① Unit : Optics ; ① Topic : Interference

* Electromagnetic Waves

- Electric & magnetic field components vibrate perpendicular to each other in EM waves.
- Travel through free space at speed of light (3×10^8 m/s)
- According to different wavelengths (energies), there are different kind of waves in Electromagnetic Spectra.

  ex - X rays, UV light rays, IR rays, Radio waves etc.

- Mostly, we are going to study the visible part of EM spectra. (400 - 700 nm wavelength region)

- Light (visible part) :

- Light behaves as a particle as well as a wave.

The phenomenon like Intereference, Diffraction etc prove wave nature of light.

* Interference :

- Christain Huygens gave the wave theory of light.
- Thomos Young's double slit expt experimentaly proved this.

Intereference :

- The phenomenon of ~~re~~ Redistribution of light energy due to superposition ~~of~~ of light waves from two or more coherent sources is called as "Interference"
- Stationary bands of alternate darkness & brightness are called as "Fringes"

* Conditions for observing good Interference pattern :

① The waves must have same frequency, same amplitude.

(ii) Waves must maintain a constant phase difference (coherent sources)

(iii) Two coherent sources must lie close to each other as small as possible & screen should be as far as possible.

* Methods to obtain Intereference :

① By division of wavefront :

- ~~A~~ single wavefront is divided, which gives coherent sources.

  ex - Young's double slit expt.

(ii) By Division of Amplitude :

- Amplitude of light is divided into two or more beams by partial reflections.

  ex - Thin film Intereference, Michelson's interferometer

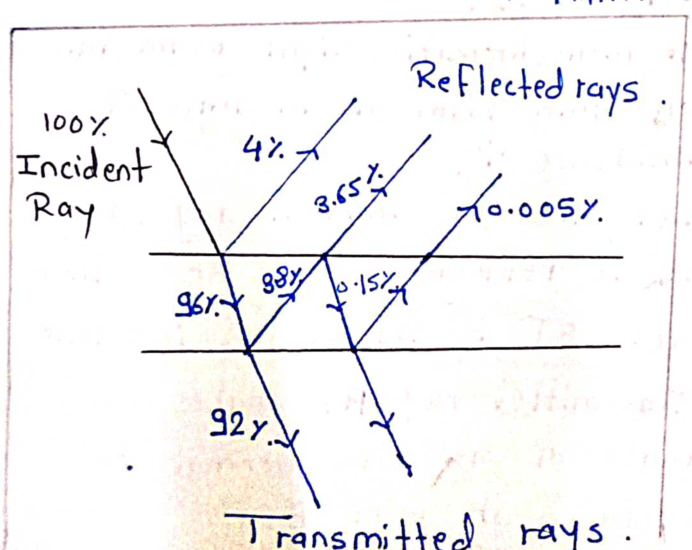
* Thin film Integeference

◦ Thin film ?

- It is an optical medium of thickness of order of 0.5 – 10 μm.
- It can be a thin sheet of transperent material like glass, mica, air film enclosed between two teansperent plates.

◦ When light is incident on such film, small part of it gets reflected from top surface & major part is transmitted into film. Again small part of transmitted component is reflected from to bottom surface & rest of it emerges out of film.

◦ Small part of light gets reflected partially several times in succession within film.



◦ For a glass film :

$\mu_{glass} = 1.54$

◦ Reflectivity $(r) = \left[\frac{(\mu - 1)}{(\mu + 1)}\right]^2$

∴ Reflectivity for glass film :

$r = 0.042 \Rightarrow \sim 4\%$

∴ Out of 100% incident ray, 4% reflected & 96% transmitted into film.

◦ Aagain out of 96% transmitted ~3.8% reflected & 92% transmitted through the film.

◦ Out of this 3.8%, ~3.65% transmitted from top surface & only 0.15% reflected.

◦ Hence after two reflections the intensity will become very small.

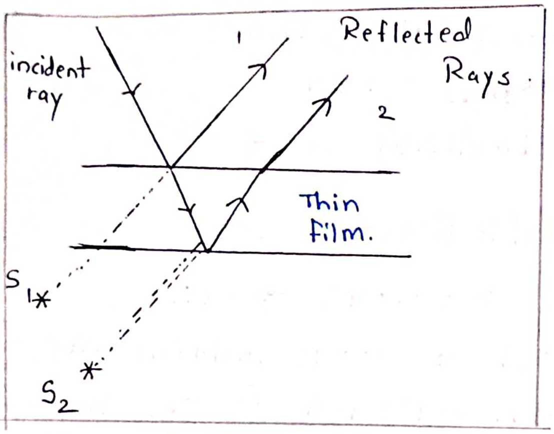
◦ At each reflection intensity & ∴ amplitude of light wave is divided into reflected & refracted component.

◦ These reflected & refracted component are brought to overlap to produce interference.

◦ Hence this type of interference by division of Amplitude.

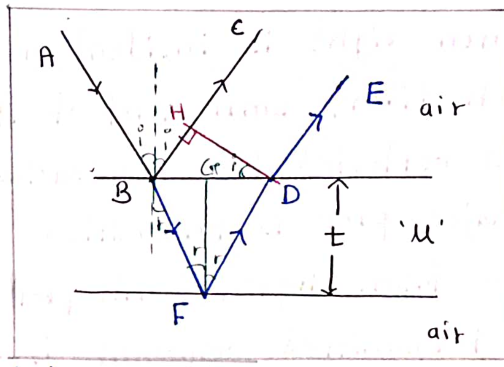
* Plane Parallel film :

◦ Transperent film of uniform thickness bounded by two 'parallel' surfaces is called Plane Parallel film.



- When light is incident on a parallel film, a small portion of it get reflected from the top surface & major portion get transmitted into film.
- Small portion of transmitted ray gets reflected from bottom surface & major portion transmitted through bottom surface.
- This reflected ray get further transmitted from top surface & emerges out (ray 2)
- Films transmit incident light strongly & reflect only weakly.
- After two reflections intensities will be negligible.
  Hence we consider first two reflected rays only.
- These two rays are obtained from a (single) same incident ray but appear to come from two sources located below film. These sources are virtual coherent sources.
- We want to know conditions for maxima & minima. For that we need to know the Geometric & optical path difference:

* Interference Due to Reflected Light:



- Let us consider a transparent film of uniform thickness 't' bounded by two parallel surfaces.
- Let refractive index of material be 'μ'
- film is surrounded by air on both sides.
- A monochromatic light beam falls on thin film at an angle of incidence 'i'.
- Let 'AB' be incident ray, ray AB is reflected along BC & other part BF is transmitted into film
- Transmitted ray BF makes an angle of 'r' with normal to surface at point B.

- Transmitted ray BF again reflected along FD. & major part will be transmitted along FK.
- Part of reflected ray FD is transmitted at upper surface along DE.
- As film surfaces are parallel, reflected rays BC & DE will be parallel to each other.
- Waves travelling along BC & BFDE are obtained from single wave AB. Hence these are coherent so & can produce interference pattern

① Geometric Path Difference:

- Let us draw DH ⊥lar to BC.
- From H & D onwards, rays HC & DE travel equal path.
- BH ~~to~~ ray travels in air while the ray BD travels in film along Ray BD of R.I. μ along path BF & FD

∴ Geometric path Difference between two rays:

$$\boxed{BF + FD - BH} \quad \text{— I}$$

② Optical Path Difference:

$$\Delta_a = \mu L.$$

$$= \mu\left[BF + FD - BH\right]$$

$$\Delta_a = \mu(BF + FD) - BH \quad \text{-II}$$

$$(\because \mu_{air} = 1)$$

- In ΔBFD, $\angle BFG = \angle GFD = \angle r$

Also $BF = FD$

$$\cos r = \frac{FG}{BF} \quad \therefore BF = \frac{FG}{\cos r}$$

$$\therefore BF = \frac{t}{\cos r} \quad (\because FG = t)$$

$$\therefore \boxed{BF + FD = \frac{2t}{\cos r}} \quad \text{— III}$$

- Also, $BG = GD \Rightarrow BD = 2\,BG$

$$\tan r = \frac{BG}{FG} \Rightarrow BG = t \cdot \tan r.$$

$$\therefore BD = 2t \cdot \tan r$$

- Now, ΔBHD, $\angle HBD = (90 - i)^\circ$

$$\angle BHD = 90^\circ \quad \therefore \angle BDH = i$$

$$\therefore BH = BD \sin i = 2t \cdot \tan r \cdot \sin i$$

Snell's law $\Rightarrow$ $\sin i = \mu \sin r$

$$\therefore BH = 2 \cdot t \cdot \tan r \cdot \mu \sin r$$

$$= 2t\,\frac{\sin r}{\cos r} \cdot \mu \sin r$$

$$\therefore \boxed{BH = \frac{2\mu t}{\cos r} \sin^2 r} \quad \text{—IV}$$

Substitute eq^n III & IV in II

$$\therefore \Delta_a = \mu\,\frac{2t}{\cos r} - \frac{2\mu t}{\cos r}\sin^2 r$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$\boxed{\Delta_a = 2\mu t \cos r} \quad \text{— V}$$

- When ray is reflected at boundary of rarer to denser medium path change of $\frac{\lambda}{2}$ occurs for BC ray.

- There is no path difference due to transmission at D.

$\therefore$ $\boxed{\Delta_{true} = 2\mu t \cos r - \lambda/2}$ - VI

④ Conditions for maxima (Brightness) & minima (darkness) :

- Maxima will occur when the optical path difference is equal to intergral multiple of λ

$\therefore$ $\Delta = m\lambda$

m : integer : 1, 2, - - .

$\therefore 2\mu t \cos r - \frac{\lambda}{2} = m\lambda$

$\therefore$ $\boxed{2\mu t \cos r = \left(\frac{2m+1}{2}\right)\lambda}$

... Condition for Maxima

- Minima will occur when

$\Delta = (2m+1)\frac{\lambda}{2}$

$\therefore 2\mu t \cos r - \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}$

$\therefore$ $2\mu t \cos r = (m+1)\lambda$

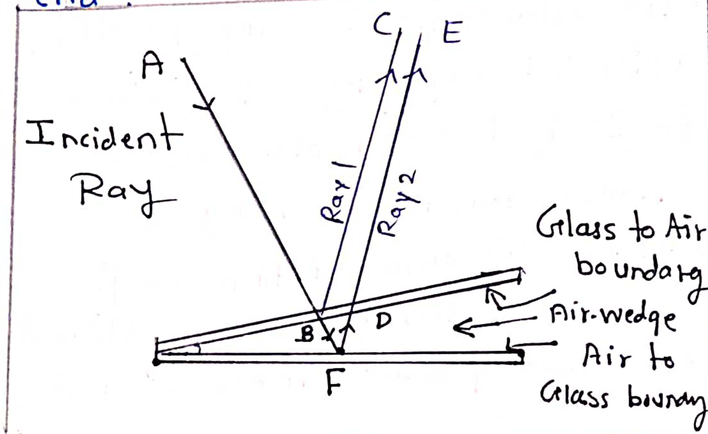
If we add / subtract a full wave there is no any change in phase relationship.

$\therefore$ $\boxed{2\mu t \cos r = m\lambda}$

... condition for minima.

* Wedge-Shaped (Variable Thickness) Film :

- A thin film of varying thickness having a zero thickness at one end & increasing to a particular thickness at other end.



* Interference pattern in wedge shaped air film :

- Parallel beam of monochromatic light ~~is~~ incident on wedge shaped air film from above, rays reflected from two bounding surfaces will not be parallel
- They appear to diverge from a point near film.
- Due to change in thickness of film at different points there is path difference between the rays reflected from upper & lower surfaces of the air film.
- Hence alternate dark & bright Friges are observed.

- Let AB be incident ray, incident ray will get partly reflected from glass-to-air boundary (ray BC) & part of light transmitted through air film & get reflected partly at air-to-glass boundary (ray FE)
- For small thickness film, rays interfere producing dark & bright fringes.

We know,
Optical path difference betⁿ BC & FE ⇒

$$\Delta = 2\mu t \cos r + \cancel{\frac{\lambda}{2}}$$

We know conditions for maxima & minima :

$$2\mu t \cos r + \cancel{\frac{\lambda}{2}} = \left(\frac{2m+1}{2}\right)\lambda \ \&$$

$$2\mu t \cos r - \cancel{\frac{\lambda}{2}} = m\lambda.$$

- Consider this condⁿ for minima:

$$2\mu t \cos r = m\lambda$$

For normal incidence, $\cos r = 1$

thickness of air film $= t_1$

$\therefore$ at A : $2\mu t = m\lambda$.

nex dark fringe will occur at 'C' where thickness $CL = t_2$

$\therefore$ at C → $2\mu t_2 = (m+1)\lambda$.

Subtracting :

$$2\mu(t_2 - t_1) = \lambda$$

But $t_2 - t_1 = BC$

$\therefore 2\mu\, BC = \lambda$

$\Rightarrow BC = \frac{\lambda}{2\mu}$.

ΔABC, $\angle CAB = \theta$ & $BC = AB\tan\theta$

$\therefore (AB)\tan\theta = \frac{\lambda}{2\mu}$.

$\therefore$ Fring width (distance betⁿ cosecutive dark fringes)

$$\beta = \frac{\lambda}{2\mu \tan\theta}$$

For small θ

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

$\therefore$ As θ increases, fringe separation increases.

* Salient Features of the Interference pattern :

(i) Fringe at apex (point of contact of two plates) is dark
(ii) Fringes are straight & parallel
(iii) Fringes are equidistant
(iv) Fringes ~~of~~ of equal thickness

* Applications of Interference:

- Interference phenomenon is used widely.

ex. for measuring wavelength, R.I of liquids & gases, etc.

① Testing of Flatness of Surfaces :

- If you want to use machine component for ~~some~~ particular application, you must know irregularities in that component as these will lead to cracks when stress is applied.
- The machine components which are going to be ~~sut~~ subjected to high stress & load, required to have a smooth surface finish.
- The smoothness/flatness can be found by keeping an optical flat on the component at an angle & incident monochromatic light on it.
- There is formation of air wedge film between optical flat & machine component.
- If surface is smooth it will produce straight & equidistant fringes.
- If surface is concave → fringes are curved towards contact angle
- If surface is convex → fringes are curved away.

* Testing a lens surface :

- Lens are used widely in telescopes & other instruments. ~~man~~ To check if lens are smooth & polished, Newton's rings are used
- If lens is perfect & polished we will get circular fring pattern if not then we can say there are defects in lens.

② Anti-Reflecting coatings :

- In optical instruments like telescopes & cameras multi component glass lenses are used.
- When light incident on such lenses lots of light is lost due to reflection & quality of image produced by device is poor.
- In case of solar cells, if light is lost in reflection amount of current generated will be less.
- To avoid these losses due to reflection we can coat the surface with thin film of sutaible refractive index Such coatings are Anti reflecting coatings. (AR coatings)
- Scientist Alexander Smakula discoved that reflections from a surface can be reduced by coating the surface with a thin transperent dielectric material.
- The film to act as AR coating should follow :

(i) Phase condition :

- To produce destructive interference, waves reflected from top surfaces & bottom surfaces are in opposite phase

(ii) Amplitude Condition :

- Waves have equal Amplitudes

So depending on these conditions we can select accordingly material of particular R.I. & thickness of the film.

* Phase condition & minimum thickness of film :

- Let thickness be 't' & R.I μ_f, phase condn (out of phase / 180° phase differene) For that optical path difference must be $\frac{1}{2}$ or an odd number of waves

$$\therefore \Delta = 2\mu_f t \cos r - \frac{\lambda}{2} - \frac{\lambda}{2}$$

corresponds to π change at topsurface

corresponds to π change at film toglass (bottom) surface.

For normal incidence, $\cos r = 1$

$$\therefore \Delta = 2\mu_f t - \lambda = 2\mu_f t$$

As addn / subtraction of a full wave do not affect phase relation.

For Destructive interference :

$$\Delta = (2m+1)\frac{\lambda}{2}$$

$$\therefore 2\mu_f t = (2m+1)\frac{\lambda}{2}$$

A film to be transperent, thickness should be minimum it will happen if $m=0$

$$\therefore 2\mu_f t_{min} = \frac{\lambda}{2}$$

$$\therefore \boxed{t_{min} = \frac{\lambda}{4\mu_f}}$$

$\Rightarrow$ Thickness of AR film Coating should be one-quarter ($\frac{1}{4}$th) of wavelength.

$\therefore$ The film of $\frac{1}{4}^{th}\lambda$ thickness suppress reflections & causes light to pass through it.

* Amplitude Condition :

- Amplitude of reflected rays - equal

$$\therefore E_1 = E_2$$

Requires :

$$\left[\frac{\mu_f - \mu_a}{\mu_f + \mu_a}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2$$

μ_a - R.I of air,

μ_f - R.I of film

μ_g - R.I of glass subtrate.

- as $\mu_a = 1$

$$\therefore \left[\frac{\mu_f - 1}{\mu_f + 1}\right]^2 = \left(\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right)^2$$

Expanding :

$$4\mu_f^3 \mu_g + 4\mu_f \mu_g = 4\mu_f^3 + 4\mu_f \mu_g^2$$

Dividing by $4\mu_f$ & rearrange :

$$\mu_f^2 - \mu_g \mu_f + \mu_g^2 - \mu_g = 0$$

$$\Rightarrow \mu_f^2 = \mu_g (1 + \mu_f^2 - \mu_g)$$

$$\Rightarrow \mu_f^2 \cong \mu_g \text{ (as } \mu_f \approx \mu_g)$$

$$\therefore \boxed{\mu_f \simeq \sqrt{\mu_g}}$$

$\Rightarrow$ R.I. of film should be less than that of substrate & possibly near its square root.

- In case of glass, $\mu_g = 1.5$

$\therefore \mu_f \simeq 1.22$

Materials having R.I. near this value are magnesium fluride (MgF_2) - 1.38

Cryolite, $3NaF \cdot AlF_3$ ($\mu = 1.36$)

In addition of RI wee need to consider other properties of material it should be durable, scratch prouf & insoluble in ordinary solvents etc.

MgF_2 & cryolite satisfy these conditions & MgF_2 is cheaper hence widely used in AR coatings.

- Thickness condition is satisfied at a particular wavelength

- Normally we ~~use~~ choose ~~ω~~ λ of 5500 Å, which is most sensitive to human eye
- wavelength near yellow-green portion of visible spectrum.

* Multilayer AR coating :

- Single layer AR coating is effective only at particular wavelength.
- To cover more than a single wavelength we can use multilayer coatings.
- In practice three layer coatings are widely used & are highly effective over most of visible spectrum.

(i) A glass microscope lens ($\mu = 1.5$) is coated with magnesium floride ($\mu_f = 1.38$) film to increase transmission of normally incident light $\lambda = 5800$ Å. What is the minimum film thickness that should be deposited on film :

$$\text{4} \quad t_{min} = \frac{\lambda}{4\mu_f} = \frac{5800 \times 10^{-10}}{4 \times 1.38}$$

$$\boxed{t_{min} = 1051 \text{ Å}}$$

(ii) Can a thin film of water ($\mu = 1.33$) formed on a glass window plane ($\mu_f = 1.52$) act as non-reflecting film.

~~Interference~~ Interference

~~e light~~

If so, how thick should be water film?

↳ $\mu_f = \sqrt{\mu}$

$\therefore \sqrt{1.52} = 1.233$

As. μ_{water} is nearer to water film $\sqrt{\mu}$ can act as non-reflecting film on glass

$\therefore$ Minimum thickness of film:

$$t_{min} = \frac{\lambda}{4\mu_f}$$

Human eye is more sensitive to green $\therefore$ assume $\lambda = 5500 Å$

$$\therefore t_{min} = \frac{5500 \times 10^{-10}}{4 \times 1.33}$$

$$\boxed{t_{min} = 1034 Å}$$

* A material of RI 1.3 used to coat a piece of glass. What should be minimum thickness of film to mininize reflected light at a wavelength of 500nm & what should be RI of glass for best effects?

$\mu_f = 1.3$, $\lambda = 500 \times 10^{-9} m$

$$t_{min} = \frac{\lambda}{4\mu_f} = \frac{500 \times 10^{-9}}{4 \times 1.3}$$

$t_{min} = 96.15 \times 10^{-9} m$

$t_{min} = 96.15$ ~~m~~ nm.

R.I of glass:

$\mu_f = \sqrt{\mu_g} \Rightarrow \mu_g = \mu_f^2$

$\mu_g = 1.69$

— x — x — x —

Diffraction

- Bending of waves around an obstacle & deviation from a rectilinear path is called as Diffraction.

* Dependence of diffraction on wavelength:

- When light (waves) pass through an opening (slit):
- If slit is of large width, waves do not bend around edges.
- If slit is of small width, waves ~~bends ar~~ bend around edges & it is noticeable
- Hence, Diffraction ~~of~~ effect is observed (is noticeable) only when size of obstacle is comparable to a wavelength of light used.

* Diffraction Pattern:

- When light is passed through a small opening, alternate dark & bright fringes are observed with ~~d~~ intensity decreasing from centre to peripheri.
- Central position is known as 'central maxima' & there are secondary maxima & minima on both of its sides.
- Maxima & minima are produced by ~~interferfereace~~ interference of diffracted light waves.

* Distinction between interference & Diffraction:

Interference	Diffraction
• Observed as a result of interaction of light waves coming from different sources (different wavefronts)	• Observed as a result of interaction of ~~light~~ wave fronts from different parts of same wavefront.
• Fringes may or may not be of same width	• Fringes are not of same width.
• Regions of minimum intensity are perfectly dark	• Points of minimum intensity are not perfectly dark.
• All bright fringes are of same intensity	• All bright fringes are not of same intensity.

* Types of Diffraction:

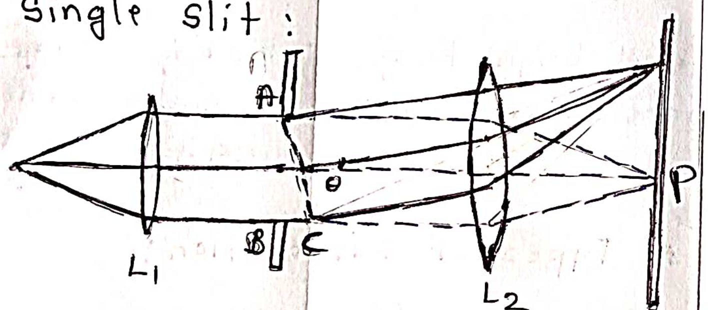
① Fresnel Diffraction:

- Source of light & screen are effectively at finite distances from obstacle
- Lenses are not used to make ray parallel or convergent.
- As incident wave front is not planar, phase of secondary wavelets is not same at all points in the plane ~~of~~ of obstacle.

② Fraunhoffer Diffraction:

- Source of light & screen are effectivly at infinite distances from obstacle.
- Conditions required for diffraction are achieved by using two convex lens : one to make the light from source parallel & other to focus the light on screen, after diffraction.
- Incident wavefront is a plane wavefront hence secondary wave-lets are in same phase at every point in the plane of obstacle.

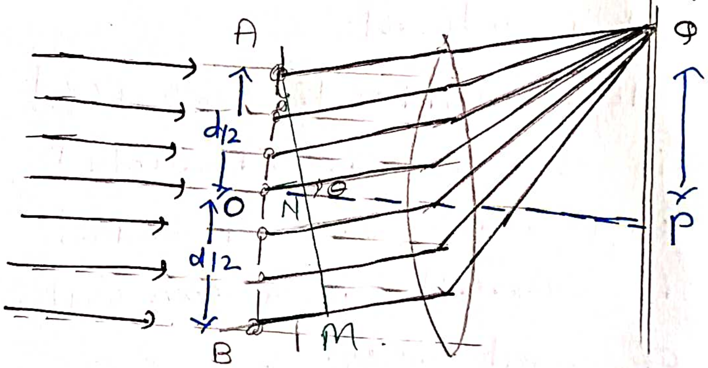
* Fraunhoffer Diffraction at a Single slit :



- Consider above arrangement, Lenses L_1 & L_2, ~~Kep~~ keep the source & screen effectively at infinity.
  L_1 will make rays parallel & L_2 will converge rays at a point P on screen.
- Consider slit AB of width d perpendicular to plane of page. It is illuminated by a parallel beam of monochromatic light of wavelength 'λ'.
- According to optics principles, a sharp image at point 'P' should be obtained.
- However, we obtain on screen is slit image of maximum brightness at centre, followed by secondary maxima on either sides gradually decreasing intensities, with distance.
- Intensity distribution on the screen is Fraunhofer Diffraction pattern.

Lets try to understand why this happens :

* Formation of maxima & minima:



- A plane parallel wavefront (parallel rays) incident on slit AB.
- Each point on AB acts as a source of secondary wavelets.
- We can say silt AB as a string of point sources.
- As A points on AB are coherent / in phase, so the point sources will be coherent

- Hence light from one point source will interfere with light from other point source & resultant intensity will depend on direction of θ
- Secondary wavelets travelling parallel to OP come to focus at point P, waves from points equidistant from 'O' from portion OA & OB start in phase.

They will travel same distance reaching at P

$\therefore$ Optical path difference is zero & waves will be in phase

They interfere & ~~give~~ produce maximum intensity at 'P'

Centre of diffraction pattern & called zero order central maxima.

- For points other than OP ~~ex.~~ suppose 'Q', light from different parts travel different distances
- Path difference between waves reaching points from different parts increases gradually
- Secondary waves traveling in a direction making an angle 'θ' with OP

These 2° wavelets are brought to the focus at point Q by convex lens

It will have maximum or minimum intensity, depending on path difference between waves arriving at point Q. from differet points on AB.

- Divide wavefront AB into AO & OB
- Line AM ⊥lar to direction of diffracted rays

ON & Bm are in phase at slit.

Bm travels larger distance than ON.

$\therefore$ Path difference betn wavelets

$= ON = \frac{d}{2}\sin\theta$.

If $ON = \frac{\lambda}{2}$, 2 waves interfere destructivity & produce darkness at Q

This is true for any two waves originating at points seperated by $d/2$.,

path difference betn them will be $\lambda/2$.

$\therefore$ For every point in upper

half OA there is corresponding point in lower half OB.
& path difference bet^n them $= \lambda/2$

$\therefore$ waves from upper half AO interfere destructively with waves from lower half OB.

$$\therefore \frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

$$\therefore \sin\theta = \frac{\lambda}{d}.$$

intensity at Q is zero & a dark band is called as 1st order minimum.
Similar dark band occur at pt Q' below P.
-1^{st} order minimum.

- we can divide slit into four parts, six parts & so on..

$\therefore$ In general:

$$\boxed{\sin\theta_m = m\lambda}$$

Condition for minimum

$m = 1, 2, 3, \ldots$

$\therefore$ Condition for minima:

$$\boxed{d\sin\theta = \pm n\lambda}$$

- In addition to secondary minima, there are secondary maxima. which lie between secondary minima on either sides of central maximum.

- For maxima, path difference

$$ON = d\sin\theta = \frac{(2m+1)}{2}\lambda$$

$\therefore$ for maxima condition is:

$$\boxed{\sin\theta_m = \left(\frac{2m+1}{d}\right)\frac{\lambda}{2}}$$

- Diffraction pattern due to single slit consists of central bright maximum & secondary maxima & minima on both the sides.

* Intensity distribution in Diffraction pattern Due to single Slit:

- Let a plane wave be incident on a long narrow slit of width 'd'.
Imagine this 'd' divided into N parallel strips of width Δx

- Each width act as a source of secondary wavelets leading to charactristic distribution of intensity at point Q.
- Position of Q is fixed by angle θ & adjecent strips have a path difference of $\Delta x \sin\theta$
- Let disturbance causeal at Q by unit width of slit be :

$$y_0 = A\cos\omega t$$

- Amplitude of wavelet from width dx at A, when it reaches Q is $(A\,dx)$
- Phase of wavelet :

$$\phi = \omega t + \frac{2\pi}{\lambda} \times OQ$$

$$= \omega t + \frac{2\pi}{\lambda} \cdot x\sin\theta \cdot$$

$x \rightarrow$ distance of Q from O

- Disturbance caused at Q by wavelet :

$$dy = A\,dx \cos\left[\omega t + \frac{2\pi x \sin\theta}{\lambda}\right]$$

$\therefore$ Total distrubance at Q ~~fro~~ by all wavelets :

width of slit $AB = a$.

$$\therefore y = \int_{-a/2}^{a/2} dy =$$

$$= \int_{-a/2}^{a/2} A\cos\left[\omega t + \frac{2\pi x\sin\theta}{\lambda}\right] dx$$

Let, $\frac{2\pi\sin\theta}{\lambda} = k$.

$$\therefore \cos\left(\omega t + \frac{2\pi \sin\theta}{\lambda}\right) = \cos(\omega t + kx)$$

$$\cos(\omega t + kx) = \cos\omega t \cdot \cos kx - \sin\omega t \cdot \sin kx$$

$$\therefore Y = \int_{-a/2}^{a/2} A\left[\cos\omega t \cdot \cos kx - \sin\omega t \cdot \sin kx\right] dx.$$

$$= A\cos\omega t \int_{-a/2}^{a/2} \cos kx \, dx - A\sin\omega t \int_{-a/2}^{a/2} \sin kx \, dx.$$

$$= A\cos\omega t \left[\frac{\sin kx}{k}\right]_{-a/2}^{a/2} - A\sin\omega t \left[\frac{-\cos kx}{k}\right]_{-a/2}^{a/2}$$

$$Y = A\cos\omega t\left[\frac{\sin\left(\frac{ka}{2}\right) + \sin\left(\frac{ka}{2}\right)}{k}\right] - A\sin\omega t\left[\frac{\cos\left(\frac{ka}{2}\right) - \cos\left(\frac{ka}{2}\right)}{k}\right]$$

$$= A\cos\omega t\left[\frac{2\sin\left(\frac{ka}{2}\right)}{k}\right] = 2A\cos\omega t\left\{\frac{\sin\left(\frac{2\pi\cdot\sin\theta\cdot a}{2\lambda}\right)}{\frac{2\pi\cdot\sin\theta}{\lambda}}\right\}$$

$$\Rightarrow Y = \left\{A\frac{\sin\left(\frac{\pi\sin\theta\, a}{\lambda}\right)}{\pi\sin\theta/\lambda}\right\}\cos\omega t$$

$$= \left[Aa\frac{\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\frac{\pi a\sin\theta}{\lambda}}\right]\cos\omega t.$$

$Y = A_\theta\cos\omega t$. $\quad A_\theta$ - resultant disturbance at Q

For $\theta = 0$, $\quad A_\theta = A_0 = Aa$

$$Y = A_0\left[\frac{\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\frac{\pi a\sin\theta}{\lambda}}\right]\cos\omega t$$

let $\alpha = \dfrac{\pi a\sin\theta}{\lambda}$

$$\therefore Y = A_\theta\cos\omega t = A_0\left[\frac{\sin\alpha}{\alpha}\right]\cos\omega t$$

$$\Rightarrow \boxed{A_\theta = A_0\left[\frac{\sin\alpha}{\alpha}\right]} \rightarrow \text{Amplitude distribution.}$$

$\therefore$ Intensity distribution is .

$$\boxed{I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2}$$; I_0 - intensity of principal maximum at $\theta = 0$.

$\therefore$ Intensity at any point on the screen is proportional to $\left(\frac{\sin\alpha}{\alpha}\right)^2$

$\therefore$ A phase difference of 2π corresponds to a path difference of λ.

$\therefore$ phase difference of 2α is

$$2\alpha = \frac{2\pi}{\lambda} a \sin\theta .$$

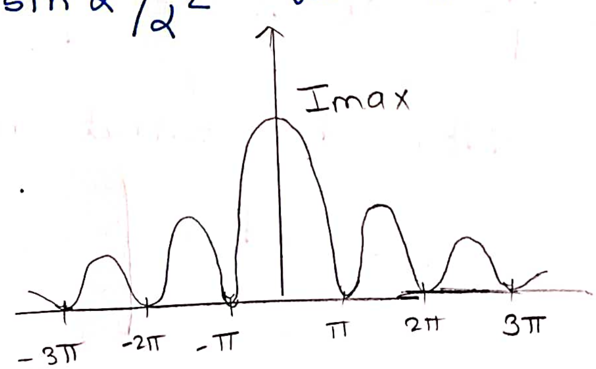
$a\sin\theta$ - path difference bet^n 2° waves from A & B.

$$\boxed{\alpha = \frac{\pi}{\lambda} a \sin\theta}$$

$\therefore$ Value of α depends on angle of diffraction (θ)

$\therefore$ Value of $\left(\frac{\sin^2\alpha}{\alpha^2}\right)$ for point values of θ gives intensity at that point.

- Graph of $\sin^2\alpha/\alpha^2$ Vs α or $\sin\theta$.



⇒ Most of the light is confined to central band & its intensity is far greater than any other maxima Intensity of secondary maxima falls off rapidly as we move away from centre.

- Intensity of 1st secondary maximum is about 1/22 & that of 2nd is 1/61 of intensity of principal maximum.
- Secondary maxima are too faint to see properly.

* Linear width of Principal Maximum:

If x is distance of 1st secondary minimum from centre of principal maximum, width of central maximum is : W = 2X.

- If lens L is very near to slit | screen is far away from lens, & f is focal length of lens, then OP = f is very large

$$\therefore \sin\theta = \frac{x}{f}$$

For 1st minimum, $\sin\theta = \frac{\lambda}{d}$

$$\frac{x}{f} = \frac{\lambda}{d} \quad \text{or} \quad x = \frac{f\lambda}{d}.$$

∴ Linear width of central maximum:

$$\boxed{W = 2x = \frac{2f\lambda}{d}}$$

↳ When slit width $d >> \lambda$,
we see uniform illumination in shape of slit, on screen.
As width is reduced, illumination starts to spread out & dark bands become visible
(We have seen this during praticals)
& width of central maximum increases as silt narrows.

- Position of first secondary maxima on either side of central maxima :

$$\sin\theta_1 = \frac{3\lambda}{2d}$$

$$\therefore \quad \frac{x}{f} = \frac{3\lambda}{2d}$$

$$\Rightarrow \quad x = \frac{3f\lambda}{2d}$$

$$\boxed{x = \frac{3f\lambda}{2d}}$$

* In single slit diffraction pattern, distance between 1st minima on either side of central zero maximum is 4.4 mm., screen is at 0.7 m distance calculate slit width :
$\lambda = 5890\ \mathring{A}$.

↳ Given :
$f = 0.7\,m$, $\lambda = 5890 \times 10^{-10}\,m$,

$x = 4.4 \times 10^{-3}$ m.

$$\therefore \; d = \frac{f\lambda}{x} = \frac{0.7 \times 5890 \times 10^{-10}\ m^2}{4.4 \times 10^{-3}\ m} = 937 \times 10^{-7} m$$

$$\boxed{d = 0.094 \text{ mm}}$$

* parallel light ($\lambda = 5000$ Å) normally incident on a single slit. 1st central maximum fans out at 30° on both sides of direction of incident light. calculate slit width?
For what slit width central maximum would spread out to 90° from direction of incident?

↳ $\lambda = 5000$ Å,

$\theta = 30^\circ$

$d = ?$

We know, $a \sin\theta = \pm m\lambda$

Angular spread, $\sin\theta = \lambda/a$.

$\theta = 30^\circ \therefore \sin\theta = 0.5$ & $\lambda = 5000$ Å

$$\therefore \; a = \frac{\lambda}{\sin\theta} = \frac{5000 \times 10^{-8}\ cm}{1/2} = 10{,}000 \times 10^{-8}\ cm$$

$$\boxed{a = 1 \times 10^{-4}\ cm}$$

For $\theta = 90^\circ$,

$$a = \frac{\lambda}{\sin 90} = \frac{5 \times 10^{-5} cm}{1} = \underline{5000 \text{ Å}}$$

So when slit width is of order (equal to) of wavelength (5000 Å) , then central maxima will spread out at 90°.

* Plane Diffraction Grating :

- Device consisting of a large number of parallel slits of equal width & separated from one another by equal opaque spaces is called as a diffraction grating. Distance between centres of two adjacent slits is known as 'Grating Period'
- ~~Let us consider transmission grating held~~
- Rowland (1848 - 1901) produced transmission grating by ruling extremely close, equidistant & parallel lines on optically plane glass plats with diamond point

  The dimond scratch scatter light & are effectively opaque while parts without scratch transmit light & act as slit.

- Fabrication is expensive & difficult. hence gratings are reproduced from original ruled gratings. Replica gratings are made by pouring a thin layer of collodion solution over surface of ruled grating & solⁿ is allowed to harden & removed afterwards. Film retains impression of original grating.

Film mounted bet^n glass plates & it acts as plane diffraction grating.

The number of lines on transmission grating is of order of 6000 lines per cm.