ASTR 792 HW 11

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December 2023

14.2a

The transition probability in general can be written as

$$P(3p \rightarrow 1s) = A_{3p->1s}\delta t$$

The two possible transitions from the $3p$ state are

$$\begin{aligned} &3p \rightarrow 2s \qquad A_{3p->2s} = 2.245 \cdot 10^{7}\ s^{-1} \\ &3p \rightarrow 1s \qquad A_{3p->1s} = 1.672 \cdot 10^{8}\ s^{-1} \end{aligned}$$

with wavelengths

$$\begin{aligned} H\alpha: \ 3p \rightarrow 2s \qquad \lambda_{3p->2s} = 656.46\ nm \\ Ly\beta: \ 3p \rightarrow 1s \qquad A_{3p->1s} = 102.57\ nm \end{aligned}$$

The transition probability from $E_u \rightarrow E_l$ is

$$p(E_u \rightarrow E_l) = A_{ul} \cdot \Delta t$$

The transition time can be computed from the frequency using the Rydberg formula. From the two possibilities above

$$\begin{aligned} \nu_{3p\rightarrow 2s} = \frac{\Delta E}{h} = \frac{13.6\ eV}{h}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 4.56 \cdot 10^{14}\ Hz \rightarrow \Delta t = 2.19 \cdot 10^{-15}\ s \\ \rightarrow p(E_{3p} \rightarrow E_{2s}) = 2.245 \cdot 10^{7}\ s^{-1} \cdot 2.19 \cdot 10^{-15}\ s \\ = 4.92 \cdot 10^{-8} \end{aligned}$$

$$\begin{aligned} \nu_{3p\rightarrow 1s} = \frac{\Delta E}{h} = \frac{13.6\ eV}{h}\left(1 - \frac{1}{3^2}\right) = 2.92 \cdot 10^{15}\ Hz \rightarrow \Delta t = 3.42 \cdot 10^{-16}\ s \\ \rightarrow p(E_{3p} \rightarrow E_{1s}) = 1.672 \cdot 10^{8}\ s^{-1} \cdot 3.42 \cdot 10^{-16}\ s \\ = 5.54 \cdot 10^{-8} \end{aligned}$$

Therefore the probability of getting a $Ly\beta$ photon is

$$
\begin{aligned}
P(3p \to 1s) &= \frac{A_{3p\to 1s} \cdot \Delta t_{3p\to 1s}}{\sum A_{3p\to l} \cdot \Delta t_{3p\to l}} \\
&= \frac{4.92 \cdot 10^{-8}}{4.92 \cdot 10^{-8} + 5.54 \cdot 10^{-8}} \\
&= .47
\end{aligned}
$$