SYSC 5001W: Project deliverable 3

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1 Data Collection

The first thing we need to consider after model translation is data collection. For the supply of components are infinite, the input I formulate here is **the output of two inspectors**. There are five channels output of two inspectors and three kinds of components totally. So I extract data from Inspector's inspection time for components. Each data point represents the time of a specific component processed. I identify the distributions of each set of data using histograms. Below is what I draw by using python matplotlib library. Obviously, the distributions fit the exponential PDF curve well.

I choose exponential distribution to do a fitting and it seems good.

2 Q-Q plots

The construction of histograms are necessary ingredients for choosing a family of distribution but it's not as useful to evaluate the fit of chosen distribution. Quantile-quantile plot is a useful tool for evaluating distribution fit.

First let $\{x_i, i = 1, 2, ..., n\}$ be sampled data and sort the from the smallest to the largest $\{y_j, j = 1, 2, ..., n\}$. The $q-q$ plot is based on that y_i is an estimate of the $(j - 1/2)/n$ quantile of sampled data. I set $\sqrt{300} = 18$ bins totally.

Try some different kinds of common distribution and I found exponential distribution fits the most. Below I choose exponential distribution as the family.

3 Chi-Square Test

3.1 Component1 from Inspector1

The following hypotheses are formed;

H_0: The random variable is exponential distributed. H_1: The random variable is not exponential distributed.

the pmf for exponential distribution is given:

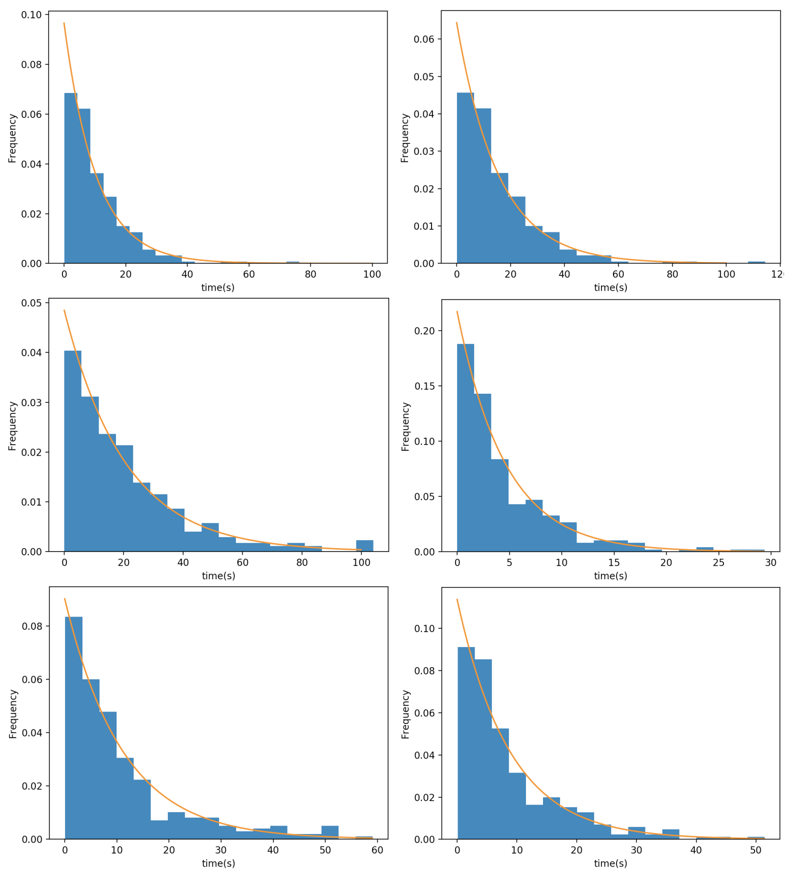

Figure 1: Data collection of 3 Inspectors and 3 Workstations

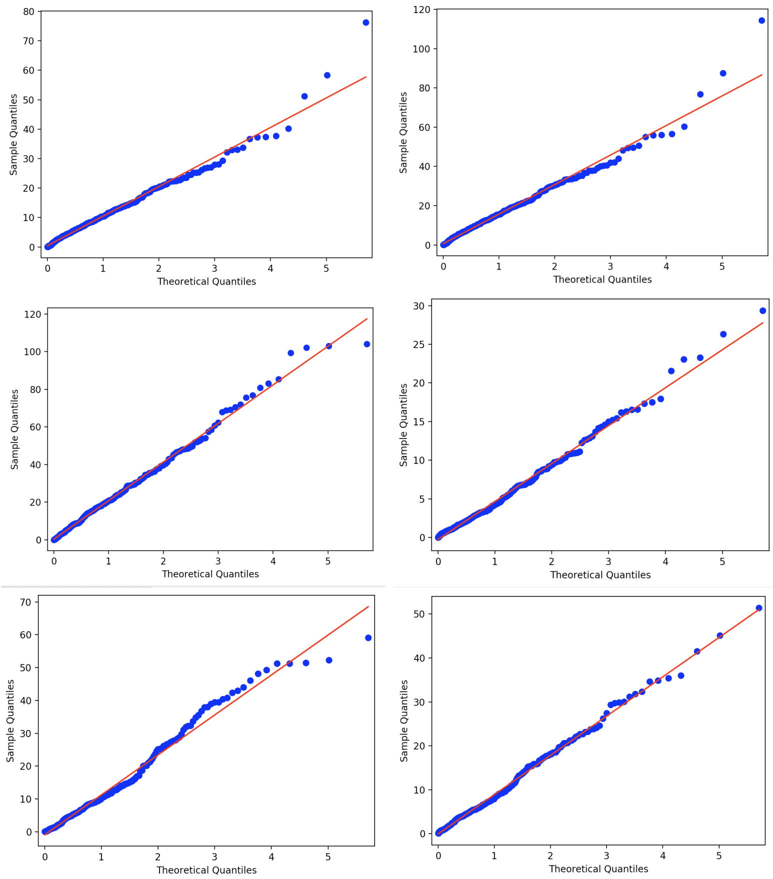

Figure 2: Fitting of normal exponential distribution with distributions of 3 Inspectors and 3 Workstations

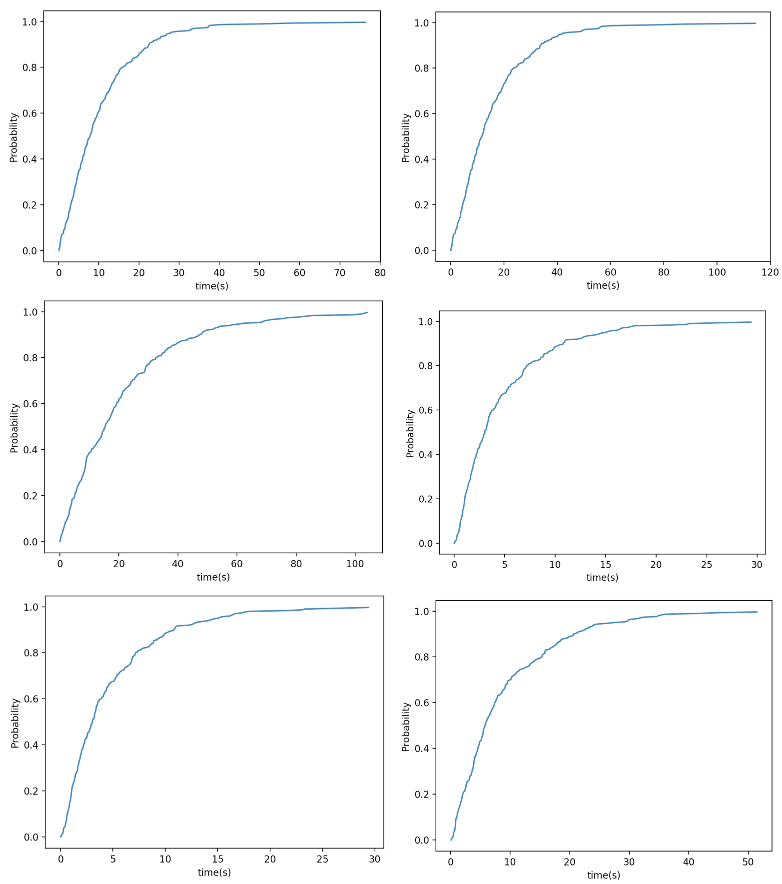

Figure 3: CDF of 3 Inspectors and 3 Workstations

$$p(x) = \begin{cases} \lambda e^{-\lambda e}, x \geq 0 \\ 0, x < 0 \end{cases} \tag{1}$$

We assume that $\hat{\lambda} = \frac{1}{X} = 0.097$

Table 1: Chi-Square Goodness-of-Fit Test

x_i	Class Interval	*Observed Frequency*	*Expected Frequency*	$\frac{(O_i-E_i)^2}{E_i}$
0	[0, 0.4238)	87	100.74	1.87
1	[4.238, 8.476)	79	66.91	2.18
2	[8.476, 12.714)	46	44.44	0.05
3	[12.714, 16.952)	34	29.52	0.68
4	[16.952, 21.19)	19	19.61	0.02
5	[21.19, 25.428)	16	13.02	0.68
6	[25.428, 29.666)	7	8.65	0.31
7	[29.666, 33.904)	4	5.75	0.53
8	[33.904, 38.142)	4	3.82	0.01
9	[38.142, 42.38)	1	2.53	0.93
10	[42.38, 46.618)	0	1.68	1.68
11	[46.618, 50.856)	0	1.12	1.12
12	[50.856, 55.094)	1	0.74	0.09
13	[55.094, 59.332)	1	0.49	0.52
14	[59.332, 63.57)	0	0.33	0.33
15	[63.57, 67.808)	0	0.22	0.22
16	[67.808, 72.046)	0	0.14	0.14
17	[72.046, 76.284]	1	0.10	8.51
18	(76.284, ∞)	0	0.19	0.19
Total		300	300	20.08

We have 18 bins and the freedom degree should be 18-1-1=16. Using a significance level of 0.05 and check the table, we find that $\chi^2_{0.05,16} = 26.3 > 20.09$. So we can't reject that the input follows exponential distribution.

3.2 Component2 from Inspector2

We assume that $\hat{\lambda} = \frac{1}{X} = 0.097$

We have 18 bins and the freedom degree should be 18-1-1=16. Using a significance level of 0.05 and check the table, we find that $\chi^2_{0.05,16} = 26.3 > 20.06$. So we can't reject that the input follows exponential distribution.

3.3 Component3 from Inspector2

We assume that $\hat{\lambda} = \frac{1}{X} = 0.048$

Table 2: Chi-Square Goodness-of-Fit Test

x_i	Class Interval	*Observed Frequency*	*Expected Frequency*	$\frac{(O_i-E_i)^2}{E_i}$
0	[0, 6.357)	87	100.74	1.87
1	[6.357, 12.714)	79	66.91	2.18
2	[12.714, 19.071)	46	44.44	0.05
3	[19.071, 25.428)	34	29.52	0.68
4	[25.428, 31.785)	19	19.61	0.02
5	[31.785, 38.142)	16	13.02	0.68
6	[38.142, 44.499)	7	8.65	0.31
7	[44.499, 50.856)	4	5.75	0.53
8	[50.856, 57.213)	4	3.82	0.01
9	[57.213, 63.57)	1	2.53	0.93
10	[63.57, 69.927)	0	1.68	1.68
11	[69.927, 76.284)	0	1.12	1.12
12	[76.284, 82.641)	1	0.74	0.09
13	[82.641, 88.998)	1	0.49	0.52
14	[88.998, 95.355)	0	0.33	0.33
15	[95.355, 101.712)	0	0.22	0.22
16	[101.712, 108.069)	0	0.14	0.14
17	[108.069, 114.426]	1	0.10	8.51
18	(114.426, ∞)	0	0.19	0.19
Total		300	300	20.08

Table 3: Chi-Square Goodness-of-Fit Test

x_i	Class Interval	*Observed Frequency*	*Expected Frequency*	$\frac{(O_i-E_i)^2}{E_i}$
0	[0, 5.7788)	70	73.28	0.15
1	[5.7788, 11.5576)	54	55.38	0.03
2	[11.5576, 17.3364)	41	41.85	0.02
3	[17.3364, 23.1152)	37	31.63	0.91
4	[23.1152, 28.894)	24	23.90	0
5	[28.894, 34.6728)	20	18.06	0.21
6	[34.6728, 40.4516)	15	13.65	0.13
7	[40.4516, 46.2304)	7	10.32	1.07
8	[46.2304, 52.0092)	10	7.80	0.62
9	[52.0092, 57.788)	5	5.89	0.14
10	[57.788, 63.5668)	3	4.45	0.47
11	[63.5668, 69.3456)	3	3.37	0.04
12	[69.3456, 75.1244)	2	2.54	0.12
13	[75.1244, 80.9032)	3	1.92	0.60
14	[80.9032, 86.682)	2	1.45	0.21
15	[86.682, 92.4608)	0	1.10	1.10
16	[92.4608, 98.2396)	0	0.83	0.83
17	[98.2396, 104.0184]	4	0.63	18.15
18	(104.0184, ∞)	0	0.01	0.01
Total		300	300	24.80

We have 18 bins and the freedom degree should be 18-1-1=16. Using a significance level of 0.05 and check the table, we find that $\chi^2_{0.05,16} = 26.3 > 24.8$. So we can't reject that the input follows exponential distribution.

3.4 Workstation1

We assume that $\hat{\lambda} = \frac{1}{\bar{X}} = 0.217$

Table 4: Chi-Square Goodness-of-Fit Test

x_i	Class Interval	*Observed Frequency*	*Expected Frequency*	$\frac{(O_i-E_i)^2}{E_i}$
0	[0, 1.6319)	92	89.53	0.07
1	[1.6319, 3.2638)	70	62.81	0.82
2	[3.2638, 4.8957)	41	44.07	0.21
3	[4.8957, 6.5276)	21	30.92	3.18
4	[6.5276, 8.1595)	23	21.69	0.08
5	[8.1595, 9.7914)	16	15.22	0.04
6	[9.7914, 11.4233)	13	10.67	0.51
7	[11.4233, 13.0552)	4	7.49	1.63
8	[13.0552, 14.6871)	5	5.25	0.01
9	[14.6871, 16.319)	5	3.69	0.47
10	[16.319, 17.9509)	4	2.59	0.77
11	[17.9509, 19.5828)	1	1.81	0.37
12	[19.5828, 21.2147)	0	1.27	1.27
13	[21.2147, 22.8466)	1	0.89	0.01
14	[22.8466, 24.4785)	2	0.63	3.01
15	[24.4785, 26.1104)	0	0.44	0.44
16	[26.1104, 27.7423)	1	0.31	1.55
17	[27.7423, 29.3742]	1	0.22	2.84
18	(29.3742, ∞)	0	0.51	0.51
Total		300	300	17.79

We have 18 bins and the freedom degree should be 18-1-1=16. Using a significance level of 0.05 and check the table, we find that $\chi^2_{0.05,16} = 26.3 > 17.79$. So we can't reject that the input follows exponential distribution.

3.5 Workstation2

We assume that $\hat{\lambda} = \frac{1}{\bar{X}} = 0.217$

We have 18 bins and the freedom degree should be 18-1-1=16. Using a significance level of 0.01 and check the table, we find that $\chi^2_{0.01,16} = 26.3 > 24.28$. So we don't reject that the input follows exponential distribution.

Table 5: Chi-Square Goodness-of-Fit Test

x_i	Class Interval	*Observed Frequency*	*Expecte Frequency*	$\frac{(O_i-E_i)^2}{E_i}$
0	[0, 3.2821)	82	76.85	0.34
1	[3.2821, 6.5642)	59	57.17	0.06
2	[6.5642, 9.8463)	47	42.52	0.47
3	[9.8463, 13.1284)	30	31.63	0.08
4	[13.1284, 16.4105)	22	23.53	0.10
5	[16.4105, 19.6926)	7	17.50	6.30
6	[19.6926, 22.9747)	10	13.02	0.70
7	[22.9747, 26.2568)	8	9.68	0.30
8	[26.2568, 29.5389)	8	7.20	0.09
9	[29.5389, 32.821)	5	5.36	0.02
10	[32.821, 36.1031)	3	3.98	0.24
11	[36.1031, 39.3852)	4	2.96	0.36
12	[39.3852, 42.6673)	5	2.20	3.55
13	[42.6673, 45.9494)	2	1.64	0.08
14	[45.9494, 49.2315)	2	1.22	0.50
15	[49.2315, 52.5136)	5	0.91	18.46
16	[52.5136, 55.7957)	0	0.67	0.67
17	[55.7957, 59.0778]	1	0.50	0.49
18	(59.0778, ∞)	0	1.46	1.46
Total		300	300	24.28

3.6 Workstation3

We assume that $\hat{\lambda} = \frac{1}{\bar{X}} = 0.217$

Table 6: Chi-Square Goodness-of-Fit Test

x_i	Class Interval	*Observed Frequency*	*Expected Frequency*	$\frac{(O_i-E_i)^2}{E_i}$
0	[0, 3.2821)	78	76.84	0.02
1	[3.2821, 6.5642)	73	57.16	4.39
2	[6.5642, 9.8463)	45	42.52	0.14
3	[9.8463, 13.1284)	27	31.63	0.68
4	[13.1284, 16.4105)	14	23.53	3.86
5	[16.4105, 19.6926)	17	17.50	0.01
6	[19.6926, 22.9747)	13	13.02	0
7	[22.9747, 26.2568)	11	9.68	0.18
8	[26.2568, 29.5389)	6	7.20	0.20
9	[29.5389, 32.821)	2	5.36	2.11
10	[32.821, 36.1031)	5	3.98	0.26
11	[36.1031, 39.3852)	2	2.96	0.31
12	[39.3852, 42.6673)	4	2.20	1.46
13	[42.6673, 45.9494)	0	1.64	1.64
14	[45.9494, 49.2315)	1	1.22	0.04
15	[49.2315, 52.5136)	1	0.91	0.01
16	[52.5136, 55.7957)	0	0.67	0.67
17	[55.7957, 59.0778]	1	0.50	0.49
18	(59.0778, ∞)	0	1.46	0.09
Total		300	300	17.57

We have 18 bins and the freedom degree should be 18-1-1=16. Using a significance level of 0.05 and check the table, we find that $\chi^2_{0.05,16} = 26.3 > 17.57$. So we don't reject that the input follows exponential distribution.

4 Generate the input

For each inspector and workstation, we choose a model based on that the sampled input. We choose 0.95 significance level for each and make sure they can all be presented as the distribution. What I do is generating random numbers first in uniform distribution. Then, I put these random numbers into exponential distribution that they are transferred random variables of time.

- Inspection time for C1:

$$f(x) = \begin{cases} 0.0965e^{-0.0965e}, x \geq 0 \\ 0, x < 0 \end{cases} \tag{2}$$

- Inspection time for C2:

$$f(x) = \begin{cases} 0.0644e^{-0.0644e}, x \geq 0 \\ 0, x < 0 \end{cases} \tag{3}$$

- Inspection time for C3:

$$f(x) = \begin{cases} 0.0485e^{-0.0485e}, x \geq 0 \\ 0, x < 0 \end{cases} \tag{4}$$

- Inspection time for W1:

$$f(x) = \begin{cases} 0.217e^{-0.217e}, x \geq 0 \\ 0, x < 0 \end{cases} \tag{5}$$

- Inspection time for W2:

$$f(x) = \begin{cases} 0.0901e^{-0.0901e}, x \geq 0 \\ 0, x < 0 \end{cases} \tag{6}$$

- Inspection time for W3:

$$f(x) = \begin{cases} 0.1137e^{-0.1137e}, x \geq 0 \\ 0, x < 0 \end{cases} \tag{7}$$

Here we have generated random numbers data.

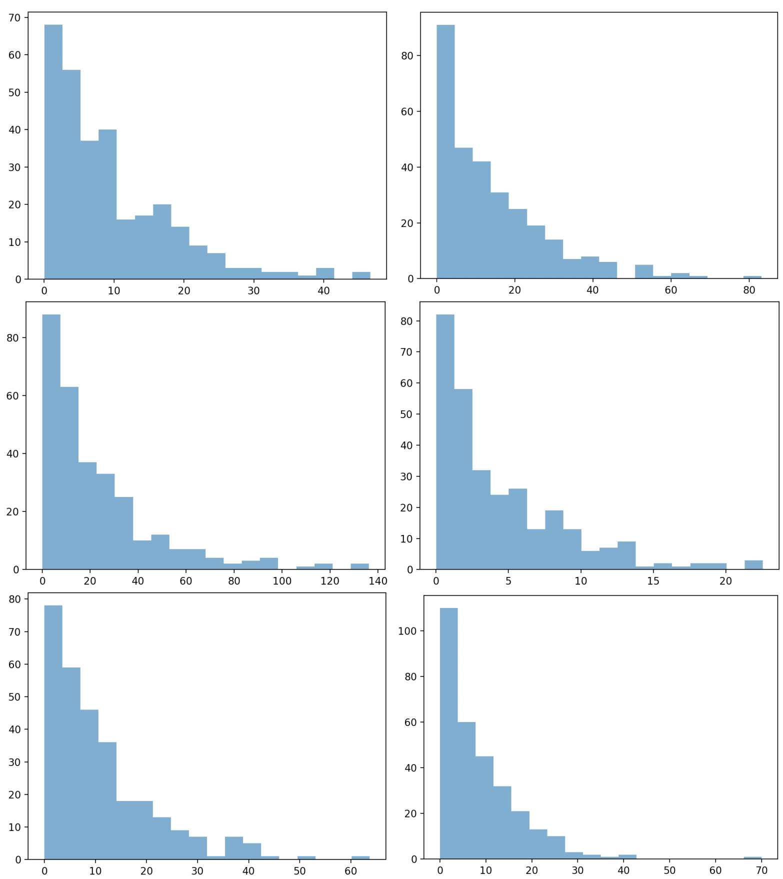

Figure 4: Generated data of 3 Inspectors and 3 Workstations