### 1(a) Carry out exploratory data analysis for the response variable with the predictors provided.

wage\_status ~ maritl:

```
Pearson's Chi-squared test

1. Never Married 2. Married 3. Widowed 4. Divorced 5. Separated
0 446 748 9 104 36 data: earnings$wage_status and earnings$marit1
1 202 1326 10 100 19 X-squared = 227.98, df = 4, p-value < 2.2e-16
```

The Chi-squared test statistic has p < 0.0001, which indicates an association between wage\_status ~ maritl.

wage\_status ~ race:

```
Pearson's Chi-squared test

1. White 2. Black 3. Asian 4. Other

0 1072 172 73 26 data: earnings$wage_status and earnings$race
1 1408 121 117 11 X-squared = 38.224, df = 3, p-value = 2.534e-08
```

The Chi-squared test statistic has p < 0.0001, which indicates an association between wage\_status ~ race.

wage\_status ~ education:

```
1. < HS Grad 2. HS Grad 3. Some College 4. College Grad 5. Advanced Degree 0 208 609 286 185 55 1 60 362 364 500 371

Pearson's Chi-squared test

data: earnings$wage_status and earnings$education X-squared = 505.86, df = 4, p-value < 2.2e-16
```

The Chi-squared test statistic has p < 0.0001, which indicates an association between wage\_status  $\sim$  education.

wage status ~ health:

The Chi-squared test statistic has p < 0.0001, which indicates an association between wage\_status  $\sim$  health.

wage\_status ~ year:

```
Pearson's Chi-squared test

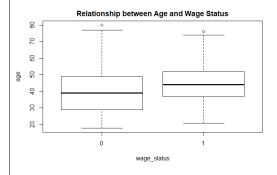
2003 2004 2005 2006 2007 2008 2009

0 262 224 202 166 173 167 149 data: earnings$wage_status and earnings$year

1 251 261 245 226 213 221 240 X-squared = 16.644, df = 6, p-value = 0.01068
```

The Chi-squared test statistic has p > 0.0001, which indicates **NO** association between wage\_status  $\sim$  year.

wage\_status ~ age:



The boxplots of both categories in the wage\_status seems to have similar variances, though the boxplot of wage\_status(0) is slightly skewed to its lower whisker. There is just one outlier for both categories in wage\_status which is negligible.

### 1(b) Develop a GLM for wage status using the predictors provided.

Single Covariate Regression Models:

Model	Pr(>Chisq)	AIC
model_maritl	< 2.2e-16	3905.995
model_race	2.561e-08	4095.755
model_education	< 2.2e-16	3596.223
model_health	1.005e-11	4083.638
model_year	0.01055	4123.280
model_age	< 2.2e-16	4007.556

The p value for all the covariates are less than 0.2, with maritl, education and age being the most significant while education also has the least AIC score.

### Multivariate Regression Models:

Model	Covariates	AIC
model1	maritl, race, education, health, year, age	3334.571
model2	maritl, education, health, year, age	3342.419
model3	maritl, education, health, age	3351.729
model4	maritl, education, age	3366.290
model5	maritl, education	3390.285

The variables are checked for the significance level of 0.05 and different models are fitted. The lowest AIC score is achieved by model 1.

### 1(c) Write down your final model equation.

$$\label{eq:loss} \begin{split} &\text{Log}\,\frac{\pi_i}{1-\pi_i} = \text{- }3.671683 \, (\text{Intercept}) + 1.210894 \, (\text{maritl2. Married}) + 0.853295 \, (\text{maritl3. Widowed}) + 0.580225 \\ &(\text{maritl4. Divorced}) + 0.388077 \, (\text{maritl5. Separated}) - 0.460736 \, (\text{race2. Black}) - 0.325503 \, (\text{race3. Asian}) - 0.502806 \, (\text{race4. Other}) + 0.710168 \, (\text{education2. HS Grad}) + 1.619020 \, (\text{education3. Some College}) + 2.298950 \, (\text{education4. College Grad}) + 3.078956 \, (\text{education5. Advanced Degree}) + 0.381752 \, (\text{health2.}) = \text{Very Good}) + 0.146517 \, (\text{year2004}) + 0.315777 \, (\text{year2005}) + 0.486721 \, (\text{year2006}) + 0.237894 \, (\text{year2007}) + 0.375433 \, (\text{year2008}) + 0.652628 \, (\text{year2009}) + 0.023955 \, (\text{age}) \end{split}$$

### 1(d) Interpret the model parameters.

An individual's wage *does not* exceed \$100k per year if the individual's race is Black, Asian or Other.

The chances of an individual's wage exceeding \$100k per year is highest when that individual has an Advanced Degree. The College Graduates follow closely behind with more chances of earning more than \$100k per year.

The marital statuses except Never Married also have a nominal positive impact on an individual's wage exceeding \$100k per year. It is surprising to see that an individual being Married has a considerable positive impact on an individual's wage exceeding \$100k per year rather than an individual who is Never Married.

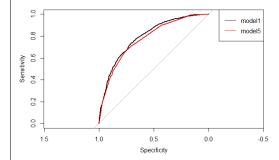
Except year 2003, the rest of the years does seem to be a token positive factor in an individual's wage exceeding \$100k per year, including the recession year 2008.

Again, an individual being in Very Good health only has little positive effect on the wage exceeding \$100k per year.

The age has a very negligible positive bearing on an individual's wage exceeding \$100k per year.

# 1(e) Obtain the ROC curve for your final model and the corresponding AUC value. Are the predictions from your final model better than tossing a coin?

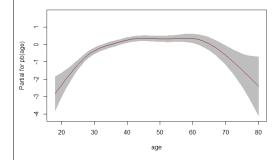
Model	<b>Area Under Curve (AUC)</b>
model1	0.7906
model2	0.7881
model3	0.7841
model4	0.7811
model5	0.7735



The ROC curve of model1 is almost near to sensitivity = specificity = 1 and the AUC is well above the minimum required 0.7 and is near 0.8 at 0.7906, indicating better predictive ability.

With model5 having the least AUC at 0.7735, model1 indeed produces predictions better than tossing a coin.

# 1(f) We are also interested in whether your final model can be improved by replacing the linear *age* term by including an additional smooth term. Using a GAM and its partial plot, investigate whether the smooth term of *age* is significant or not.



The linear part and smooth term of age looks significant.

# 2(a) Ignoring within-subject correlation, and using only semantic class, length of recipient, access of rec and length of theme as covariates, find a reasonable model for realization of recipient.

Single Covariate Regression Models:

Model	Pr(>Chisq)	AIC
model_semantic	0.1576	128.4834
model_recipient	2.134e-07	102.1925
model_access	2.648e-07	100.8120
model_theme	8.571e-07	104.8756

The p value for all the covariates are less than 0.2, with 'length of theme' being the most significant while 'access of recipient' has the least AIC score.

### Multivariate Regression Models:

Model	Covariates	AIC
model_a	semantic class, length of recipient, access of rec, length of theme	92.69076
model_b	length of recipient, access of rec, length of theme	86.83496
model_c	length of recipient, length of theme	87.29820

The variables are checked for the significance level of 0.05 and different models are fitted. The lowest AIC score is achieved by model\_b.

## 2(b) To model the correlation, add a random intercept for *speaker* i.e. the subject to your best model in (a).

## library(lme4)

model\_b\_cor <- glmer(dative\$realization\_of\_recipient ~ dative\$length\_of\_recipient + dative\$access\_of\_rec + dative\$length\_of\_theme + (1|dative\$speaker), family = binomial(), data = dative)

## 2(c) Compare your models in (a) and (b) using a model selection criterion.

Model	AIC
model_b	86.83496
model_b_cor	88.83496

The correlation model produces higher AIC score, indicating that the normal model is indeed the best model.