ASTR HW 8

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Due: November 7 2023

33.1

Let

$$n(H) = 10^2\ cm^{-3} \quad n(H_2) = 50\ cm^{-3}$$

$$.3n(C)(250\ ppm) = n(C^+) \approx 7.5 \cdot 10^{-5}\ n(H) = 7.5 \cdot 10^{-3} cm^{-3}$$

$$n(e) \approx 10^{-4}\ n(H) = .01\ cm^{-3}$$

$$\frac{n(O)}{n(H)} = .04\ cm^{-3}$$

$$T_2 \equiv \frac{T}{100\ K}$$

We will also assume that $n(h\nu) = n(e)$

$\mathbf{CH_2^+}$

Since the molecular cloud is dominated by primarily molecular hydrogen, (H_2), we can consider interactions

$$C^+ + H_2 \rightarrow CH_2^+ + h\nu \qquad k_{33.6} = 5 \cdot 10^{-6}\ T_2^{-.20}\ cm^3\ s^{-1}$$
$$CH_2^+ + e^- \rightarrow CH + H_2 \qquad k_{33.7} = 1.24 \cdot 10^{-6}\ T_2^{-.60}\ cm^3\ s^{-1}$$

the creation rate for CH_2^+ is

$$\text{Formation rate of } n(CH_2^+) = k_{33.6} \cdot n(C^+) \cdot n(H_2)$$

the destruction rate for CH_2^+ is

$$\text{destruction rate of } n(CH_2^+) = k_{33.7} \cdot n(CH_2^+) \cdot n(e^-)$$

Therefore

$$k_{33.6} \cdot n(C^+) \cdot n(H_2) = k_{33.7} \cdot n(CH_2^+) \cdot n(e^-)$$
$$n(CH_2^+) = \frac{k_{33.6} \cdot n(C^+) \cdot n(H_2)}{k_{33.7} \cdot n(e^-)}$$
$$n(CH_2^+) = \frac{5 \cdot 10^{-6}\ T_2^{-.20}\ cm^3\ s^{-1} \cdot 7.5 \cdot 10^{-3} cm^{-3} \cdot 50\ cm^{-3}}{1.24 \cdot 10^{-6}\ T_2^{-.60}\ cm^3\ s^{-1} \cdot .01\ cm^{-3}}$$
$$= 151\ T^{40}\ cm^{-3}$$

CH

As above. the formation rate can be obtained by

$$CH_2^+ + h\nu \to CH + H^+ \qquad k_{33.11} = 1.38 \cdot 10^{-10}\ cm^3\ s^{-1}$$
$$CH_2^+ + e^- \to CH + H_2 \qquad k_{33.7} = 1.24 \cdot 10^{-6}\ T_2^{-.60}\ cm^3\ s^{-1}$$
$$\to \text{Formation rate of } CH = k_{33.11} \cdot n(CH) \cdot n(h\nu) + k_{33.7} \cdot n(CH) \cdot n(e^-)$$

The destruction rate is

$$\to CH + h\nu \to \begin{cases} CH^+ + e^- \quad (47\%) \\ C + H \quad (53\%) \end{cases} \qquad k_{33.13} = 1.62 \cdot 10^{-9}\ cm^3\ s^{-1}$$
$$\text{destruction rate of } n(CH) = k_{33.13} \cdot n(CH) \cdot n(h\nu)$$

Therefore

$$k_{33.11} \cdot n(CH) \cdot n(h\nu) + k_{33.7} \cdot n(CH) \cdot n(e^-) = k_{33.13} \cdot n(CH) \cdot n(h\nu)$$
$$n(CH) = \frac{k_{33.11} \cdot n(CH_2^+ \cdot n(h\nu) + k_{33.7} \cdot n(CH_2^+) \cdot n(e^-)}{k_{33.13} \cdot n(h\nu)}$$
$$= \frac{1.38 \cdot 10^{-10}\ cm^3\ s^{-1} \cdot .01\ cm^{-3} + 1.24 \cdot 10^{-6}\ T_2^{-.60}\ cm^3\ s^{-1} \cdot .01\ cm^{-3}}{1.62 \cdot 10^{-9}\ cm^3\ s^{-1} \cdot .01\ cm^{-3}} \cdot n(CH_2^+)$$
$$= 765.43\ T^{-60} \cdot n(CH_2^+)$$
$$= 1.16 \cdot 10^5 \cdot T^{-20}\ cm^{-3}$$

CO

The formation rate is

$$CH + O \to CO + H \qquad k_{33.8} = 6.6 \cdot 10^{-11}\ cm^3\ s^{-1}$$
$$\text{creation rate of } n(CO) = k_{33.8} \cdot n(CH) \cdot n(O)$$

The rate of destruction is

$$CO + h\nu \to C + O \qquad k_{33.9} = 2.3 \cdot 10^{-10}\ cm^3\ s^{-1} \cdot f_{shield}(CO)$$
$$\text{destruction rate of } n(CO) = k_{33.9} \cdot n(CO) \cdot n(h\nu)$$

Therefore

$$k_{33.8} \cdot n(CH) \cdot n(O) = n(CO) = k_{33.9} \cdot n(CO) \cdot n(h\nu)$$

$$n(CO) = \frac{k_{33.8} \cdot n(CH) \cdot n(O)}{k_{33.9} \cdot n(h\nu)}$$

$$\frac{6.6 \cdot 10^{-11}\ cm^3\ s^{-1} \cdot .04\ cm^{-3}}{2.3 \cdot 10^{-10}\ cm^3\ s^{-1} \cdot f_{shield}(CO) \cdot .01\ cm^{-3})} \cdot n(CH)$$

$$= \frac{1.15}{f_{shield}(CO)} \cdot n(CH)$$

$$= \frac{1.33}{f_{shield}(CO)} \cdot 10^5\ T^{-20}\ cm^{-3}$$