ASTR HW 6

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Due: October 24 2023

16.2

In general,

$$\frac{\text{collisional de-excitations}}{\text{radiative decay}} = \frac{n_{crit,u}(c)}{n}$$

$$\rightarrow \text{collisional de-excitations} = \frac{n_{crit,u}(c)}{n} \cdot \text{radiative decay} = n_{crit,u}(c)$$

The total number of excitations will be the sum of the collisional excitation and radiative decays so

$$\text{total excitations} = n_{crit,u}(c)n + n$$

So the fraction of followed by decay will be

$$f = \frac{n}{n_{crit,u}(c) + n}$$
$$= \frac{1}{\frac{n_{crit,u}(c)}{n} + 1}$$

This is a 2 level system, so $J = 1 \rightarrow 0$

$$n_{crit,u}(c) = \frac{\sum_{l<u}[1 + (n_\gamma)_{10}]A_{ul}}{\sum_{l<u} k_{10}(c)}$$

Since this is a single partner system and we assume there is no radiation present, this becomes

$$n_{crit,u}(c) = \frac{A_{10}}{k_{10}(c)}$$

Here, we can use define the decay coefficient $k_{01} = \frac{g_1}{g_0} k_{10} e^{-(E_1 - E_0)/kT}$. Therefore,

$$n_{crit,u}(c) = \frac{A_{10}}{\frac{g_1}{g_0} k_{10} e^{-(E_1 - E_0)/kT}}$$
$$= \frac{g_0 A_{10}}{g_1 k_{10}} e^{(E_1 - E_0)/kT}$$

The degeneracies for the 2 level system are $g_j = 2j+1$, so we get $g_0 = 1$ and $g_1 = 5$. The energy levels are $E_j = J(J+1)\hbar^2/2mr^2$, so $E_0 = 0$ and $E_1 = \hbar^2/mr^2$, so

$$= \frac{A_{10}}{5k_{10}} e^{\hbar^2/mr^2kT}$$

Therefore

$$f = \frac{n}{n_{crit,u}(c) + n}$$
$$= \frac{1}{\frac{A_{10}}{5k_{10}} e^{\hbar^2/mr^2kT} + 1}$$