decision rule (randomized on whiteboard)

$$P(E) = P(E|B_0)P(B_0) + P(E|B_1)P(B_1)$$

$$= P(A_1 \cap \hat{A}_0 \mid B_0)P(B_0) + P(A_0 \cap \hat{A}_1 \mid B_0)P(B_0)$$

$$+ P(A_1 \cap \hat{A}_0 \mid B_1)P(B_1) + P(A_0 \cap \hat{A}_1 \mid B_1)P(B_1)$$

$$= P(A_1|B_0)P(\hat{A}_0|B_0)P(B_0)$$

$$+ P(A_0|B_0)P(\hat{A}_1|B_0)P(B_0)$$

$$+ P(A_1|B_1)P(\hat{A}_0|B_1)P(B_1)$$

$$+ P(A_0|B_1)P(\hat{A}_1|B_1)P(B_1)$$

$$① = P(A_1 \cap B_0) \cdot P(\hat{A}_0|B_0) + P(A_0 \cap B_0)P(\hat{A}_1|B_0)$$

$$+ P(A_1 \cap B_1) \cdot P(\hat{A}_0|B_1) + P(A_0 \cap B_1)P(\hat{A}_1|B_1)$$

substituting all values: $P(A_0) = \frac{2}{5}$ $P(A_1) = \frac{3}{5}$

then $① = \frac{3}{5} \times \frac{1}{6} \times 70\% + \frac{2}{5} \times \frac{7}{8} \times 30\%$

$$+ \frac{5}{6} \times \frac{3}{5} \times 20\% + \frac{1}{8} \times \frac{2}{5} \times 80\%$$