1. For each of the following problems construct a deterministic finite automaton that describes or recognizes the language given. Write all DFAs in the form of a transition diagram. The underlying alphabet is $\Sigma = \{a, b\}$.

   Be sure to give a DFA for each part, and not an NFA unless the problem asks for one. Do **not** use any of the notational conveniences or shortcuts shown in lecture.

   The notation $\#a(w)$ is used below to refer to the number of a's occurring in the string w. For example, $\#a(bbaba) = 2$.

   Note: the first seven parts are the languages from the second question in the prior practice problem set. It would be instructive to compare the DFA and the regular expression for these languages.

   a. $\{\, w \mid w \text{ begins with } abab \,\}$

   b. $\{\, w \mid w \text{ ends with } abab \,\}$

   c. $\{\, w \mid w \text{ begins with } ab \text{ and ends with } ba \,\}$
      Note: The string aba is in this language.

   d. $\{\, w \mid \#a(w) \equiv 2 \pmod 5 \,\}$
      Recall that $i \equiv j \pmod k$ if and only if $i - j$ is divisible by k.

   e. $\{\, w \mid \#a(w) \text{ is even or } |w| \text{ is even} \,\}$

   f. $\{\, w \mid aaa \text{ is a substring of } w \,\}$

   g. $\{\, w \mid aaa \text{ is } \textbf{not} \text{ a substring of } w \,\}$

   h. $\{\, w \mid w \text{ contains exactly one occurrence of the substring } aaa \,\}$
      Note: the string $aaaa$ has two occurrences of aaa.

   i. $\{\, w \mid \text{either } \#a(w) \text{ is divisible by 3 or } w \text{ begins with } bbb \,\}$
      Also write a regular expression that describes or recognizes this language.

   j. $\{\, w \mid \#a(w) \equiv 1 \pmod 3 \text{ and } \#b(w) \text{ is odd} \,\}$

   k. $\{\, w \mid \text{neither } aa \text{ nor } bb \text{ is a substring of } w \,\}$

   l. Write an NFA for the language in part #1i.

2. Consider the following language:

   $\{\, w \mid w \in \{0, 1\}^* \text{ and } w \text{ contains an even number of 0s, and } w \text{ does not contain three consecutive 1s} \,\}$

   Determine whether each of the following DFAs correctly describes or recognizes this language or not. Identify why each incorrect DFA is wrong– give a string that the DFA doesn't give the right results for, and identify what result the DFA should give for that string, and what result it actually gives.

a.

1 1 1

0 0

0

dead
state

0

1

b.

1 1 1

0 0

1

0

dead
state

0

1

c.

1 1 1

0 0 1 0 0 1 0 0 1

dead
state

0

1

d.

0

1 1 1

0

0

1

0

1

dead
state

0

1

0

0

e.

f.

g.

h.

3