Unit 2 Seminar: Sets, Set Theory, Truth Tables and Logic

1. Read Partee et al (1993) Chapter 1 and then attempt exercises 1 and 4, located at the end of the chapter.

1- Given the following sets:

A = {a,b,c,2,3,4}

B = {a, b}

C = {c,2}

D = {b,c}

E = {a,b,{c}}

F = ∅

G = {{a, b},{c, 2}}

classify each of the following statements as true or false

(a) $c \in A$	TRUE
(b) $c \in F$	FALSE
(c) $c \in E$	FALSE
(d) $\{c\} \in E$	TRUE
(e) $\{c\} \in C$	FALSE
(f) $B \subseteq A$	TRUE
(g) $D \subset A$	TRUE
(h) $A \subseteq C$	FALSE
(i) $D \subseteq E$	FALSE
(j) $F \subseteq A$	TRUE
(k) $E \subseteq F$	FALSE
(l) $B \in G$	TRUE
(m) $B \subseteq G$	FALSE
(n) $\{B\} \subseteq G$	TRUE
(o) $D \subseteq G$	FALSE
(p) $\{D\} \subseteq G$	FALSE
(q) $G \subseteq A$	FALSE
(r) $\{\{c\}\} \subseteq E$	TRUE

4. Consider the following sets:

Sl = {{∅},{A},A}

S2 = A

S3 = {A}

S4 = {{A}}

S5 = {{A},A}

S6 = ∅

S7 = {∅}

S8 = {{∅}}

S9 = {∅,{∅}}

Answer the following questions. Remember that the members of a set are the items separated by commas, if there is more than one, between the outermost braces only; a subset is formed by enclosing within braces zero or more of the members of a given set, separated by commas.

(a) Of the sets S1- S9 which are members of S1?

S2, S3, S6

(b) which are subsets of S1?

S1, S2, S3, S5, S6

(c) which are members of S9?

S6, S7

(d) which are subsets of S9?

S6, S7, S9

(e) which are members of S4?

S3

(f) which are subsets of S4?

S3, S4

2. Read the wiki at Sharma et al (2022) and then attempt the exercises below:

i. For each clause (a) - (f) below, create truth tables for each to answer the question of when each statement is false.

a. ¬ P

P	**¬P**
T	F
F	T

b. P ∧ Q

P	**Q**	**P ∧ Q**
T	T	T
T	F	F
F	T	F
F	F	F

c. P ∨ Q

P	**Q**	**P ∨ Q**
T	T	T
T	F	T
F	T	T
F	F	F

d. P → Q

P	**Q**	**P ⇒ Q**
T	T	T
T	F	F
F	T	T
F	F	T

e. P ←→ Q

P	**Q**	**P ⇔ Q**
T	T	T
T	F	F
F	T	F
F	F	T

f. P → (¬ Q)

P	**Q**	**¬Q**	**P ⇒ (¬Q)**
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

ii. Consider the statement (¬ Q) ⇒ (¬ P).

a. When is it false?

When Q is false and P is true.

b. Now consider P → Q. When is it false?

When P is true and Q is false.

c. Do you believe these two compound statements mean the same thing?

No, because the direction of implication is opposite.

d. Construct the truth table for the statement (¬ Q) ⇒ (¬ P). Then revisit your answer to (c).

Q	**¬Q**	**P**	**¬P**	**(¬Q) ⇒ (¬P)**
T	F	T	F	T
T	F	F	T	T
F	T	T	F	F
F	T	F	T	T

OK, yes – it is the same. The direction of implication is cancelled by both being negations.

iii. Construct the truth table for P XOR Q.

P	**Q**	**P XOR Q**
T	T	F
T	F	T
F	T	T
F	F	F

iv. Construct truth tables for the following statements.

a. ¬ (P ∧ Q)

P	**Q**	**P ∧ Q**	**¬ (P ∧ Q)**
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

b. P ∨ (Q ∧ R)

P	Q	R	(Q ∧ R)	P ∨ (Q ∧ R)
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

c. P ∨ (Q ∨ R)

P	Q	R	(Q ∨ R)	P ∨ (Q ∨ R)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	F

d. (P ∨ Q) ∨ R (Compare to the previous statement.)

P	Q	R	(P ∨ Q)	(P ∨ Q) ∨ R
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

Same as the previous one because the parenthesis make no difference with disjunction.

e. (P → Q) ∧ (Q → P)

P	Q	P → Q	Q → P	(P → Q) ∧ (Q → P)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

This is the same result as P ⇔ Q.

References

Partee, B.H., Ter Meulen, A. & Wall, R.E. (1993) *Mathematical Methods in Linguistics. Studies in Linguistics and Philosophy.* Dordrecht: Springer Netherlands.

Sharma, G., Lo, N. & Pilling, G. (2022) Truth Tables.