

ASTR 792 HW 11

Craig Brooks

November 2023

15.10

$$\begin{aligned}d &= 2.6 \text{ pc} \\T_{eff} &= 25200 \text{ K} \\R &= .0081 R_{\odot} \\R_{\odot} &= 6.96 \cdot 10^8 \text{ m} \\\sigma_{sb} &= 5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}\end{aligned}$$

a

The luminosity of Sirius B is

$$\begin{aligned}L &= 4\pi R^2 \sigma_{sb} T^4 \\&= 4\pi (.0018 \cdot 6.96 \cdot 10^8 \text{ m})^2 (5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}) (25200 \text{ K})^4 \\&= 9.1323 \cdot 10^{24} \text{ W}\end{aligned}$$

b

The rate of H Ionizing photons is

$$\dot{N} = \frac{L}{\langle h\nu \rangle}$$

Let $\langle h\nu \rangle = 2.71kT = 2.71 \cdot 1.38 \cdot 10^{-23} \text{ JK}^{-1} \cdot 25299 \text{ K} = 9.42 \cdot 10^{-18} \text{ J}$.
therefore

$$\begin{aligned}\dot{N} &= \frac{9.1323 \cdot 10^{24} \text{ W}}{9.42 \cdot 10^{-18} \text{ J}} \\&= 9.49 \cdot 10^{41} \text{ photons s}^{-1}\end{aligned}$$

Since 42.7% of photons have $I_H < \langle h\nu \rangle$, then

$$\begin{aligned}Q_0 &= .427 \dot{N} = .427 * 9.49 \cdot 10^{41} \text{ photons s}^{-1} \\&= 4.05 \cdot 10^{41} \text{ photons s}^{-1}\end{aligned}$$

c

The Stromgren radius is defined as

$$R_s \equiv \left(\frac{3Q}{4\pi\alpha n_H^2} \right)^{1/3}$$

Here, we can use Q_0 we calculated above. To calculate α_B , we use

$$\begin{aligned} \alpha_B &= 2.56 \cdot 10^{-13} T_4^{-.83} \text{ cm}^3 \text{ s}^{-1} \\ &= 2.56 \cdot 10^{-13} (7000 \text{ K})^{-.83} \text{ cm}^3 \text{ s}^{-1} \\ &= 1.65 \cdot 10^{-16} \text{ cm}^3 \text{ s}^{-1} \end{aligned}$$

Therefore

$$\begin{aligned} R_s &\equiv \left(\frac{3 \cdot 4.05 \cdot 10^{41} \text{ photons s}^{-1}}{4\pi \cdot 1.65 \cdot 10^{-16} \text{ cm}^3 \text{ s}^{-1} \cdot (.05 \text{ cm}^{-3})^2} \right)^{1/3} \\ &= 6.17 \cdot 10^{19} \text{ cm} \end{aligned}$$

d

$$\Delta R = \frac{1}{n_H \sigma_{p.i.}}$$

We can calculate $\sigma_{p.i.}$ by

$$\begin{aligned} \sigma_{p.i.} &= 6.304 \cdot 10^{-18} Z^{-2} \text{ cm}^2 \left(\frac{h\nu}{Z^2 I_H} \right) \\ \rightarrow n_H \sigma_{p.i.} &= .05 \cdot 6.304 \cdot 10^{-18} \cdot .23 \\ &= 7.25 \cdot 10^{-20} \text{ cm}^{-1} \\ \rightarrow \Delta R &= 1.38 \cdot 10^{19} \text{ cm} \end{aligned}$$

where $Z = 1$, $I_H = 2.18 \cdot 10^{-18} \text{ J}$, and $h\nu = 9.42 \cdot 10^{-18} \text{ J}$