Задача 1

а) $$\min_{x\in[-10,10]^d} \max_{y\in[-10,10]^d} f(x,y) = (x-b_x)^T A(y-b_y) + \frac{\lambda}{2}\|x-b_x\|^2 - \frac{\lambda}{2}\|y-b_y\|^2$$

$$\max_{y\in\Delta^d} f(x_0,y) = \max_{y\in\Delta^d} (x_0-b_x)^T A(y-b_y) + \frac{\lambda}{2}\|x_0-b_x\|^2 - \frac{\lambda}{2}\|y-b_y\|^2$$

$$\frac{\partial f}{\partial y} = \left((x_0-b_x)^T A\right)^T + 0 - \frac{\lambda}{2}\cdot 2(y-b_y) = 0 \qquad (\lambda = 1)$$

$$A^T(x_0-b_x) = y - b_y$$

$$x_0 \longrightarrow y = A^T(x_0-b_x) + b_y \quad \text{– реш. max}$$

$$\min_{x\in[-10,10]^d} (x-b_x)^T A\cdot A^T(x-b_x) + \frac{1}{2}\|x-b_x\|^2 - \frac{1}{2}\|A(x-b_x)\|^2 =$$
$$= \frac{1}{2}\|A(x-b_x)\|^2 + \frac{1}{2}\|x-b_x\|^2$$

$$\frac{\partial f}{\partial x} = A(x-b_x) + (x-b_x) = 0 \quad \Rightarrow \quad \boxed{\begin{aligned} x_{opt} &= b_x \\ y_{opt} &= b_y \end{aligned}}$$

$$f(x, \underset{\text{param}}{y_0}) = (x-b_x)^T A(y_0-b_y) + \frac{\lambda}{2}\|x-b_x\|^2 - \frac{\lambda}{2}\|y_0-b_y\|^2$$

$$\nabla f(x) = A(y_0-b_y) + \frac{\lambda}{2}2(x-b_x)$$

$$\nabla^2 f(x) = \operatorname{diag}\{\lambda\} \Rightarrow \underline{\lambda\text{– сильно вып. по } x}$$

$$f(x_0,y) = (x_0-b_x)^T A(y-b_y) + \frac{\lambda}{2}\|x_0-b_x\|^2 - \frac{\lambda}{2}\|y-b_y\|^2$$

$$\nabla f(y) = A^T(x_0-b_x) - \frac{\lambda}{2}2(y-b_y)$$

$$\nabla^2 f(y) = \operatorname{diag}\{-\lambda\} \Rightarrow \underline{\lambda\text{– сильно вогнута по } y}$$

$$F(z) = F(x,y) = \begin{pmatrix} \nabla_x f(x,y) \\ -\nabla_y f(x,y) \end{pmatrix} \qquad \nabla f(x,y) = \left(\begin{array}{cc|cc} \lambda & & & \\ & \ddots \; \lambda & & 0 \\ \hline & 0 & -\lambda & \\ & & & \ddots \; -\lambda \end{array}\right)$$

$$\forall z_1, z_2 \quad \langle F(z_1) - F(z_2), z_1 - z_2 \rangle \overset{?}{\geq} \frac{\mu}{2} \| z_1 - z_2 \|^2$$

$z = \begin{pmatrix} x \\ y \end{pmatrix}$:

$$\left\langle \begin{pmatrix} \nabla_x f(x_1, y_1) - \nabla_x f(x_2, y_2) \\ -(\nabla_y f(x_1, y_1) - \nabla_y f(x_2, y_2)) \end{pmatrix}, \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix} \right\rangle \geq \frac{\mu}{2} \left\| \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix} \right\|^2$$

$$\left(\nabla_x f(x_1, y_1)(x_1 - x_2) - \nabla_x f(x_2, y_2)(x_1 - x_2) \right) - \left(\nabla_y f(x_1, y_1)(y_1 - y_2) - \nabla_y f(x_2, y_2) \right) \overset{?}{\geq} \frac{\mu}{2} \left(\|x_1 - x_2\|^2 + \|y_1 - y_2\|^2 \right)$$

$$\langle \nabla f(x_1) - \nabla f(x_2), x_1 - x_2 \rangle \geq \frac{\mu}{2} \| x_1 - x_2 \|^2$$

$$f(x_2) - f(x_1) - \langle \nabla f(x_1), x_1 - x_2 \rangle \geq \frac{\mu}{2} \| x_1 - x_2 \|^2$$

$$f(x_2) - f(x_1) = \langle \nabla f(x_2), x_2 - x_1 \rangle$$

$$\geq \frac{\mu_x}{2} \| x_1 - x_2 \|^2 + \frac{\mu_y}{2} \| y_1 - y_2 \|^2 \geq$$

$$\geq \frac{2 \min(\mu_x, \mu_y)}{2} \left(\|x_1 - x_2\|^2 + \|y_1 - y_2\|^2 \right)$$

Да, сильно монотонный с $\mu = 2 \min(\mu_x, \mu_y)$

У нас: $\mu = 2 \min(\lambda, \lambda) = 2\lambda = 2 = 4$