

# RAYLEIGH



## User Manual

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## RAYLEIGH USER MANUAL

### 1.1 Grid Specification

Rayleigh solves the fluid equations in spherical-shell geometry. As the poles are included, the grid is fully specified by providing four pieces of information:

- The coordinates of the computational domain's radial boundaries of the domain,  $r_{\min}$  and  $r_{\max}$
- The number of radial grid points,  $N_r$
- The number of latitudinal grid points,  $N_\theta$

The number of longitudinal grid points,  $N_\phi$ , is always twice  $N_\theta$ . The total number of gridpoints for a Rayleigh simulation is then given by  $2N_r N_\theta^2$ . Note that both  $N_r$  and  $N_\theta$  must be even. Rayleigh's computational grid is specified using the `problemsize` namelist in the `main_input` file. A quick reference for all `problemsize`-namelist variables is provided in the [namelist documentation](#). In this section, we discuss in detail how to define Rayleigh's grid using these variables.

#### 1.1.1 Standard grid specification

We begin by discussing how to define a grid employing a single Chebyshev domain in radius, meaning that a single Chebyshev expansion is carried out over the domain  $r_{\min} \leq r \leq r_{\max}$ . This is probably the most common grid setup employed in Rayleigh.

The `problemsize` variables `n_r` and `n_theta` provide values for  $N_r$  and  $N_\theta$  respectively. Similarly, `rmin` and `rmax` define the value for  $r_{\min}$  and  $r_{\max}$ . If we wanted to define a spherical shell extending from  $r=1.0$  to  $r=2.0$ , with  $N_r = 48$  and  $N_\theta = 96$ , our `problemsize` namelist should look like:

```
&problemsize_namelist
  n_r = 48
  n_theta = 96
  rmin = 1.0
  rmax = 2.0
/
```

Note that  $N_r$  and  $N_\theta$  may also be specified at the command line using the flags `-nr` and `-ntheta`, e.g.:

```
mpiexec -np 8 ./rayleigh.opt -nr 48 -ntheta 96
```

Doing so will override any values supplied via `main_input`. This can be particularly useful when scripting

performance analyses on a new machine.

If desired, a user may instead specify the radial domain bounds in terms of the shell aspect ratio  $\chi = r_{\min}/r_{\max}$ , and the shell depth  $r_{\max} - r_{\min}$ . This is accomplished using the `aspect_ratio` and `shell_depth` problemsize variables. The example below describes a grid equivalent to the one described above.

```
&problemsize_namelist
  n_r = 48
  n_theta = 96
  aspect_ratio = 0.5
  shell_depth = 1.0
/
```

Rayleigh's horizontal resolution ( $N_\theta \times N_\phi$ ) may alternatively be described in terms of spherical harmonics. The maximum Legendre degree employed in Rayleigh's truncated spherical harmonic expansion is denoted by  $\ell_{\max}$ , and the total number of degrees by  $N_\ell$ . These two variables are related to  $N_\theta$  via

$$N_\ell = \ell_{\max} + 1 = \frac{2}{3}N_\theta,$$

and they are described by the problemsize variables `n_l` and `l_max`. Thus, the examples

```
&problemsize_namelist
  n_r = 48
  l_max = 63
  rmin = 1.0
  rmax = 2.0
/
```

and

```
&problemsize_namelist
  n_r = 48
  n_l = 64
  aspect_ratio = 0.5
  shell_depth = 1.0
/
```

both describe a grid extending from  $r=1.0$  to  $r=2.0$ , with  $N_r = 48$  and  $N_\theta = 96$ .

### 1.1.2 Defining multiple Chebyshev domains

In some instances, it may be advantageous to describe the radial grid using multiple Chebyshev domains. The most common use case probably occurs when the system under consideration is characterized by layers subject to different physical conditions. For instance, models that include regions that are both superadiabatically and subadiabatically stratified might employ a different Chebyshev expansion within each domain. Similarly so for geodynamo models that include the solid inner core.

When describing a grid with  $N$  Chebyshev domains, the `main_input` file must first supply  $N+1$  points  $r_i$  that define the bounds of these domains. The  $i$ th Chebyshev domain will span the interval  $r_i \leq r \leq r_{i+1}$ , and

the global domain bounds are defined such that

$$r_0 \equiv r_{\min} \quad \text{and} \quad r_{N+1} \equiv r_{\max}.$$

I was here. The next example is good. Note that we use `ncheby`. Note that a radial point will be repeated.

It is possible to run Rayleigh with multiple, stacked domains in the radial direction. Each of these is discretized using their own set of Chebyshev polynomials. The boundaries and number of polynomials can be set for each domain individually, which makes it possible to control the radial resolution at different radii.

To use this feature the problem size has to be specified using `domain_bounds` and `ncheby` instead of `rmin`, `rmax`, and `n_r`. `ncheby` takes a comma-separated list of the number of radial points to use in each domain. `domain_bounds` takes a comma-separated list of the radii of the domain boundaries, starting with the smallest radius. It has one element more than the number of domains. This is an example of two radial domains, one covering the radii 1 to 2 with 16 radial points, the other the radii 2 to 4 with 64 radial points.

```
&problemsize_namelist
  domain_bounds = 1.0, 2.0, 4.0
  ncheby = 16, 64
/
```

Radial values in the diagnostic output will be repeated at the inner domain boundaries. Most quantities are forced to be continuous at these points.

### 1.1.3 Controlling radial dealiasing



## MAIN\_INPUT NAMELISTS

### 2.1 Problemsize

This namelist is used to specify the grid.

**n\_r** Number of radial points in model grid

**rmin** Radius of the inner domain boundary,  $r_{\min}$

**rmax** Radius of the outer domain boundary,  $r_{\max}$

**aspect\_ratio**  $r_{\min}/r_{\max}$

**shell\_depth**  $r_{\max} - r_{\min}$

**n\_theta** Number of theta points in the model grid,  $N_{\theta}$

**l\_max** Truncation degree  $\ell_{\max}$  used in the spherical harmonic expansion

**n\_l**  $\ell_{\max} + 1$

**nprow** Number of MPI ranks within each row of the 2-D process grid

**npcol** Number of MPI ranks within each column of the 2-D process grid

**ncheby** Comma-separated list indicating number of Chebyshev polynomials used in each radial subdomain (e.g., 16, 32, 16). Default: n\_r [ single domain]

**dealias\_by** Comma-separated list indicating number of Chebyshev modes dealiased to zero. Default is 2/3 ncheby.

**domain\_bounds** The domain bounds defining each Chebyshev subdomain

**n\_uniform\_domains** Number of uniformly-sized Chebyshev domains spanning the depth of the shell. Default: 1

**uniform\_bounds** When set to .true., each chebyshev subdomain will possess the same radial extent. Default: .false.



## 2.2 Numerical Controls

This namelist provides access to Rayleigh’s run-time optimization options.

**band\_solve** For use with models employing at least three Chebyshev domains. In those models, the rows of the normally dense matrices used in the Crank-Nicolson scheme may be rearranged into a block-banded form. Setting this variable to `.true.` will perform this rearrangement, and Rayleigh will execute a band, rather than dense, solve during each timestep. Using the band-solve approach can help save memory and may yield performance gains. No benefit is gained for models using one or two Chebyshev domains. The default behavior is to use a dense solve (`band_solve = .false.`).

**static\_transpose** When set to `.true.`, buffer space used during Rayleigh’s transposes is allocated once at runtime. The default behavior (`static_transpose=.false.`) is to allocate and deallocate buffer space during each transpose. On some machines, avoiding this cycle of allocation/deallocation has led to minor performance improvements.

**static\_config** When set to `.true.`, sphericalbuffer configurations (e.g., `p3a`, `s2b`) are allocated once at runtime. The default behavior (`static_config=.false.`) is to save memory by deallocating memory associated with the prior configuration space following a transpose. If memory is not an issue, this may lead to minor performance improvements on some systems.

**pad\_alltoall** When set to `.true.`, transpose buffers are padded throughout with zeros to enforce uniform message size, and a standard alltoall is used for each transpose. The default behavior (`pad_alltoall=.false.`) uses alltoallv and variable message sizes. Depending on the underlying alltoall algorithms in the MPI implementation used, performance may differ between these two approaches.

## 2.3 Physical Controls

This namelist controls the physical effects used in a Rayleigh simulation.

**magnetism** When set to `.true.`, the MHD approximation is employed. The default (`magnetism=.false.`) is to omit the effects of magnetism.

**nonlinear** When set to `.false.`, all nonlinear terms are omitted in the model. The default (`nonlinear=.true.`) is to include those terms.

**momentum\_advection** When set to `.false.`,  $\mathbf{v} \cdot \nabla \mathbf{v} = 0$ . This flag is primarily for debugging purposes. The default value is `.true.`.

**inertia** When set to `.false.`, the material derivative of velocity is omitted ( $\frac{D\mathbf{v}}{Dt} = 0$ ). This option is primarily intended for mantle convection models. The default value is `.true.`.

**rotation** When set to `.true.`, the Coriolis term is included in the momentum equation. The default behavior is to omit rotation in a Rayleigh model (`rotation = .false.`).

**lorentz\_forces** Set this debugging/development flag to `.false.` to disable the Lorentz force. Default value is `.true.`, but this flag is ignored entirely when `magnetism = .false.`

**viscous\_heating** Determines whether viscous heating is included in the thermal energy equation. Default value is `.true.` Note that the user-supplied value of this variable is ignored entirely for Boussinesq models run with `reference_type = 1`. In those models, `viscous_heating` is set to `.false.`

**ohmic\_heating** Determines whether ohmic heating is included in the thermal energy equation. Default value is `.true.` Note that the user-supplied value of this variable is ignored entirely for Boussinesq

models run with `reference_type = 1`. In those models, `ohmic_heating` is set to `.false`.

**advect\_reference\_state** Determines whether the reference-state entropy is advected. The default is `.true`. When set to `.false`., the  $v_r \frac{\partial \bar{S}}{\partial r}$  term is omitted in the thermal energy equation. Note that this variable has no impact on models with an adiabatic background state.

**benchmark\_mode** When set to a positive value in the interval  $[1,4]$ , an accuracy benchmark will be performed. The default is 0 (no benchmarking). Boussinesq benchmarks are performed for values of 1 (nonmagnetic) and 2 (magnetic). Anelastic benchmarks are performed if `benchmark_mode` has a value of 3 (nonmagnetic) or 4 (magnetic).

**benchmark\_integration\_interval** Determines the interval (in timesteps) between successive benchmark snapshot analyses.

**benchmark\_report\_interval** Determines the interval (in timesteps) between successive benchmark report outputs. Each output contains an average over all benchmark snapshot analyses performed since the previous report.

## 2.4 Temporal Controls

This namelist controls timing, time-stepping, and checkpointing in Rayleigh.

**alpha\_implicit** Determines the value of  $\alpha$  used in the Crank-Nicolson semi-implicit time-stepping scheme employed for linear terms. The default value is 0.5, which ensures second-order accuracy of the algorithm. A value of 1 (0) describes a fully implicit (explicit) algorithm.

**max\_iterations** Maximum number of timesteps for which to evolve a single instance of Rayleigh before exiting the program. Note that this value does not describe the maximum number of timesteps a model can be run for. Instead, it determines the maximum number of timesteps Rayleigh will run for during a given session (i.e. following a single call to `mpiexec/mpirun`). The default value is 1,000,000.

**max\_time\_minutes** Maximum walltime (in minutes) for which to run a single instance of Rayleigh before exiting. As with `max_iterations`, this is specific to a given Rayleigh session. Default is  $10^8$  minutes (essentially, unlimited).

**max\_simulated\_time** The maximum time, in simulation units, for which to evolve a Rayleigh model. Restarting a model that has already reached this limit will result in running for a single time step before exiting. The default is effectively unlimited, with a value of  $10^{20}$ .

**save\_last\_timestep** When set to `.true`. (default), Rayleigh will checkpoint before exiting normally. Note that this generally occurs when the maximum time or iterations is reached. This does not apply when a job is terminated by the MPI job scheduler.

**checkpoint\_interval** Number of iterations between successive checkpoint outputs. Default value is -1 (no checkpointing).

**check\_frequency** (deprecated) Same as `checkpoint_interval`.

**quicksave\_interval** Number of iterations between successive quicksave outputs. Default value is -1 (no quicksaves).

**num\_quicksaves** Number of quicksave slots (i.e., rapid, rolling checkpoint folders) to use for a given simulation. Default value is 3.

**quicksave\_minutes** Time in minutes between successive quicksaves. If this variable is set to a positive value

(default is -1), the value of `quicksave_interval` will be ignored.

**max\_time\_step** The maximum allowed time step. This value will be respected even when the CFL constraint admits a larger time-step size. Default value is 1.0.

**min\_time\_step** The minimum allowable time step. If the CFL constraint forces a time-step size that falls below this value, Rayleigh will exit.

**cflmin** Used for adaptive timestep control. Rayleigh ensures that the time-step size never falls below  $cflmin \times t_{CFL}$ , where  $t_{CFL}$  is the minimum timestep allowed by the CFL constraint. The default value is 0.4.

**cflmax** Used for adaptive timestep control. Rayleigh ensures that the time-step size never exceeds  $cflmax \times t_{CFL}$ , where  $t_{CFL}$  is the minimum timestep allowed by the CFL constraint. The default value is 0.6.

**new\_iteration** If desired, a simulation's iteration numbers may be reset upon restarting from a checkpoint. Set this value to the new iteration number to use (must be greater than zero), and the old iteration number contained in the checkpoint file will be ignored. The default value is 0.

## 2.5 IO Controls

This namelist provides various options to control Rayleigh's input and output cadence and structure.

**stdout\_file** If desired, set this variable to the name of a file to which Rayleigh's text output is redirected. This can be useful for monitoring run progress and time-step size on systems that otherwise don't produce the text output until a run has completed. The default value is 'nofile,' which indicates that Rayleigh should not redirect stdout to a file.

**stdout\_flush\_interval** Number of lines to cache before writing to the `stdout_file` if used. This prevents excessive disk access while a model is evolving. The default value is 50.

**jobinfo\_file** Set this variable to the name of a file, generated during Rayleigh's initialization, that contains the values assigned to each namelist variable, along with compiler and Git hash information. The default filename is 'jobinfo.txt'

**terminate\_file** The name of a file that, if found in the top-level simulation directory, indicates Rayleigh should terminate execution. This can be useful when trying to exit a run cleanly before the scheduled wall time runs out. The default filename is 'terminate'.

**terminate\_check\_interval** Number of iterations between successive checks for the presence of the job termination file. The default value is 50.

**statusline\_interval** Number of iterations between successive outputs to stdout indicating time step number and size. The default value is 1, so that iteration number and time-step size are printed during every time step.

**outputs\_per\_row** Determines the number of process columns that participate in MPI-IO during checkpointing and diagnostic outputs. Acceptable values fall in the range [1,nprow], with a default value of 1.

**integer\_output\_digits** Number of digits to use for all integer-based filenames (e.g., G\_Avgs/00000001). The default value is 8.

**integer\_input\_digits** Number of digits for integer-based checkpoint names to be read during a restart. The default value is 8.

**decimal\_places** Number of digits to use after the decimal point for those portions of Rayleigh's text output

that displayed in scientific notation. The default value is 3.

## 2.6 Boundary Conditions

This namelist provides those options necessary to determine the boundary conditions employed in a Rayleigh model.

**fix\_tvar\_top** Logical flag indicating whether thermal variable (T,S) should be fixed on the upper boundary. Default = .true.

**fix\_tvar\_bottom** Logical flag indicating whether thermal variable (T,S) should be fixed on the lower boundary. Default = .true.

**fix\_dtdr\_top** Logical flag indicating whether the radial derivative of thermal variable (T,S) should be fixed on the upper boundary. Default = .false.

**fix\_dtdr\_bottom** Logical flag indicating whether the radial derivative of thermal variable (T,S) should be fixed on the lower boundary. Default = .false.

**T\_top** Value of thermal variable (T,S) at the upper boundary. Default = 0.

**T\_bottom** Value of thermal variable (T,S) at the lower boundary. Default = 1.

**dTdr\_top** Value of radial derivative of thermal variable (T,S) at the upper boundary. Default = 0.

**dTdr\_bottom** Value of radial derivative of thermal variable (T,S) at the lower boundary. Default = 0.

**adjust\_dTdr\_top** Logical flag indicating that dTdr\_top should be set based on the values of heating\_integral (or luminosity) and the value of dTdr\_bottom. Default value is .false. When .true., this flag only has an effect when fix\_dtdr\_top = .true. and heating\_type > 0. When active, dTdr\_top is set such that the integrated flux passing through the upper boundary is equal to the sum of those due to internal heating and any flux passing through the lower boundary due to fixed dTdr\_bottom.

**no\_slip\_top** When .true., a no-slip condition on the horizontal velocity field is enforced at the upper boundary. Default = .false.

**no\_slip\_bottom** When .true., a no-slip condition on the horizontal velocity field is enforced at the lower boundary. Default = .false.

**stress\_free\_top** When .true., a stress-free condition on the horizontal velocity field is enforced at the upper boundary. Default = .true.

**stress\_free\_bottom** When .true., a stress-free condition on the horizontal velocity field is enforced at the lower boundary. Default = .true.

**no\_slip\_boundaries** When .true., both no\_slip\_top and no\_slip\_bottom are set to .false. Default = .false.

**strict\_L\_Conservation** In some cases, typically rotating models employing MHD or thick shells, angular momentum can leak into/out of the domain even when using stress-free boundaries. When .true., this flag replaces the upper boundary condition with an integral constraint on the  $\ell = 1$  toroidal streamfunction that enforces strict conservation of angular momentum. Note that the upper boundary is neither stress-free nor no-slip in this case. Default = .false.

**T\_top\_file** Generic-input file containing a custom, fixed (T,S) upper boundary condition.

**T\_bottom\_file** Generic-input file containing a custom, fixed (T,S) lower boundary condition.

**dTdr\_top\_file** Generic-input file containing a custom, fixed ( $\partial T/\partial r$ ,  $\partial S/\partial r$ ) upper boundary condition.

**dTdr\_bottom\_file** Generic-input file containing a custom, fixed  $(\partial T/\partial r, \partial S/\partial r)$  lower boundary condition.

**C\_top\_file** Generic-input file containing a custom upper boundary condition for the poloidal flux function  $C$ .

**C\_bottom\_file** Generic-input file containing a custom lower boundary condition for the poloidal flux function  $C$ .

## 2.7 Reference

This namelist provides options to control the properties of Rayleigh's background state.

### reference\_type

**Determines the fluid approximation and background state used by Rayleigh.**

- type 1: Boussinesq + nondimensional
- type 2: Anelastic + polytropic background state (dimensional)
- type 3: Anelastic + polytropic background state (non-dimensional)
- type 4: Custom reference-state (read from file)

**poly\_n** The polytropic index used to describe the background state for reference types 2 and 3.

**poly\_Nrho** Number of density scaleheights spanning the interval  $r_{\min} \leq r \leq r_{\max}$  for reference types 2 and 3.

**poly\_mass** Mass interior to  $r_{\min}$ , used in defining the polytropic reference state for reference types 2 and 3.

**poly\_rho\_i** Specifies the value of density at the inner boundary  $r = r_{\min}$  for the polytropic reference states of reference types 2 and 3.

**pressure\_specific\_heat** Determines the value of the specific heat at constant pressure,  $c_p$  for reference types 2 and 3.

### heating\_type

**Integer value that determines the form of the internal heating function  $Q(r)$ . The default value is 0, which indicates no heating.**

- type 1:  $Q(r) \propto \bar{\rho}(r)\bar{T}(r)$ .
- type 4:  $Q(r)$  is a constant function of radius.

**heating\_integral** Determines the heating normalization  $L$ , defined such that  $L = 4\pi \int_{r_{\min}}^{r_{\max}} Q(r)r^2 dr$ .

**luminosity** Same as heating\_integral. If both are specified, the value of heating\_integral will be used.

**angular\_velocity** Determines the frame rotation rate  $\Omega$  for rotating models employing reference type 2.

**rayleigh\_number** Sets the value of the Rayleigh number  $Ra$  for reference type 1.

**ekman\_number** Sets the value of the Ekman number  $Ek$  for reference types 1 and 3.

**prandtl\_number** Sets the value of the Prandtl number  $Pr$  for reference types 1 and 3.

**prandtl\_number** Sets the value of the magnetic Prandtl number  $Pm$  for reference types 1 and 3.

**dissipation\_number** Sets the value of the dissipation number  $Di$  for reference type 3.

**modified\_rayleigh\_number** Sets the value of the modified Rayleigh number  $Ra^*$  for reference type 3.

**gravity\_power** Specifies the value of  $n$  (real number) used to determine the radial variation of gravitational acceleration  $g$  in reference type 1, where  $g \propto \left(\frac{r}{r_{\max}}\right)^n$ .

**ra\_constants** Indicates the desired value of specified constant coefficients when reading the value from main\_input instead of from a custom-reference file. For use with override\_constants or override\_constant flags. Syntax is:

```
&Reference_Namelist
...
ra_constants( 2) = 1.0
ra_constants(10) = 14.0
...
/
```

**with\_custom\_constants** Comma separated list of integers indicating which constant coefficients should be read from a custom-reference file when with\_custom\_reference is true.

**with\_custom\_functions** Comma separated list of integers indicating which non-constant coefficients should be read from a custom-reference file when with\_custom\_reference is true.

**with\_custom\_reference** Logical flag that indicates some constant and non-constant coefficients should be read from a custom-reference file and used to overwrite those values otherwise assigned for reference\_Types 1–3. Default value is .false.

**custom\_reference\_file** Name of file from which to read custom-reference-state information when using reference\_type 4 or when augmenting reference types 1–3.

**override\_constants** When true, ALL constant coefficients specified in the custom-reference file will be ignored, and those specified in main\_input will be used instead. Constant coefficients not specified in main\_input will be assigned a value of zero. Default value is .false.

**override\_constant** Indicates that particular constant coefficients, rather than all, should be overridden using main\_input values when using reference\_type 4. Multiple constant overrides can be specified, one per line, with the syntax:

```
&Reference_Namelist
...
override_constant( 2) = T
override_constant(10) = T
...
/
```

## 2.8 Transport

This namelist enables control of Rayleigh's diffusivities.

**{nu,kappa,eta}\_type**

**Determines the radial profile of the associated diffusion coefficient.**

- type 1 : no radial variation
- type 2 : diffusivity profile varies as  $\rho^n$  for some real number  $n$ .
- type 3 : diffusivity profile is read from a custom-reference-state file

**{nu,kappa,eta}\_top**

**Specifies the value of the associated diffusion coefficient at the upper boundary. This is primarily used for dimensionless quantities.**

- reference\_type 1:  $\nu_{\text{top}} = 1$ ,  $\kappa_{\text{top}} = 1/\text{Pr}$ ,  $\eta_{\text{top}} = 1/\text{Pm}$
- reference\_type 3:  $\nu_{\text{top}} = \text{Ek}$ ,  $\kappa_{\text{top}} = \text{Ek}/\text{Pr}$ ,  $\eta_{\text{top}} = \text{Ek}/\text{Pm}$

**{nu,kappa,eta}\_power** Denotes the value of the exponent  $n$  in the  $\rho^n$  variation associated with diffusion type 2.

**hyperdiffusion**

**Set this to variable to .true. to enable hyperdiffusion. The default value is .false. When active, diffusivities are modified as follows:**

$$\bullet \{\nu, \kappa, \eta\} \rightarrow \{\nu, \kappa, \eta\} \left(1 + \alpha \left(\frac{\ell-1}{\ell_{\text{max}}-1}\right)^\beta\right)$$

**hyperdiffusion\_alpha** Determines the value of  $\alpha$  when hyper diffusion is active.

**hyperdiffusion\_beta** Determines the value of  $\beta$  when hyper diffusion is active.