ASTR 792 HW 6

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19.2

Let the partition function

$$Z_{rot} = \sum_{J=0}^{\infty} 2(J+1)e^{-J(J+1)B_0/kT_{exc}}$$

We can transform this into an integral by integrating over J

$$Z_{rot} = \int_{J=0}^{\infty} 2(J+1)e^{-J(J+1)B_0/kT_{exc}}\ dJ$$

Let $u = -J(J+1)B_0/kT_{exc} \rightarrow du = -2(J+1)B_0/kT_{exc}\, dJ$. This transforms the integral into

$$\begin{aligned} Z_{rot} &= -\frac{kT_{exc}}{B_0}\int_{J=0}^{\infty} e^u\ du \\ &= -\frac{kT_{exc}}{B_0}e^{-J(J+1)B_0/kT_{exc}}\Big|_0^{\infty} \\ &= \frac{kT_{exc}}{B_0} \end{aligned}$$

In the limit $kT_{exc} \gg B_0$, then Z_{rot} becomes kT_{exc}