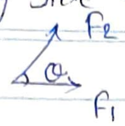
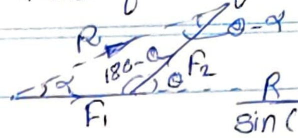
Triangle law may be stated as if two forces acting on a body ~~body~~ are represented one after another by the sides of a triangle, then resultant is represented by the closing side as the triangle taken from first point to the last.

* <u>Triangle Law:</u>

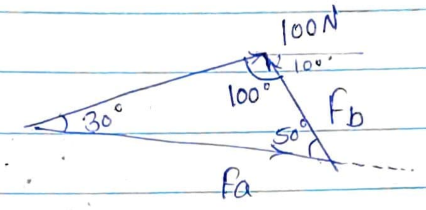


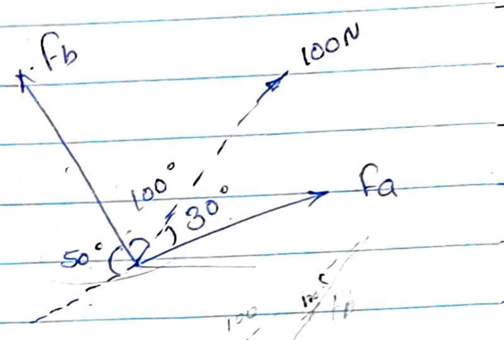


$$\frac{R}{\sin(180-\theta)} = \frac{F_2}{\sin\alpha} = \frac{F_1}{\sin(\theta-\alpha)}$$

① ~~Two forces P & Q applied at point A of a hook support as shown in figure.~~

① Find components F_a & F_b of the 100N force along the directions shown in fig. using sine rule.





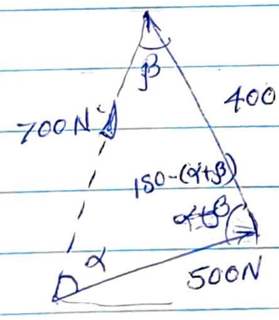
Using triangle law.

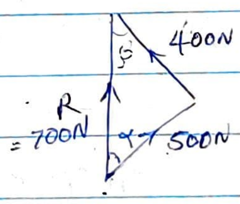
$$\frac{F_a}{\sin 100} = \frac{F_b}{\sin 30} = \frac{100}{\sin 50}$$

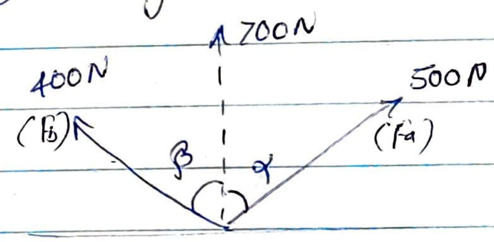
$$F_a = \frac{100}{\sin 50} \times \sin 100 = 128.557\,N$$

$$F_b = \frac{100}{\sin 50} \times \sin 30 = 65.270\,N$$

② If the resultant of the two forces shown in fig. is 700N directed vertically upwards. find the angles α & β using ~~rectangular components~~ triangle law.







Apply Cosine Rule,

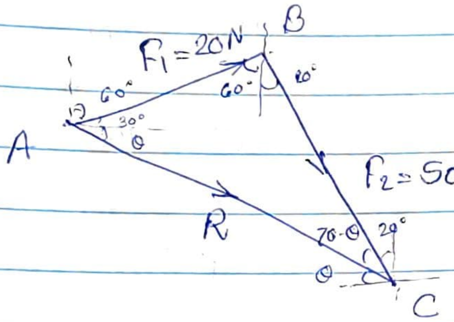
$$\cos\alpha = \frac{500^2 + 700^2 - 400^2}{2(500)(700)} = \frac{(\text{Add. of adjacent side})^2 - (\text{Opposit side})^2}{2.(\text{adjacent side})}$$

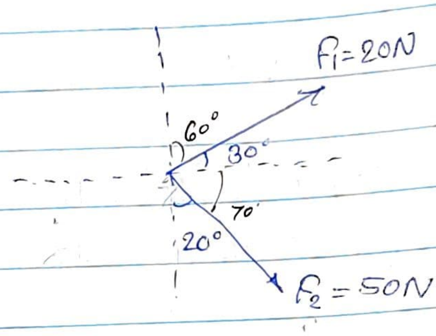
$$= 0.828$$

$$\alpha = 34.047^\circ$$

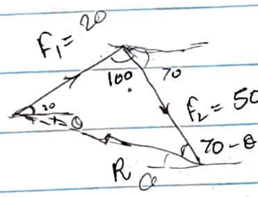
$$\cos\beta = \frac{400^2 + 700^2 - 500^2}{2(400)(700)} = 0.714 \qquad \beta = 44.45^\circ$$

③ Find resultant of the two forces shown in fig. using triangle law.





Using Cosine Rule,



$$\cos B = \frac{F_1^2 + F_2^2 - R^2}{2F_1F_2}$$

$$\cos 80 = \frac{20^2 + 50^2 - R^2}{2 \times 50 \times 20}$$

$$347.296 = 20^2 + 50^2 - R^2$$

$$R^2 = 2552.704$$

$$\boxed{R = 50.524\,N}$$

Using Sine Rule.

$$\frac{F_1}{\sin(70-\theta)} = \frac{R}{\sin 80} = \frac{F_2}{\sin(\theta + 30)}$$

$$\therefore \frac{20}{\sin(70-\theta)} = \frac{50.524}{\sin 80}$$

$$19.696 = 50.527 \sin(70-\theta)$$

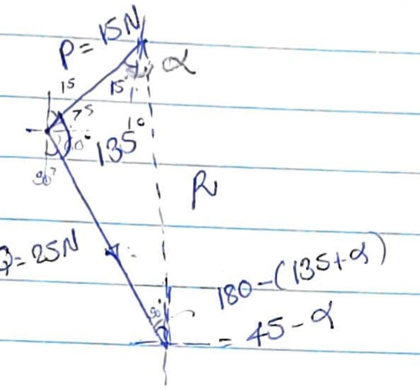
$$0.389 = \sin(70-\theta)$$

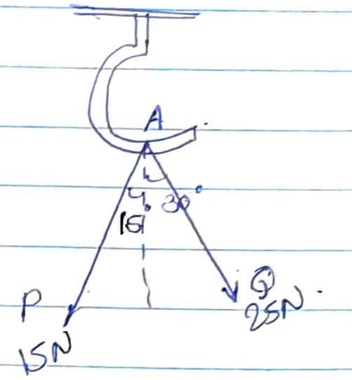
$$70-\theta = 22.942$$

$$\boxed{\theta = 47.057^\circ}$$

③

④ Two forces P & Q are applied as shown at A of a hook support. Knowing that P = 15N & Q = 25N, determine graphically, the magnitude & direction of their resultant using parallelogram & triangle law.





Cosine Rule

$$\cos 135 = \frac{15^2 + 25^2 - R^2}{2 \times 15 \times 25}$$

$$-530.330 = 15^2 + 25^2 - R^2$$

$$R^2 = 1380.330$$

$$R = 37.152\ N$$

Sine Rule,

$$\frac{R}{\sin 135} = \frac{25}{\sin \alpha}$$

$$\sin \alpha = \frac{25}{37.152} \times \sin 135$$

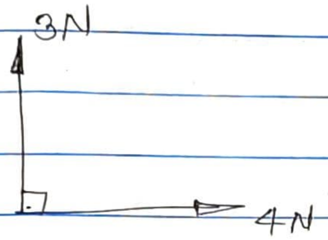
$$= 0.475 = 28.412$$

$$R^2 = 15^2 + 25^2 + 2 \times 15 \times 25 \cos 45$$

$$= 37.152N.$$

* Parallelogram & Triangle Law :-

1. Find the resultant of given forces



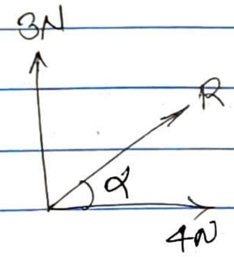
ⓐ By parallelogram law.

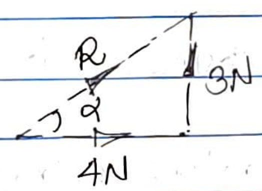
$$R = \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \cos 90}$$

$$= 5N$$

$$\tan\alpha = \frac{3 \times \sin 90}{4 + 3 \times \cos 90}$$

$$\alpha = 36.87^\circ$$





ⓑ By Triangle Law,

By cosine Rule.

$$R = \sqrt{4^2 + 3^2 - 2 \times 3 \times 4 \times \cos 90}$$

$$R = 5N$$

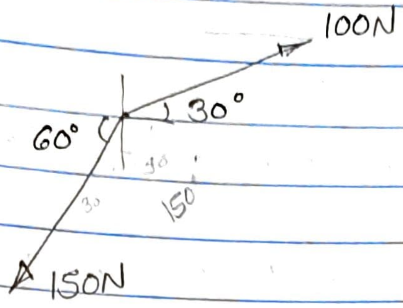
By sine Rule,

$$\frac{R}{\sin 90} = \frac{3}{\sin\alpha}$$

$$\sin\alpha = \frac{3}{5}$$

$$\alpha = 36.87^\circ$$

2. Find the resultant of the given forces.



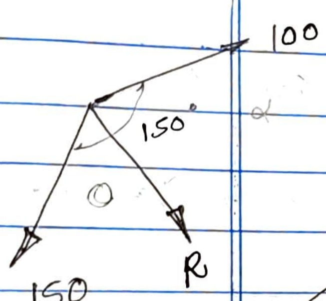
ⓐ By parallelogram law.

$$R = \sqrt{100^2 + 150^2 + 2 \times 100 \times 150 \cos 150}$$

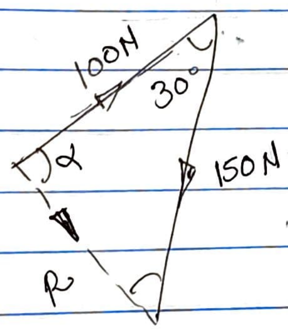
$$= 80.74\ N$$

$$\tan\alpha = \frac{150 \sin 150}{100 + 150 \cos 150}$$

$$\alpha = 68.26^\circ$$



By Cosine Rule: $\cos 30 = \frac{100^2 + 150^2 - R^2}{2 \times 100 \times 150}$



$$R = \sqrt{100^2 + 150^2 - 2 \times 100 \times 150 \cos 30^\circ}$$

$$= 80.74\ N.$$

By sine Rule.

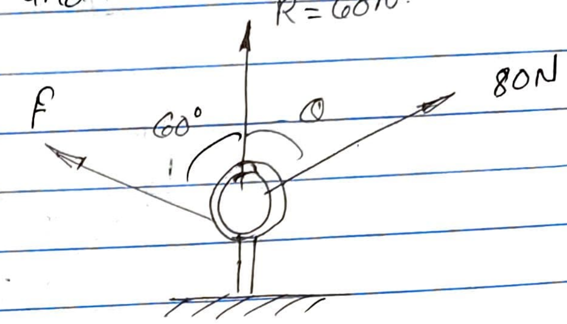
$$\frac{R}{\sin 30} = \frac{150}{\sin\alpha}$$

$$\sin\alpha = \frac{150 \sin 30}{80.74} = 0.9289$$

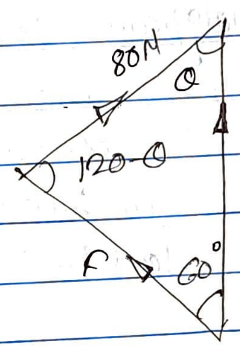
$$= 68.26^\circ$$

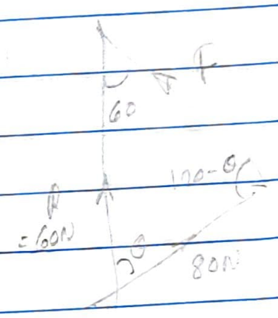
$$\frac{150}{\sin\alpha} = \frac{100}{\sin\theta} = \frac{R}{\sin 150\ 30}$$

(3). Resolve the force $R = 60N$ into two components F and $80N$ as shown in fig. Find the value of F and θ.



By Triangle law,





~~cos60 =~~

$\cos\theta = \frac{60^2 + 80^2 - F^2}{2 \times 60 \times 80}$

By Sine Rule,

$$\frac{80}{\sin 60} = \frac{F}{\sin\theta} = \frac{60}{\sin(180 - 60 - \theta)}$$

$$\frac{80}{\sin 60} = \frac{F}{\sin\theta} = \frac{60}{\sin(120 - \theta)}$$

$$\sin(120 - \theta) = \frac{60}{80} \times \sin 60$$

$$\sin(120 - \theta) = 0.649$$

$$120 - \theta = 40.505$$

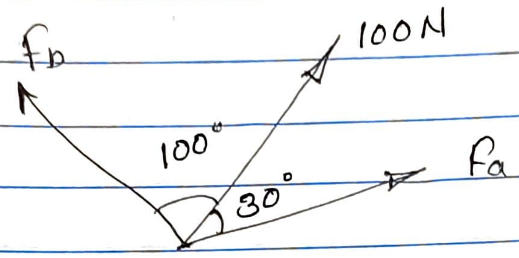
$$\therefore \theta = 79.494^\circ$$

$$\therefore F = \frac{80}{\sin 60} \times \sin 79.494$$

$$F = 90.827\,N$$

By Parallelogram law it's so difficult.

(4) Find components F_a & F_b of the 100N force along the directions shown in fig.



(a) By Parallelogram law,

$F_a = P \qquad F_b = Q$

$\theta = 130° \qquad \alpha = 30° \qquad R = 100N.$

$$\tan 30 = \frac{F_b \cdot \sin 130}{F_a + F_b \cos 130}$$

$0.577 F_a - 0.3717 F_b = 0.766 . F_b$

$0.577 F_a = 1.1371 F_b$

$\therefore F_a = 1.97 F_b \quad — (1)$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{(1.97F_b)^2 + F_b + 2 \times 1.97 F_b \times F_b \times \cos 130}$

$= 3.880 F_b + F_b^2 - 2.531 F_b^2$

$100^2 = 2.349 F_b^2$

$\therefore F_b = 65.274 N$

$F_a = 128.589 N$

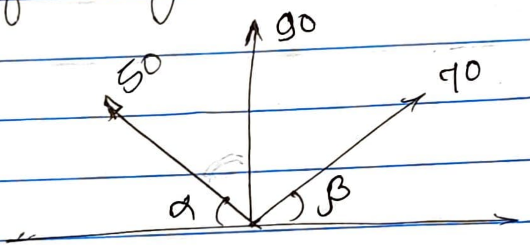
(b) By Triangle law,

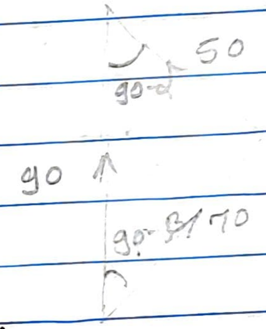
$$\frac{fa}{\sin 100} = \frac{fb}{\sin 30} = \frac{100}{\sin(180-130)}$$

$$\therefore \; fa = \frac{100}{\sin 50} \times \sin 100 = 128.557\ N$$

$$fb = \frac{100}{\sin 50} \times \sin 30 = 65.270\ N$$

(5) If the resultant of two forces shown in fig. is 90kN directed vertically upward, find angles α & β.





$$\cos(90-\alpha) = \frac{50^2+90^2-70^2}{2\times 50\times 90}$$

$$\cos(90-\alpha) = 0.633$$

~~cos(~~

$$90-\alpha = 50.703$$

$$\alpha = 39.296^\circ$$

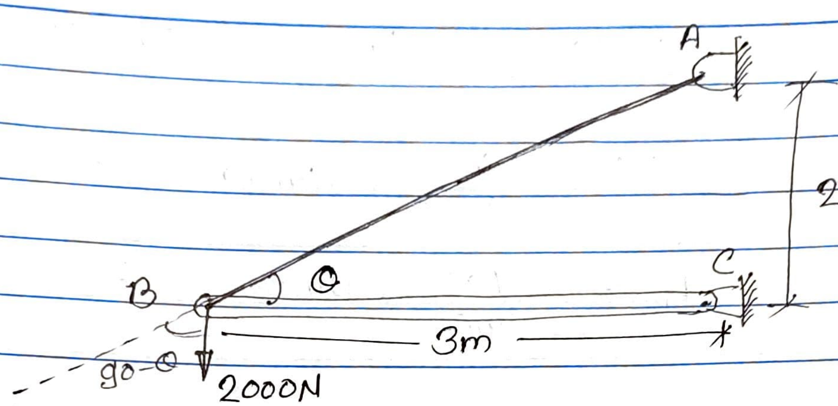
$$\cos(90-\beta) = \frac{90^2+70^2-50^2}{2\times 90\times 70}$$

$$\cos(90-\beta) = 0.833$$

$$90-\beta = 33.557^\circ$$

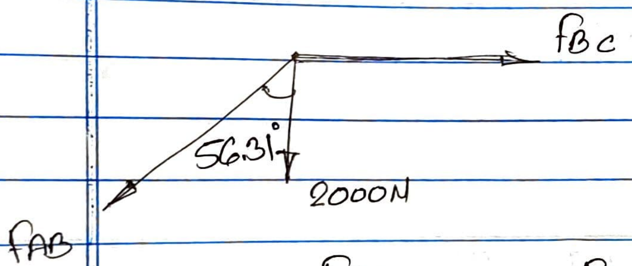
$$\beta = 56.442^\circ$$

6. Resolve the 2000 N force into two oblique components one acting along AB & the other acting along BC.



$$\tan\theta = \frac{2}{3}$$

$$\theta = 33.69^\circ$$



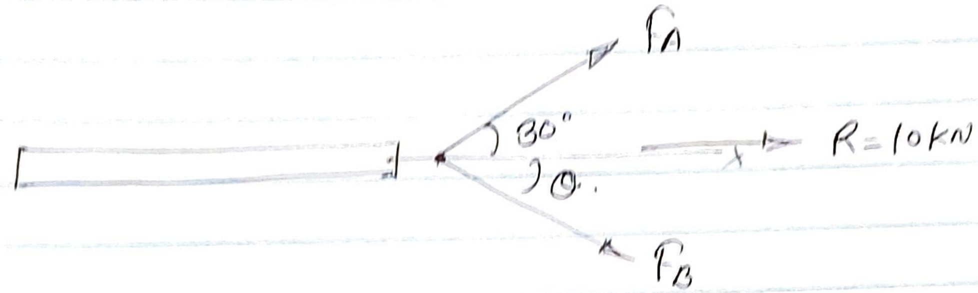
$$\frac{f_{BC}}{\sin 56.31} = \frac{f_{AB}}{\sin 90} = \frac{2000}{\sin(180-90-56.31)}$$

$$\frac{f_{BC}}{\sin 56.31} = \frac{f_{AB}}{\sin 90} = \frac{2000}{\sin 33.69}$$

$$f_{BC} = \frac{2000}{\sin 33.69} \times \sin 56.31 = 3000 \text{ N}$$

$$f_{AB} = \frac{2000}{\sin 33.69} \times \sin 90 = 3605.557 \text{ N}$$

7. The log is being towed by two tractors A and B. If the resultant F_R of the two forces acting on the log is to be directed along +ve x-axis and have a magnitude of 10kN, determine the angle θ of the cable attached to B such that the force F_B in this cable is a minimum. What is the ~~mg~~ magnitude of force in each cable for this situation?



By
(a) Force F_B is minimum, if forces F_A and F_B are ⊥ to each other,

∴ From fig,

$$150 - \theta = 90$$

$$\theta = 60°$$

(b) By Sine Rule,

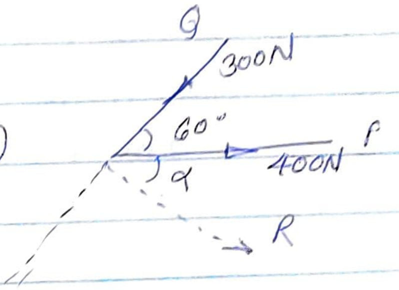
$$\frac{10}{\sin(150-\theta)} = \frac{F_A}{\sin\theta} = \frac{F_B}{\sin 30}$$

$$F_A = \frac{10}{\sin 90} \times \sin 60 = 8.66N$$

$$F_B = \frac{10}{\sin 90} \times \sin 30 = 5N.$$

Examples Based on Law Of Parallelogram Of Forces:-

① The angle between the two forces of magnitude 400N & 300 N is 60°, the 400N force being horizontal. Determine the resultant in magnitude & direction if the 300N force is a push & 400N force is pull.



$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$= \sqrt{400^2+300^2+2\times400\times300\times\cos(180-60)} \quad (120^\circ)$$

$$R = 360.555N$$

$$\tan\alpha = \frac{Q.\sin\theta}{P+Q\cos\theta} = \frac{300\sin(180-60)}{400+300\cos(180-60)} = \cancel{0.472}\ 1.039$$

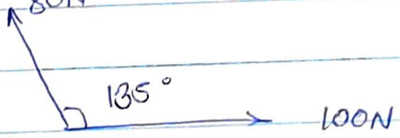
$$\alpha = 46.102^\circ$$

② ① Two forces 100N & 80N respectively having included angle of 135° are acting on a particle. Find the resultant in magnitude & direction when

ⓐ Both forces are pulling (acting away)

ⓑ The force of 100N is pull but 80N is push.

→ ⓐ Both forces are pulling.



$$R = \sqrt{P^2+Q^2+2PQ.\cos\theta}$$

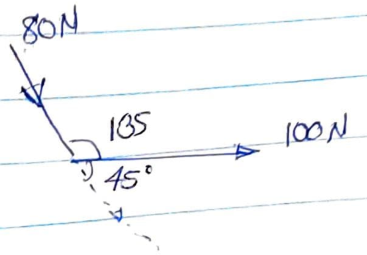
$$= \sqrt{100^2+80^2+2\times100\times80\times\cos135}$$

$$= 71.318N$$

$$\tan\alpha = \frac{Q.\sin\theta}{P+Q\cos\theta} = \frac{80\sin135}{100+80\cos135} = 1.302$$

$$\alpha = 52.484^\circ$$

ⓑ Force 100N is pull but 80N is push

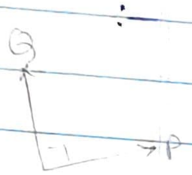


$$R = \sqrt{100^2+80^2+2\times100\times80\times\cos45}$$

$$= 166.474N$$

$$\tan\alpha = \frac{80\sin45}{100+80\cos45} = 0.361 \qquad \alpha = 19.864^\circ$$

③ Find the magnitude of forces P & Q such that if they act at right angle, then resultant is $\sqrt{34}$ N. If they act at an angle of 60°, their resultant is 7N.



$R = \sqrt{34}$

: Let P & Q are the magnitudes of the two forces.

when $\theta = 90°$. $R = \sqrt{34}$ N

when $\theta = 60°$ $R = 7N$

ⓐ when $\theta = 90°$ $R = \sqrt{34}$ N

From the parallelogram law of forces,

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$34 = P^2 + Q^2 + 2PQ\cos 90$$

$$\therefore P^2 + Q^2 = 34 \quad \text{——— } ①$$

ⓑ Similarly when $\theta = 60°$ $R = 7N$

$$49 = (P^2 + Q^2) + 2PQ\cos 60$$

from ①

$$49 = 34 + 2PQ\cos 60$$

$$\therefore PQ = 15 \quad \text{or} \quad P = \frac{15}{Q}$$

put in ①

$$P^2 \quad \left(\frac{15}{Q}\right)^2 + Q^2 = 34$$

$$\frac{225}{Q^2} + Q^2 = 34$$

$$225 + Q^4 - 34Q^2 = 0$$

$$Q^4 - 34Q^2 + 225 = 0$$

$$Q^2 = 25 \text{ or } 9$$

$$\therefore Q = 5N \text{ or } 3N$$

so if $Q = 3N$

then $P = 5N$ Or vice versa.

Home work

(4) The resultant of two forces is 8kN & its direction is inclined at 60° to one of the force whose magnitude is 4kN. Find the magnitude & direction of the other force.

→ $R = 8kN$

$\alpha = 60^\circ \quad P = 4kN \quad Q = ?$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$8^2 = 4^2 + Q^2 + 2\times 4Q \cdot \cos\theta$

$64 - 16 = Q^2 + 8Q\cos\theta$

$Q^2 + 8Q\cdot\cos\theta - 48 = 0$

$48 - Q^2 = 8Q\cos\theta \quad \text{—— (1)}$

$\tan\alpha = \dfrac{Q\cdot\sin\theta}{P + Q\cdot\cos\theta}$

$\tan 60 = \dfrac{Q\cdot\sin\theta}{4 + Q\cdot\cos\theta}$

$4 + Q\cdot\cos\theta = \dfrac{Q\cdot\sin\theta}{\tan 60} \quad \text{—— (2)}$

squaring on both side

$16 + 2\times 4\times Q\cdot\cos\theta + Q^2\cos^2\theta = \dfrac{Q^2\sin^2\theta}{3}$

$16 + \underline{8Q\cdot\cos\theta} + Q^2\cos^2\theta = \dfrac{Q^2\sin^2\theta}{3}$

$16 + 48 - Q^2 + Q^2\cos^2\theta = 0.33\,Q^2\sin^2\theta$

$64 - Q^2(1 - \cos^2\theta) = 0.33\,Q^2\sin^2\theta$

$64 - Q^2\sin^2\theta = 0.33\,Q^2\sin^2\theta$

$64 = 1.33\,Q^2\sin^2\theta$

$Q\cdot\sin\theta = 6.936 \quad \text{—— (3)}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = (1 - \cos^2\theta)$

put in (2)

$4 + Q\cdot\cos\theta = \dfrac{6.936}{\tan 60}$

$4 + Q\cdot\cos\theta = 4.$

$Q\cdot\cos\theta = 0 \quad \text{—— (4)}$

$\cos 60 =$

$32 = 80 - Q^2$

$3.464 =$

$\theta = 90^\circ$

$\therefore$ eq^n ① becomes

$$8 \times 0 = 48 - Q^2$$

$$Q^2 = 48$$

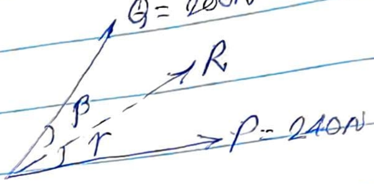
$$\boxed{Q = 6.926\,N}$$

from eq^n ④

$$Q.\cos\theta = 0$$

$$\therefore \cos\theta = 0 \qquad \therefore \boxed{\theta = 90^\circ}$$

⑤ Two concurrent coplaner forces system having magnitude 240N & 200N & there angle between them is 60°. Determine the magnitude of the resultant force & angle made by each force with resultant



$\because$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{240^2 + 200^2 + 2 \times 240 \times 200 \times \cos 60}$$

$$= 381.575\,N$$

Now applying Sine Rule.

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin 60}$$

$$\frac{240}{\sin\beta} = \frac{200}{\sin\gamma} = \frac{381.575}{\sin 60}$$

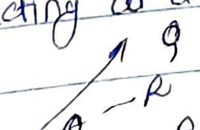
$$\sin\beta = \cancel{\frac{381.575}{\sin 60}}\ \frac{240\sin 60}{381.575} = 0.544$$

$$\beta = 33^\circ$$

$$\sin\gamma = \cancel{381.575\sin 60}\ \frac{200\sin 60}{381.575} = 0.4539$$

$$\gamma = 27^\circ$$

⑥ Determine the maximum & minimum resultant of two forces P & Q acting at a point.



We know,

$$R = \sqrt{P^2 + Q^2 + 2PQ.\cos\theta}$$

In the above expression R will be maximum when θ is max

$\cos\theta = 1$ where $\theta = 0^\circ$

$$R_{max} = \sqrt{P^2+Q^2+2PQ}$$

$$= \sqrt{(P+Q)^2}$$

$$R_{max} = P+Q$$

Again R will be minimum, when $\cos\theta = -1$ i.e $\theta = 180^\circ$ Hence force P & Q act along the same line but in opposite sense.

$$R_{min} = \sqrt{P^2+Q^2-2PQ} = \sqrt{(P-Q)^2}$$

$$\therefore R_{min} = P-Q$$

7) The forces having magnitude equal to 2P & P are concurrent coplaner. *If the first force be doubled & the second is increased by 12N & the direction of resultant remains unaltered (same). Find the value of force P

$\therefore$ θ be the included angle between two forces 2P & P.

α be the angle made by the resultant with 2P.

As the direction of resultant remains unaltered when the force 2P is doubled & P is increased by 12.

Direction of resultant is given by

$$\tan\alpha = \frac{P.\sin\theta}{2P+P.\cos\theta} \quad \text{--- (1)}$$

Again by using second condition,

$$\tan\alpha = \frac{(P+12)\sin\theta}{4P+(P+12)\cos\theta} \quad \text{--- (2)}$$

As the direction of resultant is same, we can equate eq^n (1) & (2)

$$\frac{P\cancel{\sin\theta}}{2P+P.\cos\theta} = \frac{(P+12)\cancel{\sin\theta}}{4P+(P+12)\cos\theta}$$

$$\frac{1}{2+\cos\theta} = \frac{P+12}{4P+P.\cos\theta+12\cos\theta}$$

$$4P+P\cos\theta+12\cos\theta = (P+12)(2+\cos\theta)$$

$$4P+P.\cos\theta+12\cos\theta = 2P+24+P\cos\theta+12\cos\theta$$

$$\therefore 2P = 24N$$

$$\therefore P = 12N$$

*(3) (8) The sum of magnitudes of two forces is 20N & their resultant which is ⊥er to the smaller force is of 12N. Find the magnitude of the ~~forces~~. larger

:-

Resultant's inclination is $90° = \alpha$ to the smaller force.

$P+Q = 20N$ —— (1)

$\tan\alpha = \frac{Q.\sin\theta}{P+Q.\cos\theta}$ as $\tan 90 = \frac{1}{0}$

$P+Q.\cos\theta = 0$ —— (2)

from (1) $P = 20 - Q$

∴ (2) becomes

$20-Q+Q.\cos\theta = 0$

$Q(1-\cos\theta) = 20$ —— (3)

$20-Q+Q.\cos\theta = 0$

$20-Q(1-\cos\theta) = 0$

$20-Q(1-\cos\theta) = 0$

$20 = Q(1-\cos\theta)$

$R^2 = P^2+Q^2+2PQ.\cos\theta$

$R^2 = (P+Q)^2 + 2PQ.\cos\theta - 2PQ$

$12^2 = 20^2 - 2PQ(1-\cos\theta)$

$144 = 20^2 - 2P\times20$

$144 = 400 - 40P$

$P = 6.40N$

$Q = 13.60N$

*(2) (9) The two forces, one of which is double the other, has resultant of 260N. If the direction of the larger force is reversed & other remains unaltered reduce to 180N. Determine the magnitude of the forces & the angle between them.

:- P & 2P be the two forces & θ be the angle betn them.

Using first condition,

$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$260^2 = \sqrt{P^2+4P^2+2P\times2P\cos\theta}$

$67600 = 5P^2+4P^2\cos\theta$

$16900 = 1.25P^2+P^2\cos\theta$

$16900 = P^2(1.25+\cos\theta)$ —— (1)

Using second condition.

$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$180^2 = P^2 + 4P^2 - 4P^2\cos\theta$

$32400 = 5P^2 - 4P^2\cos\theta \Rightarrow \frac{32400}{4} = \frac{5}{4}P^2 - P^2\cos\theta$

$8100 = 1.25P^2 - P^2\cos\theta$ — (2)

from (1) & (2)

$$\frac{16900}{1.25+\cos\theta} = \frac{8100}{1.25-\cos\theta}$$

$169(1.25-\cos\theta) = 81(1.25+\cos\theta)$

$211.25 - 169\cos\theta = 101.25 + 81\cos\theta$

$110 = 250\cos\theta$

$\cos\theta = 0.44$

$\theta = 63.896^\circ$

Magnitude of other force say Q is double of other force P.

from (2)

$8100 = 1.25P^2 - P^2\, 0.44$

$8100 = 0.81P^2$

$P^2 = 10000 \quad \therefore P = 100N$

$\therefore Q = 200N$

(6) Two forces act at angle of 120°. The greater is represented by 80N & the resultant is at right angle to the smaller. Find the latter force

→ Let P & Q are the greater & smaller forces resp.

$\tan\alpha = \frac{Q.\sin\theta}{P+Q.\cos\theta}$

$\tan 90 = \frac{Q.\sin 120}{80+Q.\cos 120} = \frac{1}{0}$

$80 + Q.\cos 120 = 1$

$Q.\cos 120 = -80$

$Q = 160N$

$\tan\alpha = \frac{P.\sin\theta}{Q+P.\cos\theta}$

$\tan 90 = \frac{80\sin 120}{Q+80\cos 120} = \frac{1}{0}$

$Q - 40 = 1$

$Q = 40N$

$R = 138.56$

(11) The sum of two concurrent forces P & Q is 270N & their resultant is of 180N. If resultant is perpendicular to P. Find P & Q

$P + Q = 270$ ——— ①

$R = 180$ $\qquad \alpha = 90°$

$\tan\alpha = \dfrac{Q.\sin\theta}{P + Q.\cos\theta} = \tan 90 = \dfrac{1}{0}$

$\therefore P + Q.\cos\theta = 0$ ——— ②

Now $\qquad Q\cos\theta = -P$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$(180)^2 = P^2 + Q^2 + 2P(-P)$ from ①

$180^2 = Q^2 - P^2$

$180^2 = (Q-P)(P+Q)$

$180^2 = (Q-P)\,27$

$\therefore Q - P = 120$

$P + (120 + P) = 270$

$\therefore P = 75N$

$Q = 195N$

(12) For two forces P & Q acting at a point, maximum resultant is 2000N & minimum magnitude of ~~two forces~~ resultant is 800N. Find values of P & Q

$R^2 = P^2 + Q^2 + 2PQ.\cos\theta$

for maximum value of R $\theta = 0$

$\therefore R_{max} = P + Q$

$\therefore P + Q = 2000N$ ——— ①

For minimum value of R, $\theta = 180°$

$\therefore R_{min} = P - Q$

$\therefore P - Q = 800$ ——— ②

from ① & ②

$P + (P - 800) = 2000$

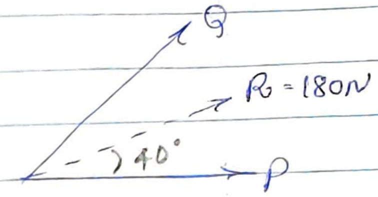
$2P = 2800$

$\therefore P = 1400N$

$Q = 600N$

(13) The resultant of two forces is 180N. It makes an angle of 40° with each force other. Find magnitude of two forces.

Here $\theta = 80°$, $\alpha = 40°$ $R = 180N$



$R^2 = P^2 + Q^2 + 2PQ \cos\theta$

$180^2 = P^2 + Q^2 + 2PQ \cos 80°$

$180^2 = P^2 + Q^2 + 0.347PQ$ —— (1)

$\tan\alpha = \frac{Q.\sin\theta}{P + Q.\cos\theta}$

$\tan 40 = \frac{Q.\sin 80}{P + Q.\cos 80}$

$0.84 = \frac{0.98Q}{P + 0.17Q}$

$P + 0.17Q = 1.167Q$

$P = 0.997Q$

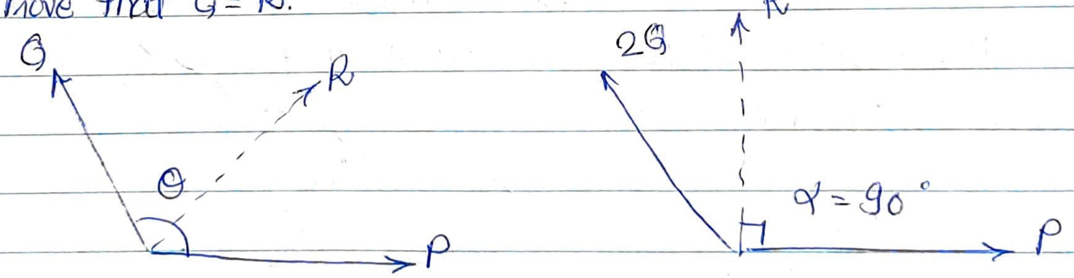
$\therefore P = Q$ —— (2)

Putting in (1) we get.

$180^2 = P^2 + P^2 + 0.347P^2$

$P = Q = 117.494N$

(14) The resultant of two forces P & Q is R. If Q is doubled the new resultant ler to P. Prove that Q = R.



$R^2 = P^2 + Q^2 + 2PQ\cos\theta$ —— (1)

$\tan 90 = \frac{2Q.\sin\theta}{P + 2Q.\cos\theta} = \infty = \frac{1}{0}$

$\therefore P + 2Q.\cos\theta = 0$

$2Q.\cos\theta = -P$

from (1)

$R^2 = P^2 + Q^2 - P^2$

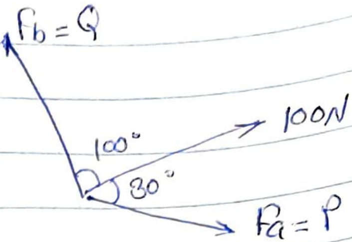
$R = Q$ Proved

(15) Find components F_a & F_b of the 100N force along the directions shown in fig. using parallelogram law

(A)

Let $P = F_a$ & $Q = F_b$

$\therefore R = 100\,N$



$\alpha = 30°$

$\theta = 130°$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$100^2 = F_a^2 + F_b^2 + 2 \cdot F_a \cdot F_b \cos 130°$$

$$100^2 = F_a^2 + F_b^2 - 1.285\, F_a \cdot F_b \quad \text{——— (1)}$$

Now $\tan\alpha = \dfrac{Q \cdot \sin\theta}{P + Q\cos\theta}$

$$\tan 30 = \frac{F_b \cdot \sin 130}{F_a + F_b \cos 130}$$

$$0.577\, F_a - 0.371\, F_b = 0.766\, F_b$$

$$0.577\, F_a = 1.1371\, F_b$$

$$F_a = 1.97\, F_b \quad \text{——— (2)}$$

Put in (1)

$$100^2 = (1.97\, F_b)^2 + F_b^2 - 1.285\,(1.97\, F_b)\, F_b$$

$$100^2 = 4.880\, F_b^2 - 2.531\, F_b^2$$

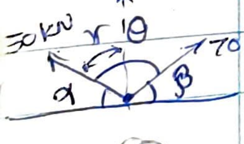
$$100^2 = 2.349\, F_b^2$$

$$\cancel{100 = 1.532\, F_b}$$

$F_b = 65.274\ N$

$F_a = 1.97 \times 65.274 = 128.589\ N$

(16) If the resultant of two forces shown in fig. is 90 kN directed vertically upwards, find the angles α & β using rectangular component



(A)

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$90^2 = 50^2 + 70^2 + 2 \times 50 \times 70 \cos\theta$$

$\cos\theta = 0.1 \qquad \theta = 84.260°$

$\therefore \alpha + \beta = 180 - \theta = 180 - 84.260 = 95.739$

$$\tan\gamma = \frac{70 \sin 84.260}{50 + 70\cos 84.260}$$

$\tan\gamma = 1.221 \qquad \gamma = 50.702°$

$\alpha = 90 - 50.702 = 39.298°$

$\beta = 95.739 - 39.298 = 56.441°$

Using Cosine Rule.

$90 - \beta = 180 - \alpha - 180 - \alpha - \beta$

$R = 90 = -\gamma - \beta$

$$\cos(90 - \alpha) = \frac{50^2 + 90^2 - 70^2}{2 \times 50 \times 90}$$

$= 0.633$

$90 - \alpha = 50.703$

$\alpha = 39.296$

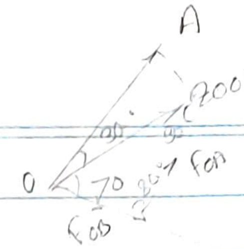
$$\cos(90 - \beta) = \frac{70^2 + 90^2 - 50^2}{2 \times 70 \times 90}$$

$= 0.833$

$90 - \beta = 33.557$

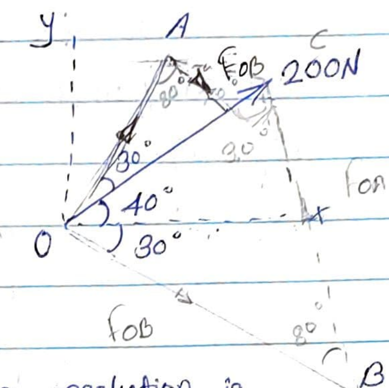
$\beta = 56.442°$

Non-Ler Resolution



Examples:–

① Resolve 200N force into components in ~~ⓐ x & y directions.~~
① ⓑ Along A & B directions. Ref. fig.



ⓐ x & y components:

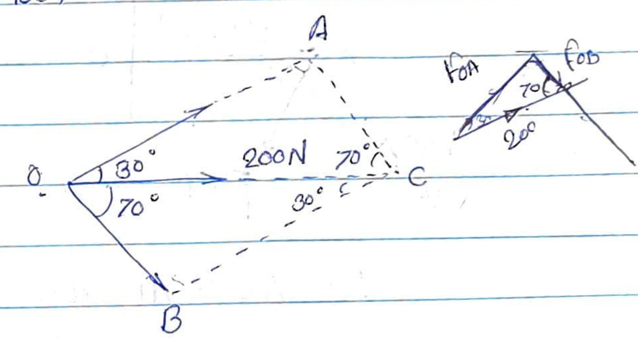
x component $= 200 \cos 40 = 153.208$ N

y component $= 200 \sin 40 = 128.557$ N

ⓑ Components along A & B

As angle between OA & OB is $100°$, so resolution is non-perpendicular.

In $\triangle OBC$, by sine rule



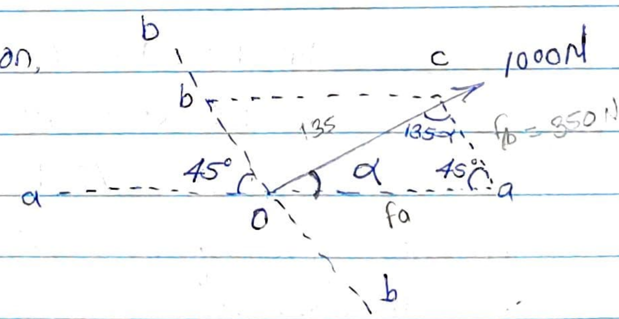
$$\frac{200}{\sin 80°} = \frac{F_B}{\sin 30} = \frac{F_A}{\sin 70}$$

$$\therefore F_A = \frac{200 \times \sin 70}{\sin 80} = 190.837\,N$$

$$F_B = \frac{200 \sin 30}{\sin 80} = 101.542\,N$$

② A force of 1000N is to be resolved into two components along line a-a & a-b as shown in fig. If the component along line b-b is 350N. Find angle
② α & component along a-a.

:– This is non-perpendicular resolution.



In $\triangle OAC$, by sine rule,

$$\frac{1000}{\sin 45} = \frac{F_a}{\sin(135-\alpha)} = \frac{F_b}{\sin \alpha}$$

But $F_b = 350$ N --- Given

$$\therefore \frac{1000}{\sin 45} = \frac{350}{\sin \alpha}$$

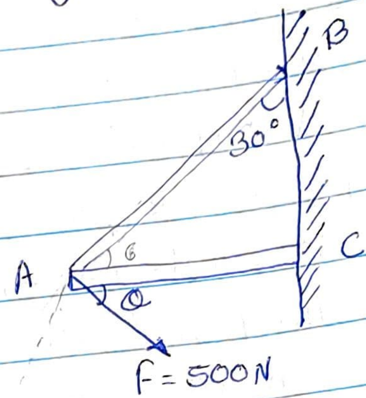
$\sin \alpha = 0.247$ $\quad$ $\alpha = 14.328°$

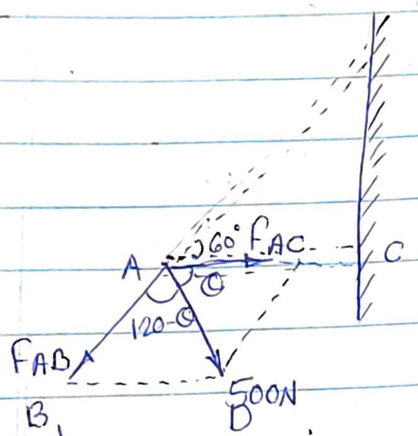
$$F_a = \frac{1000}{\sin 45} \times \sin(135 - 14.328)$$

$= 1216.367$ N

(3) The force F acting on the frame has a magnitude of 500N & it is to be resolved into two components acting along strut AB & AC. Determine the angle θ, so that component F_{AC} is directed from A towards C & has a magnitude of 400N

Ref. fig.





As AC & AB are non-perpendicular so, this is resolution into non-perpendicular components.

In $\triangle ADB$, by sine rule,

$$\frac{F_{AB}}{\sin\theta} = \frac{F_{AC}}{\sin(120-\theta)} = \frac{500}{\sin 60}$$

but $F_{AC} = 400N$ given.

$$\therefore \frac{500}{\sin 60} = \frac{400}{\sin(120-\theta)}$$

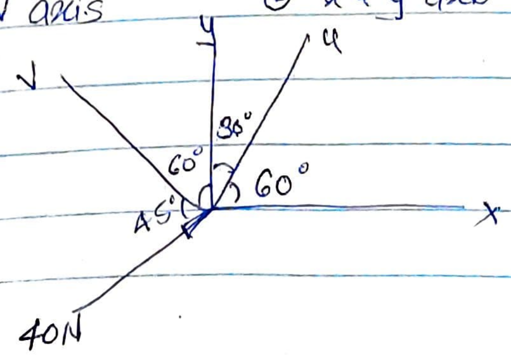
$$\sin(120-\theta) = 0.692$$

$$120-\theta = 43.853$$

$$\therefore \theta = 76.146^\circ$$

(4) Resolve given force of 40N along

(a) u & v axis (b) x & y axis



ⓐ Resolving along u & v axis.

$\theta = 90 + 45 = 135^\circ$

~~$F_u = F_v = 28.28$~~

$F_u = 40 \cos 135 = 28.284\ N$

$F_v = 40 \sin 135 = 28.284$

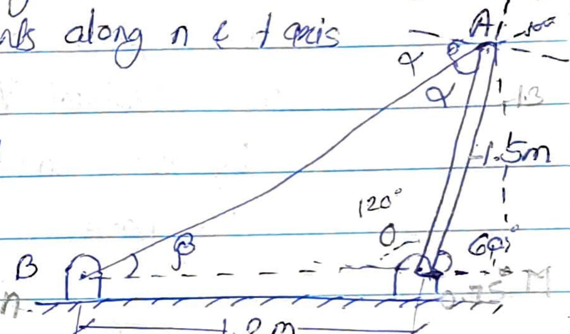
ⓑ Resolving along x & y axis.

~~$F_x =$~~ $\theta = 90 + 60 + 45 = 195^\circ$

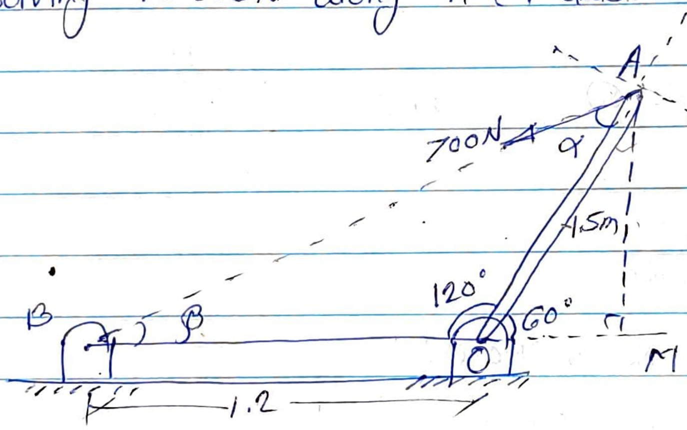
$F_x = 40 \cos 195 = -38.637\ N$

$F_y = 40 \sin 195 = -$ ~~$68.637\ N$~~ $-10.352\ N$

⑤ The cable AB prevents rod OA from rotating clockwise about hinge 'O'. If the cable tensions is 700N. Find components along n & t axis



Resolving T = 700N along n & t axis.



from geometry of ΔOMA

$AM = 1.5 \sin 60 = 1.3m$

$OM = 1.5 \cos 60 = 0.75m$

Now in ΔBMA, $\tan\beta = \dfrac{AM}{BM} = \dfrac{1.3}{1.95}$ (1.2 + 0.75)

$\therefore \beta = 33.69^\circ$

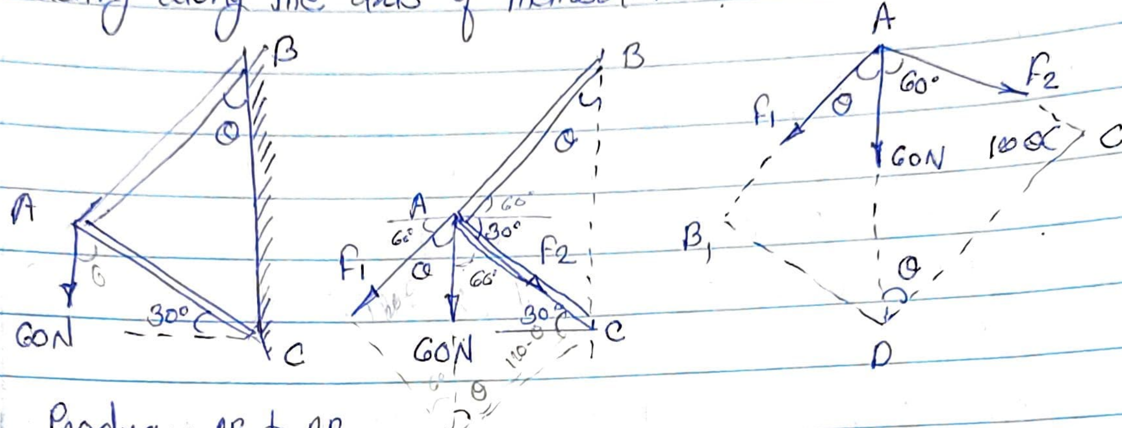
$\alpha = 180^\circ - (120^\circ + 33.69)$

$\alpha = 26.31^\circ$

t-component $= -700 \cos 26.31 = -627.486\ N$

n-component $= -700 \sin 26.31 = -310.259\ N$

(6) A force of 60N acts downward at A. Determine the angle θ ($0 \le \theta \le 90°$) of member AB so that component of F acting along the axis of AB is 80N. What is the magnitude of the force component acting along the axis of member AC?



∴ Producing AB to AB_1

The component of force must be along AC & AB_1 as shown in fig.

Now construct the parallelogram.

By sine rule in $\triangle ACD$,

We have,
$$\frac{60}{\sin(120-\theta)} = \frac{F_1}{\sin 60} = \frac{F_2}{\sin\theta}$$

but $F_1 = 80N$

$$\therefore \frac{60}{\sin(120-\theta)} = \frac{80}{\sin 60}$$

$$60 \sin 60 = 80 \sin(120-\theta)$$

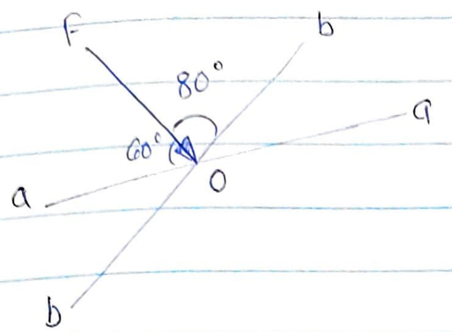
$$120-\theta = \sin^{-1}(0.65)$$

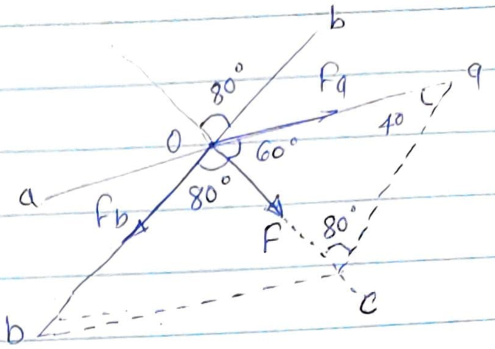
$$\therefore \theta = 79.46°$$

$$\therefore F_2 = \frac{80 \sin 79.46}{\sin 60} = 90.82N$$

∴ Force along AC is 90.82N

(7) The component of a force F acting along line a-a is 30N. Determine F and its component along line b-b.





As per principle of transmissibility transferring the forces as shown in fig.

In ΔOCa by sine rule,

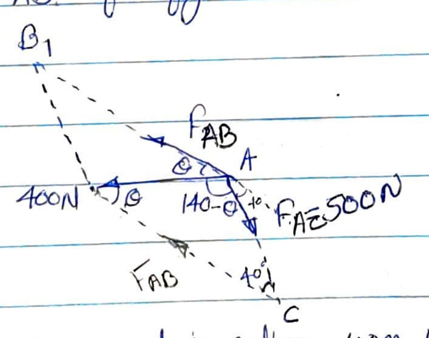
$$\frac{F_a}{\sin 80} = \frac{F_b}{\sin 60} = \frac{F}{\sin 40}$$

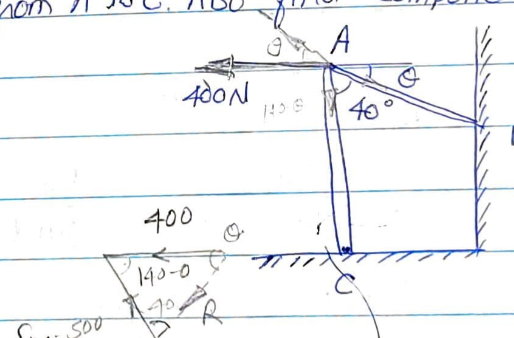
$F_a = 30N$ given $\therefore \frac{30}{\sin 80} = \frac{F_b}{\sin 60} = \frac{F}{\sin 40}$

$$\therefore F_b = \frac{30}{\sin 80} \times \sin 60 = 26.38\ N$$

$$F = \frac{30}{\sin 80} \times \sin 40 = 19.58N$$

(8) Determine the angle θ for strut AB so that 400N horizontal force has a component of 500N acting from A to C. Also find component along member AB. Refer fig.





As one component is acting from A to C, the component along AB must be from B to A i.e. A to B_1 as shown in fig.

By sine rule, $\frac{400}{\sin 40} = \frac{F_{AC}}{\sin \theta} = \frac{F_{AB}}{\sin(140-\theta)}$

but $F_1 = F_{AC} = 500N$

$$\frac{400}{\sin 40} = \frac{500}{\sin \theta} = \frac{F_2}{\sin(140-\theta)}$$

$$\sin\theta = \frac{500 \sin 40}{400} = 0.803$$

$$\theta = 53.46^\circ$$

$$F_2 = \frac{400 \sin(140 - 53.46)}{\sin 40}$$

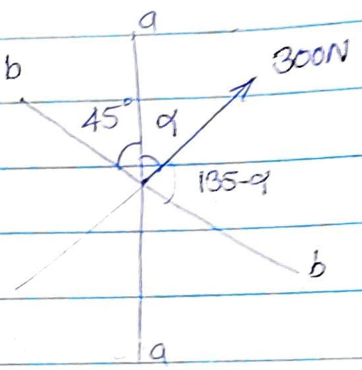
$$= 621.15\,N$$

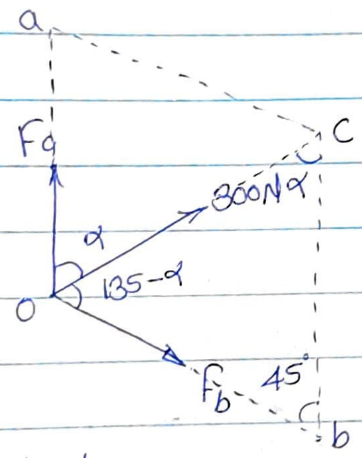
9) The 300N force is to be resolved into components along line a-a & b-b.

ⓐ Determine the angle α if components along a-a is to be 200N

ⓑ What is the corresponding value of component along b-b?

Refer fig.





We have,

$$\frac{F_a}{\sin(135-\alpha)} = \frac{F_b}{\sin\alpha} = \frac{300}{\sin 45}$$

but $F_a = 200N$ given

$$\frac{200}{\sin(135-\alpha)} = \frac{F_b}{\sin\alpha} = \frac{300}{\sin 45}$$

$$\frac{200}{\sin(135-\alpha)} = \frac{300}{\sin 45}$$

$$200 \sin 45 = 300 \sin(135-\alpha)$$

$$0.471 = \sin(135-\alpha)$$

$$135 - \alpha = 28.125$$

$$\alpha = 106.874^\circ$$