P1: 1. $P(A\cup B) = 1 - P(\overline{A\cup B}) = 1 - 0.35 = 0.65$

2. $P(A\cap B)$

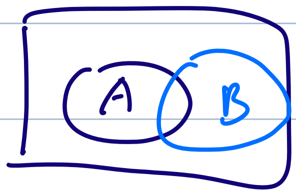
$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow P(A\cap B) = P(A) + P(B) - P(A\cup B)$

$= 0.4 + 0.3 - 0.65 = 0.05$

3. $P(A\cap B^c) = P(A) - P(A\cap B)$ using venn diagram.

$= 0.4 - 0.05 = 0.35$



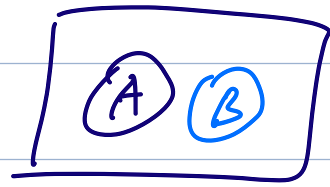
Alternatively:

$P(A) = P(A\cap B) + P(A\cap B^c)$

$\Rightarrow P(A\cap B^c) = P(A) - P(A\cap B)$

4. mutually exclusive:

$P(A\cup B) = P(A) + P(B)$



$P(A\cup B) = 0.65$

$P(A) + P(B) = 0.75.$

$\therefore$ A, B are not mutually exclusive.

5. $P(A\cap C) = P(A)\cdot P(C)$ when A, C are independent

$= 0.4 \times 0.2 = 0.08$

$P(A\cap C^c) = P(A)\cdot P(C^c) = 0.4 \times 0.8 = 0.32$

Question: A, C independent $\Leftrightarrow$ A, C^c independent

proof: $P(A\cap C) = P(A)P(C)$

$P(A) = P(A\cap C^c) + P(A\cap C)$

$\Rightarrow P(A) = P(A\cap C^c) + P(A)\cdot P(C)$

(because A and C are independent)

$$\Rightarrow P(A \cap C^c) = P(A) - P(A) \cdot P(C)$$
$$= P(A)[1 - P(C)]$$
$$= P(A) \cdot P(C^c)$$

$\Rightarrow$ A. C^c are independent.

Problem 2. 1. $P(A) = \frac{13}{52}$ because 13 cards are spade.

$$P(B) = P(\text{Jack}) + P(\text{Queen}) + P(\text{King})$$
$$= \frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{12}{52} = \frac{3}{13}$$

$P(A \cap B) = P(\text{spade and (Jack or queen or king)})$
$= \frac{3}{52}$

2. if A and B are independent.

$$P(A) \cdot P(B) = P(A \cap B)$$

$P(A) = \frac{13}{52}$

$P(B) = \frac{3}{13}$

$\Rightarrow P(A) \cdot P(B) = \frac{13}{52} \times \frac{3}{13}$

$= \frac{3}{52} = P(A \cap B)$

Yes. A, B are independent.

3. if A, B are exclusive.
$P(A \cap B) = P(\phi) = 0$. which is not the case
$\therefore A, B$ are not exclusive.

Problem 3.

1. let A_{10}: the top face be 4 at least once in 10 rolls.

let B_i: the top face is a 4 at the i-th roll)

$$P(A_{10}) = 1 - P(A_{10}^c) = 1 - P(B_1^c \cap B_2^c \cap B_3^c \cdots \cap B_{10}^c)$$

$$= 1 - P(B_1^c) \cdot P(B_2^c) \cdot P(B_3^c) \cdots P(B_{10}^c)$$

$$= 1 - \left(\frac{5}{6}\right)^{10}$$

Because. the probability of not 4 in one roll of die is $\frac{5}{6}$.

2. $P(A_{20}) = 1 - P(A_{20}^c) = 1 - P(B_1^c \cap B_2^c \cdots \cap B_{20}^c)$
$= 1 - P(B_1^c) \times P(B_2^c) \cdots \times P(B_{20}^c) = 1 - \left(\frac{5}{6}\right)^{20}$

3. let A_k be the probability of " top face

be 4 at least once on k rolls"

$$P(A_k) = 1-\left(\frac{5}{6}\right)^k$$

if $P(A_k) \geqslant \underbrace{90\%}$ call this "P"

$$\Rightarrow 1-\left(\frac{5}{6}\right)^k \geqslant \underbrace{90\%}_{P}.$$

$$\Rightarrow \left(\frac{5}{6}\right)^k \leq 1-P$$

$$k \geqslant \log_{\frac{5}{6}}(1-P)$$

$$k \geqslant \frac{\log(1-P)}{\log\left(\frac{5}{6}\right)}$$

when $P=0.9$, $k=13$