Electrostatic :- Unit II

Coloumb's Law:-

Like charges repel each other unlike charges attract eachother

The force between two very small charges Q_1 and Q_2 seperated by a distance d is directly proportional to product of this two charges and inversly proportion to square of distance between them.

$$\boxed{F \propto \frac{Q_1 \cdot Q_2}{d^2}} \qquad \boxed{F = K\frac{Q_1 Q_2}{d^2}}$$

$F \rightarrow$ force
$K \rightarrow$ Coloumb's law constant
$d \rightarrow$ distance between two charges

$$\boxed{F = \frac{1}{4\pi \varepsilon_0 \varepsilon_r} \frac{Q_1 Q_2}{d^2}} \qquad \boxed{K = \frac{1}{4\pi\varepsilon_0} \times \frac{1}{\varepsilon_r} = 9\times10^{9} \quad [\varepsilon_r = 1]}$$

Permitivity:- Property of medium wich affect the magnitude of force between two point charges.

$$\boxed{K = \frac{1}{4\pi\varepsilon_0}}$$

$\varepsilon \rightarrow$ Absolute permittivity of medium

$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$

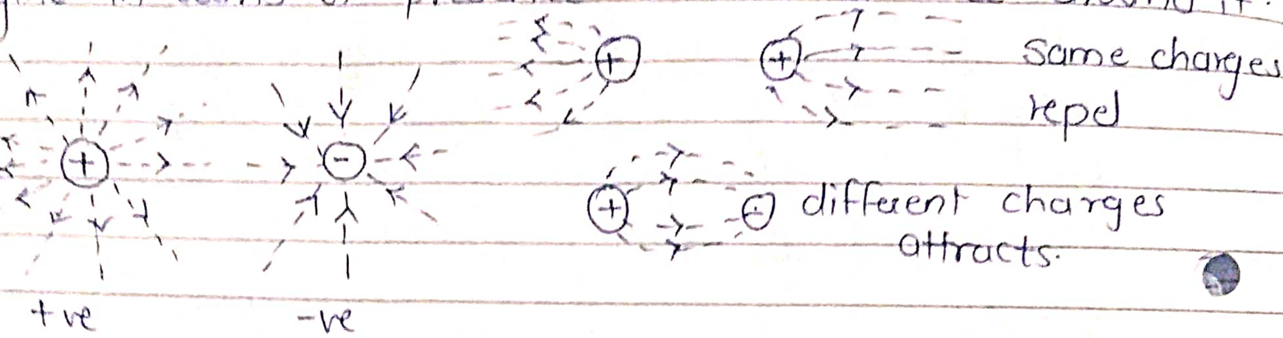
$\varepsilon_0 \rightarrow$ permitivity of free space
$\varepsilon_r \rightarrow$ Relativity ~~of~~ permitivity of medium

$$\boxed{\varepsilon_0 = 8.854\times10^{-12}\ F/m} \qquad \boxed{\varepsilon_{r(air)} = 1}$$

* Electric field:- The region around charged body where another charge experiences mechanical ~~field~~ force.

* Electric Lines of force :- Electric field around a charge is imagine in terms of presence of lines of force around it.

properties:-
parallel,
never intersect



* Electric Flux:- The total no. of lines of forces of flux in any particular electric field. (Ψ) unit:- Coloumb (C)

$\Psi = Q$ .... charge.

* Electric Flux Density (D):- Electric flux passing at right angle to unit area of surface.

$$\boxed{D = \frac{\Psi}{A} = \frac{Q}{A}} \quad \ldots \frac{C}{m^2}$$

(E) * Electric field Intensity /Strength:- Force experienced by a unit positive charge placed at any point in the electric field.

$$\boxed{E = \frac{F}{Q}} \quad \ldots\ldots \frac{N}{C}$$

$$\boxed{E = \frac{V}{d}}$$ in derivation of ‖ plates
Volt/metre

* Relationship between (D) & (E)

$$\boxed{D = \varepsilon_0 \varepsilon_r E} \qquad \boxed{D = \varepsilon E}$$

(ε) * Absolute permitivity :- Ratio of Electric flux density to Electric flux Intensity at any point.

$$\boxed{\varepsilon = \frac{D}{E}} \;\dots\dots\; \frac{F}{m}$$

* Permitivity of free space (ε_0) :- Ratio of Electric flux density in a free space (D_0) to Electric Intensity (E)

$$\boxed{\varepsilon_0 = \frac{D_0}{E}} \qquad \varepsilon_0 = 8.854 \times 10^{-12}\ F/m$$

* Relative Permitivity (ε_r) :- Ratio of Absolute permitivity to permitivity of free space.

$$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}} \qquad \text{no unit}$$

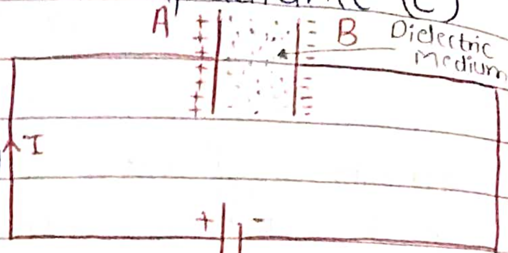
* Electric Potential (E.P) :- Electric Potential at any point in an electric field is amount of work done in bringing a unit positive charge.

$$\boxed{E.P = \frac{\text{Work}}{\text{Charge}} = \frac{W}{Q}} \;\dots\; \frac{J}{C^2}$$

* Frequency :- Measure of the rate of oscillation measured in the no. of changes per second. (Hz)

$$\boxed{f = \frac{1}{T} \qquad f = \frac{\omega}{2\pi}}$$

* Capacitor :- Capacitor is nothing but two conducting surface seperated by an insulating medium (dielectric)
* Has ability to store charg' unit 'capacitance' (C)



Working :- When capacitor is connected across ~~batty~~ battery an electric field develops having +ve to one plate and -ve to other plate.

* Capacitance :- Amount of charge Q required to attain a potential (V) of 1V between the parallel plates. $\boxed{Q = CV}$

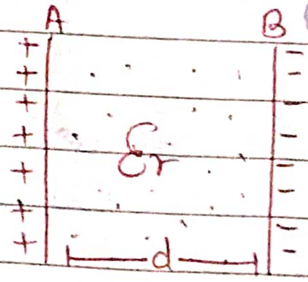
* Relation between charge and applied voltage :- $\boxed{C = \frac{Q}{V}}$

* Dielectric Strength :- Maximum voltage required to produce a dielectric breakdown through the material and expressed as volts per unit thickness.

$$\boxed{E = \frac{V}{d}} \ \cdots \ \frac{V}{m}$$

(Derivation)

* Capacitance of parallel plate capacitor :-



Consider two plates of A and B the surface area of each plate is A m^2 and seperated by ~~dm~~ distance dm.
Er is dielectric medium
Let Q bethe charge on this plate

Electric flux density is given as:

$$D = \frac{\Psi}{A} = \frac{Q}{A} \quad \dots (1)$$

Electric field intensity is given as:

$$E = \frac{V}{d} \quad \dots (2)$$

We know that

$$D = \varepsilon_0 \varepsilon_r E$$

from (1) and (2)

$$\frac{Q}{A} = \varepsilon_0 \varepsilon_r \frac{V}{d}$$

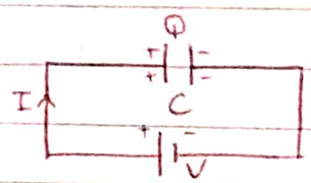
$$\frac{Q}{V} = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$\therefore Q = CV$$

$$\therefore \boxed{C = \frac{A\varepsilon_0 \varepsilon_r}{d}} \qquad \boxed{C = \frac{A\varepsilon_0 \varepsilon_r}{d}} \text{ Farad}$$

(Derivation)

* Energy stored in capacitor:- consider charging of capacitor by potential difference 'V' volts where the charge on the capacitor in 'q' coloumb



Now, If charge of capacitor is raised by small value dq then $d\omega$ is given by $d\omega = v\,dq$

where,

$$v = \frac{q}{c}$$

Workdone is finally stored in capacitor in the form of en
and hence total workdone is given by

$$W = \frac{1}{C} \int_0^Q q\, dq$$

$$W = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q$$

$$= \frac{1}{C} \times \frac{Q^2}{2}$$

$$W = \frac{Q^2}{2C}$$

$$\therefore Q = CV$$

$$W = \frac{C^2 V^2}{2C}$$

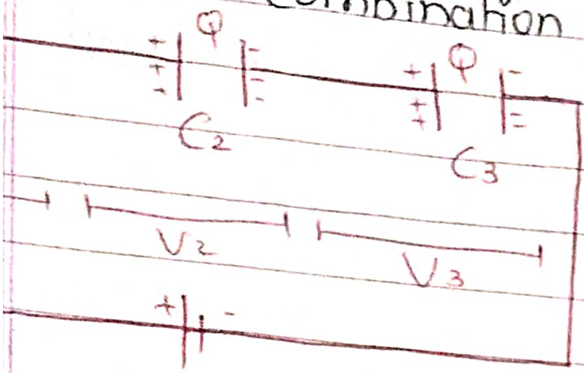
$$W = \frac{1}{2} CV^2 \qquad \boxed{W = \frac{1}{2} CV^2} \text{ --- Joules}$$

Energy stored by capacitor $\boxed{\therefore W = E}$ $\boxed{E = \frac{1}{2} QV = \frac{1}{2} CV^2}$ --- Joules

Series ⇒ Q constant constant

(Derivation)

Series Combination of Capacitor :- Consider the three capacitors C_1, C_2, C_3 in series across voltage V ∴ charge remains same



Let V_1, V_2, V_3 be potential difference or voltage across capacitor C_1, C_2, C_3

$$\therefore Q = C_1V_1 = C_2V_2 = C_3V_3$$

The voltage across capacitor

$$V_1 = \frac{Q}{C_1} \qquad V_2 = \frac{Q}{C_2} \qquad V_3 = \frac{Q}{C_3}$$

$$\therefore V = V_1 + V_2 + V_3$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad \text{— (1)}$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots}$$ series capacitor

$$\boxed{\frac{1}{C_s} = \frac{\cancel{C_1+C_2}}{\cancel{C_1}} \quad \frac{C_1+C_2}{C_1C_2}}$$ for two capacitor

Equivalent capacitance

$$V = \frac{Q}{C_{eq}} \quad \text{— (2)}$$

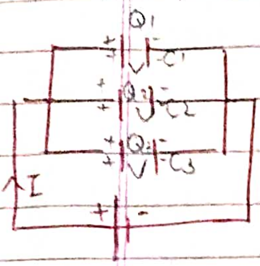
$$\therefore \frac{Q}{C_{eq}} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

from (1) and (2)

$$\boxed{\therefore \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

(Derivation)

* Parallel Combination of Capacitor :- Consider a capacitor C₁, ~~is common~~ C_2, C_3 are connected in parallel across voltage V



Voltage across capacitor is same but charge is different.

let, Q_1, Q_2, Q_3 be charge on C_1, C_2, C_3 capacitors.

$\therefore Q_1 = C_1 V \qquad Q_2 = C_2 V \qquad Q_3 = C_3 V$

$$Q = Q_1 + Q_2 + Q_3$$

$$Q = C_p V$$

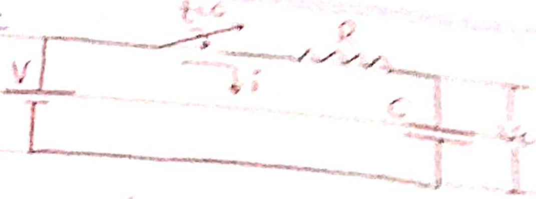
$\boxed{C_p = C_1 + C_2 + C_3}$ ... parallel capacitor

$$C_p V = V(C_1 + C_2 + C_3)$$

$$\therefore C_p = C_1 + C_2 + C_3$$

Imp

* Charging of Capacitor :- Consider a capacitor C is connected in series with resistance R and switch and battery.



...e capacitor has no charge initially when switch S is closed at ...tant t=0 then the charge get accomodated on a capacitor ... current starts flowing.

- This current at the instant of closing the switch is high as the voltage across capacitor at start is zero and ... increase gradually.

This current is given by

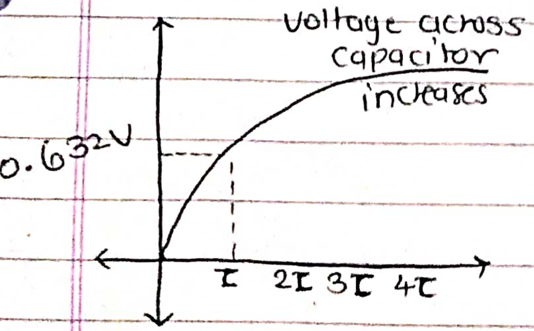
$$i = \frac{V - V_c}{R}$$

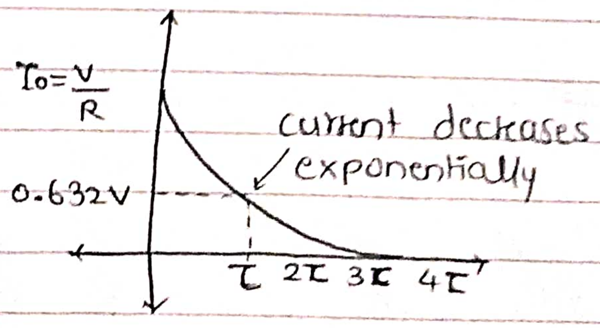
$$i = \frac{V}{R} e^{-t/CR}$$

$$\boxed{V_c = V(1 - e^{-t/CR})}$$

Graphical reprentation of charging voltage and current.

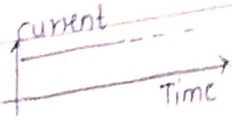
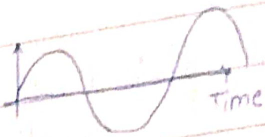
$\boxed{V = iR}$



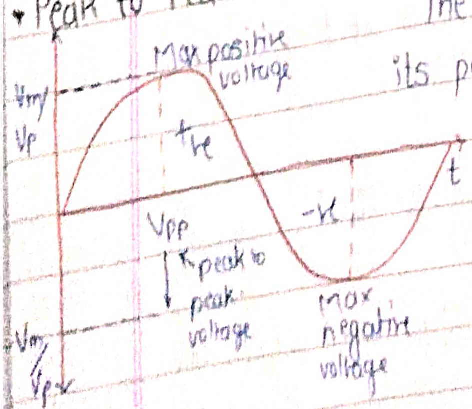


Time constant :- (τ) "The product of R and C is called as Time constant (τ); unit is seconds". $\tau = R \times C$ (sec)

* AC Fundamentals:- Alternating current is the current whose magnitude and direction changes with respect to time.
* Advantages:- • Generation of AC is easy compare to DC.
  cheaper than DC
  • AC can be converted to DC
  • transmitted over long distance

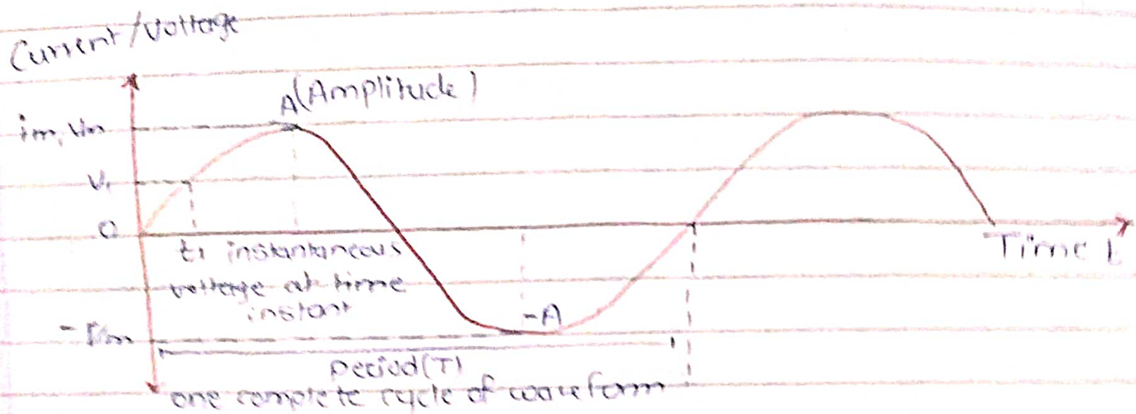
Parameter	D.C.	A.C.
Definition	It is the current having constant magnitude and direction	It is the current whose magnitude and direction is opposite
Waveform	 	
Generation	Generation of DC is not easy like AC and costly	Generation is easy and cheaper
Frequency	zero	50 Hz (India)

* Peak to Peak value:- The value of an alternating quantity from its positive peak to negative peak is called peak to peak value.



$\boxed{V_{p\text{-}p} = 2 \times \text{Maximum Value}}$ volts

① Wave form :- Graph of magnitude of alternating quantity versus time."



② Instantaneous value :- The value of alternating quantity at a particular time instant.

③ Cycle :- Repetation of 1 +ve and 1 -ve curve of waveform is called cycle.

④ Time period (T) :- Time taken by alternating quantity to complete one cycle is called period. $\boxed{T = 2\pi \text{ radian}}$

⑤ Amplitude :- Maximum value attained by alternating quantity is called amplitude (Peak value).

⑥ Frequency :- The number of cycle completed by alternating quantity per second $\boxed{F = 1/T}$ $T = 1/F$

⑦ Electric Angle :- Angle measured in electrical degree or radian

⑧ Angular frequency / Angular velocity :- (ω) :- Frequency expressed in electrical radian per second

$\omega = 2\pi f$ or $\omega = \frac{2\pi}{T}$ ... $\frac{\text{rad}}{\text{sec}}$

$\cos\theta = \sin(90+\theta)$

* Imp Equations

$e = E_m \sin(\omega t)$ $\qquad$ $i = I_m \sin(\omega t)$

$e = E_m \sin(\omega t + \phi)$ $\qquad$ $i = I_m \sin(2\pi f t)$

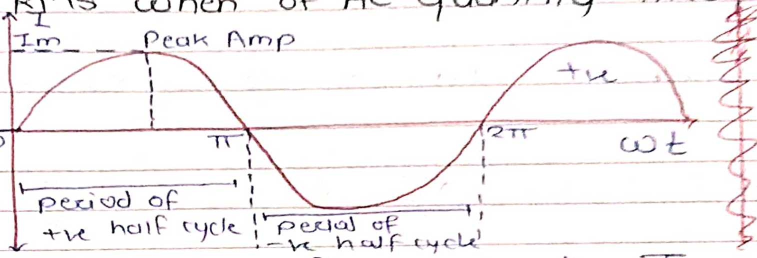
$e = E_m \sin(2\pi f t)$ $\qquad$ $i = I_m \sin\left(\frac{2\pi t}{T}\right)$

$e = E_m \sin\left(\frac{2\pi t}{T}\right)$

(Derivation)

* Effective value of RMS when of AC quantity like voltage/current

→ Consider a sinasoidal current waveform



The peak (maximum) value of current is I_m.

$I = I_m \sin\omega t$ ......(1) $\qquad$ I_m = maximum value peak value

squaring

$I^2 = I_m^2 \sin^2\omega t$ ......(2)

$\cancel{\sin^2\omega t = 1 - \cos 2\omega t}$

$I^2 = I_m^2 \left[\frac{1-\cos(2\omega t)}{2}\right]$ .... $[\cos 2\theta = 1 - 2\sin^2\theta$

$I^2 = \frac{I_m^2}{2}\left[1 - \cos(2\omega t)\right]$ ...(3)

$V_{av} = 0.637\, V$

The average value I^2 is denoted by I_{av}^2. It is obtain by Integrating I^2 for positive half cycle (0 to π) and then dividing by period of the half cycle (0 to π).

$$I_{av}^2 = \frac{\int_0^{\pi} I^2 d\omega t}{\pi} = \frac{\int_0^{\pi} \frac{I_m^2}{2}[1-\cos(2\omega t)]\, d\omega t}{\pi}$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} (1-\cos(2\omega t))\, d\omega t$$

$$= \frac{I_m^2}{2\pi}\left[\int_0^{\pi} 1\, d\omega t - \int_0^{\pi} \cos(2\omega t)\, d\omega t\right]$$

$$= \frac{I_m^2}{2\pi}\left[(\omega t)_0^{\pi} - \left(\frac{\sin(2\omega t)}{2}\right)_0^{\pi}\right]$$

$$= \frac{I_m^2}{2\pi}\left[(\pi - 0) - \frac{[\sin(2\pi) - \sin(0)]}{2}\right]$$

$$= \frac{I_m^2}{2\pi}[\pi - 0] \qquad I_{av}^2 = \frac{I_m^2}{2}$$

RMS value

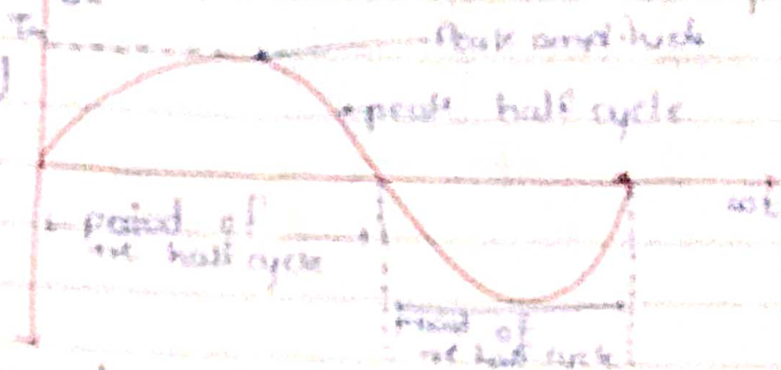
$$\therefore I_{rms} = \sqrt{\frac{I_m^2}{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad \boxed{I_{rms} = \frac{I_m}{\sqrt{2}}} = 0.707\, I_m$$

Volts or ampere

e value of sinusoidal current in terms of its peak value

Consider a sinusoidal waveform. The peak value of current is



e value I_{av}: obtained by integrating positive half cycle (0 to π) and then dividing it by period half cycle(π)

$$I_{av} = \int_0^{\pi} \frac{I}{\pi} d\omega t = \int_0^{\pi} \frac{I_m \sin(\omega t) d\omega t}{\pi}$$

$$I_{av} = \frac{I_m}{\pi} \int_0^{\pi} \sin(\omega t)\, d\omega t$$

$$= \frac{I_m}{\pi} \left[-\cos \omega t\right]_0^{\pi}$$

$$I_{av} = \frac{I_m}{\pi} \left[-\cos(\pi) - (-\cos(0))\right]$$

$$I_{av} = \frac{I_m}{\pi} [1+1] \Rightarrow \frac{2I_m}{\pi}$$

$$I_{av} = 0.637 I_m$$

$$\boxed{I_{av} = \frac{2I_m}{\pi} = 0.637\, I_m}$$ Volts or ampere

$V_{av} = 0.637\ V_m$

$I_{av} = 0.637\ I_m$

* Form Factor (K_f) :- Ratio of rms value to average value of an alternating quantity is called. (K_f)

$$\boxed{K_f = \frac{\text{rms value}}{\text{average value}}}$$

~~K_f~~ $\boxed{K_f = 1.11}$
Unitless

* Peak factor or Crest factor (K_p) :- Maximum (peak) value of alternating quantity is called as crest value. The ratio of peak (crest) value to the rms value of an alternating quantity is called. (K_p)

$$\boxed{K_p = \frac{\text{Peak value or crest value}}{\text{rms value}}}$$

$\boxed{K_p = 1.414}$
Unitless

* Phasor Representation of Sinusoidal Quantities :- simple method to represent alternating quantities such as sine wave or cosine wave."

$$\boxed{I_{max} = \sqrt{2}\ \cancel{E_{RMS}}\ I_{RMS}}$$

* Phase :- A position of phasor at a particular instant is ~~experenced~~ expressed by term phase. ϕ .... radian / degree

* Phase Angle :- Angle made by a phasor with respect to reference axis

$$V = V_m \sin(\omega t \pm \phi)$$

Anticlockwise $\longrightarrow$ + ve
Clockwise $\longrightarrow$ − ve

* Phase Difference :- diffrence of phase angle between two alternating quantities

Representation

Equ^n

$$V = V_m \sin(\omega t + \phi)$$

V_m = Peak value voltage

ϕ = phase of voltage

polar repsesentation

$r, \angle \phi$

Rectangular Representation

$V(t) = V_m \cos\phi + jV_m \sin\phi$

rectan form

Addition / Substraction is done in (Rectangular form)

$A = x_1 + jy_1 \qquad B = x_2 + jy_2$

$\therefore A+B = (x_1+x_2) + j(y_1+y_2)$

$\therefore A-B = (x_1-x_2) + j(y_1-y_2)$

Multiplication / Division is done in (Polar form)

$A = r_1 \angle\phi_1 \qquad B = r_2 \angle\phi_2$

$A \cdot B = r_1 \cdot r_2 \cdot \angle(\phi_1 + \phi_2)$

$\frac{A}{B} = \frac{r_1}{r_2} \angle(\phi_1 - \phi_2)$