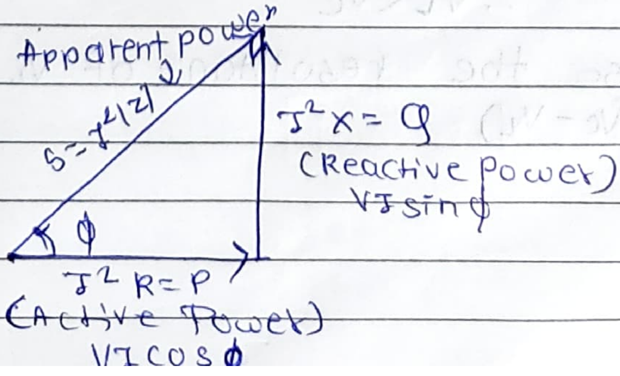
⇩ Divide Each side by I



[Impedance triangle]

* Power triangle is obtain by multiplying each side of vtg triangle by I^2

⇩



from vtg triangle

$$V^2 = V_R^2 + (V_L - V_C)^2$$

Taking square root of both sides

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Now $V_R = I \cdot R$, $V_L = IX_L$ and $X_C = IX_C$

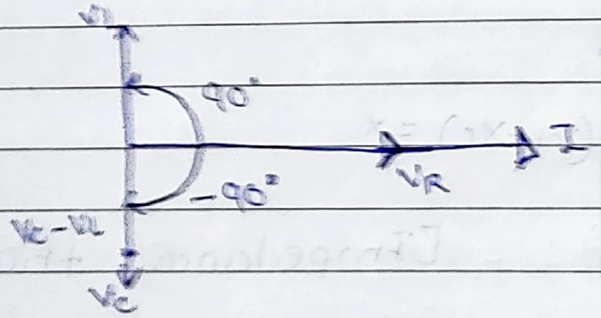
$$V = \sqrt{I^2R^2 + (IX_L - IX_C)^2} = \sqrt{I^2R^2 + I^2(X_L - X_C)^2}$$

~~Tex~~ $$V = \sqrt{I^2}\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

2. Phasor Diagram for $X_L < X_C$

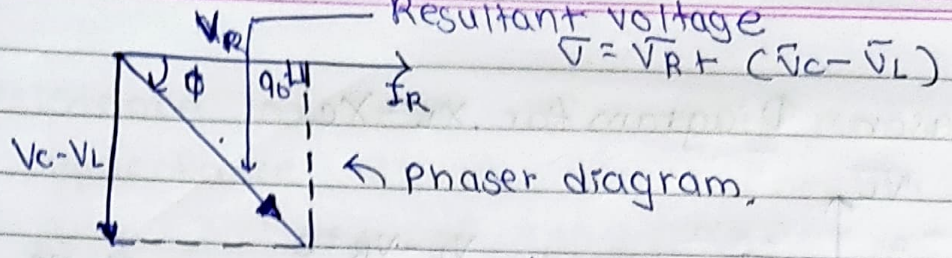


Since $X_L < X_C$ ∴ Voltage drop across Inductor is less than voltage drop across capacitance.

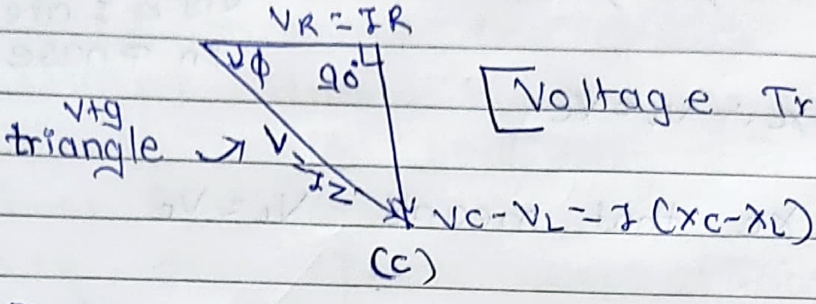
$$\therefore V_L < V_C$$

capacitive circuit

$V_C > V_L$, so the resultant of V_L and V_C is $(V_C - V_L)$



⇓



[Voltage Triangle]

(c)

From vtg triangle

$$V^2 = V_R^2 + (V_C - V_L)^2$$
$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

Here,

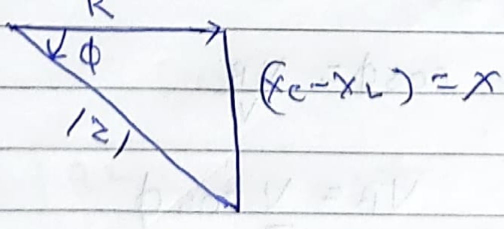
$$V_R = I.R,\ V_C = I.X_C \text{ and } V_L = I.X_L$$

$$V = \sqrt{I^2R^2 + (IX_C - IX_L)^2} = \sqrt{I^2R^2 + I^2(X_C - X_L)^2}$$
$$V = \sqrt{I^2}\sqrt{R^2 + (X_C - X_L)^2}$$
$$V = I\sqrt{R^2 + (X_C - X_L)^2}$$
$$\left[\therefore Z = \sqrt{R^2 + (X_C - X_L)^2}\right]$$

$$\tan\phi = \frac{-(X_C - X_L)}{R} = \frac{-(V_C - V_L)}{R}$$

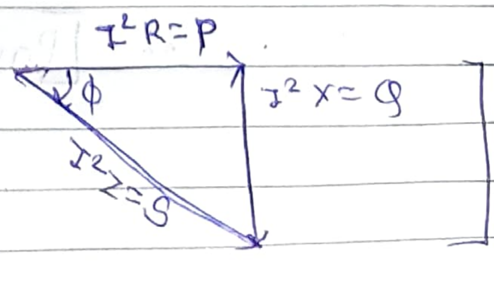
$$\left[\phi = \tan^{-1}\frac{(V_L - V_C)}{R}\right]$$

* Impedance triangle.

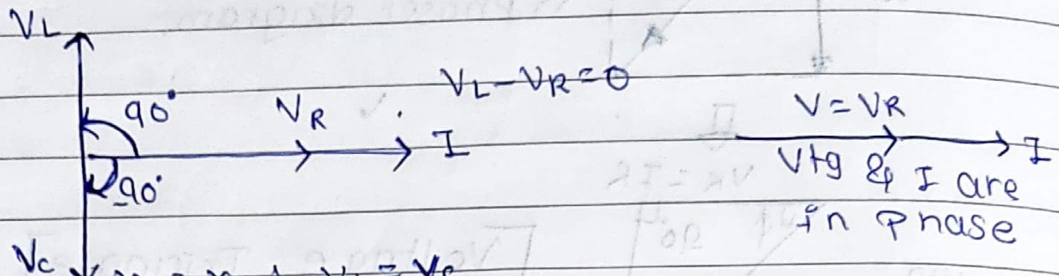


* Power triangle

[AcP =
RP
AP =]



3. Phasor Diagram for $X_L = X_C$



$X_L = X_C \therefore V_L = V_C$

Since $X_L = X_C$ that means $V_L = V_C$

V_L & V V_C will cancel each other and its resultant will be zero.

$\therefore V_L - V_C = 0$

★ Power consumed by series RLC circuit.

> The avrage power supplied to series RLC circuit is the add^n of power consumed by resistor (R) inductor (L) and capacitor (C).

> But pure inductor and capacitor do not consume any power.

$$\therefore P_{av} = I^2 R = I \cdot (I \cdot R)$$

$$\therefore \boxed{P_{av} = I \cdot V_R}$$

from vtg Δ

$$\cos\phi = \frac{V_R}{V}$$

$$V_R = V \cos\phi$$

$$\therefore \boxed{P_{av} = V \cdot I \cdot \cos\phi}$$

Page No.:
Date:

Q A Resistance of 20Ω Inductance ~~and~~ of 0.05H and capacitance $50\mu F$ are connected in series a supply voltage of 230V, 50Hz is connected across this series combination. Calculate

1) Impedance of the circuite
2) Current drawn by the circuit
3) Phase angle betn vtg and current
4) Power factor of circuit
5) vtg drop across capacitance
6) Active and Reactive power P, Q

Ans Given

$R = 20\Omega$

$L = 0.05 H$

$C = 50\mu F = 50\times10^{-6} F$

$V_{rms} = 230V$

$f = 50Hz$

→ $X_L = 2\pi f L$

$= 2\pi\times50\times0.05$

$\boxed{X_L = 15.707\Omega}$

iii) → $X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi\times50\times50\times10^{-6}}$

$\boxed{X_c = 63.660}$

As, we can see $X_c > X_L$ Hence complete circuit is capacitive in nature,

i) $Z = \sqrt{R^2 + (X_c - X_L)^2}$

$\boxed{Z = 51.96}$

Page No.:
Date:

$I = \frac{V}{Z} = \frac{230}{51.96}$

$\boxed{I = 4.426\,A}$

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$= \tan^{-1}\left(\frac{15.70 - 63.66}{20}\right)$

$\phi = -67.36$

P.F = $\cos\phi$

P.F = 0.3849

$V_C = I \times X_C$

$= 4.426 \times 63.66$

$\boxed{V_C = 281.75V}$

$P = VI\cos\phi = 230 \times 4.426 \cos\phi$

$= 230 \times 4.426 \times 0.3849$

$P = 391.86$ watt

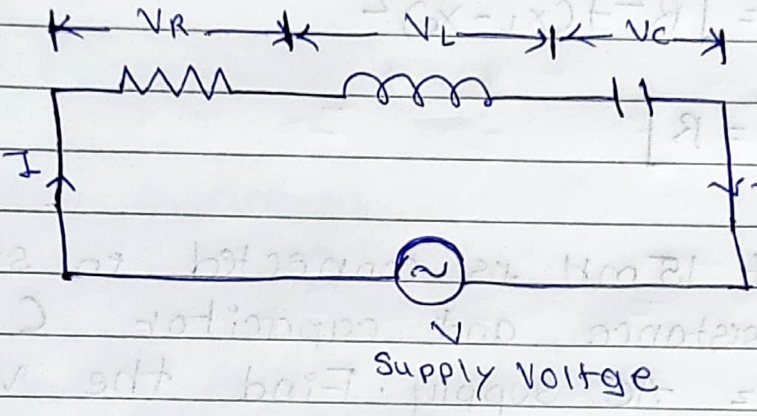
$Q = VI\sin\phi = 230 \times 4.426 \sin(-67.36)$

$Q = -939.53$ VAR.

Page No.:
Date:

★ Resonance in series R-L-C circuit

> The series resonance circuit consists of resistor (R), inductance (L) and capacitance (C) connected in series with supply vtg (V).



★ Resonant frequency

Inductive reactance X_L and capacitive reactance X_C are the funct^n of supply frequency. If the supply frequecy vary both X_C & X_L varies.

At certain frequency X_L becomes equal to X_C is called series resonance & a frequency at which resonance occur is called as resonant frequency.

> It is denoted by f_r

> Under this resonance condition current reaches the maximum value such that $I_{max} = \frac{V}{R}$ & impedance of this circuit i.e Z falls to a minimum value Hence Z=R,

As $X_L = X_C$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

Page No:
Date:

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}\ \text{Hz}}$$

$X_L > X_C$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\boxed{Z = R}$$

Q) A coil of 15 mH is connected in series with 25 Ω Resistance and capacitor C across 230 V, 50 Hz AC supply. Find the value of this capacitor so that circuit has maximum current. Also find the power factor & power consumed of this circuit.

Soln Given

$L = 15\,mH = 15 \times 10^{-3}\,H$

$R = 25\,\Omega$

$V = 230\,V$

$f = 50\,Hz$

As current reaches to a maximum value it is a resonance condition.

Under Resonance Condition

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$C = \frac{1}{(2\pi f)^2 \times L}$$

$$C = \frac{1}{4\pi^2 \times 50^2 \times 15 \times 10^{-3}}$$

$C = 6.7547 \times 10^{-4} J$

$Z = R$

$Z = 25\ \Omega$

Power factor $= \cos\phi = 1$

Power consumed

$P = VI\cos\phi$

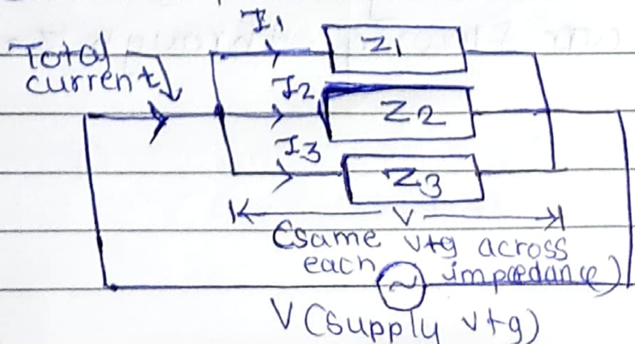
$I_{max} = \frac{V}{R} = \frac{230}{25} = 9.2A$

$P = 230(9.2)$

$[P = 2116\ W]$

★ Basic concept of Parallel AC circuits.

- A parallel AC circuit ~~is~~ a consists of two or more impedance connected in parallel across supply voltage 'V'
- Each impedance is called branch of this parallel circuit.
- The voltage across all the impedance is same as the supply voltage V
- ~~According to~~ ~~K~~ But the current flowing through each impedance is different



According to krichoffs current Law total current is the addition of individual current.

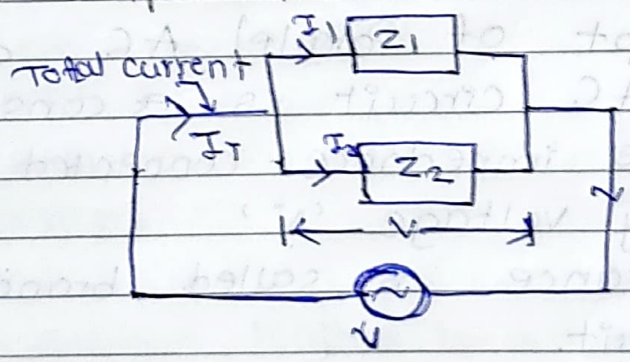
i.e $I_{total} = I_1 + I_2 + I_3$

$$\frac{V}{Z} = \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3}$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_2}$$

Z_{eq} - is the equivalent impedance of the circuit

* Two impedance in Parallel



Consider two impedance Z_1 and Z_2 are connected in parallel across supply vtg V.

The different current

I_T = Total current

I_1 = curr flowing through Z_1

I_2 = curr flowing through Z_2

The current is given as.

$$I = \frac{V}{Z}$$

Thus I_1 & I_2 can be written as.

$I_1 = \frac{\bar{V}}{Z_1}$ and $I_2 = \frac{\bar{V}}{Z_2}$

Now $I_T = \bar{I}_1 + \bar{I}_2$

$$I_T = \frac{V}{Z_T}$$

$$\therefore \frac{\bar{V}}{Z_T} = \frac{V}{Z_1} + \frac{V}{Z_2}$$

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$I_T = I_1 + I_2$

$I_1 = I_T \times \frac{Z_2}{Z_1+Z_2}$

$I_2 = I_T \times \frac{Z_1}{Z_1+Z_2}$

Q) Two impedance $Z_1 = 30\angle 45^\circ \Omega$ & $Z_2 = 45\angle 30^\circ \Omega$ are connected in parallel across 230 V single phase supply calculate total impedance of this parallel circuit Find total current then Power factor of this circuit, & power consumed by this circuit.

Soln Given,

$Z_1 = 30\angle 45 \Omega = 21.21 + j21.21$

$Z_2 = 45\angle 30^\circ \Omega = 38.97 + j22.5$

$$Z_{total} = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

$$= \frac{(30 \times 45)\angle(45+30)}{(21.21 + 38.97) + j(21.21 + 22.5)}$$

$$= \frac{1350 \angle 75}{60.18 + j43.71}$$

$$= \frac{1350 \angle 75}{74.37 \angle 35.99}$$

$$= (1350 \div 74.37) \angle (75 - 35.99)$$
$$= 18.15 \angle 39.01$$

$$I_{total} = \frac{V}{Z_{total}} = \frac{230 \angle 0^\circ}{18.15 \angle 39.01}$$
$$= 12.67 \angle (-39.01)$$

$PF = \cos\phi$

$PF = \cos(-39.01)$

$[PF = 0.777]$

$P = VI\cos\phi$

$P = 230 \times \quad \times 0.777$

$[P = \quad]$

★ Concept of Admittance

It is defined as the reciprocal of impedance (Z)".

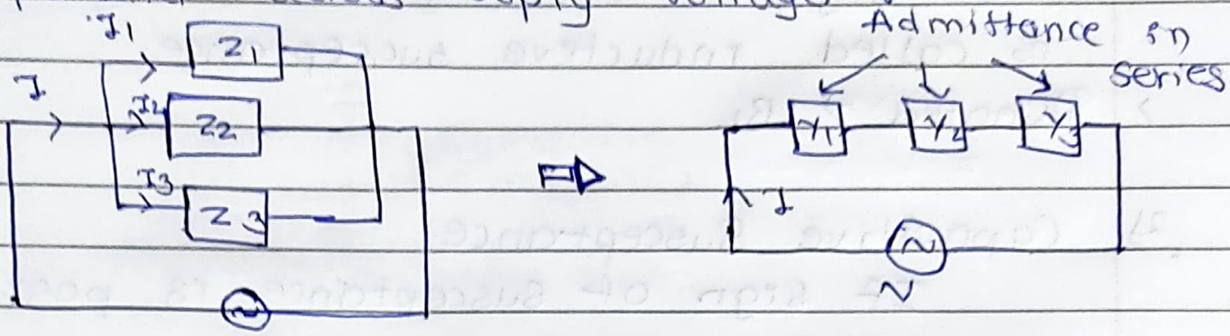
$$\therefore Y = \frac{1}{Z}$$

> Unit is Siemens (S) or mho ($\mho$)

↓ One Siemens

(It is admittance of network having impedance 1 Ω)

> Consider three impedance Z_1, Z_2 & Z_3 connected in parallel across suply voltage V



$$I_{total} = I_1 + I_2 + I_3$$

$$\frac{V}{Z} = \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3} \qquad \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\boxed{Y = Y_1 + Y_2 + Y_3} \qquad \because Y = \frac{1}{Z}$$

> When impedances are connected in ~~series~~ parallel then admittances is in series
> When impedances are connected in series then admittances are in parallel.

* Conductance

It is the ratio of resistance (R) to the square of impedance (Z^2)."

$$\therefore G = \frac{R}{Z^2}$$

> Unit = Siemens (S) or mho (℧)

* Susceptance :-

It is the ratio of reactance to the square of impedance (Z^2)."

$$\therefore B = \frac{X}{Z^2}$$

Unit (S) or (℧)"

Types:-

1) Inductive Susceptance;
If sign of susceptance is negative, it is called inductive susceptance,

> Denoted → B_L

2) Capacitive Susceptance
If sign of susceptance is positive, it capacitive susceptance.

> Denoted → B_C

★ Admittance Triangle

The equation of admittance is

$$Y = G \mp jB$$

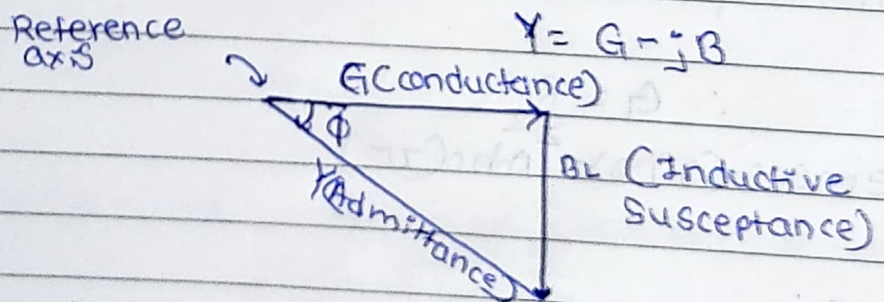
G ↓ Real term; jB → Imaginary term

> The conductance (G) is real part of admittance, so conductance (G) is consider as reference axis

> Each side of triangle reprents admittance conductance and susceptance.

For Inductive susceptance (B_L), equation of admittance is,

$$Y = G - jB$$

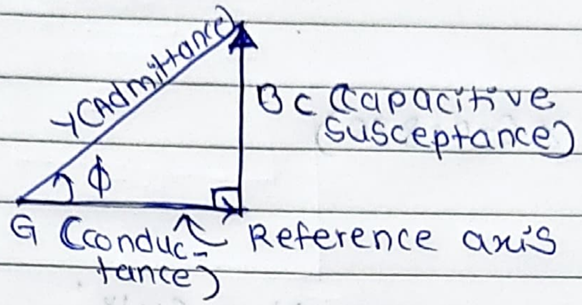


Admittance triangle of inductive suseptance

Page No.:
Date:

> For capacitive susceptance (B_C)

$$Y = G + jB$$



Q) Two impedance $Z_1 = 6 + j8\ \Omega$ and $Z_2 = 5 + j12\ \Omega$ are connected in series across 100 V 50 Hz supply calculate equivalent impedance of this circuit, power factor & P, Q, S.

Soln Given,

$$Z_1 = 6 + j8\ \Omega$$
$$Z_2 = 5 + j12\ \Omega$$

$$Z_{eq} = Z_1 + Z_2$$

$$Z_{eq} = 11 + j20\ \Omega = 22.82 \angle 61.18^\circ$$

$$PF = \cos\phi$$
$$= \cos 61.18^\circ$$
$$\boxed{PF = 0.48}$$

$$I = \frac{V}{Z} = \frac{100\angle 0}{22.82\angle 61.18}$$

$$I = 4.38 \angle -61.18$$

$$P = VI\cos\phi$$
$$Q = VI\sin\phi$$
$$S = VI$$

$P = 100 \times (4.38 \times 0.48$

$P = 210.24$

$Q = VI \sin \phi$

$= 100 \times 4.38 \times (-0.876)$

$= -383.68$

$S = 438$