ASTR 792 HW 10

Craig Brooks

November 2023

24.3

a

$$Q_{abs} \propto \lambda^{-2}, \lambda > 1\mu m$$
$$\lambda_{max} = 140 \cdot 10^{-6}\ m$$
$$T_{ss} = 18\ K$$

Assume $\langle Q_{abs} \rangle \propto \lambda^{-2}$. so

$$\langle Q_{abs} \rangle \propto \left(\frac{\lambda}{\lambda_0}\right)^{-\beta} \propto \left(\frac{T_{ss}}{T_g}\right)^{-\beta}$$

Therefore, $\beta = 2$.
At equilibrium,

$$U = \frac{\lambda I_\lambda}{\lambda I'_\lambda} \propto \left(\frac{T_{ss}}{T_g}\right)^{-\beta}$$

Therefore

$$U = \left(\frac{T_{grain}}{T_{ss}}\right)^{\beta}$$
$$\sqrt{U} = \frac{T_g}{18\ K}$$
$$\rightarrow T_g = 18\ K \cdot \sqrt{U}$$

b

Given

$$U = \frac{\lambda I_\lambda}{\lambda I'_\lambda} = 10^3$$

By Wien's displacement law, and $T = \sqrt{U} \cdot 18\ K$

$$\begin{aligned}
\lambda_{max} &= \frac{.0029\ m \cdot K}{T} \\
&= \frac{.0029\ m \cdot K}{(10^{3/2})(18\ K)} \\
&= 5.1 \cdot 10^{-6}\ m \\
&= 5.1\ \mu m
\end{aligned}$$