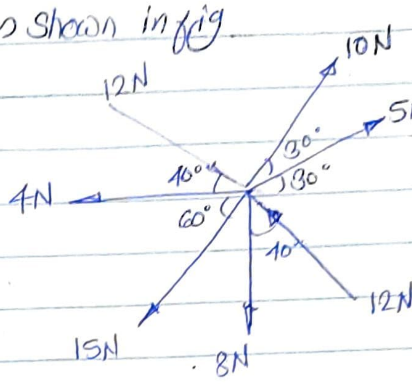
1st Resolution.

Examples Based on Resultant of Coplaner Concurrent forces:-

(1) Determine resultant of following force system as shown in fig.



(1) $\Sigma Fx = 10\cos 60 + 5\cos 30 - 12\cos 50 - 15\cos 60 - 4 + 12\cos 40$

$= -0.6907\ N$

$\Sigma Fy = 10\sin 60 + 5\sin 30 + 12\sin 50 - 8 - 15\sin 60 - 12\sin 40$

$= -8.3510\ N$

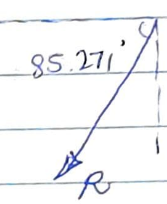
$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2} = \sqrt{(-0.6907)^2 + (-8.3510)^2}$

$R = 8.379\ KN$

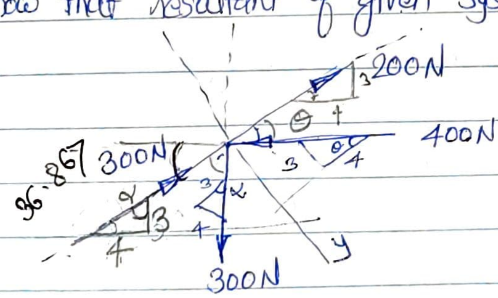
$\tan\theta = \dfrac{\Sigma Fy}{\Sigma Fx} = \dfrac{-8.3510}{-0.6907} = 12.09$

$\therefore \theta = 85.271°$

R is acting in 2nd quadrant.



(2) Show that resultant of given system as shown in figure is zero.



$\tan\theta = \dfrac{3}{4} = 0.75$ $\theta = 36.869$

$\tan\alpha = \dfrac{4}{3} = 1.33$

$\alpha = 53.133$

$\Sigma Fx = 200\cos 36.869 - 400 + 300\cos ~~53.133~~$ 36.867

$= ~~-60.01~~\ 0$

$\Sigma Fy = 200\sin 36.869 + 300\sin ~~53.133~~ - 300$ 36.867

$= ~~60~~\ 0$

$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$

$= \sqrt{(-60.01)^2 + (60)^2}$

$= 0$

Along x & y direction as shown

$Fx = 200 + 300 - 400\cos 36.869 - 300\cos 53.133$

$= 0$

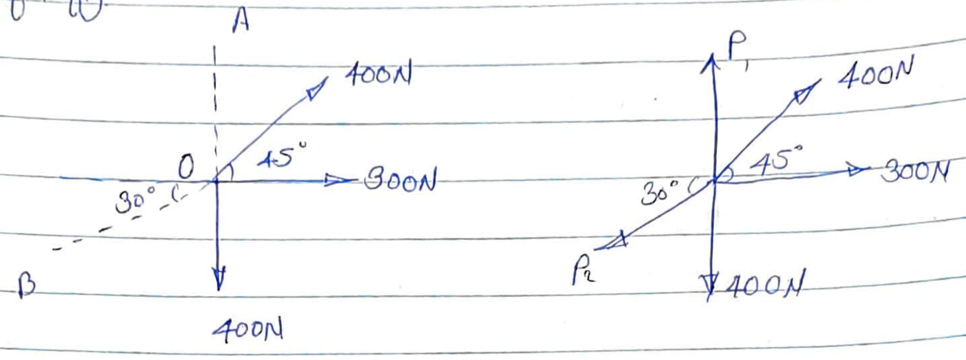
$\Sigma Fy = 400\sin 36.869 - 300\sin 53.133$

$= 0$

$\therefore R = \sqrt{\Sigma Fx^2 + \Sigma Fy}$

$R = 0$ ... Proved.

③ Three concurrent, coplanar forces act on a body at point O. Determine two additional forces, along OA & OB such that the resultant of the five forces is zero. Refer fig.



Let P_1 & P_2 be the additional forces acting along OA & OB respt.

As resultant is zero.

$\Sigma f_x = 0$ & $\Sigma f_y = 0$

$\therefore \quad \Sigma f_x = 400\cos 45 + 300 - P_2 \cos 30 = 0$

$\therefore P_2 = 673N$

$\Sigma f_y = P_1 + 400\sin 45 - 400 - P_2 \sin 30$

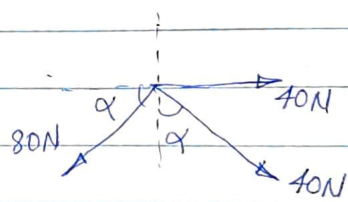
$0 = P_1 + 400\sin 45 - 400 - 673\sin 30$

$P_1 = 453.657\ N$

④ Determine ⓐ angle α for which resultant of three forces is vertical.

ⓑ Also find corresponding magnitude of R.

Refer fig.



Here, R is vertical but direction is not given, so assume that R is acting downward.

$\Sigma f_x = 0$ & $\Sigma f_y = -R$

① $\Sigma f_x = 0$

$40 + 40\cos(90-\alpha) - 80\cos\alpha = 0$

$40 - 80\cos\alpha + 40\sin\alpha = 0$

$\therefore 1 + \sin\alpha = 2\cos\alpha$

$1 + 2\sin\alpha + \sin^2\alpha = 4\cos^2\alpha$

$1 + 2\sin\alpha + \sin^2\alpha = 4(1 - \sin^2\alpha)$

$5\sin^2\alpha + 2\sin\alpha - 3 = 0$

$\sin\alpha = 0.6$ or -1

$\alpha = 36.869°$ or $\alpha = -90°$

Now using $\Sigma F_y = -R$

$-80 \sin\alpha - 40 \sin(90-\alpha) = -R$

$-80 \sin 36.87 - 40 \sin(90 - 36.87) = -R$

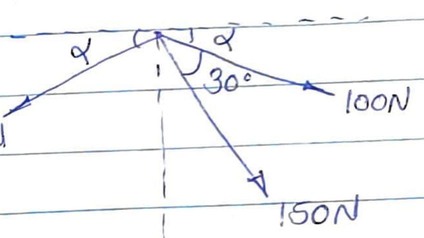
$-80 = -R$

$\therefore R = 80N \downarrow$

+ve answer means assumption for direction is correct.

⑤ For given system determine ⓐ the required value of α if resultant of three ③ forces is to be vertical ⓑ The corresponding value of resultant.

Assume that R is acting downward.



Conditions

① $\Sigma F_x = 0$

$100 \cos \cancel{30}\alpha + 150 \cos(30+\alpha) - 200 \cos\alpha = 0$

$100 \cos\alpha = 150 \cos(\alpha+30)$

$\cos\alpha = 1.5 [\cos\alpha . \cos 30 - \sin\alpha . \sin 30]$

$\cos\alpha = 1.3 \cos\alpha - 0.75 \sin\alpha$

~~tan α =~~ $0.75 \sin\alpha = 0.3 \cos\alpha$

$\tan\alpha = \frac{0.3}{0.75}$

$\alpha = 21.8°$

② $\Sigma F_y = -R$

$-200 \sin\alpha - 150 \sin(\alpha+30) - 100 \sin\alpha = -R$

$-200 \sin 21.8 - 150 \sin(21.8+30) - 100 \sin 21.8 = -R$

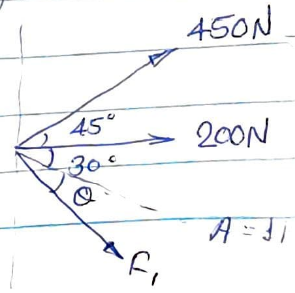
$\therefore R = 229.288 \text{ N} \downarrow$

★★ Possitive answer means assumption is correct.

⑥ Three forces act as shown in fig. Determine magnitude & ~~ma~~ direction θ of F_1 so that resultant is directed along axis A & has magnitude of 1 kN

④ As resultant of 1000N is acting along A axis

We know that when R is inclined.



① x component of $R = \Sigma F_x$
② y component of $R = \Sigma F_y$

① $1000 \cos 30 = 450 \cos 45 + 200 + F_1 \cos(\theta + 30)$

$\therefore F_1 \cos(\theta + 30) = 347.83$ —— ⓐ

② $-1000 \sin 30 = 450 \sin 45 - F_1 \sin(\theta + 30°)$

$\therefore F_1 \sin(\theta + 30) = 818.198$ —— ⓑ

from ⓐ & ⓑ

$$\frac{F_1 \sin(\theta + 30)}{F_1 \cos(\theta + 30)} = \frac{818.198}{347.830}$$

$$\tan(\theta + 30) = 2.352$$

$$\theta + 30 = 66.968$$

$$\theta = 36.968°$$

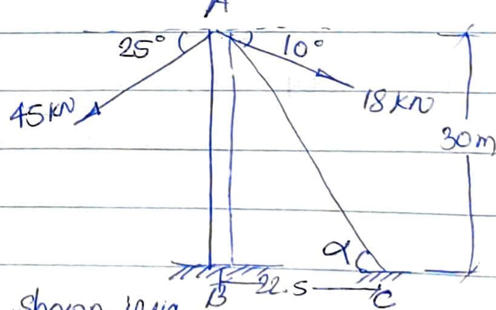
Substituting in eqⁿ ⓐ we get

$$F_1 \cos(\theta + 30) = 347.83$$

$$F_1 = \frac{347.83}{\cos(36.968 + 30)} =$$

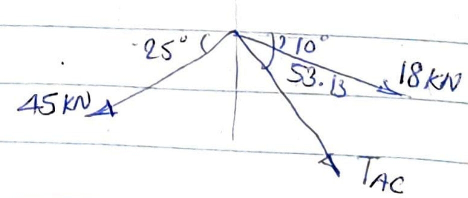
$$F_1 = 889.033\ N$$

⑦ Two cables are attached at top of a vertical pole. Determine tension in guy cable AC if resultant of three forces at A is vertical



The three forces acting at point A are shown in fig.
Let α be the angle of AC with horizontal.

$$\therefore \tan\alpha = \frac{30}{22.5}$$



$$\alpha = 53.130°$$

Assume R to be acting downwards

Conditions: $\Sigma F_x = 0$, $\Sigma F_y = -R$

① Using $\Sigma F_x = 0$

$$18\cos 10 + T_{AC}\cos 53.13 - 45\cos 25 = 0$$

$$\therefore T_{AC} = 38.428 \text{ kN}$$

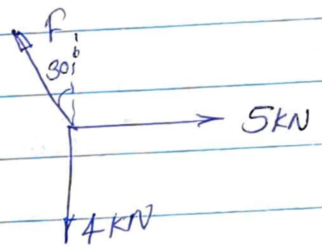
② $\Sigma F_y = -R$

$$-18\sin 10 - T_{AC}\sin 53.130 - 45\sin 25 = \cancel{0} - R$$

$$22.143 + 38.428\sin 53.130 = R$$

$$R = 52.885 \text{ kN} \downarrow$$

⑧ Determine the magnitude of force F so that the magnitude of resultant of three forces is as small as possible. Also find minimum magnitude of R.



$$\Sigma F_x = 5 - F\cos 60$$
$$= 5 - F/2$$

$$\Sigma F_y = F\sin 60 - 4$$
$$= \frac{\sqrt{3}}{2}F - 4$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$R^2 = \left(5 - \frac{F}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}F - 4\right)^2 \quad \text{———— ①}$$

Differentiate w.r.to. F.

$$2R\frac{dR}{dF} = 2\left(5 - \frac{F}{2}\right)\left(\frac{-1}{2}\right) + 2\left[\frac{\sqrt{3}}{2}F - 4\right]\cdot\frac{\sqrt{3}}{2}$$

For R to be as small as possible $\frac{dR}{dF} = 0$

$$0 = -5 + \frac{F}{2} + \cancel{2}\left(\frac{3F}{2} - 4\sqrt{3}\right)$$

$$\therefore 2F = 5 + 4\sqrt{3}$$

$$\therefore F = \frac{5 + 4\sqrt{3}}{2} = 5.96 \text{ kN}$$

Substituting this value in eqⁿ ① we get

$$\therefore R_{(min)} = \sqrt{\left(5 - \frac{5.96}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\times 5.96 - 4\right)^2}$$

$$R_{(min)} = 2.330 \text{ kN}$$

⑨ A member of a crane is attached by 3 cables. If $T_{AC} = 8\ kN$, $T_{AD} = 10.4\ kN$. Determine tension in cable A_E if resultant of three tensions at A must be directed along AB. Also find corresponding resultant.

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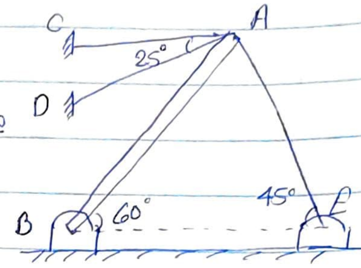
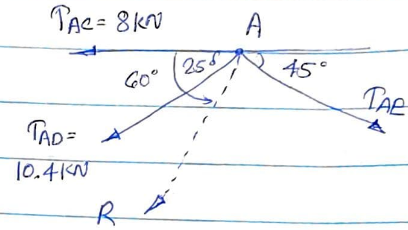


The three forces acting at point A are T_{AC}, T_{AD} & T_{AE}. Assume direction of resultant from A to B.

As R is inclined the conditions are.

① x component of $R = \Sigma F_x$



$$-R\cos 60 = -10.4\cos 25 - 8 + T_{AE}\cdot\cos 45.$$

$$0.5R + 0.707\,T_{AE} = 17.425 \quad \text{— ①}$$

Now using y component of $R = \Sigma F_y$

$$-R\sin 60 = -10.4\sin 25 - T_{AE}\sin 45$$

$$0.87R - 0.707\,T_{AE} = 4.395 \quad \text{— ②}$$

Solving ① & ②

we get $R = 15.927\ kN$

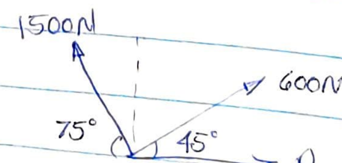
$T_{AE} = 13.382\ kN$

Here answer of R is passitive so assume direction is correct.

⑩ ✗ Three forces acting at a point. Determine the range of values of magnitude of P so that the magnitude of resultant does not exceed 2500N. Force P always directed to the right.

:→

$R = 2500N$



$\Sigma F_x = P + 600\cos 45 - 1500\cos 75$

$= P + 36.03$

$\Sigma F_y = 600\sin 45 + 1500\sin 75$

$= 1873.15\ N$

Now $R^2 = (\Sigma fx)^2 + (\Sigma fy)^2$

$(2500)^2 = (P+36.03)^2 + (1873.15)^2$

$2741.309 \times 10^3 = (P+36.03)^2$

$P + 36.03 = \pm 1655.689$

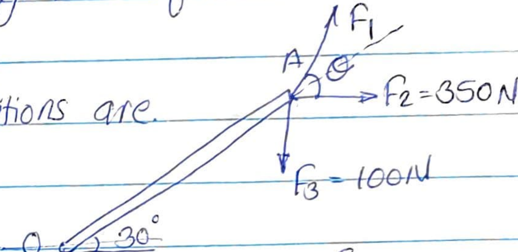
$\therefore P = 1,655.689 - 36.03$

$P = 1619.659$ N

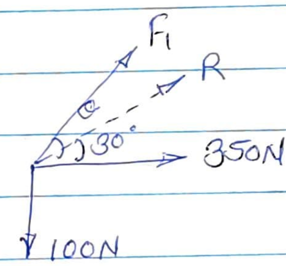
$P \cong 1.62$ kN

$\therefore$ Range of magnitude of P will be $0 \le P \le 1.62$ kN.

(11) If resultant of 3 forces is acting along the arm OA (from O to A). Determine force f_1 & its direction θ. The magnitude of resultant is 600N.



Here resultant is acting from O to A, so conditions are.



$\therefore$ x component of $R = \Sigma fx$

$R\cos 30 = f_1 \cos\theta + 350$ —— (1)

y component of $R = \Sigma fy$

$R \sin 30 = f_1 \sin\theta - 100$ —— (2)

Here $R = 600N$ given

$\therefore 600 \cos 30 = 350 + f_1 \cos\theta$

$f_1 \cos\theta = 169.615 N \rightarrow$

& $600 \sin 30 = f_1 \sin\theta - 100$

$f_1 \sin\theta = 400 N \uparrow$

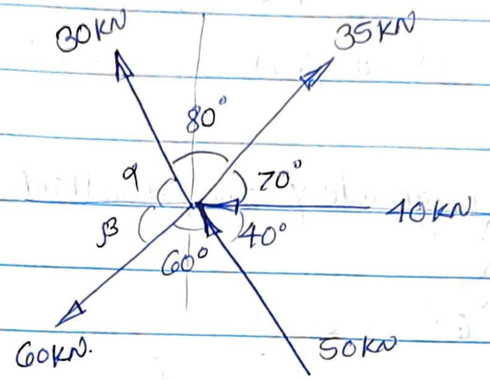
$$\frac{f_1 \sin\theta}{f_1 \cos\theta} = \frac{400}{169.615}$$

$\tan\theta = 2.358$

$\theta = 67.021°$

$$\therefore f_1 = \frac{400}{\sin 67.021} = 434.476N$$

Find the equilibriant of the force system given below.



$\alpha = 180 - 80 - 70 = 30^\circ$

$\beta = 180 - 60 - 40 = 80^\circ$

$\Sigma F_x = 35 \cos 70 - 40 - 50 \cos 40 - 60 \cos 80 - 30 \cos 30$

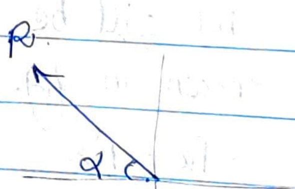
$= -102.731 \text{ kN.}$

$\Sigma F_y = 35 \sin 70 + 50 \sin 40 - 60 \sin 80 + 30 \sin 30$

$= 20.940 \text{ kN.}$

$R = \sqrt{(-102.731)^2 + (20.940)^2}$

$= 104.843^\circ$



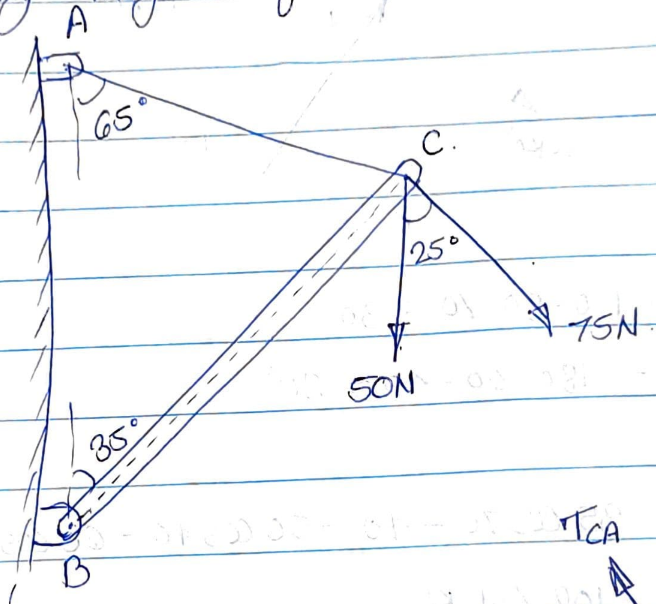
$\alpha = \tan^{-1} \dfrac{20.940}{-102.731}$

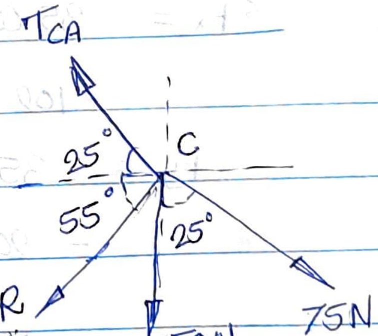
$= 11.52^\circ$

A Determine

i) The required tension in cable AC, known that the resultant of three forces exerted at point C of boom BC must be directed along BC.

(ii) The corresponding magnitude of the resultant.





The forces acting at C will be 50N, 75N, T_{CA} directed from C to A & the resultant of these forces R along BC which will be directed from C to B as shown in fig.

$\Sigma F_x = R_x$

$R_x = F_x$

$-R\cos 55 = 75\cos 65 - T_{CA}\cos 25$

$-R\cos 55 + T_{CA}\cos 25 = \cancel{43.018}\ 31.696$ —— 1

$R_y = F_y$

$-R\sin 55 = T_{CA}\sin 25 - 50 - 75\sin 65$

$+R\sin 55 + T_{CA}\sin 25 = \cancel{111.436}\ 117.973$ —— 2

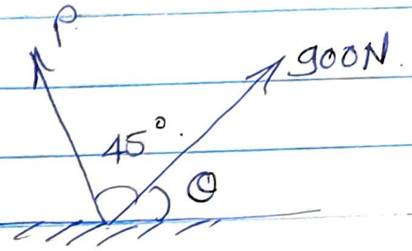
$R = 94.967\,N$

$T_{CA} = 95.074\,N$

Two forces are shown in fig. knowing that the magnitude of P is 600N, determine,

a] the required angle θ if the resultant R of the two forces is to be vertical,

b] the corresponding value of R.



Here R is vertical, $\Sigma f_x = 0$ & $\Sigma f_y = R$

i] $\Sigma f_x = 0$.

$900 \cos\theta - P\cos(180-45-\theta) = 0$.

$900\cos\theta - P\cos(135-\theta) = 0$.

$P = 600N$.

$900\cos\theta - 600[\cos 135 . \cos\theta + \sin 135 . \sin\theta] = 0$.

$900\cos\theta + 424.264\cos\theta - 424.264\sin\theta = 0$.

$1324.264\cos\theta = 424.264\sin\theta$

$\tan\theta = \dfrac{1324.264}{424.264} = 3.121$

$\theta = 72.235°$ $\boxed{\theta = 59.53}$

$\Sigma f_y = R$.

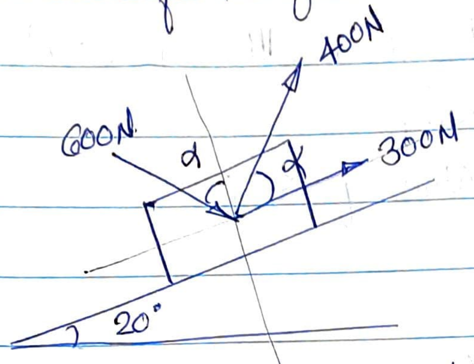
$\Sigma f_y = 900\sin\theta + 600 \sin(135-\theta) = R$.

$900\sin 72.235 + 600\sin(135-72.235) = R$

$\therefore R = 1390.566N$. $\boxed{R = 539.37}$

* a) Find Resultant of shown fig. if $\alpha = 65^\circ$
b) Find value of α if R is $\parallel$el to inclined plane.

:→



Selecting x-axis $\parallel$el to plane.

Force	Incli	x Comp	y Comp.
300	0	300	
400	65	400 cos 65	400 sin 65
600	25	600 cos 25	− 600 sin 25
		1012.83	108.95

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$= \sqrt{1012.83^2 + 108.95^2}$$

$$= 1018.67 \text{ N}$$

$$\theta = \tan^{-1} \frac{108.95}{1012.83}$$

$$= 6.14^\circ$$

with horizontal $= 6.14 + 20 = 26.14^\circ$

b]

As R is ||el to inclined plane $\Sigma F_y = 0$.

$$\Sigma F_y = 400 \sin\alpha - 600 \cos\alpha = 0$$

$$400 \sin\alpha = 600 \cos\alpha$$

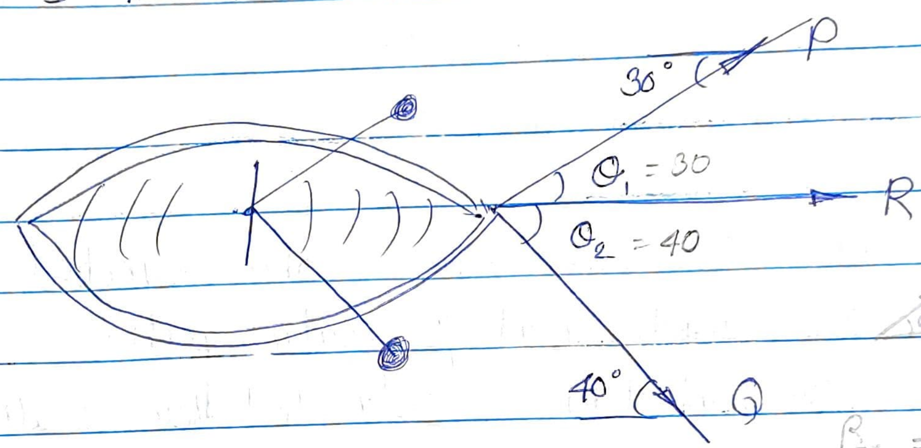
$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{600}{400}$$

$$\alpha = 56.31^\circ$$

$$R = 300 + 400 \cos 56.31 + 600 \sin 56.31$$

$$= 1021.11 N.$$

To move a boat uniformly along the river at a given speed a resultant force $R = 520N$ is required. Two men pulling with force P and Q by means of ropes to do this. The ropes makes an angle of $30°$ & $40°$ respectively with the sides of the river as shown in fig. (i) Determine the forces P & Q.
(ii) If $\theta_1 = 30°$ find the value of θ_2 such that the force in the rope Q is minimum. What is the minimum force Q ?



(i)
$$\Sigma R_x = \Sigma F_x$$
$$520 = P\cos 30 + Q\cos 40$$
$$520 = 0.866\,P + 0.766\,Q \quad \text{—— (1)}$$
$$\Sigma F_y = 0$$
$$0 = P\sin 30 - Q\sin 40$$
$$0 = 0.5P - 0.6428\,Q \quad \text{—— (2)}$$
From (1) & (2)
we get $P = 355.7\,N$ $Q = 276.88\,N$.

(ii) For the force Q to be minimum, force P & Q should be ⊥er to each other.
$$\theta_1 + \theta_2 = 90°$$
but $\theta_1 = 30°$ $\Rightarrow$ $\theta_2 = 60°$

$R_x = \Sigma F_x$

$520 = P\cos 30 + Q_{min} \cos 60$

$520 = 0.866 P + 0.5 Q_{min}$ —— (3)

$\Sigma F_y = 0.$

$0 = P\sin 30 - Q_{min} \sin 60.$

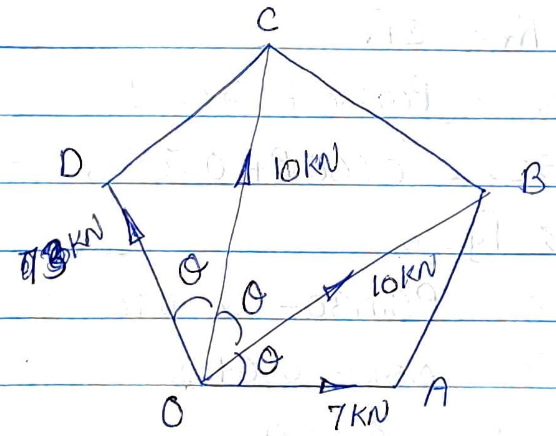
$0 = 0.5P - 0.866\ Q_{min}$ —— (4)

Solving (3) & (4)

$Q_{min} = 260 N.$

$P = 450.32 N.$

* Forces 7KN, 10KN, 10KN & 3KN respectively act at one of the angular point of regular pentagon toward the other four point taken in order. Find their resultant completely.



Regular pentagon is polygon having five sides of equal length. The point of intersection of two sides is called as angular point.

∴ Pentagon has five angular points. $(2n-4)\,90$

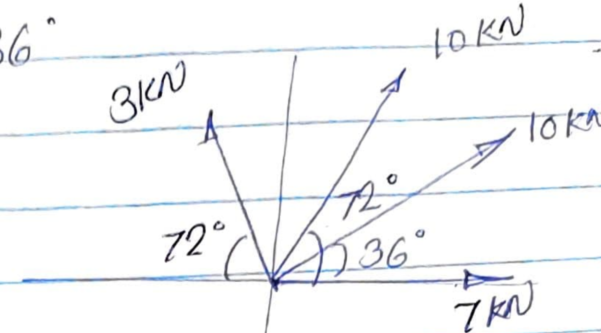
$= (2\times5-4)\times 90 = \frac{540}{5} = 108^\circ$

Included angles of any regular polygon $= 180 - \frac{360}{5}$ $\left[180 - \frac{360}{\text{No. of sides}}\right]$

$= 108^\circ$

∴ From the figure,

$$\theta = \frac{108}{3} = 36^\circ$$



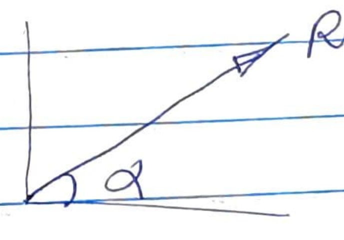
$$\Sigma f_x = 10\cos 36 + 10\cos 72 - 3\cos 72 + 7$$
$$= 17.253 \text{ KN.}$$

$$\Sigma f_y = 3\sin 72 + 10\sin 72 + 10\sin 36$$
$$= 18.241 \text{ KN.}$$

$$R = \sqrt{17.253^2 + 18.241^2}$$

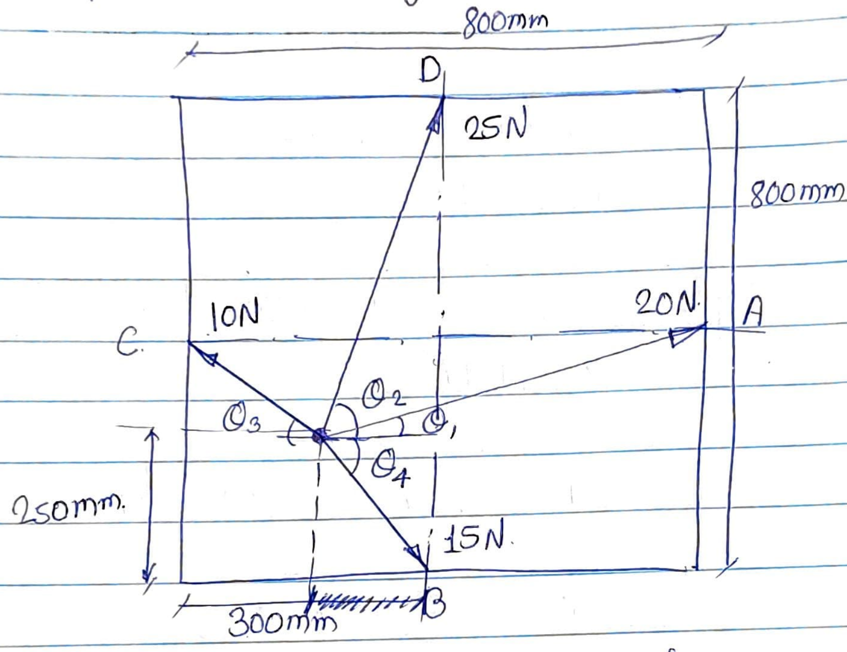
$$R = 25.107 \text{ KN}$$

$$\alpha = \tan^{-1}\frac{18.241}{17.253}$$



$$\alpha = 46.594^\circ$$

* The striker of carom board laying on the board is being pulled by four players as shown in fig. The players are sitting exactly at the centre of the four sides. Determine the resultant of forces in magnitude and direction.



$\tan\theta_1 = \frac{150}{500}$ $\quad\therefore\ \theta_1 = 16.7°$

$\tan\theta_2 = \frac{550}{100}$ $\quad\theta_2 = 79.7°$

$\tan\theta_3 = \frac{150}{300}$ $\quad\theta_3 = 26.56°$

$\tan\theta_4 = \frac{250}{100}$ $\quad\theta_4 = 68.2°$

$\Sigma F_x = 20\cos 16.7 + 25\cos 79.7 + 15\cos 68.2 - 10\cos 26.56$

$= 20.252$ kN.

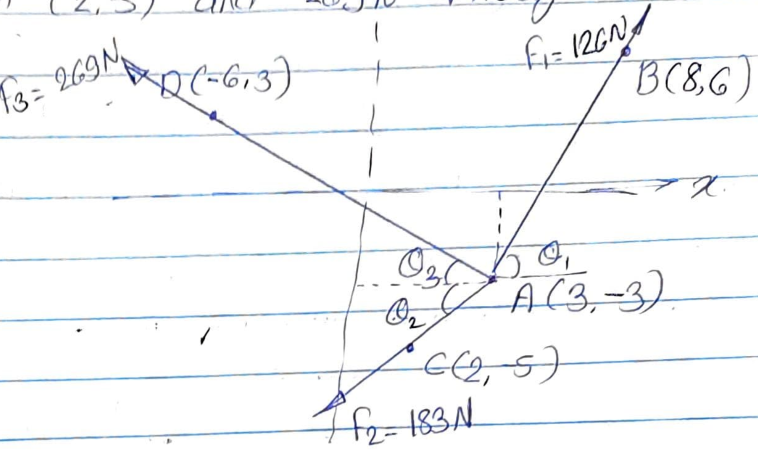
$\Sigma F_y = 20\sin 16.7 + 25\sin 79.7 + 10\sin 26.56 - 15\sin 68.2$

$= 20.888$ kN.

$R = \sqrt{20.252^2 + 20.888^2} = 29.093$ kN.

$\alpha = \tan^{-1}\frac{20.888}{20.252} = 45.885°$

* Determine the resultant of the three forces originating at point (3, -3) and passing through the point indicated: 126N through (8, 6), 183N through (2, -5) and 269N through (-6, 3)



To find θ_1, θ_2 & θ_3

$$\tan\theta = \left| \frac{y_2 - y_1}{x_2 - x_1} \right|$$

$$\theta_1 = \tan^{-1}\left| \frac{6-(-3)}{8-3} \right| = \tan^{-1}\left(\frac{9}{5}\right) = 60.945°$$

$$\theta_2 = \tan^{-1}\left| \frac{-5-(-3)}{2-3} \right| = \tan^{-1}\left(\frac{-2}{-1}\right) = 63.44°$$

$$\theta_3 = \tan^{-1}\left| \frac{3-(-3)}{-6-(3)} \right| = \tan^{-1}\left(\frac{6}{-9}\right) = 33.69°$$

$$\Sigma F_x = 126 \cos 60.945 - 183 \cos 63.44 - 269 \cos 33.69$$

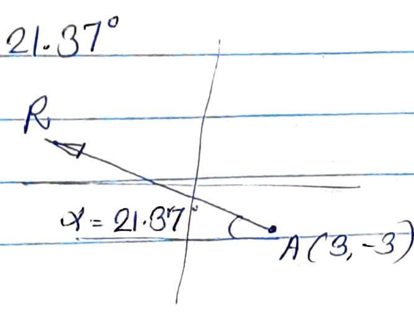
$$= -244.47 \text{ N}$$

$$\Sigma F_y = 126 \sin 60.945 - 183 \sin 63.44 + 269 \sin 33.69$$

$$= 95.68 \text{ N}.$$

$$R = \sqrt{(-244.47)^2 + 95.68^2} = 262.53 \text{ N}.$$

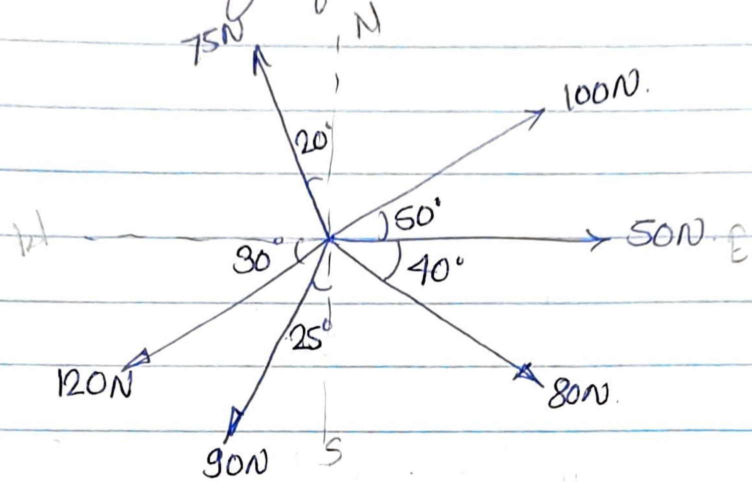
$$\alpha = \tan^{-1}\left(\frac{95.68}{-244.47}\right) = 21.37°$$



* A body is acted upon by forces as below. Find the resultant of these forces.

(i) 50N acting due East
(ii) 100N 50° North of East
(iii) 75N 20° West of North
(iv) 120N acting 30° South of West
(v) 90N acting 25° West of South
(vi) 80N acting 40° south of East

All forces acting from the point O.

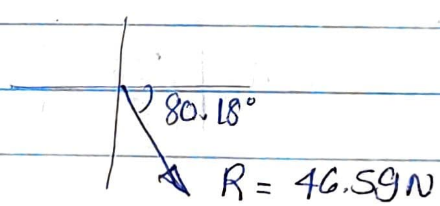


Ans: $\Sigma F_x = 7.95N \rightarrow$

$\Sigma F_y = 45.91N \downarrow$

$R = 46.59N.$

$\alpha = 80.18°$



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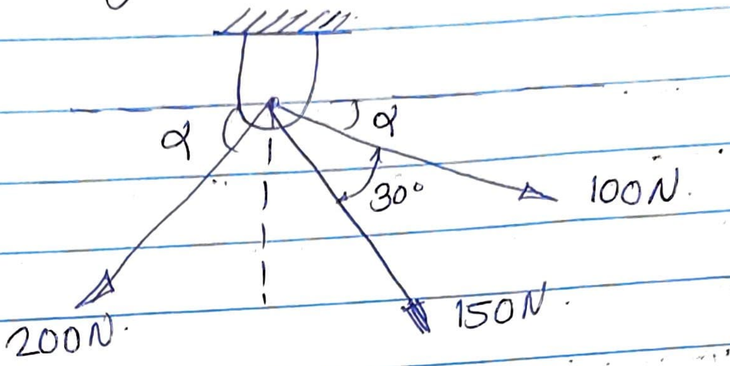
* For the system shown, determine

(i) the required value of α if resultant of three forces is to be vertical and

(ii) the corresponding magnitude of resultant.



(i) Since resultant is vertical.

$$\Sigma F_x = 0$$

$$100 \cos\alpha + 150 \cos(30+\alpha) - 200 \cos\alpha = 0$$

$$150(\cos 30 . \cos\alpha - \sin 30 . \sin\alpha) - 100 \cos\alpha = 0$$

$$130 \cos\alpha - 75 \sin\alpha - 100\cos\alpha = 0$$

$$30 \cos\alpha - 75 \sin\alpha = 0.$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{30}{75} = \tan\alpha$$

$$\therefore \alpha = 21.8^\circ$$

(ii) Resultant is vertical.

$$R = \Sigma F_y$$

$$\Sigma F_y = R$$

$$-100\sin\alpha - 150 \sin(\alpha+30) - 200\sin\alpha = 0.$$

$$R = -100\sin\alpha - 150 \sin(\alpha+30) - 200\sin\alpha = 0.$$

$$-100 \sin 21.8 - 150 \sin(21.8+30) - 200\sin 21.8 = 0$$

$$R = -229.29N.$$

$$\therefore R = 229.29 N \downarrow$$