COMP2521 19T0
Week 2, Thursday: Trees!

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recursion
trees

Recursion

Recursive Linked Lists

A linked list can be described recursively!

```
struct node {
    Item item;
    node *next;
};
```

"... this value, and the rest of the values"

Recursive Linked Lists

Functions Amenable to Recursion (I)

```
size_t list_length (node *curr)
{
    if (curr == NULL) return 0;                 // base case
    return 1 + list_length (curr->next);    // recursive case
}
```

```
int int_list_sum (intnode *curr)
{
    if (curr == NULL) return 0;                 // base case
    return curr->item +
        int_list_sum (curr->next);              // recursive case
}
```

Recursive Linked Lists

Functions Amenable to Recursion (II)

```
void int_list_print (node *curr)
{
    if (curr == NULL) return;
    printf ("%d\n", curr->item);
    int_list_print (curr->next);
}

void int_list_print_reverse (node *curr)
{
    if (curr == NULL) return;
    int_list_print_reverse (curr->next);
    printf ("%d\n", curr->item);
}
```

Divide-and-Conquer, Recursively

REMINDER divide and conquer algorithms tend to:

- divide the input into parts,
- solve the problem on the parts recursively, then
- combine the results into an overall solution.

(This is a common ‘big-data’ approach: map-reduce.)

((“There’s no such thing as ‘big data’.”))

Divide-and-Conquer, Recursively

Maximum of an Unsorted Array (I)

Iteratively:

```
int array_max (int a[], size_t n)
{
    int max = a[0];
    for (size_t i = 0; i < n; i++)
        if (a[i] > max) max = a[i];
    return max;
}
```

complexity: $O(n)$

Divide-and-Conquer, Recursively

Maximum of an Unsorted Array (II)

Recursively:

```
int array_max (int a[], size_t l, size_t r)
{
    if (l == r) return a[l];
    int m = (l + r) / 2;
    int m1 = array_max (a, l, m);
    int m2 = array_max (a, m + 1, r);
    return (m1 < m2) ? m2 : m1;
}
```

complexity: ...

Divide-and-Conquer, Recursively

Maximum of an Unsorted Array (IIa)

How many calls of `array_max` are necessary?

for length 1, $c(1) = 1$
for length $n > 1$, $c(n) = c(\frac{n}{2}) + c(\frac{n}{2}) + 1$
... overall $c(n) = 2n - 1$ calls

in each recursive call, we do $O(1)$ steps.

$$\implies O(n)$$

Binary Search, Revisited

Recursive Binary Search (I)

Iteratively:

```
ssize_t binary_search (int a[], size_t n, int key)
{
    size_t lo = 0, hi = n - 1;
    while (hi >= lo) {
        size_t mid = (lo + hi) / 2;
        if (a[mid] == key) return mid;
        if (a[mid]  > key) hi = mid - 1;
        if (a[mid] <  key) lo = mid + 1;
    }
    return -1;
}
```

complexity: $O(\log n)$

Binary Search, Revisited

Recursive Binary Search (II)

Recursively:

```
ssize_t binary_search (int a[], size_t n, int key)
{
    return binary_search_do (a, 0, n - 1, key);
}

ssize_t binary_search_do (int a[], size_t lo, size_t hi, int key)
{
    if (lo > hi) return -1;
    size_t mid = (lo + hi) / 2;
    if (a[mid] == key) return mid;
    if (a[mid]  > key) return binary_search_do (a, lo, mid - 1, key);
    if (a[mid] <  key) return binary_search_do (a, mid + 1, hi, key);
    assert (!"unreachable");
}
```

complexity: $O(\log n)$

Trees

Searching

Search is a critical operation, e.g.

- looking up a name in a phone book
- selecting records in databases
- searching for pages on the web

Characteristics of the search problem:

- typically, very large amount of data (very many items)
- query specified by keys (search terms)
- effective keys identify a small proportion of data

Searching

The Search Problem (I)

We'll abstract the problem to:
a large collection of *items*,
each containing a *key* and other data
(We can think of these as
'key/data' or 'key/value' pairs.)

```
typedef ⟨···⟩ Key;
typedef struct {
    Key key;
    ⟨··· data ···⟩
}Item;
```

The search problem:

input a key value

output item(s) containing that key

Common variations:

- keys are unique; key value matches 0 or 1 items
- multiple keys in search, items containing any key
- multiple keys in search/item, items containing all keys

We assume: keys are unique, each item has one key.
Cheap, easy gains from searching sorted data.

Searching

The Search Problem (III)

Maintaining sorted sequences is hard...
inserting into a sorted sequence is a two-step problem.

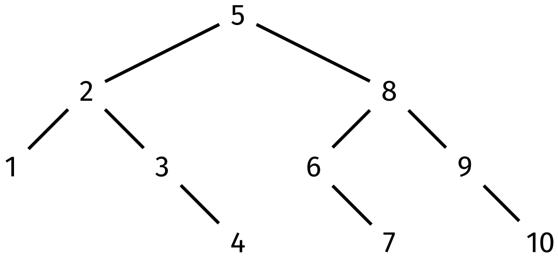
array — search $O(\log n)$, insert $O(n)$
... we have to move all the items along

linked list — search $O(n)$, insert $O(1)$
... search is *always* linear

Can we do better?

Search Trees

1	2	3	4	5	6	7	8	9	10



Trees

Terminology of Trees (I)

Trees are branched data structures,
consisting of nodes and edges, with no cycles.

Each node contains a value.
Each node has edges to $\leq k$ other nodes.
For now, $k = 2$ — binary trees

Trees can be viewed as a set of nested structures:
each node has k (possibly empty) subtrees.

Trees

Terminology of Trees (II)

A node is a parent if it has outgoing edges.
A node is a child if it has incoming edges.
The root node has no parents.
A leaf node has no children.

A node's depth or level is
the number of edges from the root to that node.
The root node has depth 0;
all other nodes have one more than their parent's depth

Trees

Terminology of Trees (III)

For a given number of nodes, a tree is said to be
balanced if it has minimal height, and
degenerate if it has maximal height.

A k-ary tree's internal nodes have k children.
A tree is ordered if data/keys in nodes are constrained.

Trees

Uses of Trees

- representing hierarchical data structures
  (e.g., expressions in a programming language)
- efficient search
  (e.g., in sets, symbol tables)

Binary Trees

For much of the course,
we'll look at binary trees (where $k = 2$).

Binary trees are either empty,
or are a node with two subtrees,
where each node has a value,
and the subtrees are binary trees.

$$\begin{array}{lcl} \text{BTree} & := & \text{Empty} \\ & \mid & \text{Node} \quad x \quad \text{BTree}\, l \quad \text{BTree}\, r \end{array}$$

Binary Trees

Properties

A binary tree with n nodes has a height of
at most $n - 1$, if degenerate; or
at least $\lfloor \log_2 n \rfloor$, if balanced.

Cost for insertion:
balanced $O(\log_2 n)$, degenerate $O(n)$
(we always traverse the height of the tree)

Cost for search/deletion:
balanced $O(\log_2 n)$, degenerate $O(n)$
(worst case, key $\notin \tau$; traverse the height)

Binary Search Trees

A binary tree!

For all nodes in the tree:
the values in the left subtree are less than the node value
the values in the right subtree are greater than the node value

Structure tends to be determined
by order of insertion:
$[4, 2, 1, 3, 6, 5, 7]$ vs $[6, 5, 2, 1, 3, 4, 7]$

Binary Search Trees

Exercise: Happy Little Trees

Starting with an initially-empty binary search tree ... show the tree resulting from inserting values in the order given, and give its resulting height —

1. $[4, 2, 6, 5, 1, 7, 3]$
2. $[5, 3, 6, 2, 4, 7, 1]$
3. $[1, 2, 3, 4, 5, 6, 7]$

Binary Search Trees

Implementation in C: The Type

```
struct btree_node {
    Item item;
    btree_node *left;
    btree_node *right;
};
```

As before: the empty tree is `NULL`.

Binary Search Trees

Implementation in C: Search

```
// return the node if found, or NULL otherwise
btree_node *btree_search (btree_node *tree, Item key)
{
    if (tree == NULL) return NULL;
    int cmp = item_cmp (key, tree->item);
    if (cmp == 0) return tree;
    if (cmp <  0) return btree_search (tree->left, key);
    if (cmp  > 0) return btree_search (tree->right, key);
}
```

EXERCISE Try writing an iterative version.

Binary Search Trees

Implementation in C: Insertion

We're (recursively) inserting value v into tree τ.

Cases:

- τ empty
  $\Rightarrow$ make a new node with v as the root of the new tree
- the root of τ contains v
  $\Rightarrow$ tree unchanged (assuming no duplicates)
- $v < \tau$`->item`
  $\Rightarrow$ do insertion into τ`->left`
- $v > \tau$`->item`
  $\Rightarrow$ do insertion into τ`->right`

EXERCISE Try writing an iterative version.

Binary Search Trees

Implementation in C: Dimensions

- `btree_size` $:: \mathrm{BTree} \to$ `size`
  return the number of nodes in a tree
- `btree_height` $:: \mathrm{BTree} \to$ `size`
  return the height of a tree

Binary Tree Traversals

‘serialisation’ of a structure:
flattening it in a well-defined way,
such that the original structure can be recovered

Depth-first:

- pre-order traversal (NLR)
  … visit node, then left subtree, then right subtree
- in-order traversal (LNR)
  … visit left subtree, then node, then right subtree
- post-order traversal (LRN)
  … visit left subtree, then right subtree, then node

Breadth-first:

- level-order traversal
  … visit node, then all its children

Binary Search Trees

Deletion

Insertion is easy!
find location, create node, link parent
Deletion is much harder!
find node, unlink and delete, ...?

One option: don't delete nodes `:-)`
instead, just mark them as deleted, and ignore them

Otherwise, we must *promote* a child (carefully).
A child with no subtrees: drop.
A child with one subtree: promote that subtree.
A child with two subtrees: ...
replace node with leftmost of right subtree