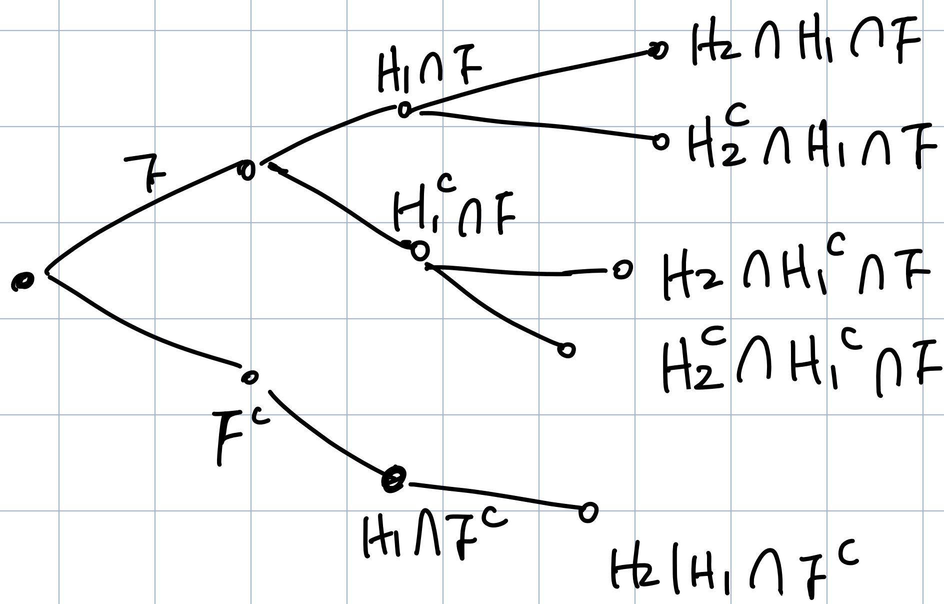
The joint event $H_1 \cap H_2 \cap F$.
has multiple ways to decompose,

consider (in class we decompose into)



F^c: the complement of fair coin. $\rightarrow$ 2-heads.

$$P(H_2 | H_1) = \frac{P(H_2 \cap H_1)}{P(H_1)} \quad \text{"conditioning"}$$

So we care about $P(H_2 \cap H_1)$

The following events satisfy $H_2 \cap H_1$:

$H_2 \cap H_1 \cap F$, $H_2 \cap H_1 \cap F^c$.

$H_1 \cap H_2$

$\therefore P(H_2 \cap H_1) = P(H_2 \cap H_1 \cap F) + P(H_2 \cap H_1 \cap F^c)$

(using chain rule)

$$= P(H_2 | H_1 \cap F) P(H_1 | F) P(F) + P(H_2 | H_1 \cap F^c) P(H_1 | F^c) P(F^c)$$

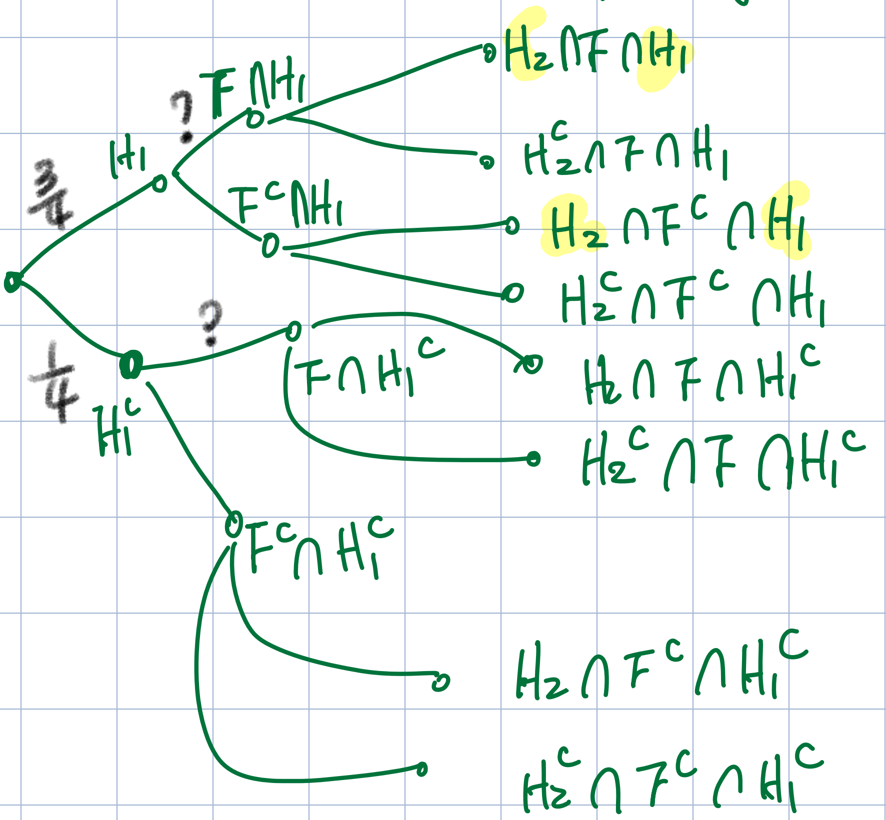
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + 1 \times 1 \times \frac{1}{2} = 0.625.$$

$$\therefore \frac{P(H_1 \cap H_2)}{P(A_1)} = \frac{0.625}{0.75} = P(H_2 | H_1)$$

However, if we decompose the event in a different order?

PC

Consider the following decomposition:



① H_1 vs H^c

② F vs F^c

③ H_2 vs H_2^c

based on this decomposition:

$$P(H_2 | H_1) = P(H_2 \cap F \cap H_1) + P(H_2 \cap F^c \cap H_1)$$

(which is the same summation as we see with the first decomposation) based on the new tree.

$$= P(H_2 \mid F \cap H_1)\, \underline{P(F \mid H_1)}\, P(H_1)$$

$$+ P(H_2 \mid F^c \cap H_1)\, \underline{P(F^c \mid H_1)}\, P(H_1)$$

we know: $P(H_2 \mid F \cap H_1) = \frac{1}{2}$

$P(H_2 \mid F^c \cap H_1) = 1$; $P(H_1) = \frac{3}{4}$

But we do not know

$P(F \mid H_1)$ $P(F^c \mid H_1)$

? can we compute these values?

Yes! $\rightarrow$ Bayes (later)