ASTR 792 HW 2

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Let

$$a = \text{radius of dust}$$
$$r_H = \text{radius of hydrogen atom} = \text{Bohr radius} = 5.3 \cdot 10^{-9}\ cm$$
$$\text{n}_H = \text{the number density of hydrogen}$$
$$T = \text{the temperature of the hydrogen gas}$$
$$\text{a} \gg r_H$$
$$\text{m}_{dust} \gg m_H$$

2a

$$< v_H >= \sqrt{8\frac{k_b T}{\pi m_H}}$$
$$= \sqrt{\frac{8 \cdot 10^{-22} J/K \cdot T}{7\pi \cdot 1.67 \cdot 10^{-27} kg}}$$

2b

Let τ_m be the mean free path for a hydrogen atom

$$\tau_m = \frac{1}{n_b \sigma_{AB}}$$
$$\sigma = \pi(a + r_H)^2$$
$$\rightarrow \tau_m = \frac{1}{n_H \pi (a + r_H)^2}$$

Letting t be the time for a grain to be hit by a single hydrogen atom, then we can find the time to encounter a hydrogen atom by taking the mean free path τ_m and dividing by the velocity of the hydrogen

$$t = \frac{\tau_m}{< v_H >}$$
$$= \frac{1}{< v_H > n_H \cdot \pi (a + r_H)^2}$$

Since this is the mean free path for a single collision, we need to determine the number of collisions it would take to accumulate the mass of a dust grain. we can calculate the mass of the dust particle by

$$N_H = \frac{m_{dust}}{m_H}$$
$$\rightarrow t_{total} = \frac{m_{dust}}{m_H} \frac{1}{< v_H > n_H \cdot \pi(a + r_H)^2}$$

2c

a $= 10^{-5}\ cm$
$n_H = 30cm^{-3}$
$\rho_{dust} = 3g\ cm^{-3}$
$T = 10^2\ K$

c1

$$< v_H >= \sqrt{\frac{8 \cdot 10^{-22} J/K \cdot 10^2\ K}{7\pi \cdot 1.67 \cdot 10^{-27} kg}}$$
$$< v_H > \approx 1.5 \cdot 10^3\ m/s$$
$$= 1.5 \cdot 10^5\ cm/s$$

c2

First, let's calculate the mass of a single dust grain

$$\rho \cdot V = m_{dust}$$
$$\rightarrow 3\ g\ cm^3 \cdot \frac{4\pi}{3}(10^{-5}\ cm)^3 = m_{dust}$$
$$\rightarrow m_{dust} = 4\pi \cdot 10^{-5}\ g$$

Now we can divide this mass by the mass of a hydrogen atom to obtain the number N_H of atoms necessary to equal the mass of a single dust grain

$$\rightarrow N_H = \frac{m_{dust}}{m_H}$$
$$N_H = \frac{4\pi \cdot 10^{-5}\ g}{1.67 \cdot 10^{-25} g}$$
$$= 7.5 \cdot 10^{20}\ H\ atoms\ per\ dust\ grain$$

Now we can plug this and 2c1 into the result from 2b

$$\begin{aligned}\rightarrow t_{total} &= \frac{m_{dust}}{m_H}\frac{1}{< v_H > n_H \cdot \pi(a+r_H)^2} \\ &= 7.5 \cdot 10^{20}\frac{1}{(1.5 \cdot 10^5\ cm/s)(30\ cm^3)\pi(10^{-5}\ cm + 5.3 \cdot 10^{-9}\ cm)^2} \\ &= 5.3 \cdot 10^{23}\ s \\ &\approx 1.7 \cdot 10^{16}\ years\end{aligned}$$