Centroid / Center of Gravity

2.

* Gravity axis :-
- Line of action of the gravitational force acting on the body.

* Center of Gravity :- (g)
- The point through which the whole mass or weight of the body acts, irrespective of it's position is known as centre of gravity.

* Center of mass :-
- The point where the entire mass of a body is concentrated.

* Centroid :- (c)
- The plane figure's like triangle, quadrilateral, circle have only area's but no mass. the center of such figure's is known as centroid.
- centroid is always in coordinate's.
- centroid is defined as the point, at which whole centroid of line is defined as the point at which whole area may be assumed to be concentrated.

* Centroid of line :- (c) :-
- Centroid of line is defined as the point at which whole length of the line may be assumed to be concentrated.

- Location of centroid of a line :-

$$\bar{x} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3} \qquad \bar{y} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3}$$

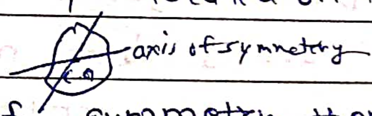
$$\bar{x} = \frac{\Sigma l x}{\Sigma l} \qquad \bar{y} = \frac{\Sigma l y}{\Sigma l}$$

∴ Coordinate's of centroid $= (\bar{x}, \bar{y})$.

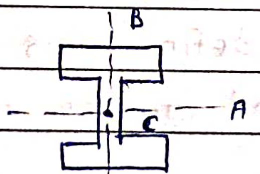
- As the figure is symmetric about x-axis, of course the x-axis is located at $y=0$.

- $\bar{x}$ is the distance of centroid from y-axis, and $\bar{y}$ is the distance from x-axis.
- Moment of line = length of line $\times$ $\perp$ distance of centroid.
- Unit of moment of line is m^2, $(\text{length})^2$.
- the centroid of line is not necessarily located on the line. If line is straigth it's centroid is at mid point of line

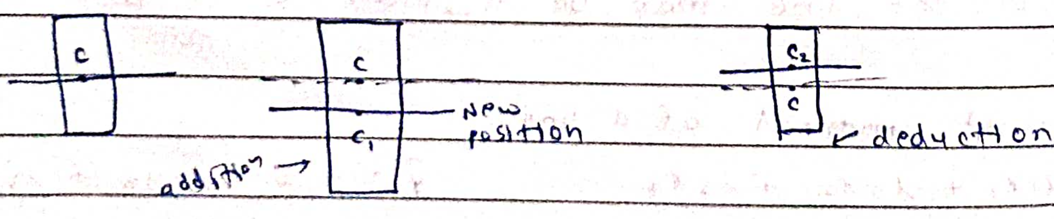
* Axis of Symmetry :-
- Axis of symmetry is defined as the line which divide's the body or figure in two equal part's, so that the moment's of these line's are equal and opposite.
- Centre of gravity or centroid is alway's located on the axis of symmetery.
- If the figure is symmetrical about x-axis then the distance of centroid from x-axis is ; $\bar{y}=0$ and vice versa.
- Centre of gravity or centroid is alway's located on the axis of symmetery.



- If the body possess two axis of symmetry then the centre of gravity or centroid must be located at intersection of two axes.



- Centre of gravity or centroid shift toward's the addition and shift's away from deduction.



- When a body is freely suspended from any point, the centre of gravity of the body is alway's located on the vertical line passing through point of suspension.

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- To locate position of centroid of Area :-
- For line centroid is :-

$$\bar{x} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3}$$

$$\bar{y} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3}.$$

- Co-ordinate's for centre of volume :-

$$\bar{x} = \frac{V_1 x_1 + V_2 x_2 + V_3 x_3}{V_1 + V_2 + V_3}$$

$$\bar{y} = \frac{V_1 y_1 + V_2 y_2 + V_3 y_3}{V_1 + V_2 + V_3}$$

$$\therefore \bar{x} = \frac{\Sigma Vx}{\Sigma V}$$

$$\therefore \bar{y} = \frac{\Sigma Vy}{\Sigma V}.$$

- Co-ordinate's of centroid of area are :-

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\therefore \bar{y} = \frac{\Sigma Ay}{\Sigma A}.$$

$$\therefore \bar{x} = \frac{\Sigma Ax}{\Sigma A}$$

- When,

$A = l \times b.$

$$\bar{y} = \frac{\Sigma lb \times y}{\Sigma lb}$$

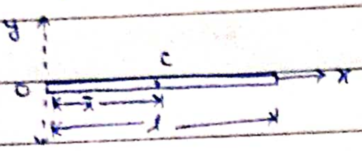
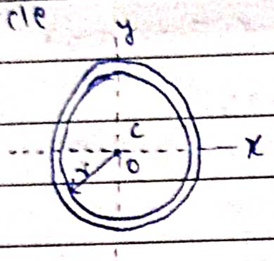
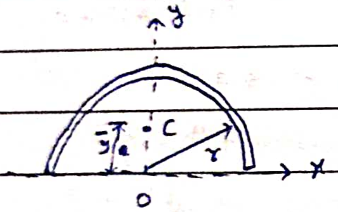
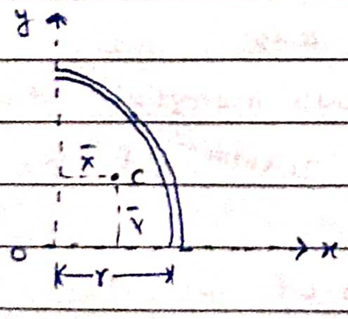
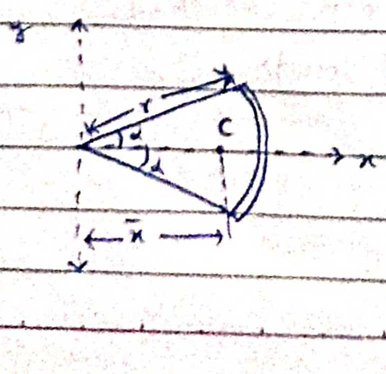
$$\bar{x} = \frac{\Sigma lb \times x}{\Sigma lb}$$

these above formulae's for homogeneous material it's centroid of line coincide's with centre of gravity of the wire.

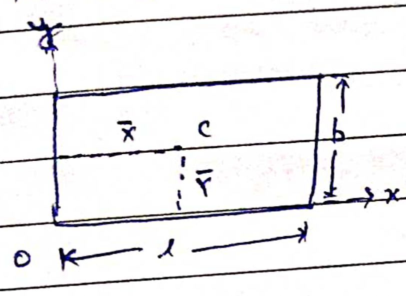
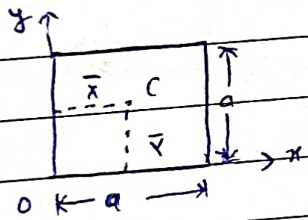
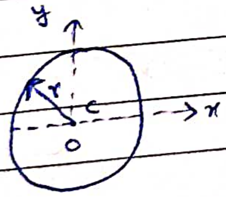
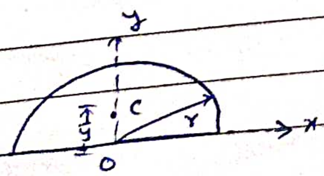
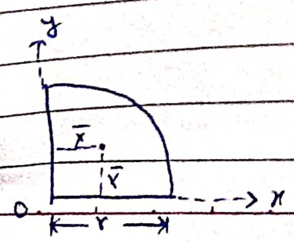
- To locate :-

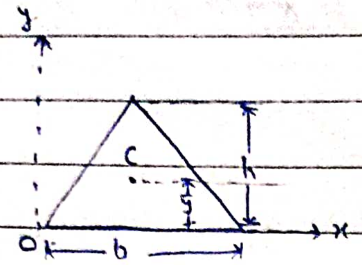
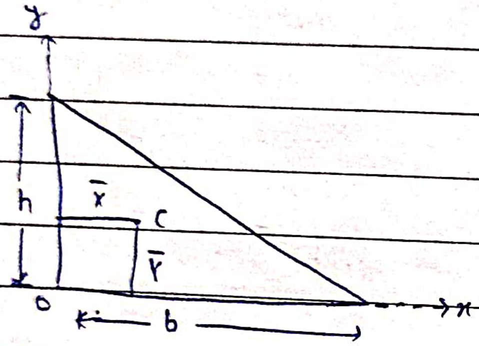
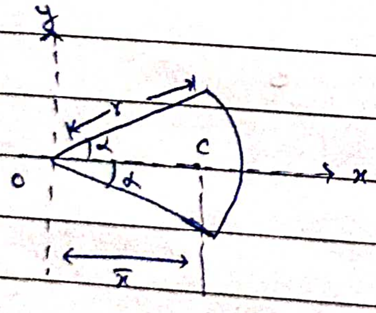
Centre of gravity :- take moment's of weight's.
Centre of Mass :- take moment's of Mass.
Centre of Volume :- take moment's of Area.
centre of area / lamina :- take moments of Area.
Centroid of area / lamina :- take moment's of length.
Centroid of line :- take moment's of length.

* Centroid of common Geometrical shape's of line :-

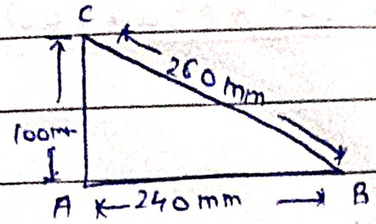
Sr. No.	Shape	length	$\bar{X}$	$\bar{Y}$
1)	Straight line 	l	$l/2$	0 (centroid is located x-axis).
2)	Circle 	$2\pi r$	0 (symmetrical).	0 (symmetrical).
3)	Semi-circular arc 	πr	0 (centroid is located at y axis).	$\frac{2r}{\pi}$.
4)	Quarter circular Arc. 	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$.
5)	Arc of a circle 			0

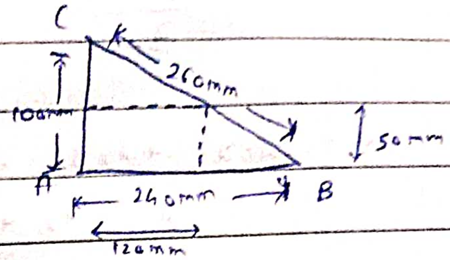
* Centroid of common geometrical shape's of Area's :-

Sr. No.	Shape	Area	$\bar{x}$	$\bar{y}$
1)	Rectangle. 	$l \times b$	$l/2$	$b/2$
2)	Square. 	l^2	$a/2$	$a/2$
3)	Circle. 	πr^2	0	0
4)	Semi-circle :- 	$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$
5)	Quarter circle:- 	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$

Shape	Area	$\bar{x}$	$\bar{Y}$
6) Triangle :- 	$\frac{1}{2} b.h$	0	$\frac{1}{3}h$
7] Right angled triangle :- 	$\frac{1}{2} b \times h$	$\frac{1}{3}b$	$\frac{1}{3}h$
8) Circular sector :- 			0

Q. A thin homogenous wire ABC is bent. determine the location of it's centroid with respect to A.





→ $\therefore l_1 = 240$ mm $x_1 = 120$ mm $y_1 = 0$

$\therefore l_2 = 260$ mm $x_2 = 120$ mm $y_2 = 50$ mm

$\therefore l_3 = 100$ mm $x_3 = 0$ $y_3 = 50$ mm

$$\therefore \bar{x} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3}$$

$$= \frac{(240 \times 120) + (260) \times (120) + (100 \times 0)}{240 + 260 + 100}$$

$\therefore \bar{x} = 100$ mm.

$$\therefore \bar{y} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3}$$

$$= \frac{(240 \times 0) + (260 \times 50) + (100 \times 50)}{240 + 260 + 100}$$

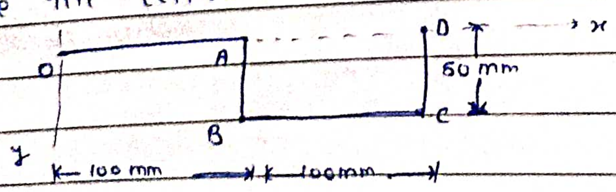
$\therefore \bar{y} = 30$ mm.

$\therefore$ location of centroid w.r.t point A is

→ $(\bar{x}, \bar{y}) = (100, 30)$.

Q. A thin rod is bend into a shape of OABCD as shown. Determine the centroid of the bend rod w.r.t origin O.

→



$\therefore$ OA	$l_{OA} = 100$ mm	$x_{OA} = 50$ mm	$y_{OA} = 0$ mm
AB	$l_{AB} = 50$ mm	$x_{AB} = 100$ mm.	$y_{AB} = -25$ mm
BC	$l_{BC} = 100$ mm	$x_{BC} = 150$ mm.	$y_{BC} = -50$ mm.
CD	$l_{CD} = 50$ mm	$x_{CD} = 200$ mm.	$y_{CD} = -25$ mm.

$$\therefore \bar{x} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3 + l_4 x_4}{l_1 + l_2 + l_3 + l_4}$$

$$= \frac{(100 \times 50) + (50 \times 100) + (100 \times 150) + (50 \times 200)}{100 + 50 + 100 + 50}$$

$$= \frac{35000}{300} = 116.67 \text{ mm}$$

$$\therefore \bar{y} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3 + l_4 y_4}{l_1 + l_2 + l_3 + l_4}$$

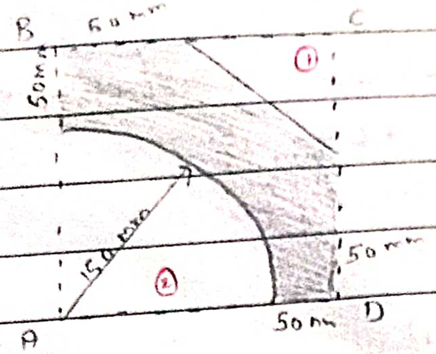
$$= \frac{(100 \times 0) + (50 \times -25) + (100 \times -50) + (50 \times -25)}{300}$$

$$= \frac{-7500}{300} = -25 \text{ mm}$$

$$\therefore (\bar{x}, \bar{y}) = (116.67, -25).$$

Q. Determine the

Q. Analyze and locate the position of centroid for the plane lamina

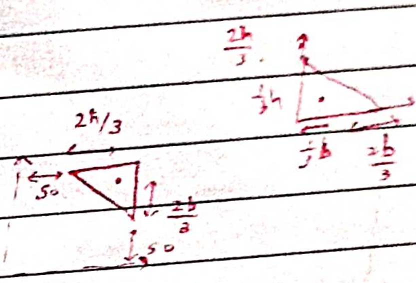


→ ① Area of square:-

$A_1 = (200)^2 = 40000\ mm^2.$

$x_1 = 100mm$

$y_1 = 100mm$



② Area of ① triangle:-

$A_2 = \frac{1}{2} \times 150 \times 150 = 11,250\ mm^2.$

$x_2 = 50 + \frac{2 \times h}{3} = 50 + \frac{2 \times 150}{3} = 150\ mm$

$y_2 = 50 + \frac{2b}{3} = 50 + \frac{2 \times 150}{3} = 150\ mm.$

③ Area of ② Quarter circle:-

$A_3 = \frac{\pi R^2}{4} = \frac{\pi \times (150)^2}{4} = 17671.45\ mm^2.$

$x_3 = \frac{4R}{3\pi} = \frac{4 \times 150}{3 \times \pi} = 63.66\ mm$

$y_3 = \frac{4R}{3\pi} = 63.66\ mm$

∴ centroid of shaded region:-

$\therefore \bar{x}_4 = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3}{A_1 - A_2 - A_3}$

$= \frac{(40000 \times 100) - (11,250 \times 150) - (17671.45 \times 63.66)}{40000 - 11,250 - 17671.45}$

$\therefore \bar{x}_4 = 107.19\ mm.$

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$$\therefore \bar{Y}_4 = \frac{A_1Y_1 - A_2Y_2 - A_3Y_3}{A_1 - A_2 - A_3}$$

$$= \frac{(40000 \times 100) - (11,250 \times 150) - (17671.45 \times 63.66)}{40000 - 11,250 - 17671.45}$$

$$\therefore \bar{Y}_4 = 107.19$$

$\therefore$ Centroid $(107.19, 107.19)$.

Moment of Inertia.

- Moment of force about any point or magnitude of force (f) and the perpendicular distance (d) called as Moment of force. (first)

$\therefore M_o = f.d.$

If (f.d) is again multiplied by distance d then it is known as second moment of force. $(f.d^2)$. also called as "Moment of inertia" of the force.

Denoted by (I).

Force is replaced by area or mass of the body that second moment is also known as moment of inertia or mass moment of inertia.

– Types of moment of inertia :-

i) Area moment of inertia :-

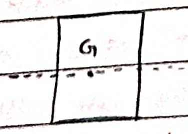
First moment $= m^2 \times m = m^3$.

Second moment $= m^3 \times m = m^4$.

ii) Mass moment of inertia:-

first moment $=$ kg m^2.

Second moment $=$ kg m^3.



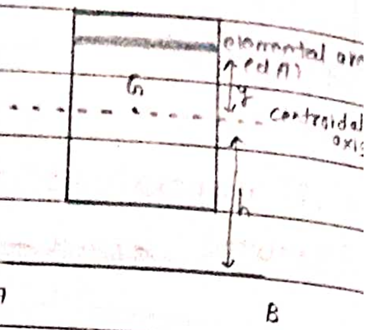
– Neutral Axis:-

- The line passing through the centroid or center of gravity of that section is known as "centroidal axis" or "Neutral Axis".

☆ Parallel Axis theorm:-

- It states that the M.I of the plane area (A) abo[ut] any axis AB which is parallel to the centroidal axis located at distance 'h'

Proof : Second moment of area of an elemental strip axis AB.



$\therefore dI_{AB} = (y+h)^2 \times dA$

for whole area :-

$\therefore \int dI_{AB} = \int (y+h)^2 \, dA$

$\therefore I_{AB} = \int y^2 dA + 2h\int y \, dA + h^2\int dA$

$\therefore I_{AB} = I_G + 0 + h^2.A$

$\boxed{\therefore I_{AB} = I_G + Ah^2.}$ (Prooved).

as $\int y^2.dA = I_G$ (M.I about centroidal axis I_G).

$2h\int y.dA = 2h \times 0 = 0.$

where,

I_{AB} = M.I of an area about the line AB. ∥ to centroidal axis

I_G = M.I of an area about centroidal axis.

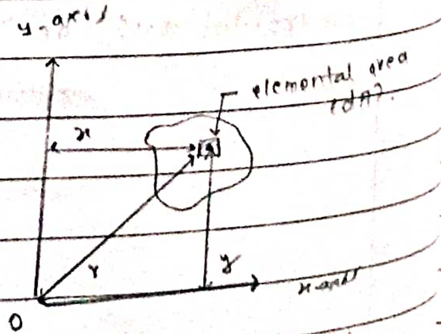
A = Total area of the figure.

h = Distance betn centroidal axis and parallel axis, AB.

☆ Perpendicular Axis theorm:-

- It state's that M.I of the plane area about an axis is perpendicular to the xy plane i.e z axis is equal to the sum of M.I of the plane area about x and y axes.

Proof:- • Let 'r' be the distance of elemental area and ⊥ to z axis.



$\therefore dI_{zz} = r^2.dA.$

for whole area. ⇒ as by resultant $\therefore r^2 = x^2 + y^2.$

$\therefore \int dI_{zz} = \int r^2.dA.$

$\therefore I_{zz} = \int (x^2+y^2).dA.$

$\therefore I_{zz} = \int x^2.dA + \int y^2.dA.$

$\boxed{\therefore I_{zz} = I_{xx} + I_{yy}.}$

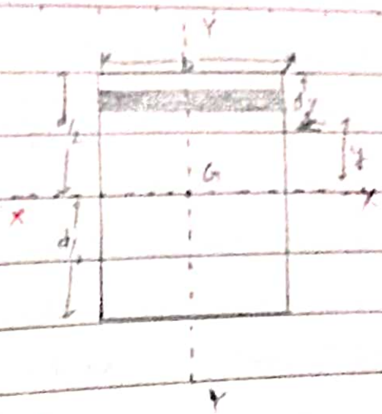
I_{xx} = M.I about y-axis.

I_{yy} = M.I about x-axis

I_{zz} = M.I about z-axis.

* M.I of standard shape's (Areas) :-

1) Rectangular area.



A) About centroidal x-axis :-

$\therefore dI_{xx} = y^2 . dA$

as $dA = b \times dy$ of elemental area.

$\therefore dI_{xx} = y^2 . b\,dy$

Total M.I about x-axis :-

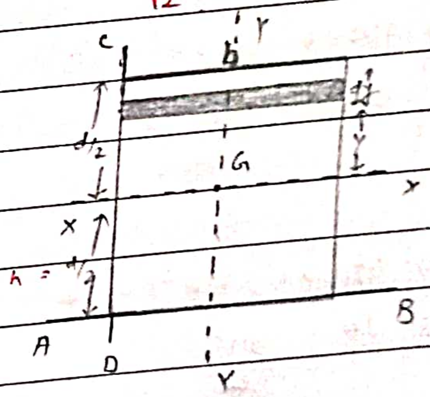
$$\therefore I_{xx} = b\int_{-d/2}^{d/2} y^2 dy$$

$$= b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b\left[\frac{d^3}{24} + \frac{d^3}{24}\right]$$

$$\therefore I_{xx} = \frac{bd^3}{12}.$$

$\Rightarrow I_{xx} = \frac{bd^3}{12}$; $I_{yy} = \frac{b^3 d}{12}$



B) About base of the Rectangle :-

As AB ∥ XX centroidal axis

$\therefore$ by parallel axis theorm :-

$$\therefore I_{AB} = I_G + Ah^2$$

$$= \frac{bd^3}{12} + bd\left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4}$$

$$\therefore I_{AB} = \frac{bd^3}{3}.$$

$\Rightarrow I_{AB} = \frac{bd^3}{3}$; $I_{CD} = \frac{b^3 d}{3}$.

2) For Hollow Rectangular area :-

$$I_{xx} = \left(\frac{BD^3}{12} - \frac{bd^3}{12}\right)$$

$$I_{yy} = \left(\frac{B^3 D}{12} - \frac{b^3 d}{12}\right).$$

3) Square :-

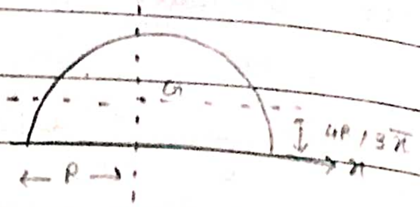
$$I_{xx} = I_{yy} = \frac{a^4}{12}.$$

• $I_{xx} = I_{yy} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$.

$\frac{\pi r^4}{4}$ $\frac{\pi D^4}{64}$

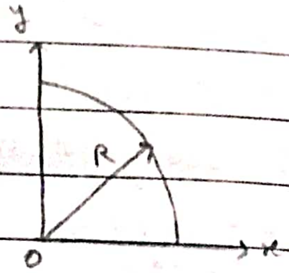
4] Semi-circle :-

• $I_{xx} = I_{yy} = \frac{\pi R^4}{8}$.



5] Quarter circle :-

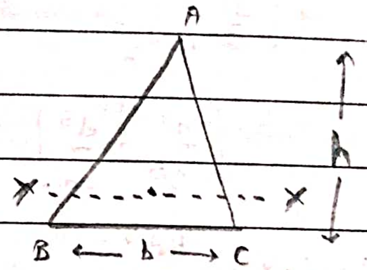
• $I_{xx} = I_{yy} = \frac{\pi R^4}{16}$



6] Triangle :-

a) About centroidal axis :-

• $I_{xx} = \frac{bh^3}{36}$.



b) About base 'BC'

• $I_{BC} = \frac{bh^3}{12}$.

c) About vertex 'A'

$I_A = \frac{bh^3}{4}$.

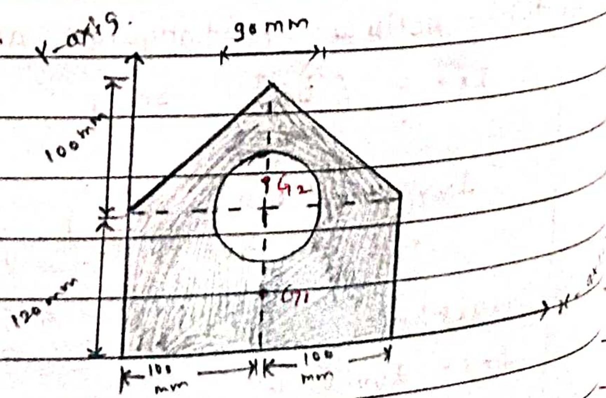
Q. Define Moment of inertia, Determine the M.I of the composite shape as shown below w.r.t to x-axis.

→ Dividing figure into 3 parts.

(1) Rectangle : 200 X 120 mm.

(2) Triangle :- 200 X 100 mm

(3) Circle - 90 mm = Diameter.



• Always add moment of inertia of shaded region and then substract unshaded area (M.O.I).

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∴ Moment of inertia about x-axis

$I_{xx} = I_{xx_1} + I_{xx_2} - I_{xx_3}$.

$I_{xx_1} = \frac{bd^3}{12}$ (Rect)

$\therefore I_{xx_1} = I_{G_1} + A_1 h_1^2$

$= \frac{200 \times (120)^3}{12} + (120 \times 200) \times (60)^2$

$= 11.52 \times 10^7 \ mm^4$.

$I_{xx_2} = \frac{bh^3}{36}$.

$\therefore I_{xx_2} = I_{G_2} + A_2 h_2^2$

$= \frac{200 \times (100)^3}{36} + \left(\frac{1}{2} \times 200 \times 100\right) \times \left(\frac{1}{3} \times (100) + 120\right)^2$.

$= 240.65 \times 10^6 \ mm^4$.

$I_{G_3} = \frac{\pi D^4}{64}$.

$\therefore I_{xx_3} = I_{G_3} + A_3 h_3^2$

$= \frac{\pi \times (90)^4}{64} + \pi \times (45)^2 \times (120)^2$

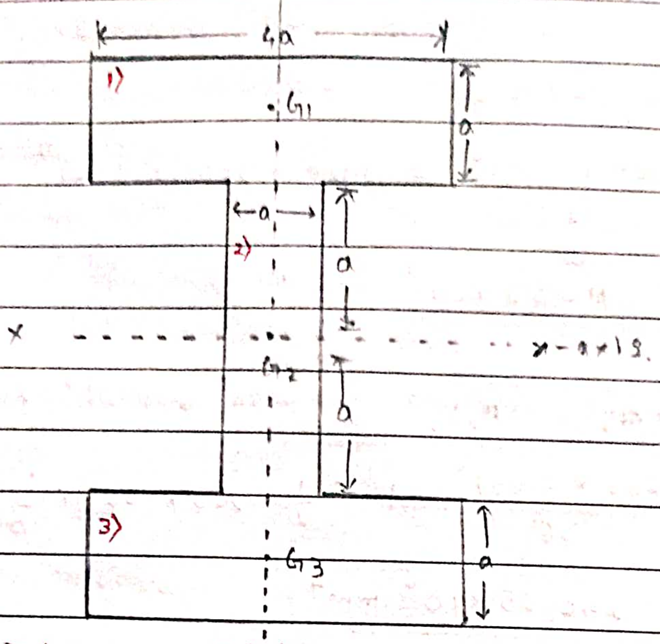
$= 9.48 \times 10^7 \ mm^4$.

$\therefore I_{xx} = 11.52 \times 10^7 + 24.065 \times 10^7 - 9.48 \times 10^7$

$= 26.105 \times 10^7$.

- for Y-axis M.I along Y-axis centroidal pt lie on it so ~~distance~~ distance betn centroidal pt & axis is zero $\boxed{h=0}$.
  $\therefore I_{YY} = I_G + A_1 k_1^2 = \underline{I_G}$.

Q. Define moment of inertia and determine the M.I of the composite figure if $a = 40$ mm w.r.t x-axis and y-axis.



→ Dividing into 3 parts. (Y'axis)

1) Rectangle : 160×40 mm.
2) Rectangle : 40×80 mm.
3) Rectangle : 160×40 mm.

$\therefore$ M.I (x-axis) $= I_{xx_1} + I_{xx_2} + I_{xx_3}$.

=

$\therefore I_{xx_1} = I_{G_1} + A_1 h_1^2$. $\qquad I_{G_1} = \frac{bd^3}{12}$,

$= \frac{160 \times (40)^3}{12} + (160 \times 40)(40 + 20)^2$

$= 2.38 \times 10^7 \text{ mm}^4$.

$I_{YY1} = I_{G2} + 0 = \frac{b^3 d}{12} = \frac{(160)^3 \times (40)}{12} = 1.36 \times 10^7$

$\therefore I_{xx2} = I_{G_2} + A_2 h_2^2$. $\qquad I_{G_2} = \frac{bd^3}{12}$. $\qquad \underline{\underline{h_2 = 0}}$

$= \frac{40 \times (80)^3}{12} \cancel{+ (40 \times 80) \times (0)}$.

$= 1.70 \times 10^6 \text{ mm}^4$.

$I_{YY2} = I_{G2} + 0 = \frac{(40)^3 \times 80}{12} = 4.26 \times 10^5$

$\therefore I_{xx3} = I_{G3} + A_3 h_3^2$.

$= \frac{160 \times (40)^3}{12} + (160 \times 40) \times (40 + 20)^2$

$= 2.38 \times 10^7 \text{ mm}^4$.

$I_{YY3} = I_{G_3} + 0 = (160)^3\ 40$

$\therefore I_{xx} = 2.38 \times 10^7 + 1.70 \times 10^6 + 2.38 \times 10^7$

$= 4.93 \times 10^7 \, mm^4$.

$\therefore I_{yy} = 1.36 \times 10^7 + 4.26 \times 10^5 + 1.36 \times 10^7$

$= 2.76 \times 10^7 \, mm^4$.

unit-2.

7 Friction.

* Friction:-

- Force that resists the sliding or rolling of one solid object over another then that force is called as "Friction force".
- Friction force will act in the direction opposite to the direction of motion.
- When there is relative motion between the two surfaces, each surface exerts friction force on the other surface.

- Types of Friction:-

1] Dry friction. ⟶ a) static
b) kinetic. ⟶ i) sliding,
ii) Rolling.

2] Fluid friction. (x)

1] Dry friction:-

- It is the friction between two dry surfaces.
- Dry friction exists because of the surface irregularities between the two bodies.

a] Static friction :-

- Friction experienced by a body when it is at rest condition or in equilibrium.

b] Kinetic friction:-

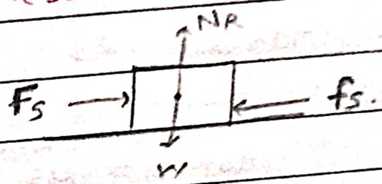
- Friction experienced by a body when it moves.
- It is the friction experienced by the surface of the body when the body is :-

Sliding over the surface (sliding friction).
Rolling over the surface (rolling friction).

* Coulomb's law of friction :-

A) For static friction :- (body is about to move instead of rest)

- Frictional force always acts in the opposite direction of motion.
- Friction force is directly proportional to Normal Reaction.
- Frictional force doesn't depends on the surface area of contact.
- Frictional force depends upon Roughness of surface.



$f_s \propto N_R$.

$\boxed{f_s = \mu_s N_R .}$

where,

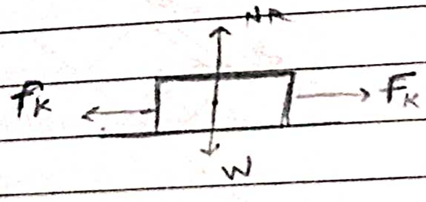
f_s = static friction.

N_R = Normal Reaction.

μ_s = proportionality constant.

B) For kinetic friction :-

- Frictional force always acts in opposite direction in which body moves.
- Frictional force is directly proportional to Normal reaction.
- Frictional force decreases with increasing speed.
- For higher speed, frictional force and vice versa.



$\therefore f_K \propto N_R$

$\boxed{\therefore F_K = \mu_K N_R}$

where,

F_K = force external body in motion.

f_K = kinetic friction

μ_K = proportionality constant.

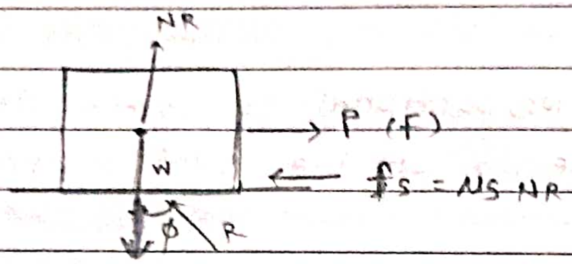
- μ_K is always less than μ_s.
  ($\mu_s > \mu_K$)

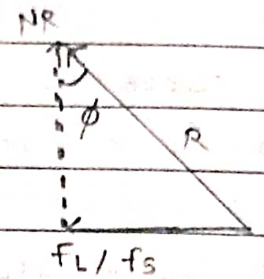
- When the motion is about to start $f_s = \mu_s N_R$. and when the motion starts $f_K = \mu_K N_R$

- The condition in which body is in impending motion is kno[wn] as "Limiting equilibrium".
- Impending Motion => About to move (static).

* Angle of friction :- (ϕ).

- When the body is in limiting equilibrium, the maximum angle between the Resulting reaction and the normal reaction is called as "angle of friction".





$$\therefore \tan\phi = \frac{f_s}{N_R} = \frac{\mu_s \cancel{N_R}}{\cancel{N_R}}$$

$$\boxed{\therefore \tan\phi = \mu_s.}$$

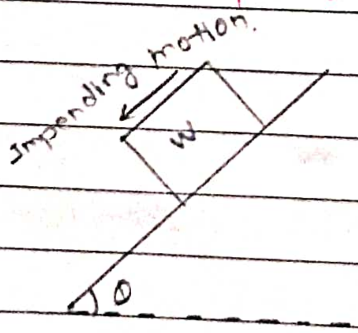
- tangent of angle of friction = coefficient of friction.

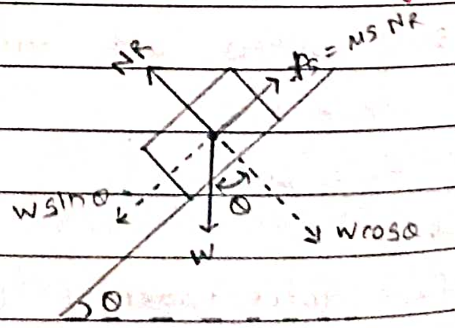
$$\therefore R^2 = F_s^2 + N_R^2$$
$$= \mu_s^2 N_R^2 + N_R^2$$
$$= N_R^2 (\mu_s^2 + 1)$$

$$\boxed{\therefore R = N_R \sqrt{\mu_s^2 + 1}.}$$

* Angle of Response :- (θ).

- It is the minimum angle of inclination of plane at which the block impend's it's motion under it's own weight.





$\therefore$ as along y-axis it is at equilibrium.

$\therefore \Sigma f_y = 0$

$\therefore N_R = W\cos\theta \quad - (1)$

as it is static (impending motion not move so equilibrium x-axis.

$\therefore \Sigma f_x = 0$

$\therefore W\sin\theta = f_s = \mu_s N_R$

$\therefore W\sin\theta = \mu_s N_R$ — ②

on dividing eq^n ② and ①;

$$\therefore \frac{W\sin\theta}{W\cos\theta} = \frac{\mu_s N_R}{N_R}$$

$$\boxed{\therefore \tan\theta = \mu_s}$$

As, $\mu_s = \tan\phi$.

$\mu_s = \tan\theta$

$\therefore \tan\phi = \tan\theta$

$$\boxed{\Rightarrow \phi = \theta.}$$

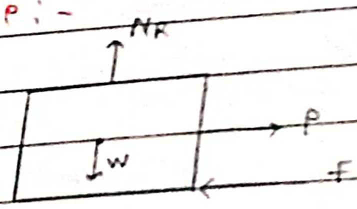
Hence, Angle of friction is equal to angle of Response.

* Applications of friction :-

Inclined planes / Horizontal planes :-

1) Inclined planes / Horizontal surface plane :-

(A) Block on rough horizontal surface plane :-



P = Applied force.

F = friction force.

N_R = Normal reaction.

W = Weight of block.

In static equilibrium :-

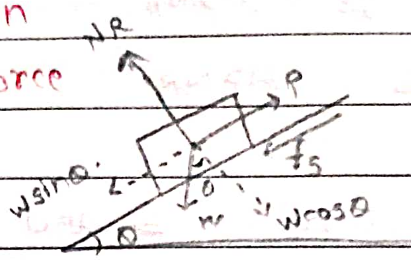
$\Sigma f_x = 0$ $\therefore F = P$.

$\Sigma f_y = 0$ $W = N_R$.

If Block is on the verge of motion (Impending motion) again same conditions are apply.

(B) Block on rough inclined plane :-

- If 'p' tends to induce motion upwards, then the friction force act's down the plane.



$\therefore \Sigma f_y = 0.$

$N_R = w\cos\theta.$

$\therefore \Sigma f_x = 0.$

$w\sin\theta + f_s = P$

If block is in impending motion :-

$\therefore f_s = P - w\sin\theta.$

$\therefore \mu_s N_R = P - w\sin\theta.$

$\therefore \cancel{P=} \mu_s(w\cos\theta) + w\sin\theta = P$

$\therefore P = w\,(\mu_s\cos\theta + \sin\theta).$