UNIT III

Equilibrium of Coplaner Force System

* Equilibrium of Bodies

∵ A body is said to be in equilibrium, its state of rest or of uniform motion in a straight line is not altered.

It is the state of rest of a body under the action of forces. When a body is subjected to different forces & even after it remains at rest position it is said to be in equilibrium.

Conditions of Equilibrium:-

For different types of force systems there are different conditions for systems to be in equilibrium:

ⓐ For co-planer concurrent forces:

For co-planer con-current forces there are two conditions of equilibrium.

$\Sigma F_x = 0$ & $\Sigma F_y = 0$ ∴ $R = 0$

ⓑ For co-planer non-concurrent forces:

For co-planer non-concurrent forces there are three conditions of equilibrium.

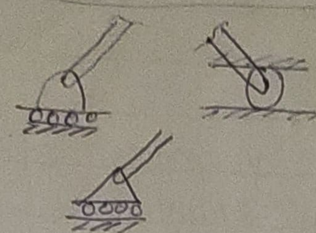
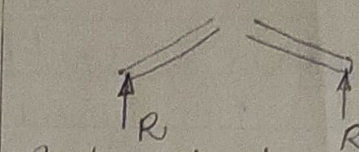
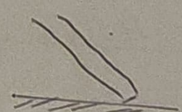
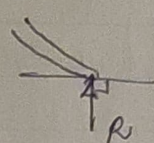
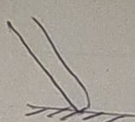
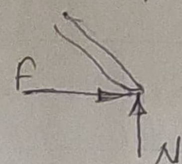
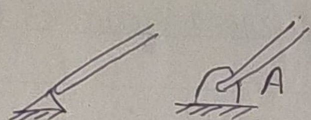
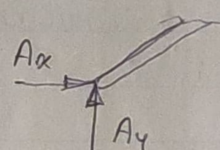
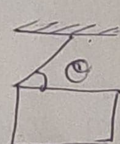
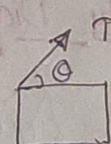
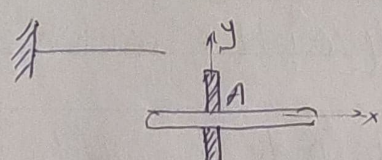
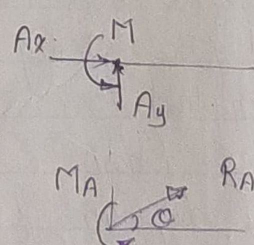
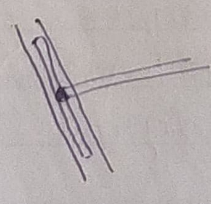
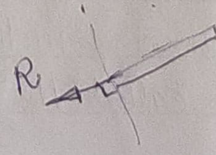
$\Sigma F_x = 0$, $\Sigma F_y = 0$ & $\Sigma M = 0$

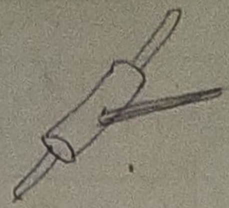
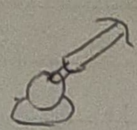
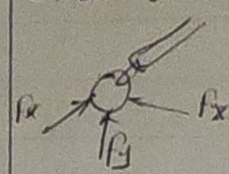
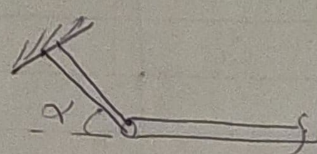
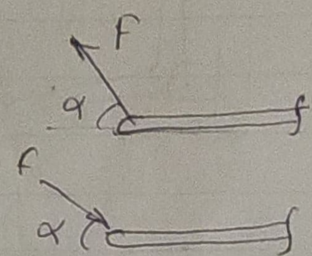
* Free Body Diagram:-

For the analysis of equilibrium condition it is necessary to isolate the body under consideration from the other bodies in contact & draw all forces acting on the body. For this first the body is drawn & then all applied forces, self weight & reactions from the other bodies in contact are drawn.

The diagram of the body in which the body under consideration is freed from all contact surfaces & is shown with all the forces on it. (Including self weight, reactions from other contact surfaces) is called the Free Body diagram (FBD.)

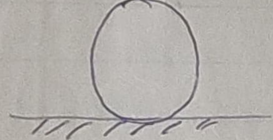
* Types of Supports & Corresponding Reactions:-

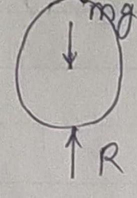
Type of Support	Symbolic Representation	Reaction Components
① Rollers		 Reactions is always ⊥r to the plane.
② Smooth surface		
③ Rough Surface		
④ Smooth pin or Hinge		
⑤ flexible cord, rope or cable of negligible weight		
⑥ Fixed Support		
⑦ Smooth pin in a slot		

⑧ A sliding collar		 Reaction is $\perp$ar to the rod along which collar is sliding without friction.
⑨ Ball & socket joint		
⑩ Weightless links		 Link can have tensile or compressive force when subjected to forces.

* Some examples of free body diagram.

①





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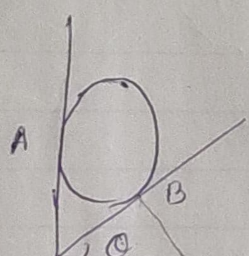

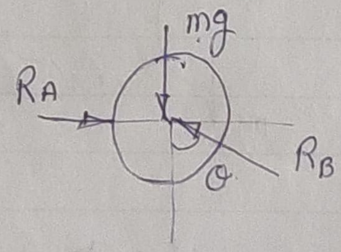

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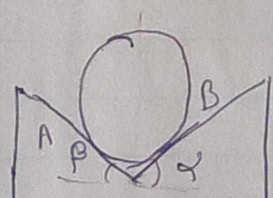

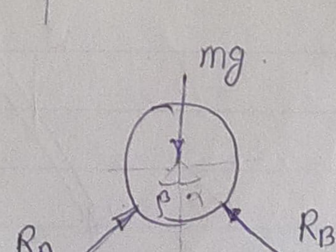

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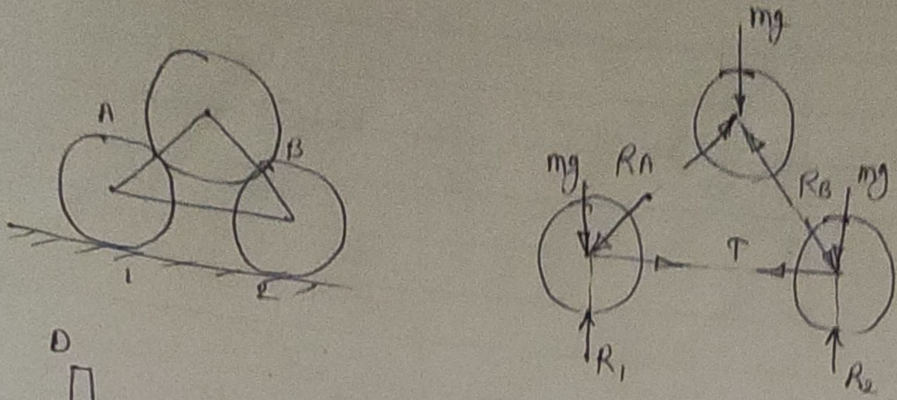
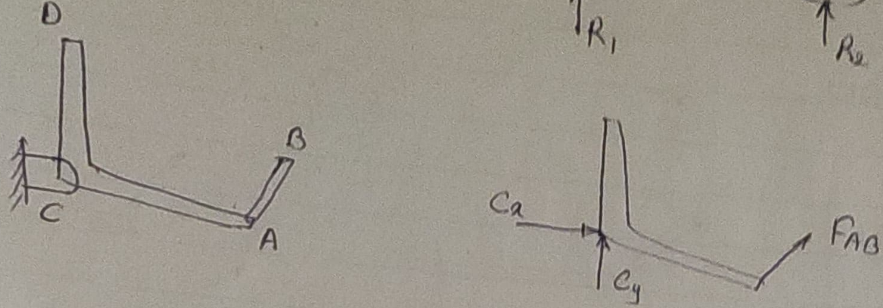
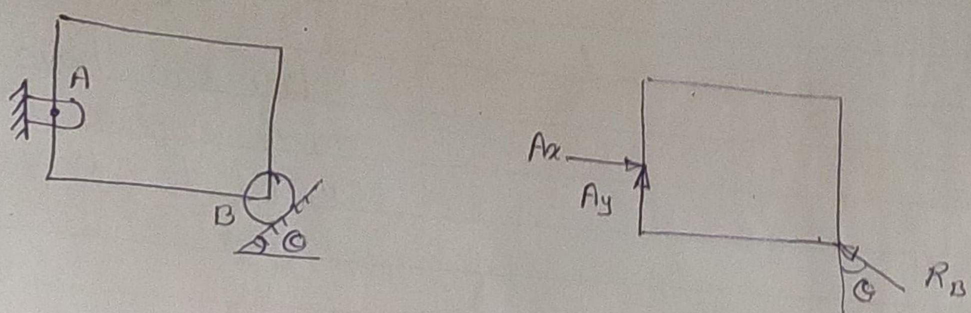

A
B
1
2
mg
RA
RB
mg
mg
T
R1
R2

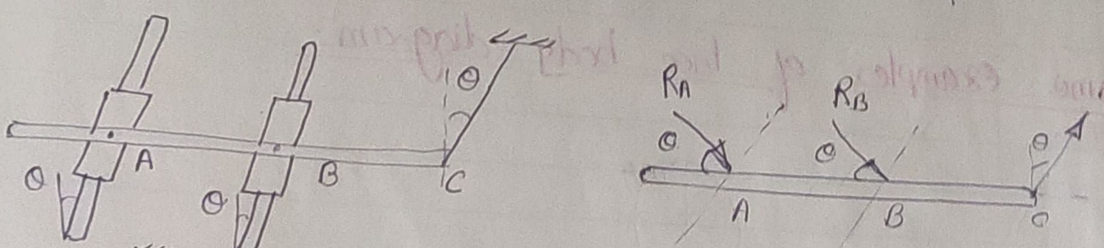
5


D
B
C
A
Cx
Cy
FAB

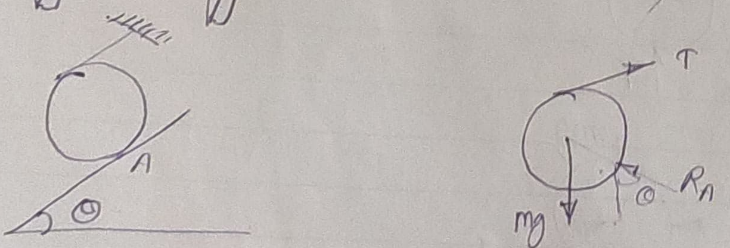
6


A
B
θ
Ax
Ay
RB
θ

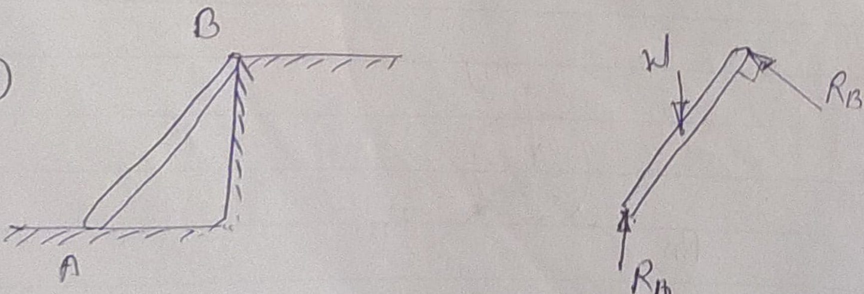
7


θ
A
θ
B
C
RA
RB
T
θ
A
B
C

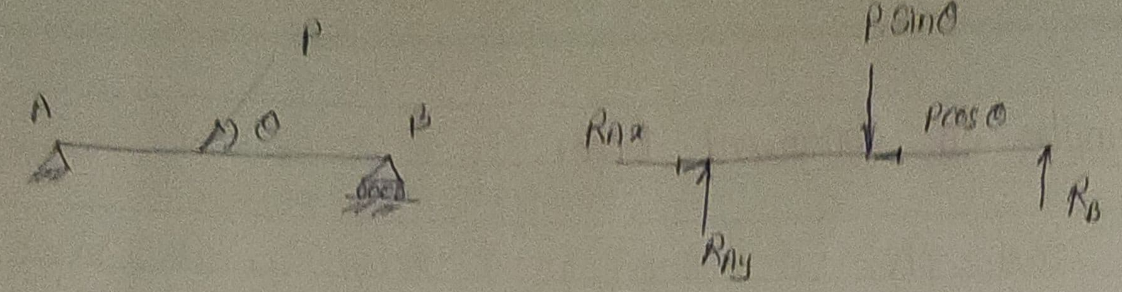
8


A
θ
T
mg
RA
θ

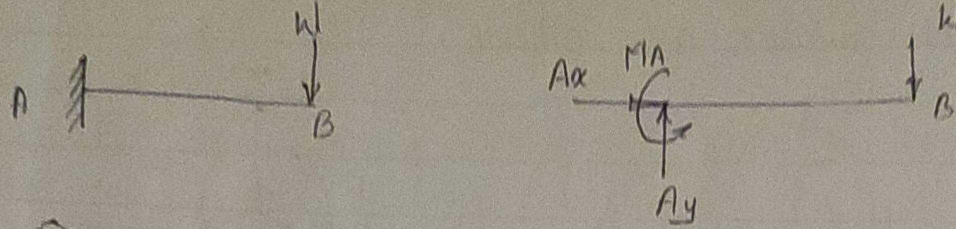
9


B
A
W
RB
RA

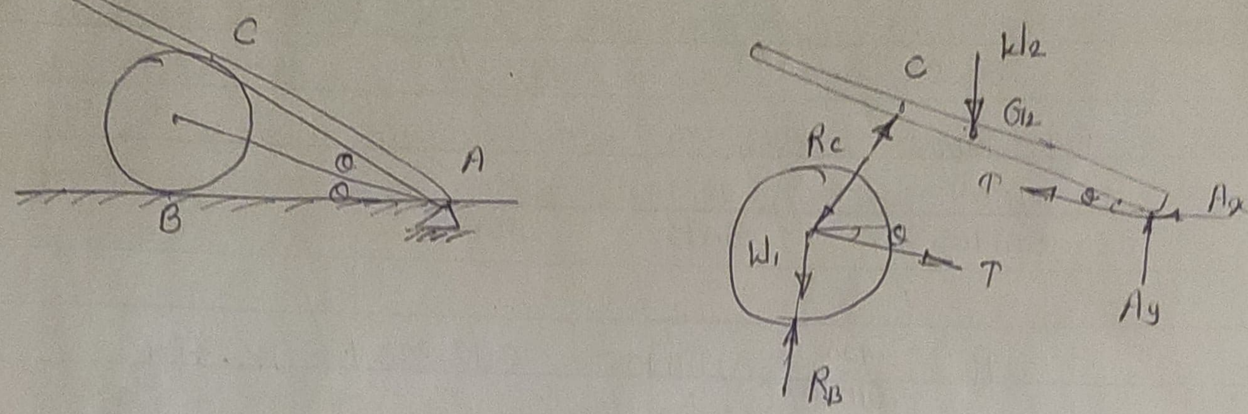
(10)



(11)



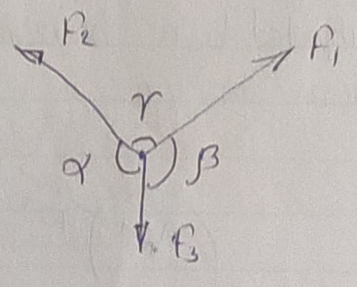
(12)



* Lamis theorem.

Lamis theorem states that if a body is in equilibrium under the action of only three concurrent forces is proportional to the sine of the angle between the other two forces. Thus for the system of forces shown in fig.

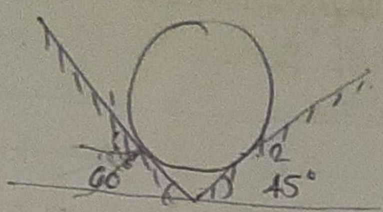
$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

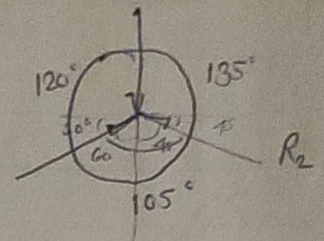


* Examples on Concurrent forces

① A 200N sphere is resting in a trough as shown in fig. Determine the reactions developed at contact surfaces. Assume all contact surfaces are smooth.

①





By lami's theorem,

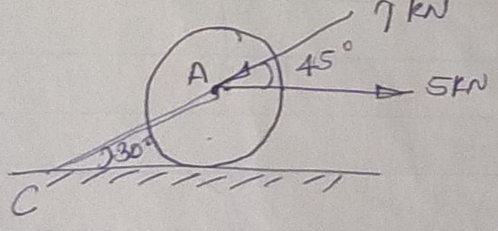
$$\frac{400}{\sin 105} = \frac{R_1}{\sin 135} = \frac{R_2}{\sin 120}$$

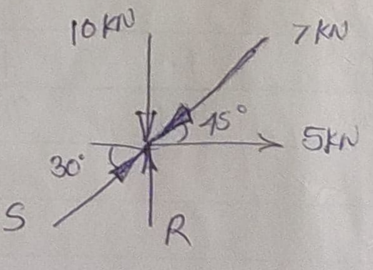
$$\therefore R_1 = \frac{400}{\sin 105} \times \sin 135 = 292.820N \quad / \quad 146.41N$$

$$R_2 = \frac{400}{\sin 105} \times \sin 120 = 358.630N \quad / \quad 179.315$$ Ximp! *

② A roller weighting 10 kN rests on a smooth horizontal floor & is connected to the floor by the bar AC as shown in fig. Determine the force in the bar AC & reaction from the floor, if the roller is subjected to a horizontal force of 5kN & an inclined force of 7kN as shown in fig.

②





$\Sigma f_x = 0$

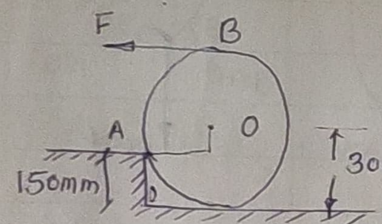
$5 - 7\cos 45 + S\cos 30 = 0$

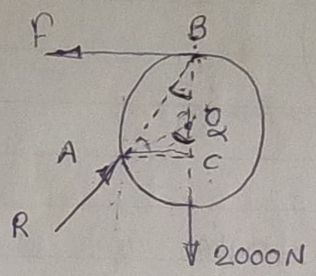
$S = -0.058$ KN

$\Sigma F_y = 0$

$$S \sin 30 + R - 10 - 7 \sin 45 = 0$$
$$(-0.058 \sin 30) + R - 10 - 7 \sin 45 = 0$$
$$R = 14.978 \text{ KN}$$

③ A roller of radius $r = 300$mm & weighing 2000N is to be pulled over a curb of height 150mm, as shown in fig, by applying a horizontal force F applied to the end of a string wound around the circumference of the roller. a) Find the magnitude of the force F required to start the roller move over the curb b) What is the least pull F through the centre of the wheel to just turn the roller over the curb.?





When the roller is about to turn over the curb, the contact with the floor is lost & hence, there is no reaction from the floor. The body is in equilibrium under the action of three forces namely,

$$\cos\alpha = \frac{OC}{OA} = \frac{300-150}{300} = \frac{1}{2}$$

$$\alpha = 60^\circ$$

Now, in ΔAOB, $\angle OAB = \angle OBA$ Since $OA = OB$ = radius of roller

but $\angle OAB + \angle OBA = \alpha$

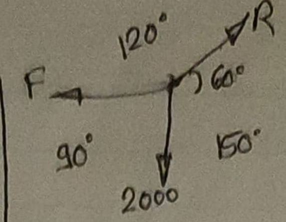
$\angle OAB + \angle OBA + 180 - \alpha = 180$

$\angle OAB + \angle OBA = \alpha$

$2\angle OBA = 60^\circ$

$\angle OBA = 30^\circ$

∴ Reaction makes 30° with the vertical.



$\Sigma V = 0$

$R \cos 30 - 2000 = 0$

$\therefore R = 2309.4\ kN$

$\Sigma H = 0$

$F - R \sin 30 = 0$

$\therefore F = 2309.4 \sin 30 = 1154.7\ kN.$

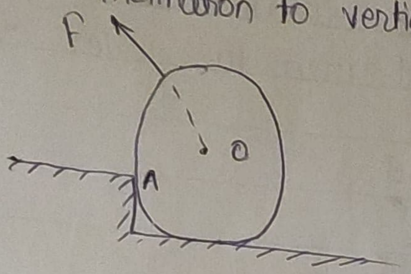
$$\frac{F}{\sin 150} = \frac{R}{\sin 90} = \frac{2000}{\sin 120}$$

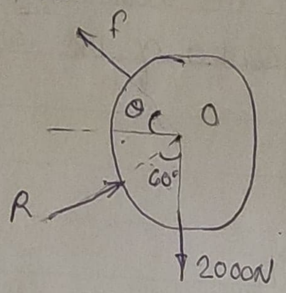
$F = 1154.700\ kN$

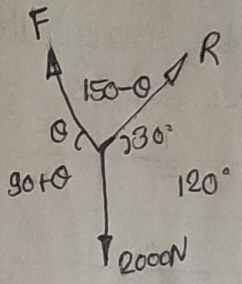
$R = 2309.40\ kN$

Least force through the centre of roller.

In this case the reaction from the curb must pass through the centre of the roller since the other two forces pass through that point. It's inclination to vertical is $\theta = 60°$







Let force F makes angle θ with the horizontal as shown in fig.

$\Sigma F_x = 0$

$F \cos\theta = R \sin 60$

$\Sigma F_y = 0$

$F.\sin\theta + R\cos 60 - W = 0$

$$F.\sin\theta + \frac{F.\cos\theta}{\sin 60} . \cos 60 = W$$

$F(\sin\theta + \cot 60 . \cos\theta) = W$

$$\sin\theta + \cot 60 . \cos\theta = \frac{W}{P}$$

$$\frac{2000}{\sin(150-\theta)} = \frac{F}{\sin 120}$$

$$F = \frac{2000}{\sin(150-\theta)} \sin 120$$

for F min denominator should be max^n.

$150 - \theta = 90$

$\therefore \theta = 60°$

for $\frac{W}{P}$ to be maximum, i.e. P to be least,

$$\frac{d}{d\theta}\left(\frac{W}{P}\right) = 0.$$

$\cos\theta + \cot 60(-\sin\theta) = 0$

$\cos\theta = \cot 60 . \sin\theta \Rightarrow \cot\theta = \cot 60 \Rightarrow \theta = 60°$

$$F = \frac{2000}{\sin(150-60)} \cdot \sin 120 = 1732.050\,N$$

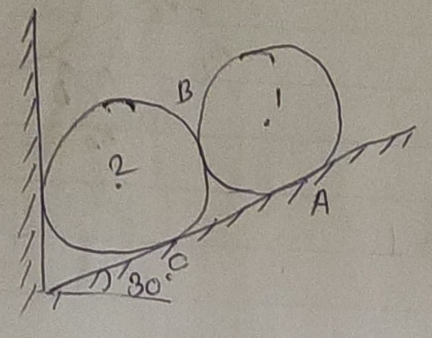
F is least when it is at right angles to the reaction R.

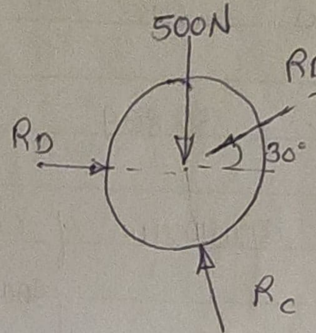
$$P_{min} = \frac{W}{\sin 60 + \cot 60 \cdot \cos 60} = \frac{2000 \sin 60}{\sin^2 60 + \cos^2 60}$$

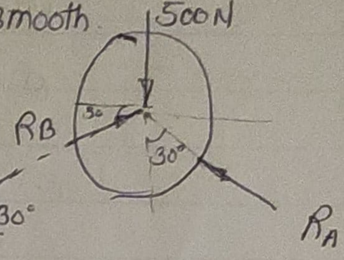
$$P_{min} = 1732\,N$$

④ Two identical cylinders, each weighing 500N are placed in a trough as shown in fig. Determine the reactions developed at contact points A, B, C & D. Assume all points of contact are smooth.

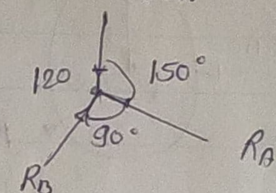
③







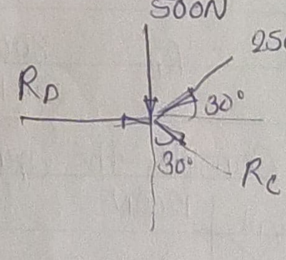
By lamis theorem,



$$\frac{500}{\sin 90} = \frac{R_A}{\sin 120} = \frac{R_B}{\sin 150}$$

$$R_A = \frac{500}{\sin 90} \times \sin 120 = 433.012\,N$$

$$R_B = \frac{500}{\sin 90} \times \sin 150 = 250\,N.$$



$\Sigma f_y = 0$

$$-500 - 250 \sin 30 + R_C \sin 60 = 0$$

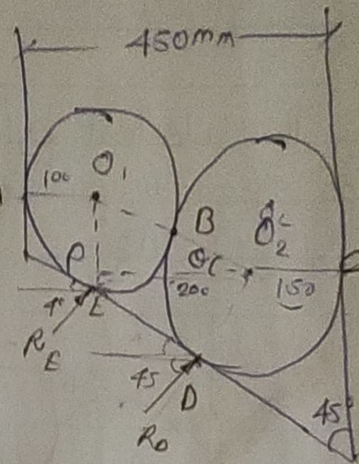
$$R_C = 721.687\,N$$

$\Sigma f_x = 0$

$$R_D - 250 \cos 30 - 721.687 \cos 60 = 0$$

$$R_D = 577.349\,N$$

(5) Cylinder 1 of diameter 200mm & cylinder of 2 of diameter 300mm are placed in a trough as shown in fig. If cylinder 1 weighs 800N & cylinder 2 weighs 1200N, determine the reactions developed at contact surfaces A, B, C & D. Assume all contact surfaces are smooth.

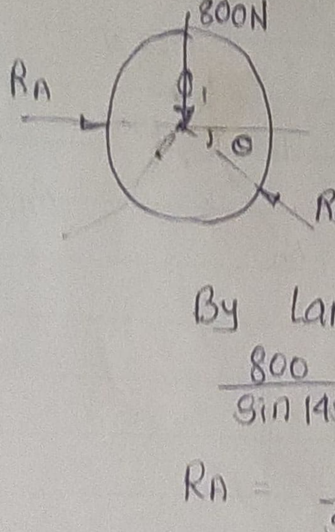


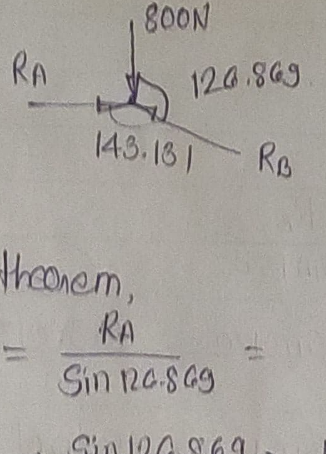
$\cos\theta = \frac{O_2P}{O_1O_2} = \frac{450-100-150}{100+150}$

$\theta = \cos^{-1}\frac{200}{250}$

$\theta = 36.869^\circ$

Consider the equilibrium of cylinder 1.





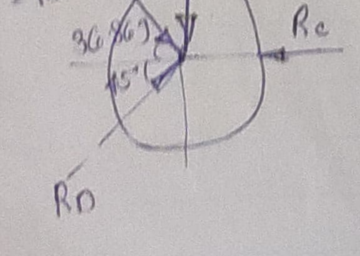
By Lami's theorem,

$\frac{800}{\sin 143.131} = \frac{R_A}{\sin 126.869} = \frac{R_B}{\sin 90}$

$R_A = \frac{800}{\sin 143.131} \times \sin 126.869 = 1066.701\,N$

$R_B = \frac{800}{\sin 143.131} \times \sin 90 = 1333.361\,N$

Consider the equilibrium of cylinder 2.



$\Sigma F_y = 0$

$R_D \cos 45 - R_B \sin 36.869 - 1200 = 0$

$R_D = 2828.426\,N$

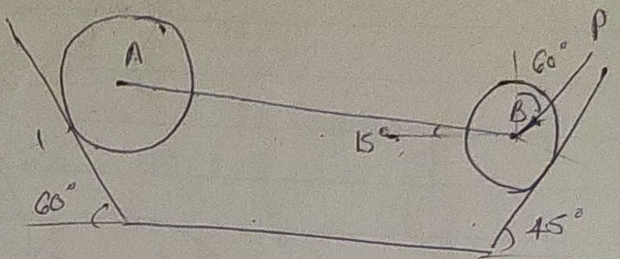
$\Sigma F_x = 0$

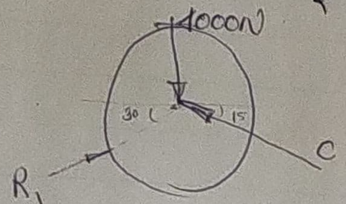
$2828.426 \cos 45 + 1333.36 \cos 36.869 - R_C = 0$

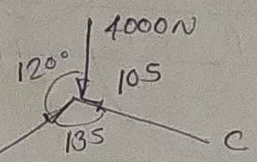
$R_C = 3066.69\,N$

⑥ Cylinder A weighting 4000N & cylinder B weighing 2000N rest on smooth inclines as shown in fig. ④ They are connected by a bar of negligible weight hinged to geometric centres of the cylinders by smooth pins. Find the force P to be applied as shown in fig. such that it will hold the system in the given position.

④





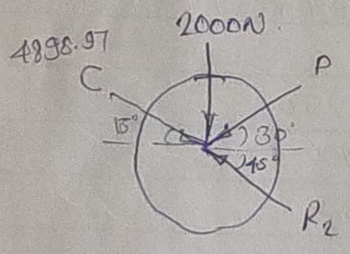


By lami's theorem

$$\frac{4000}{\sin 135} = \frac{R_1}{\sin 105} = \frac{C}{\sin 120}$$

$$R_1 = \frac{4000}{\sin 135} \times \sin 105 = 5464.101\ N$$

$$C = \frac{4000}{\sin 135} \times \sin 120 = 4898.97\ N.$$



$$\Sigma f_x = 0$$

$$4898.97 \cos 15 - P\cos 30 - R_2 \cos 45 = 0$$

$$P.\cos 30 + R_2 \cos 45 = +4732.041 \quad \text{— ①}$$

$$\Sigma f_y = 0.$$

$$R_2 \sin 45 - P.\sin 30 - 2000 - 4898.97 \sin 15 = 0$$

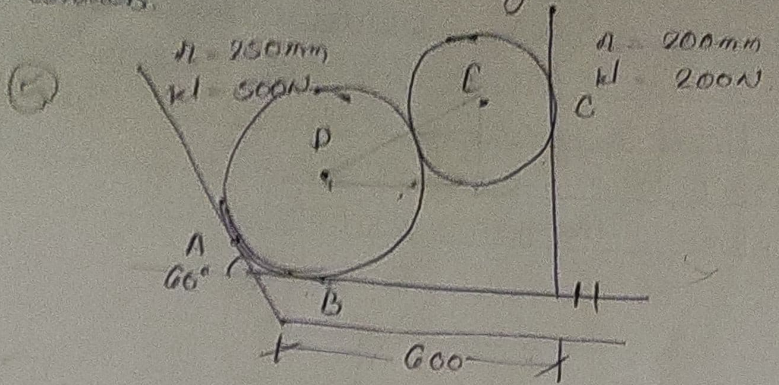
$$-P\sin 30 + R_2 \sin 45 = 3267.946 \quad \text{— ②}$$

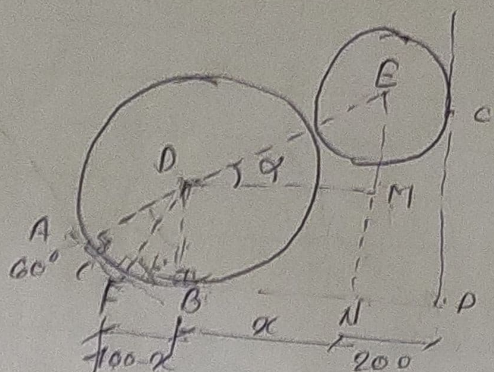
$P = \cancel{5556.330}$ 1071.791 N. 1068.67 N.

$R_2 = 5379.444\ N$

(9) Two spheres rest in a smooth hough as shown in fig. Find forces at points of contacts.

(5)





$DE = 250 + 200 = 450$ mm

Let $DM = x, = BN$

$\therefore FB = 400 - x$

$\angle B$ & $\angle A = 90°$

If we join DF it will be angle bisector which is bisecting $\angle AFB = 120°$

In $\triangle DBF$, $\tan 60 = \frac{DB}{BF} = \frac{250}{400 - x}$

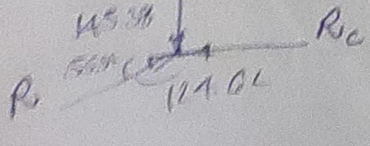
$\therefore x = 255.66$ mm

In $\triangle DME$

$\cos\alpha = \frac{DM}{DE} = \frac{255.66}{450}$

$\alpha = 55.38°$

at point E

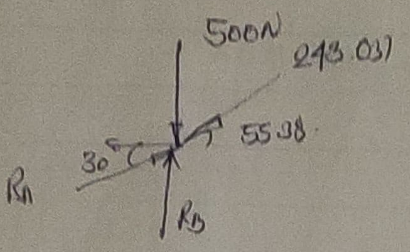


$\frac{200}{\sin 124.62} = \frac{R_c}{\sin 145.38} = \frac{R}{\sin 90}$

$R_c = \frac{200}{\sin 124.62} \times \sin 145.38 = 138.073\,N$

$R = \frac{200}{\sin 124.62} \times \sin 90 = 243.031\,N$

at D



$\Sigma F_x = 0$

$R_A \cos 30 - 243.031 \cos 55.38 = 0$

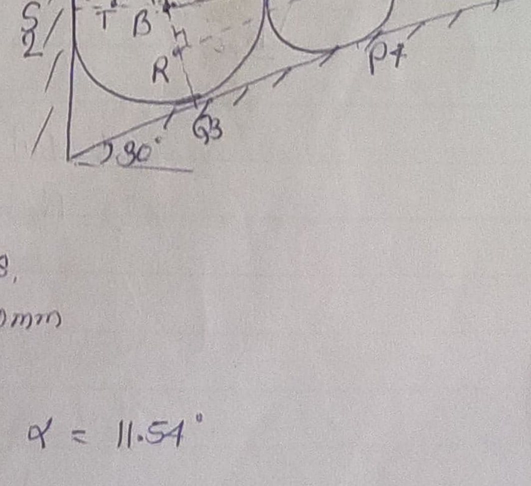
$R_A = 159.433\,N$

$\Sigma F_y = 0$

$R_B - 500 + 159.433 \sin 30 - 243.031 \sin 55.38 = 0$

$R_B = 620.282\,N$

(8) Three homogeneous smooth spheres A, B & C of weights 300N, 600N & 300N & having diameters 800mm, 1200mm & 800mm respectively are placed in a bench as shown in fig. Determine the reactions at all point of contact.



Join centres AB & BC.

From A draw a line AT parallel to plane.

In Δ ATB,

$AB = 400 + 600 = 1000$ mm

$BT = BS - ST = 600 - 400 = 200$ mm

$\cos\theta = \frac{BT}{AB} = \frac{200}{1000}$

$\theta = 78.46°$

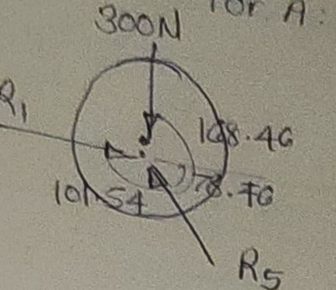
From C draw a line CR parallel to plane.

In Δ CRB, $BC = 600 + 400 = 1000$ mm

$BR = 600 - 400 = 200$ mm

$\sin\alpha = \frac{BR}{BC} = \frac{200}{1000}$ $\Rightarrow$ $\alpha = 11.54°$

For A:



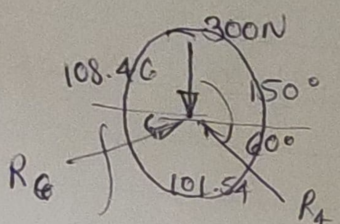
By lami's theorem.

$$\frac{300}{\sin 101.54} = \frac{R_1}{\sin 168.46} = \frac{R_5}{\sin 90}$$

$$R_1 = \frac{300}{\sin 101.54} \times \sin 168.46 = 61.253\,N$$

$$R_5 = \frac{300}{\sin 101.54} \times \sin 90 = 306.189\,N$$

For C:



$30 - 11.54 = 18.46$

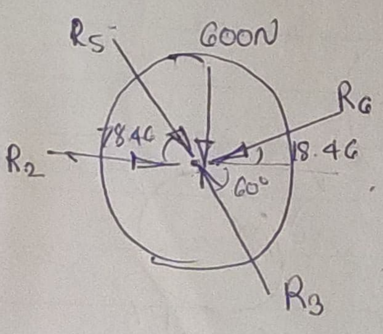
By lami's theorem.

$$\frac{300}{\sin 101.54} = \frac{R_4}{\sin 108.46} = \frac{R_6}{\sin 150}$$

$$R_4 = \frac{300}{\sin 101.54} \times \sin 108.46 = 290.434\,N$$

$$R_6 = \frac{300}{\sin 101.54} \times \sin 150 = 153.094\,N$$

For B:



$\Sigma F_x = 0$

$R_2 + 306.189 \cos 78.46 - R_3 \cos 60 - 153.094 \cos 18.46 = 0$

$R_2 - 0.5R_3 = 83.98$ —— ①

$\Sigma F_y = 0.$

$-600 - 306.189 \sin 78.46 - 153.094 \sin 18.46$

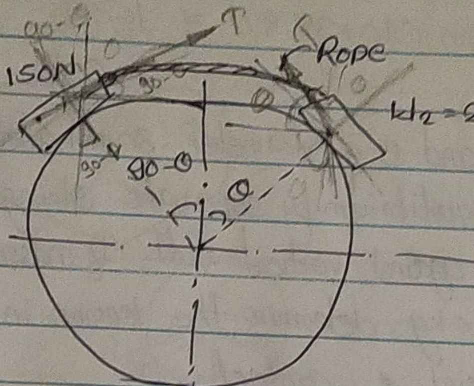
$+ R_3 \sin 60 = 0.$

$\therefore R_3 = 1095.12\,N.$

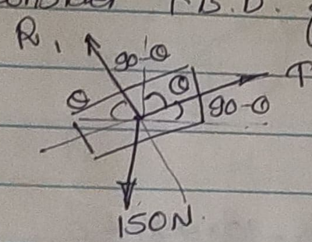
Put in ①

$R_2 = 631.54\,N$

Two bodies weighing 150N and 200N respectively rest on a cylinder and are connected by a rope as shown in fig. Find the reaction of cylinder on the bodies, the tension in rope and angle θ. Assume all surfaces to be smooth.



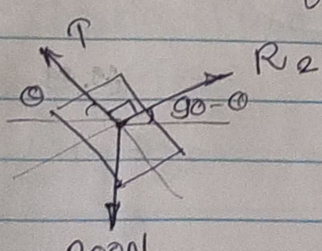
Consider F.B.D. of $W_1 = 150N$.



$$\frac{150}{\sin 90} = \frac{T}{\sin(90+\theta)} = \frac{R_1}{\sin(180-\theta)}$$

$$\therefore T = 150 \cdot \sin(90+\theta) \quad \text{---} \ (1)$$

Consider F.B.D. of $W_2 = 200N$



$$\frac{200}{\sin 90} = \frac{T}{\sin(180-\theta)} = \frac{R_2}{\sin(90+\theta)}$$

$$\therefore T = 200 \cdot \sin(180-\theta) \quad \text{---} \ (2)$$

From (1) & (2)

$$150 \sin(90+\theta) = 200 \sin(180-\theta)$$

$$150 \cos\theta = 200 \sin\theta$$

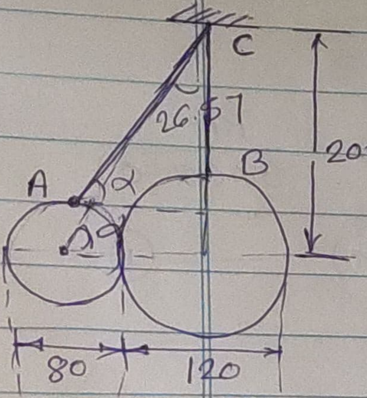
$$\tan\theta = \frac{150}{200} \quad \therefore \theta = 36.87^\circ$$

$\therefore T = 120N$

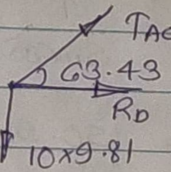
$R_1 = 150 \sin(180 - \overset{36.87}{\cancel{\theta}}) = 90N$

$R_2 = 200 \sin(90 + 36.87) = 160N.$

Two spheres A and B of diameter 80mm and 120mm respectively are held in equilibrium by separate strings as shown in fig. Sphere B rests against vertical wall. If masses of spheres A & B are 10kg and 20kg, determine the tension in the string and reactions at point of contact.



$\alpha = \tan^{-1}\frac{200}{100} = 63.434°$



$\Sigma F_y = 0$

$T_{AC} \sin 63.43 - 10 \times 9.81 = 0$

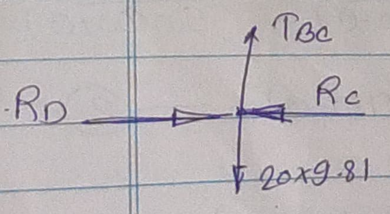
$T_{AC} = 125.61N$ $(109.68$

$\Sigma F_x = 0$

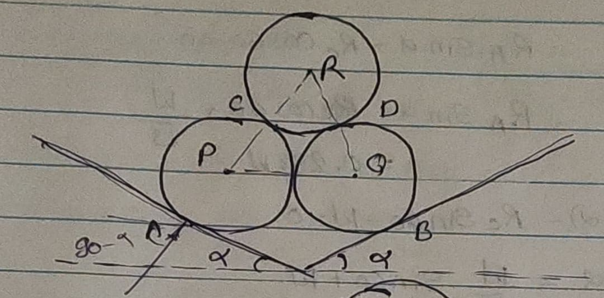
$R_D + T_{AC} \cos 63.43 = 0$

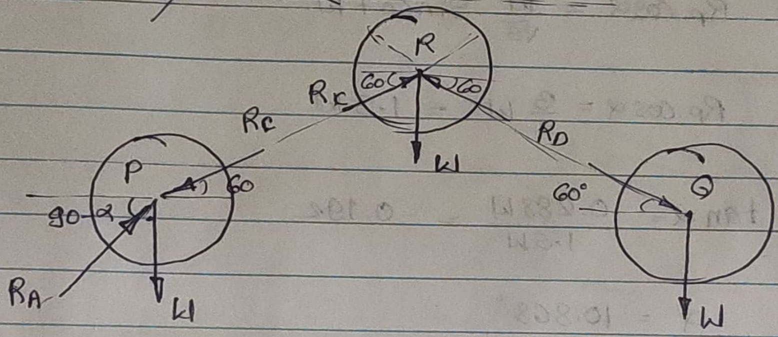
$R_D = -125.61N$

49.09



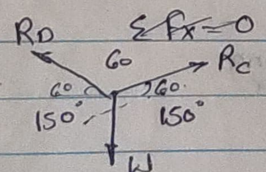
Three identical sphere P, Q, R of weight 'W' are arranged on smooth inclined surfaces as shown in fig. Determine the angle α which will prevent the arrangement from collapsing.





When the arrangement is about to collapse, P & Q tends to move away from each other. As a result, reaction between P & Q tends to zero.

Consider F.B.D. of sphere R.



$\Sigma F_x = 0$

$$\frac{W}{\sin 60} = \frac{R_D}{\sin 150} = \frac{R_C}{\sin 150}.$$

$$\therefore R_C = \frac{W}{\sin 60} \times \sin 150$$

$$R_C = \cancel{1.732}\ 0.577\ W$$

$\Sigma F_y = 0.$

$R_D \sin 60 + R_C \sin 60 - W = 0.$

$2 R_C \sin 60 = W$

$$\therefore R_C = \frac{1}{2 \sin 60}$$

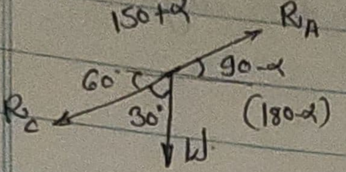
$R_C = 0.577 \, W$

$\Sigma F_x = 0$

$R_D \cos 60 - R_C \cos 60 = 0$

$\therefore R_D = R_C$

From FBD. of P:



$\Sigma F_x = 0$

$R_A \cos(90-\alpha) - R_C \cos 60 = 0$

$\therefore R_A \sin\alpha - R_C \cos 60 = 0$

$\therefore R_A \sin\alpha = \cancel{R_C} \cos 60 \times \frac{W}{\sqrt{3}}$

$= 0.288\,W$

$\Sigma F_y = 0$

$R_A \sin(90-\alpha) - R_C \sin 60 - W = 0.$

$R_P \cos\alpha = \frac{W}{\sqrt{3}} \cdot \sin 60 + W$

$R_P \cos\alpha = \frac{3}{2} W = 1.5\,W$

$\tan\alpha = \frac{0.288\,W}{1.5\,W} = 0.192$

$\alpha = 10.868°$

$\frac{W}{\sin(150+\alpha)} = \frac{R_C\,(0.577\,W)}{\sin(180-\alpha)} = \frac{R_A}{\sin 30}$

$R_C = \frac{W}{\sin(150+\alpha)} \cdot \cancel{\cos\alpha}\, \sin\alpha$

$0.577\,W = \frac{W}{\sin 150 \cdot \cos\alpha + \cancel{\cos\alpha} \cdot \sin\alpha \cdot \cos 150} \cdot \frac{\sin\alpha}{\cancel{\cos\alpha}}$

$= \frac{W\,\cancel{\cos\alpha}\,\sin\alpha}{0.5 \cdot \cos\alpha - 0.866 \sin\alpha}$

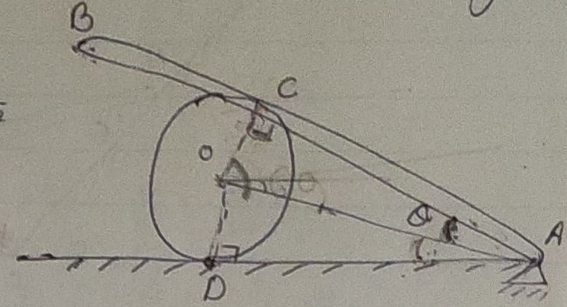
$0.577\,W\,(0.5\cos\alpha - 0.866\sin\alpha) = W \cdot \cancel{\cos\alpha}\,\sin\alpha$

$0.5\cos\alpha - 0.866\sin\alpha = 1.733\sin\alpha$

$\cancel{0.288\cos\alpha = 0.5\sin\alpha} \Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{0.5}{2.599} \therefore \alpha = 10.886$

$0.5\cos\alpha = 2.599\sin\alpha$

⑨ A smooth cylinder of radius 500mm rests on a horizontal plane & is kept from rolling by inclined string of length 1000mm. A bar AB of length 1500mm & weight 1000N is hinged at A & placed against the cylinder of negligible weight. Determine the tension in the string



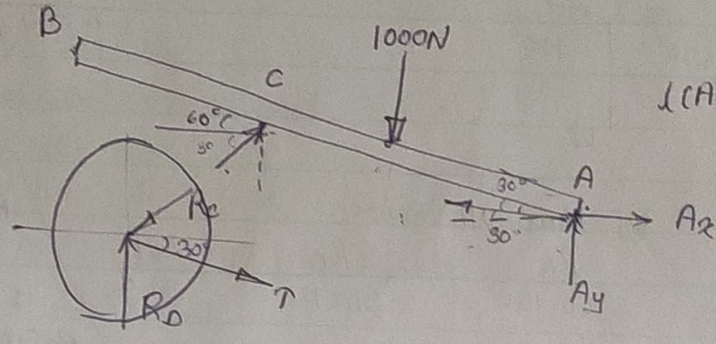
In ΔACO

$$AC = \sqrt{AO^2 - OC^2} = \sqrt{1000^2 - 500^2}$$

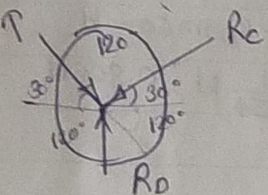
$$AC = 866.02 \text{ mm}$$

$$\sin\theta = \frac{OC}{OA} = \frac{500}{1000}$$

$$\theta = 30°$$



l(AB) = 1500



By lami's theorem,

$$\frac{R_c}{\sin 120} = \frac{T}{\sin 120} = \frac{R_D}{\sin 120}$$

$$\therefore R_c = T = R_D.$$

For bar,

$\Sigma M_A = 0$

$- R_c \times 866.02$

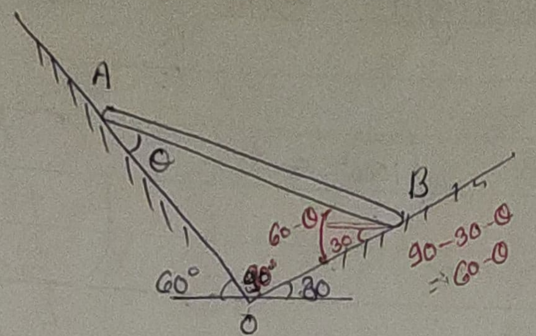
$$1000 \times 750 \cos 60 - R_c \sin 30 \times 866.02 \cos 60 - R_c \cos 30 \times 866.02 \sin 60 = 0$$

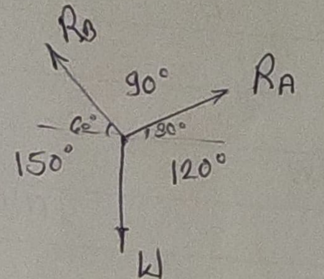
$$1000 \times 750 \cos 60 - R_c \times 866.02 = 0$$

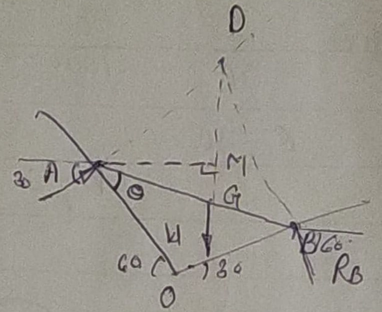
$$R_c = 433.015 N$$

$$\therefore T = R_c = R_D = 433.015 N.$$

(16) A uniform bar AB of length L & weight W as lies in a vertical plane with its ends resting on two smooth surfaces on OA & OB. Find angle θ for equilibrium of bar







By lam's theorem

$$\frac{R_A}{\sin 150} = \frac{R_B}{\sin 120} = \frac{W}{\sin 90}$$

$$\therefore R_B = \frac{W \sin 120}{\sin 90} = 0.87W$$

$\Sigma F_x = 0$

$R_A \cos 30 - R_B \cos 60 = 0.$

$\therefore R_A = 0.577 R_B$

$\Sigma F_y = 0.$

$R_A \sin 30 + R_B \sin 60 = W$

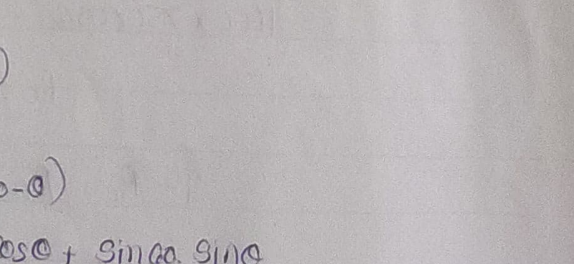
$\therefore R_B = \frac{1}{1.154} W = 0.866 W$

In Δ AOB , $OB = L\sin\theta$

$AD = L\sin\theta$

In Δ AMG

$$AM = \frac{L}{2}\cos(60-\theta)$$



$\Sigma M_A = 0$

$R_B \times AD - W(AM) = 0$

$R_B \times L\sin\theta = W.\frac{L}{2}\cos(60-\theta)$

$R_B.\sin\theta = \frac{W}{2}\cos(60-\theta)$

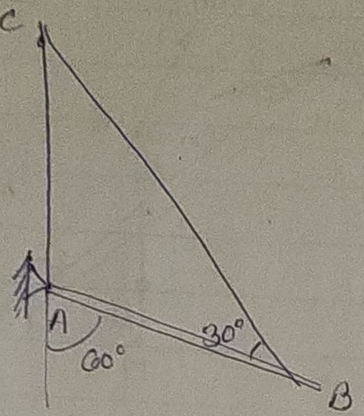
$0.87W\sin\theta = \frac{W}{2}\cos(60-\theta)$

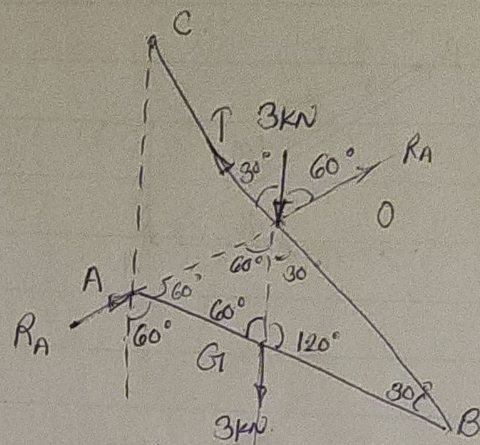
$1.74\sin\theta = \cos 60.\cos\theta + \sin 60.\sin\theta$

$0.866\sin\theta = 0.5\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{0.5}{0.866} \Rightarrow \theta = 30°$

(11)

A prismatic bar AB of length l & weight 3kN is hinged to a wall & supported by a cable BC. Find hinge reaction & tension in cable BC.





Here, $AG = GB = l/2$

Δ OBG is isoceles Δle.

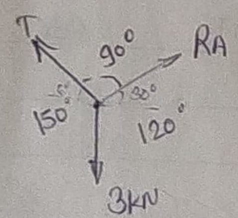
$\therefore \angle GOB = 30°$

$\angle AGO = 60°$ & $AG = OG = \frac{l}{2}$

$\therefore$ In ΔAOG is an equilateral Δle,

$\angle AOG = 60°$

At point O.



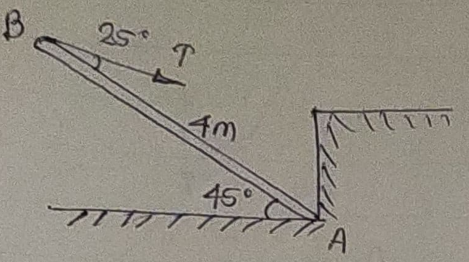
By lami's theorem,

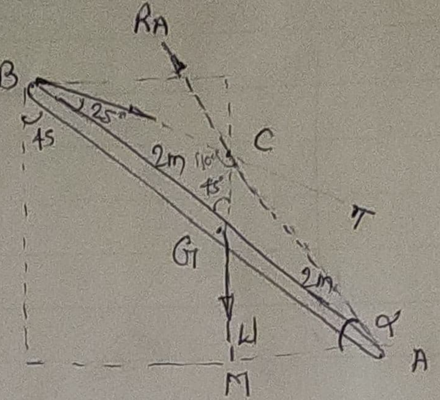
$$\frac{R_A}{\sin 150} = \frac{3}{\sin 90} = \frac{T}{\sin 120}$$

$$R_A = \frac{3}{\sin 90} \times \sin 150 = 1.5\ kN$$

$$T = \frac{3}{\sin 90} \times \sin 120 = 2.6\ kN.$$

(12) A man supports a pole 20kg as shown in fig. Find tension and reaction at A.





In Δ BCG using sine rule,

$$\frac{2}{\sin 110} = \frac{CG}{\sin 25} = \frac{BC}{\sin 45}$$

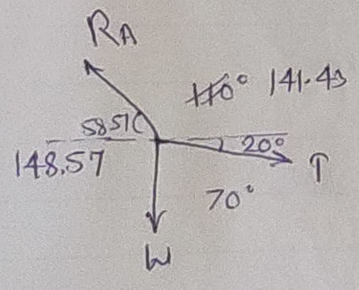
$$CG = \frac{2}{\sin 110} \times \sin 25 = 0.899\ m$$

$$BC = \frac{2}{\sin 110} \times \sin 45 = 1.504\ m$$

In Δ AMC,

$$\tan\alpha = \frac{CM}{AM} = \frac{CG + GM}{AM} = \frac{0.9 + 2\sin 45}{2\cos 45} = 1.636$$

$$\alpha = 58.57^\circ$$



By lami's theorem,

$$\frac{R_A}{\sin 70} = \frac{T}{\sin 148.57} = \frac{20 \times 9.81}{\sin 141.43}$$

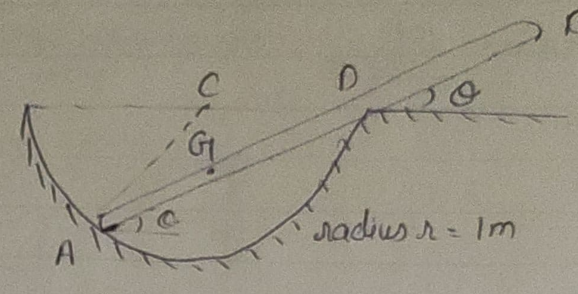
$$\therefore R_A = \frac{20 \times 9.81}{\sin 141.43} \times \sin 70 = 295.712\ N$$

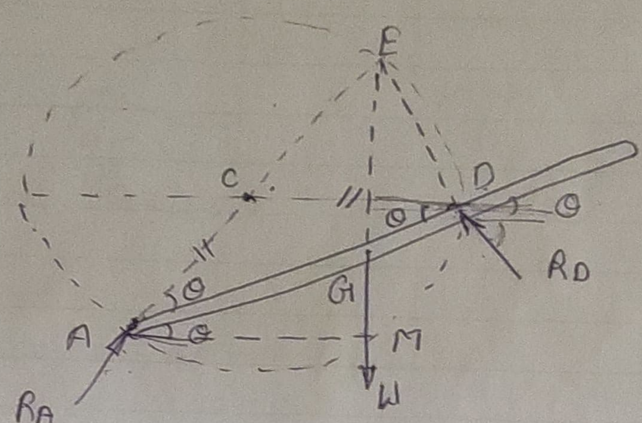
R

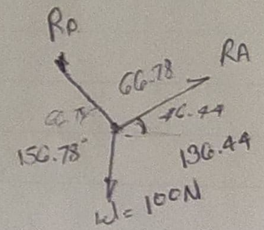
$$T = \frac{20 \times 9.81}{\sin 141.43} \times \sin 148.57 =$$

$$T = 164.09\ N$$

(13) A uniform bar of length 3m is placed inside a hemispherical bowl of radius 1m. Determine the angle made by the bar with horizontal corresponding to equilibrium. Take weight of bar as 100N acting at G as shown in fig.







$AC = CD = r$

ΔACD is isoceles Δle ∴ $\angle CAD = \theta$

ΔGMA. $\angle GAM = \theta$

ΔAME. $\cos 2\theta = \frac{AM}{AE} = \frac{AM}{2R}$

$\therefore AM = 2R\cos 2\theta$ ——— ①

ΔAMG, $\cos\theta = \frac{AM}{AG} = \frac{AM}{L/2}$

$AM = \frac{L}{2}\cos\theta$ ——— ②

Equating ① & ②

$2R.\cos 2\theta = \frac{L}{2}\cos\theta$ $\longrightarrow$ $R = 1m$, $L = 3m$.

$2\cos 2\theta = \frac{3}{2}\cos\theta$

$2[2\cos^2\theta - 1] = 1.5\cos\theta$ $\rightarrow$ $4\cos^2\theta - 1.5\cos\theta - 2 = 0$

$\cos\theta = 0.919$ or -0.544

$\theta = 23.22°$

$$\frac{R_A}{\sin 156.78} = \frac{100}{\sin 66.78} = \frac{R_D}{\sin 136.44}$$

$$R_A = \frac{100}{\sin 66.78} \times \sin 156.78 = 42.901 N$$

$$R_D = \frac{100}{\sin 66.78} \times \sin 136.44 = 74.985 N$$