CMSC 330 Formal languages and regular expressions Fall 2012

1. Consider the languages or sets of strings A $= \{ a,\ aa,\ aaa \}$ and B $= \{ bb \}$. Show the languages denoted by each of the following:

   a. A^1

   b. A^2

   c. $A \cup A^2$

   d. A^*

   e. B^1

   f. B^2

   g. B^3

   h. $B \cup B^2 \cup B^3$

   i. B^*

   j. $(AB)^2$

   k. $(A \cup B)^2$

2. Write a formal regular expression (**not** a Ruby regular expression) that describes or recognizes each of the following languages. Formal regular expressions may use **only** the three operations concatenation, alternation, and Kleene closure, as defined in lecture. Use ϵ to denote the empty string. The underlying alphabet for each part is $\Sigma = \{a,\ b\}$.

   The notation $\#a(w)$ is used below to refer to the number of a's occurring in the string w. For example, $\#a(bbaba) = 2$.

   a. $\{ w \mid w$ begins with $abab \}$

   b. $\{ w \mid w$ ends with $abab \}$

   c. $\{ w \mid w$ begins with ab and ends with $ba \}$
   Note: The string aba is in this language.

   d. $\{ w \mid \#a(w) \bmod 5 = 2 \}$
   Recall that $i \bmod k = j$ if and only if $i - j$ is divisible by k.

   e. $\{ w \mid \#a(w)$ is even or $|w|$ is even $\}$

   f. $\{ w \mid aaa$ is a substring of $w \}$

   g. $\{ w \mid aaa$ is **not** a substring of $w \}$

3. Consider the following language:

   $\{ w \mid w \in \{0,\ 1\}^*$ and w contains an even number of 0s, and w does not contain three consecutive 1s $\}$

   Determine whether each of the following regular expressions correctly describes or recognizes this language or not. Identify why each incorrect regular expression is wrong– give a string that the regular expression doesn't give the right results for, and identify what result the regular expression should give for that string, and what result it actually gives.

   a. $\big(0\,(\epsilon|1|11)\,0\big)^*$

   b. $\big((0\,(\epsilon|1|11)\,0)^* \mid 1 \mid 11\big)$

   c. $\big((0\,(\epsilon|1|11)\,0)^* \mid 1 \mid 11\big)^*$

   d. $\Big(\big((\epsilon|1|11)\,0\,(\epsilon|1|11)\,0\,(\epsilon|1|11)\big)^* \mid 1 \mid 11\Big)$

   e. $\Big(\big((\epsilon|1|11)\,0\,(\epsilon|1|11)\,0\big)^* \mid 1 \mid 11\Big)$

   f. $(\epsilon|1|11)\,\big(0\,(\epsilon|1|11)\,0\big)^*\,(\epsilon|1|11)$

   g. $\big((\epsilon|1|11)\,0\,(\epsilon|1|11)\,0\big)^*\,(\epsilon|1|11)$

   h. $\big((0 \mid 01 \mid 011 \mid 10 \mid 101 \mid 1011 \mid 110 \mid 1101 \mid 11011)\,0\big)^*\,(\epsilon|1|11)$