Problem 1 (10 pt)

$$P(\text{accepted}) = \frac{96}{100} \times \frac{95}{99} \times \frac{94}{98} \times \frac{93}{97} \times \frac{92}{96} = 0.8119$$

Problem 2 (10 pt)

$$P(C_1^c) = P(C_2^c) = p$$
$$P(C_1) = P(C_2) = 1-p$$

$$P(C \mid C_1 \cap C_2) = 1$$
$$P(C \mid C_1^c \cap C_2) = \frac{1}{2}$$
$$P(C \mid C_1 \cap C_2^c) = \frac{1}{2}$$
$$P(C \mid C_1^c \cap C_2^c) = 0$$

$$P(C) = P(C \mid C_1 \cap C_2) * P(C_1 \cap C_2) + P(C \mid C_1^c \cap C_2) * P(C_1^c \cap C_2) + P(C \mid C_1 \cap C_2^c) * P(C_1 \cap C_2^c) + P(C \mid C_1^c \cap C_2^c) * P(C_1^c \cap C_2^c)$$
$$= 1 \times (1-p)^2 + \frac{1}{2} \times p(1-p) + \frac{1}{2} \times p(1-p) + 0 \times p^2$$
$$= 1-p$$

same as $P(C_1)$ or $P(C_2)$

Problem 3 (20 pt)

(a) $P(\text{defective} \mid A) = 0.001$ $\quad P(A) = P(B) = P(C) = \frac{1}{3}$

$P(\text{defective} \mid B) = 0.004$

$P(\text{defective} \mid C) = 0.01$

$$P(\text{defective}) = P(\text{defective} \mid A) * P(A) + P(\text{defective} \mid B) * P(B) + P(\text{defective} \mid C) * P(C)$$
$$= 0.001 \times \frac{1}{3} + 0.004 \times \frac{1}{3} + 0.01 \times \frac{1}{3}$$
$$= 0.005 \quad (4\text{ pt})$$

(b) $$P(A \mid \text{defective}) = \frac{P(\text{defective} \mid A) * P(A)}{P(\text{defective})} = \frac{0.001 \times \frac{1}{3}}{0.005} = 0.0667 \quad (2\text{pt})$$

$$P(B \mid \text{defective}) = \frac{P(\text{defective} \mid B) * P(B)}{P(\text{defective})} = \frac{0.004 \times \frac{1}{3}}{0.005} = 0.2667 \quad (2\text{pt})$$

$$P(C \mid \text{defective}) = \frac{P(\text{defective} \mid C) * P(C)}{P(\text{defective})} = \frac{0.01 \times \frac{1}{3}}{0.005} = 0.6667 \quad (2\text{pt})$$

(c) $P(A) = 0.5$ $\quad P(B) = 0.4$ $\quad P(C) = 0.1$

$$P(\text{defective}) = P(\text{defective} \mid A) * P(A) + P(\text{defective} \mid B) * P(B) + P(\text{defective} \mid C) * P(C)$$

$= 0.001 \times 0.5 + 0.004 \times 0.4 + 0.01 \times 0.1$

$= 0.0031$ (4pt)

(d) $P(A \mid \text{defective}) = \frac{P(\text{defective} \mid A) * P(A)}{P(\text{defective})} = \frac{0.001 \times 0.5}{0.0031} = 0.1613$ (2pt)

$P(B \mid \text{defective}) = \frac{P(\text{defective} \mid B) * P(B)}{P(\text{defective})} = \frac{0.004 \times 0.4}{0.0031} = 0.5161$ (2pt)

$P(C \mid \text{defective}) = \frac{P(\text{defective} \mid C) * P(C)}{P(\text{defective})} = \frac{0.01 \times 0.1}{0.0031} = 0.3226$ (2pt)

Problem 4 (10pt)

(a) $P(S_1) = \frac{13}{52} = \frac{1}{4} = 0.25$ (5pt)

(b) $P(S_2) = P(S_2 \mid S_1) * P(S_1) + P(S_2 \mid S_1^c) * P(S_1^c)$

$= \frac{12}{51} \times \frac{1}{4} + \frac{13}{51} \times \frac{3}{4}$

$= \frac{1}{4} = 0.25$ (5pt)