

COMP2521 19T0

Week 6, Tuesday: Order! Order! (I)

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basic sorting algorithms
more sorting algorithms

MYEXPERIENCE now open!
`myexperience.unsw.edu.au`

PRAC EXAM #1 results look pretty good
a majority of people passed the exam!
no problem required >10 LoC;
if you just threw code at the wall,
consider a different strategy next time.

ASSIGNMENT 2 part 1 is underway!
views due **20 Jan 2019**, no extensions.
view dryruns out now — how does your code do?
hunt spec to be released during week07tue lecture

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

Sorting

Sorting

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Sorting
... arranging a collection of items in order,

Sorting

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Sorting

... arranging a collection of items in order,
... based on some property of an item (a 'key'),

Sorting

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Sorting

... arranging a collection of items in order,
... based on some property of an item (a 'key'),
... using an ordering relation on that property.

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Why are we interested?

- speeds up subsequent searches;
- arranges data in useful ways (human- or otherwise)
... *e.g.*, a list of students in a tutorial
- provides useful intermediate for other algorithms
... *e.g.*, duplicate detection/removal; DBMS operations

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What contexts?

- arrays, linked lists (in-memory, internal)
- files (external, on-disk)
- ... distributed across a network (map/reduce)

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What contexts?

- arrays, linked lists (in-memory, internal)
- files (external, on-disk)
- ... distributed across a network (map/reduce)

We'll focus on sorting **arrays** (and lists)

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Pre-conditions:

array $a[N]$ of Items

lo, hi are valid indices on a
(roughly, $0 \leq lo < hi \leq N - 1$)

Post-conditions:

array $a'[lo..hi]$ contains same values

$a'[lo] \leq a'[lo + 1] \leq a'[lo + 2] \leq \dots \leq a'[hi]$

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Properties: **stable sorts**

let $x = a[i]$, $y = a[j]$, where $\text{KEY}(x) \equiv \text{KEY}(y)$

let the 'precedes' relation be that index $i \leq j$.

if x 'precedes' y in a , then x 'precedes' y in a'

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Properties: **adaptive sorts**

where the algorithm's behaviour or performance
is affected by the input data —

that best/average/worst case performance differs

... and can take advantage of *existing order*

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Properties: **adaptive sorts**

where the algorithm's behaviour or performance
is affected by the input data —
that best/average/worst case performance differs
... and can take advantage of *existing order*

Properties: **in-place sorts**

sort data within original structure,
using only a constant additional amount of space

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```
// we deal with generic `Item's
```

```
typedef int Item;
```

```
// abstractions to hide details of items
```

```
#define key(A) (A)
```

```
#define less(A,B) (key(A) < key(B))
```

```
#define eq(A,B) (key(A) == key(B))
```

```
#define swap(A,B) { Item t; t = A; A = B; B = t; }
```

```
#define cas(A,B) { if (less (A, B)) swap (A, B); }
```

```
// cas == Compare And Swap, often hardware assisted
```

```
/// Sort a slice of an array of Items.
```

```
void sort (Item a[], int lo, int hi);
```

```
/// Check for sortedness (to validate functions).
```

```
bool sorted_p (Item a[], int lo, int hi);
```

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This framework can be adapted by...

- defining a different data structure for `Item`;
- defining a method for extracting sort keys;
- defining a different ordering (`less`);
- defining a different swap method for different `Item`

```
typedef struct { char *name; char *course; } Item;
#define key(A) (A.name)
#define less(A, B) (strcmp(key(A), key(B)) < 0)
#define swap(A,B) { Item t; t = A; A = B; B = t; }
// ... works because struct assignment works in C
```

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In analysing sorting algorithms:

- N : the number of items ($h_i - l_o + 1$)
- C : the number of comparisons between items
- S : the number of times items are swapped

(We usually aim to minimise C and S .)

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In analysing sorting algorithms:

- N : the number of items ($hi - lo + 1$)
- C : the number of comparisons between items
- S : the number of times items are swapped

(We usually aim to minimise C and S .)

Cases to consider for input order:

- random order: Items in $a[lo..hi]$ have no ordering
- sorted-ascending order: $a[lo] \leq a[lo + 1] \leq \dots \leq a[hi]$
- sorted-descending order: $a[lo] \geq a[lo + 1] \geq \dots \geq a[hi]$

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- Bubble Sort (oblivious and early-exit)
- Selection Sort
- Insertion Sort

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Shell

Values 'bubble up' the array.

4	2	5	1	6	3
---	---	---	---	---	---

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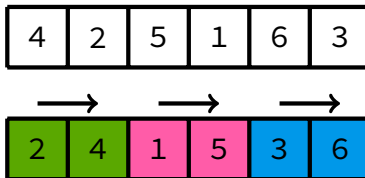
Bubble EE

Selection

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Shell

Values 'bubble up' the array.



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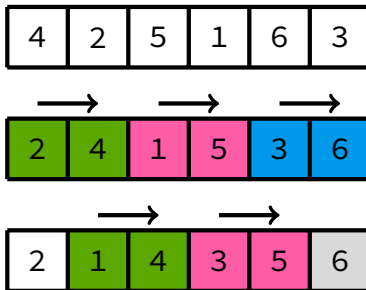
Bubble EE

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Values 'bubble up' the array.



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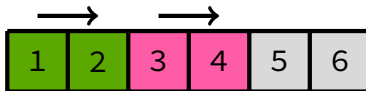
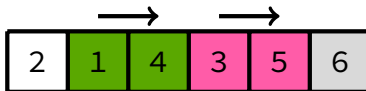
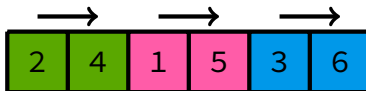
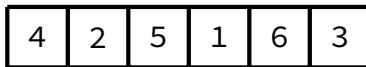
Bubble EE

Selection

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Shell

Values 'bubble up' the array.



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Shell

```
void sort_bubble (Item items[], size_t lo, size_t hi)
{
    for (size_t i = hi; i > lo; i--)
        for (size_t j = lo + 1; j <= i; j++)
            if (less (items[j], items[j - 1]))
                swap_idx (items, j, j - 1);
}
```

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- Outer loop (C_0)

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- Outer loop (C_0)

```
for (size_t i = n - 1; i > 0; i--)
```

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- Outer loop (C_0) $\Rightarrow N$
`for (size_t i = n - 1; i > 0; i--)`

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- Outer loop (C_0) $\Rightarrow N$
`for (size_t i = n - 1; i > 0; i--)`
- Inner loop (C_1)

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Shell

- Outer loop (C_0) $\Rightarrow N$
`for (size_t i = n - 1; i > 0; i--)`
- Inner loop (C_1)
`for (size_t j = 1; j <= i; j++)`

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- Outer loop (C_0) $\Rightarrow N$
`for (size_t i = n - 1; i > 0; i--)`
- Inner loop (C_1) $\Rightarrow N + (N - 1) + (N - 2) + \cdots + 2 = (N^2 + N)/2 - 1$
`for (size_t j = 1; j <= i; j++)`

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Shell

- Outer loop (C_0) $\Rightarrow N$
`for (size_t i = n - 1; i > 0; i--)`
- Inner loop (C_1) $\Rightarrow N + (N - 1) + (N - 2) + \cdots + 2 = (N^2 + N)/2 - 1$
`for (size_t j = 1; j <= i; j++)`
- Comparisons (C_2)

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Shell

- Outer loop (C_0) $\Rightarrow N$
`for (size_t i = n - 1; i > 0; i--)`
- Inner loop (C_1) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 2 = (N^2 + N)/2 - 1$
`for (size_t j = 1; j <= i; j++)`
- Comparisons (C_2) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 1 + 0 = (N^2 - N)/2$

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Shell

- Outer loop (C_0) $\Rightarrow N$
`for (size_t i = n - 1; i > 0; i--)`
- Inner loop (C_1) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 2 = (N^2 + N)/2 - 1$
`for (size_t j = 1; j <= i; j++)`
- Comparisons (C_2) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 1 + 0 = (N^2 - N)/2$
- Swaps (C_3)
(assuming the worst case: we *always* have to swap)

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Shell

- Outer loop (C_0) $\Rightarrow N$
`for (size_t i = n - 1; i > 0; i--)`
- Inner loop (C_1) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 2 = (N^2 + N)/2 - 1$
`for (size_t j = 1; j <= i; j++)`
- Comparisons (C_2) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 1 + 0 = (N^2 - N)/2$
- Swaps (C_3) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 1 + 0 = (N^2 - N)/2$
(assuming the worst case: we *always* have to swap)

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- Outer loop (C_0) $\Rightarrow N$
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- Inner loop (C_1) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 2 = (N^2 + N)/2 - 1$
`for (size_t j = 1; j <= i; j++)`
- Comparisons (C_2) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 1 + 0 = (N^2 - N)/2$
- Swaps (C_3) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 1 + 0 = (N^2 - N)/2$
(assuming the worst case: we *always* have to swap)

$$T(n) = NC_0 + \left(\frac{N^2 + N}{2} - 1 \right) C_1 + \frac{N^2 - N}{2} C_2 + \frac{N^2 - N}{2} C_3$$

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Shell

- Outer loop (C_0) $\Rightarrow N$
`for (size_t i = n - 1; i > 0; i--)`
- Inner loop (C_1) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 2 = (N^2 + N)/2 - 1$
`for (size_t j = 1; j <= i; j++)`
- Comparisons (C_2) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 1 + 0 = (N^2 - N)/2$
- Swaps (C_3) $\Rightarrow N + (N - 1) + (N - 2) + \dots + 1 + 0 = (N^2 - N)/2$
(assuming the worst case: we *always* have to swap)

$$T(n) = NC_0 + \left(\frac{N^2 + N}{2} - 1 \right) C_1 + \frac{N^2 - N}{2} C_2 + \frac{N^2 - N}{2} C_3$$

$$\Rightarrow O(N^2)$$

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How many steps does it take
to sort a collection of N elements?

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Shell

How many steps does it take
to sort a collection of N elements?

For the i th iteration, we have
 $N - i$ comparisons and
best 0, worst $N - i$ swaps
(depending on sortedness.)

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Shell

How many steps does it take
to sort a collection of N elements?

For the i th iteration, we have

$N - i$ comparisons and
best 0, worst $N - i$ swaps
(depending on sortedness.)

Bubble sort is $O(n^2)$.
Stable, in-place, non-adaptive.

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Shell

‘oblivious’ bubble-sort continues, even if the list is sorted
so what’s a better stopping-case than ‘we ran out of array’?

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Shell

‘oblivious’ bubble-sort continues, even if the list is sorted
so what’s a better stopping-case than ‘we ran out of array’?

if we complete a whole pass without swaps, we’re ordered!
this is **bubble sort with early exit**, or **adaptive bubble sort**

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Shell

```
void sort_bubble_ee (Item items[], size_t lo, size_t hi)
{
    bool no_swaps = false;
    for (size_t i = hi; i > lo && !no_swaps; i--) {
        no_swaps = true;
        for (size_t j = lo + 1; j <= i; j++)
            if (less (items[j], items[j - 1])) {
                swap_idx (items, j, j - 1);
                no_swaps = false;
            }
    }
}
```

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How many steps does it take
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Shell

How many steps does it take
to sort a collection of N elements?

Each traversal does N comparisons.

Best case: exit after one iteration
(if the collection is already sorted.)

Worst case: N traversals still necessary.

$$T_{\text{worst}}(N) = N - 1 + N - 2 + \cdots + 1 \approx N^2$$
$$T_{\text{best}}(N) = N$$

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Shell

How many steps does it take
to sort a collection of N elements?

Each traversal does N comparisons.

Best case: exit after one iteration
(if the collection is already sorted.)

Worst case: N traversals still necessary.

$$T_{\text{worst}}(N) = N - 1 + N - 2 + \cdots + 1 \approx N^2$$
$$T_{\text{best}}(N) = N$$

Bubble-sort with early exit is *still* $O(N^2)$.
Stable, in-place, adaptive (!).

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Select the smallest element.
Swap it with the first position.

Select the next smallest element.
Swap it with the second position.

... continue until sorted!

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jashankj@

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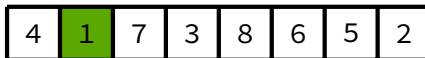
Bubble

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Shell



cs2521@
jashankj@

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Shell

4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2

cs2521@
jashankj@

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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4

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jashankj@

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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4
1	2	3	7	8	6	5	4

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jashankj@

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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4
1	2	3	7	8	6	5	4
1	2	3	4	8	6	5	7

cs2521@
jashankj@

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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4
1	2	3	7	8	6	5	4
1	2	3	4	8	6	5	7
1	2	3	4	5	6	8	7

cs2521@
jashankj@

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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4
1	2	3	7	8	6	5	4
1	2	3	4	8	6	5	7
1	2	3	4	5	6	8	7
1	2	3	4	5	6	8	7

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jashankj@

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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4
1	2	3	7	8	6	5	4
1	2	3	4	8	6	5	7
1	2	3	4	5	6	8	7
1	2	3	4	5	6	8	7
1	2	3	4	5	6	7	8

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jashankj@

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4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	2	7	3	8	6	5	4
1	2	3	7	8	6	5	4
1	2	3	4	8	6	5	7
1	2	3	4	5	6	8	7
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8

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```
void sort_selection (Item items[], size_t lo, size_t hi)
{
    for (size_t i = lo; i < hi; i++) {
        size_t low = i;
        for (size_t j = i + 1; j <= hi; j++)
            if (less (items[j], items[low]))
                low = j;
        swap_idx (items, i, low);
    }
}
```

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How many steps does it take
to sort a collection of N elements?

... picking the minimum of a sequence of N elements: N steps.
... inserting at the right place: 1.

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How many steps does it take
to sort a collection of N elements?

... picking the minimum of a sequence of N elements: N steps.

... inserting at the right place: 1.

$$T(N) = N + (N - 1) + (N - 2) + \cdots + 1 = \frac{1}{2}N(N + 1)$$

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

How many steps does it take
to sort a collection of N elements?

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... inserting at the right place: 1.

$$T(N) = N + (N - 1) + (N - 2) + \cdots + 1 = \frac{1}{2}N(N + 1)$$

Selection sort is $O(N^2)$.

Unstable, in-place, oblivious.

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

Take the first element, insert into the first position.
This starts our 'sorted sublist'.

Take the next element.
Insert it into the sorted sublist
in the right spot!

Repeat until sorted!

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

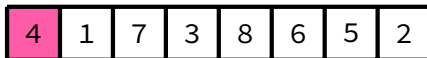
Bubble

Bubble EE

Selection

Insertion

Shell



cs2521@
jashankj@

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2

cs2521@
jashankj@

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
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Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	4	7	3	8	6	5	2
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cs2521@
jashankj@

Sorting

Problem

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Bubble EE

Selection

Insertion

Shell

4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	7	8	6	5	2

cs2521@
jashankj@

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

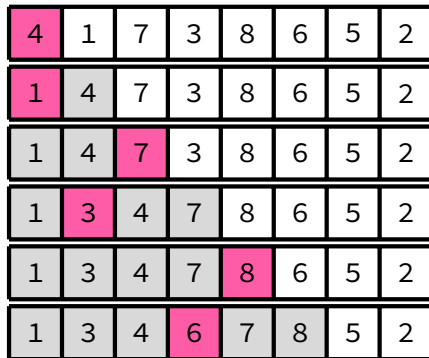
Bubble

Bubble EE

Selection

Insertion

Shell



Sorting

Problem

Formally

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Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	6	7	8	5	2
1	3	4	5	6	7	8	2

cs2521@
jashankj@

Sorting

Problem

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Selection

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cs2521@
jashankj@

Sorting

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Formally

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Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

4	1	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	4	7	3	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	7	8	6	5	2
1	3	4	6	7	8	5	2
1	3	4	5	6	7	8	2
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8

Sorting

Problem

Formally

Concretely

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Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

```
void sort_insertion (Item items[], size_t lo, size_t hi)
{
    for (size_t i = lo + 1; i <= hi; i++) {
        Item item = items[i];
        size_t j = i;
        for (/* j */; j > lo; j--) {
            if (! less (item, items[j - 1])) break;
            items[j] = items[j - 1];
        }
        items[j] = item;
    }
}
```

Sorting

Problem

Formally

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Bubble

Bubble EE

Selection

Insertion

Shell

How many steps does it take
to sort a collection of N elements?

For every element (of N elements):
1 step to pick an element;
insert into a $N' \leq N$ sequence: up to N steps.

$$T_{\text{worst}}(N) = 1 + 2 + \cdots + N = \frac{N}{2}(N + 1)$$

$$T_{\text{best}}(N) = 1 + 1 + \cdots + 1 = N$$

Sorting

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to sort a collection of N elements?

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insert into a $N' \leq N$ sequence: up to N steps.

$$T_{\text{worst}}(N) = 1 + 2 + \cdots + N = \frac{N}{2}(N + 1)$$

$$T_{\text{best}}(N) = 1 + 1 + \cdots + 1 = N$$

Insertion sort is $O(N^2)$.
Stable, in-place, adaptive.

Sorting

Problem

Formally

Concretely

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Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

Bubble- and Insertion-Sort
really only consider *adjacent* elements.

If we make longer-distance exchanges,
can we be more efficient?

Sorting

Problem

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Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

Bubble- and Insertion-Sort
really only consider *adjacent* elements.

If we make longer-distance exchanges,
can we be more efficient?

What if we consider elements that are some distance apart?
... sort sublists of mod- h indices,
for decreasing h until $h = 1$?

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
unsorted	4	1	7	3	8	6	5	2

Sorting

Problem

Formally

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Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
unsorted	4	1	7	3	8	6	5	2
$h = 3$ passes	3			4			5	

Sorting

Problem

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Bubble

Bubble EE

Selection

Insertion

Shell

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
unsorted	4	1	7	3	8	6	5	2
$h = 3$ passes	3			4			5	
		1			2			8

Sorting

Problem

Formally

Concretely

Complexity

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Bubble

Bubble EE

Selection

Insertion

Shell

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
unsorted	4	1	7	3	8	6	5	2
$h = 3$ passes	3			4			5	
		1			2			8
			6			7		

Sorting

Problem

Formally

Concretely

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Bubble

Bubble EE

Selection

Insertion

Shell

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
unsorted	4	1	7	3	8	6	5	2
$h = 3$ passes	3			4			5	
		1			2			8
			6			7		
3-sorted	3	1	6	4	2	7	5	8

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
unsorted	4	1	7	3	8	6	5	2
$h = 3$ passes	3			4			5	
		1			2			8
			6			7		
3-sorted	3	1	6	4	2	7	5	8
$h = 2$ passes	2		3		5		6	

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
unsorted	4	1	7	3	8	6	5	2
$h = 3$ passes	3			4			5	
		1			2			8
			6			7		
3-sorted	3	1	6	4	2	7	5	8
$h = 2$ passes	2		3		5		6	
		1		4		7		8

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
unsorted	4	1	7	3	8	6	5	2
$h = 3$ passes	3			4			5	
		1			2			8
			6			7		
3-sorted	3	1	6	4	2	7	5	8
$h = 2$ passes	2		3		5		6	
		1		4		7		8
2-sorted	2	1	3	4	5	7	6	8

Sorting

Problem

Formally

Concretely

Complexity

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Bubble

Bubble EE

Selection

Insertion

Shell

	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
unsorted	4	1	7	3	8	6	5	2
$h = 3$ passes	3			4			5	
		1			2			8
			6			7		
3-sorted	3	1	6	4	2	7	5	8
$h = 2$ passes	2		3		5		6	
		1		4		7		8
2-sorted	2	1	3	4	5	7	6	8
$h = 1$ pass	1	2	3	4	5	6	7	8

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

```
void sort_shell (Item items[], size_t lo, size_t hi)
{
    size_t h;
    for (h = 1; h <= (n - 1) / 9; h = (3 * h) + 1);

    for (/* h */; h > 0; h /= 3) {
        // when `h' = 1, this is an insertion sort.
        for (size_t i = h; i < n; i++) {
            Item item = items[i];
            size_t j = i;
            for (/* j */; j >= h && item < items[j - h]; j -= h)
                items[j] = items[j - h];
            items[j] = item;
        }
    }
}
```

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

The exact complexity-class depends on the h -sequence.

Probably safe to assume that $O(\leq n^2)$,
because otherwise what's the point?

Lots of h -value sequences are $O(n^{\frac{3}{2}})$.

No 'general' analysis exists.

Sorting

Problem

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Complexity

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Shell

The exact complexity-class depends on the h -sequence.

Probably safe to assume that $O(\leq n^2)$,
because otherwise what's the point?

Lots of h -value sequences are $O(n^{\frac{3}{2}})$.

No 'general' analysis exists.

Shell Sort is $O(\leq n^2)$.

It is unstable, adaptive, in-place.

Sorting

Problem

Formally

Concretely

Complexity

Elementary Sorts

Bubble

Bubble EE

Selection

Insertion

Shell

Bubble traverse list; if $\text{curr} > \text{next}$, swap.

Selection delete selected element, insert at head of sorted list.

Insertion delete first element, do order-preserving insertion.

Shell (*screaming*)