

1. Because we need more nonterminals than the number of letters between S and Z, we just continue using nonterminals A and B.

$$S \rightarrow \text{array } T \text{ of } Y;$$

$$T \rightarrow [U] \mid V$$

$$U \rightarrow U, W \mid W$$

$$V \rightarrow V[W] \mid [W]$$

$$W \rightarrow X..X \mid Y..Y \mid Z..Z \mid A..A$$

$$X \rightarrow \text{true} \mid \text{false}$$

$$Y \rightarrow Ya \mid Yb \mid Yc \mid a \mid b \mid c$$

$$Z \rightarrow 'x' \mid 'y' \mid 'z'$$

$$A \rightarrow +B \mid -B \mid B$$

$$B \rightarrow B0 \mid B1 \mid B2 \mid B3 \mid B4 \mid B5 \mid B6 \mid B7 \mid B8 \mid B9 \mid 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$$

2. a.  $S \rightarrow aaSbb \mid \epsilon$

b.  $S \rightarrow TW$

$$T \rightarrow Uc$$

$$U \rightarrow aUcc \mid V$$

$$V \rightarrow Vb \mid \epsilon$$

$$W \rightarrow Wd \mid \epsilon$$

c.  $S \rightarrow aSa \mid bSb \mid aa \mid bb$

- d. Rewrite the language as  $a^m b^n a^n a^m$ , which makes it easier to see how the grammar can be written.

$$S \rightarrow aSa \mid T$$

$$T \rightarrow bTa \mid ba$$

- e. Rewrite the language as  $a^{n-m} a^m b^m a^{n-m}$ , which makes it easier to see how the grammar can be written.

$$S \rightarrow aSa \mid T$$

$$T \rightarrow aTb \mid \epsilon$$

f.  $S \rightarrow (S)S \mid \epsilon$

g.  $S \rightarrow aT \mid bU \mid \epsilon$

$$T \rightarrow bS \mid aTT$$

$$U \rightarrow aS \mid bUU$$

h.  $S \rightarrow UT \mid TV$

$$T \rightarrow aTb \mid \epsilon$$

$$U \rightarrow Ua \mid a$$

$$V \rightarrow Vb \mid b$$

i.  $S \rightarrow aSd \mid aTc \mid bUd \mid bVc \mid \epsilon$

$$T \rightarrow aTc \mid bVc \mid \epsilon$$

$$V \rightarrow bVc \mid \epsilon$$

$$U \rightarrow bUd \mid bVc \mid \epsilon$$

3.  $S \rightarrow T \mid V$

$$T \rightarrow UU$$

$$U \rightarrow aUb \mid ab$$

$$V \rightarrow aVb \mid aWb$$

$$W \rightarrow bWa \mid ba$$

To show the grammar is ambiguous show two leftmost derivations, or two rightmost derivations, or two parse trees, for a string such as  $abab$ . Here are two leftmost derivations:

$$S \Rightarrow T \Rightarrow UU \Rightarrow abU \Rightarrow abab$$

$$S \Rightarrow V \Rightarrow aWb \Rightarrow abab$$

You can construct the corresponding parse trees.