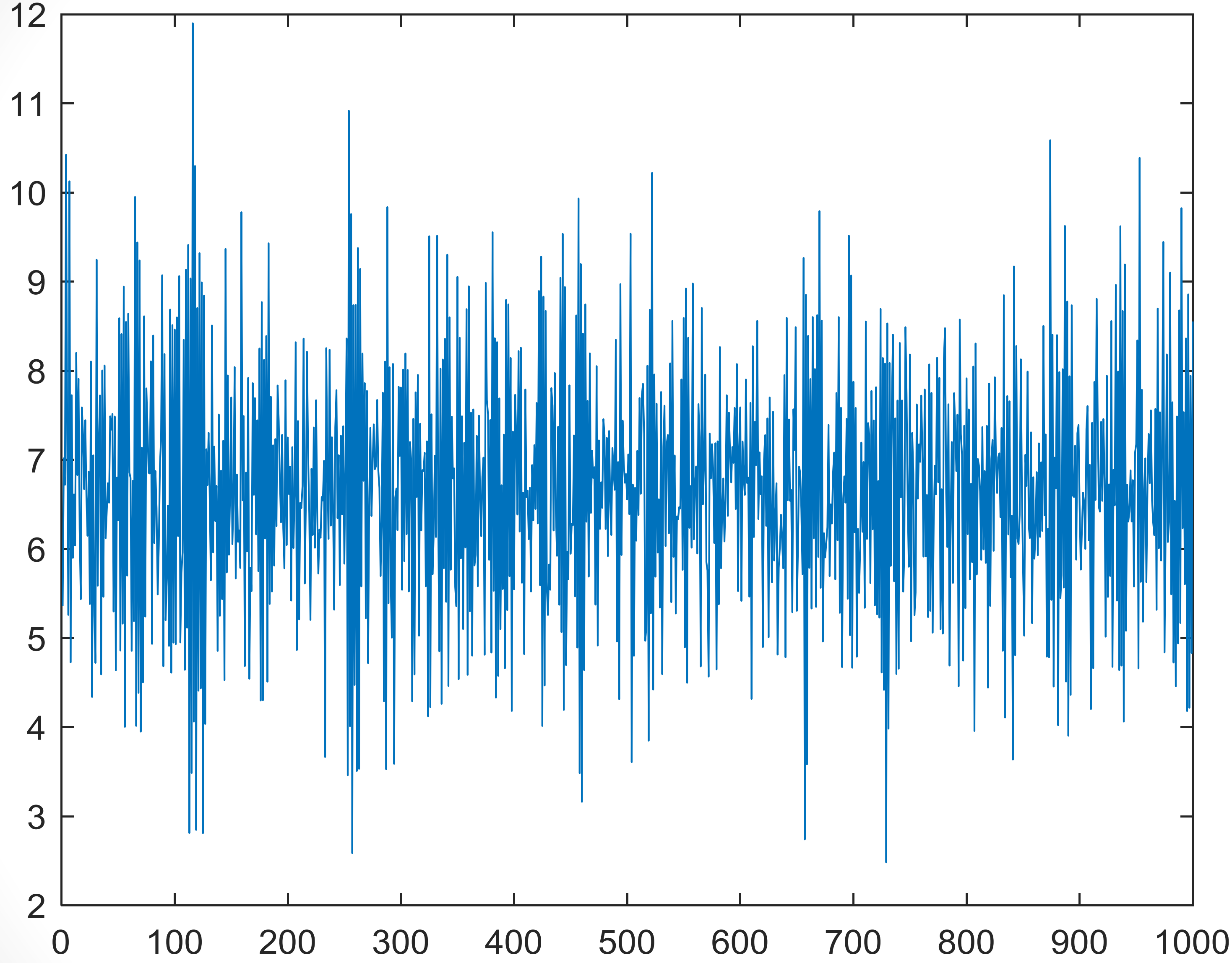
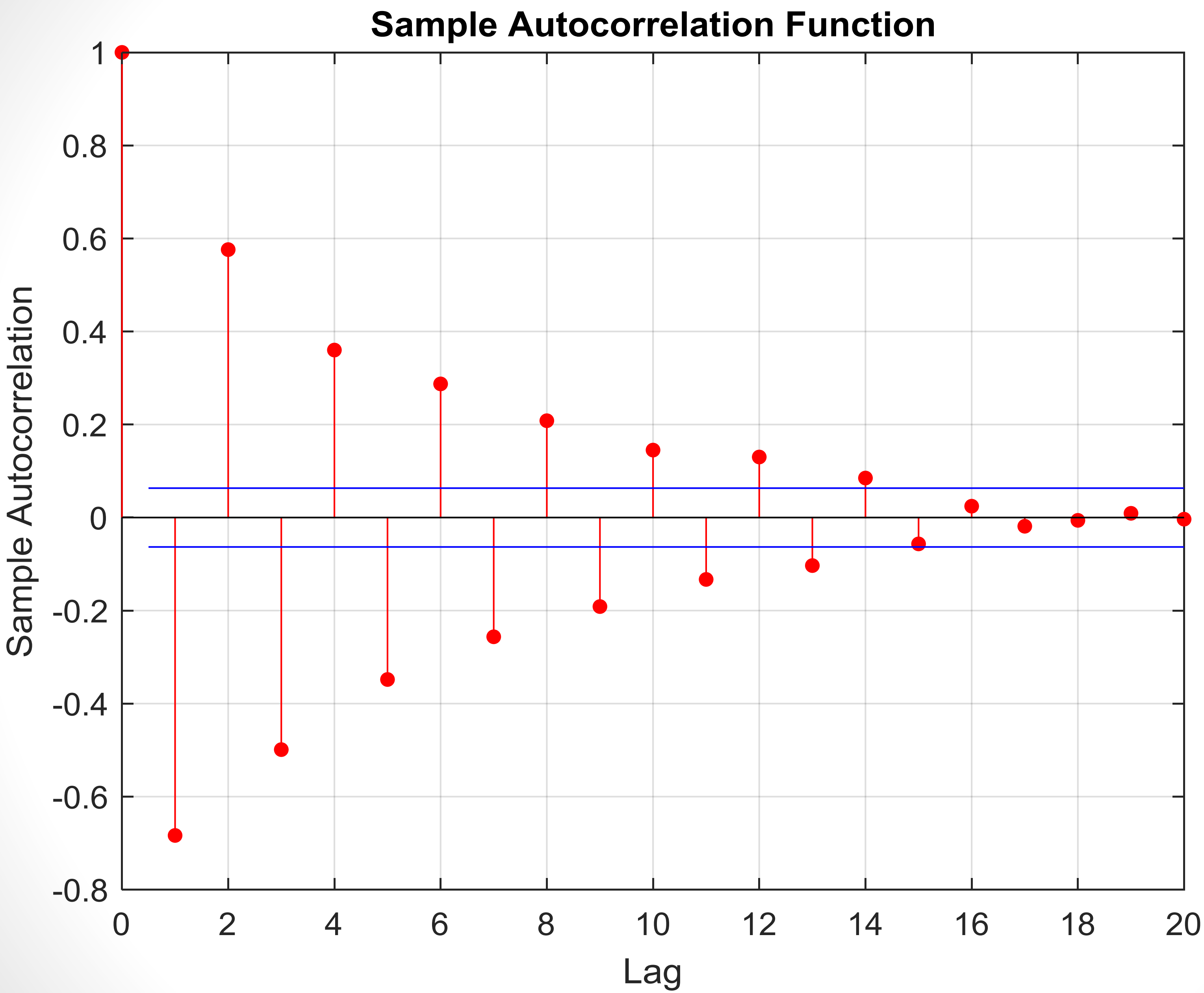
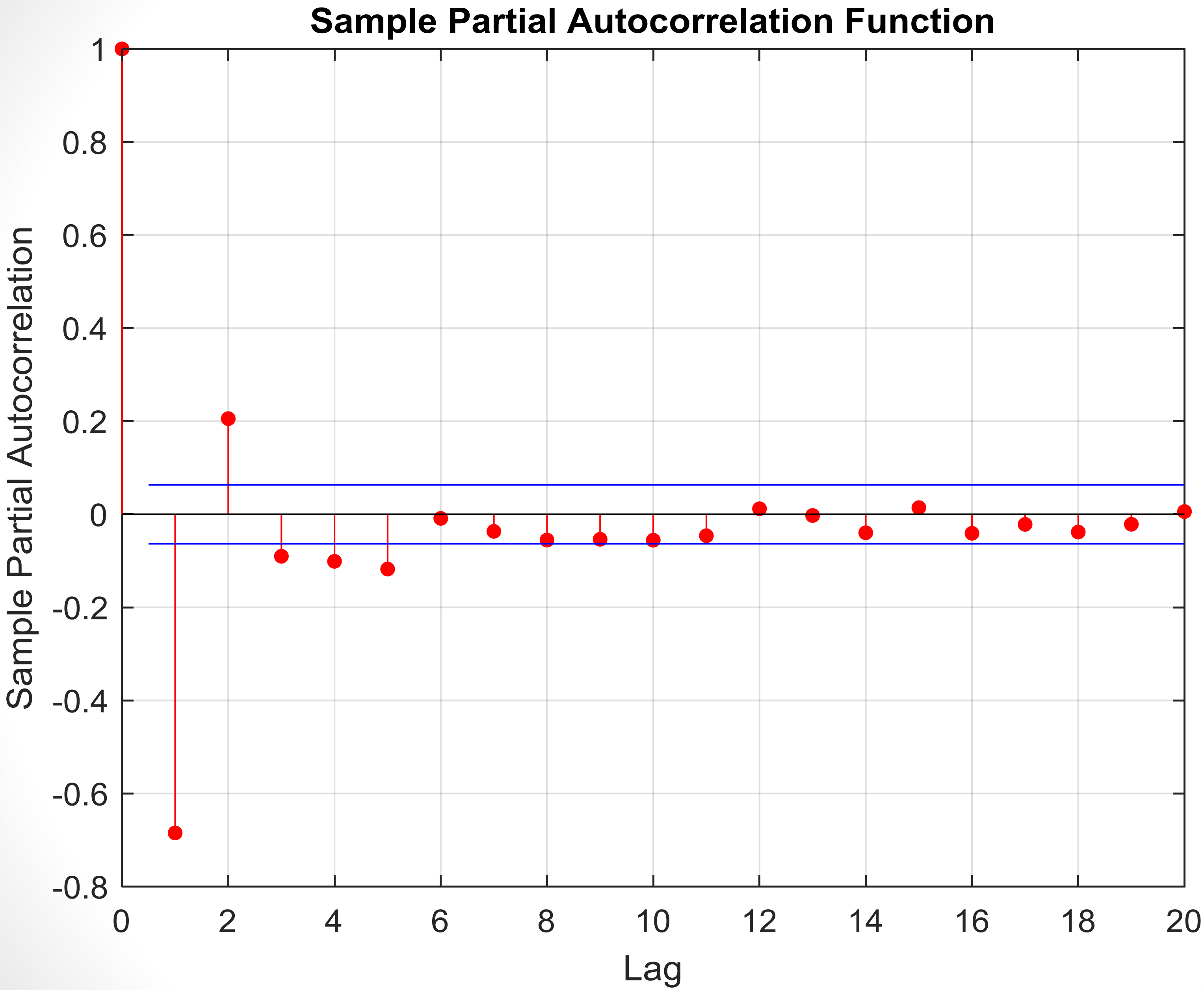
ARMA Example

$$Y_t = 0.3Y_{t-1} + 0.6Y_{t-2} - 0.2Y_{t-3} + 2 + e_t - 0.9e_{t-1}$$

```
arma31=arima('c',2, 'var' ,1, 'ar',[0.3 0.6 -0.2],'ma',-0.9)
y=simulate(arma31,1000);
plot(y)
autocorr(y)
parcorr(y)
```


12
11
10
9
8
7
6
5
4
3
2
0
100
200
300
400
500
600
700
800
900
1000


Sample Autocorrelation Function
Sample Autocorrelation
Lag
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
2
4
6
8
10
12
14
16
18
20



Sample Partial Autocorrelation Function
Sample Partial Autocorrelation
Lag
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
2
4
6
8
10
12
14
16
18
20

Estimation and Model Selection

```
m1=arima(1,0,0);
m2=arima(2,0,1);
m3=arima(3,0,1);
m4=arima(4,0,2);
```

Estimation and Model Selection

```
[m1Est,m1VM,m1LLF]=estimate(m1,y);
[m2Est,m2VM,m2LLF]=estimate(m2,y);
[m3Est,m3VM,m3LLF]=estimate(m3,y);
[m4Est,m4VM,m4LLF]=estimate(m4,y);
```

summarize(m1Est)

ARIMA(1,0,0) Model (Gaussian Distribution)

Effective Sample Size: 1000
Number of Estimated Parameters: 3
LogLikelihood: -1450.83
AIC: 2907.66
BIC: 2922.39

	Value	StandardError	TStatistic	PValue
Constant	11.217	0.15044	74.558	0
AR{1}	-0.6853	0.022144	-30.948	2.7152e-210
Variance	1.0659	0.045716	23.315	3.138e-120

summarize(m2Est)

ARIMA(2,0,1) Model (Gaussian Distribution)

Effective Sample Size: 1000
Number of Estimated Parameters: 5
LogLikelihood: -1426.88
AIC: 2863.75
BIC: 2888.29

	Value	StandardError	TStatistic	PValue
Constant	11.682	1.5113	7.7299	1.0762e-14
AR{1}	-0.79254	0.13158	-6.0231	1.7114e-09
AR{2}	0.037289	0.096791	0.38526	0.70005
MA{1}	0.26239	0.12879	2.0373	0.041618
Variance	1.016	0.04317	23.535	1.7874e-122

summarize(m3Est)

ARIMA(3,0,1) Model (Gaussian Distribution)

Effective Sample Size: 1000
Number of Estimated Parameters: 6
LogLikelihood: -1411.39
AIC: 2834.78
BIC: 2864.23

	Value	StandardError	TStatistic	PValue
Constant	2.9002	0.58865	4.9269	8.3565e-07
AR{1}	0.27219	0.056747	4.7966	1.614e-06
AR{2}	0.57238	0.041742	13.712	8.5904e-43
AR{3}	-0.28038	0.030233	-9.2742	1.79e-20
MA{1}	-0.82243	0.051248	-16.048	5.899e-58
Variance	0.98501	0.04112	23.955	8.2556e-127

summarize(m4Est)

ARIMA(4,0,2) Model (Gaussian Distribution)

Effective Sample Size: 1000
Number of Estimated Parameters: 8
LogLikelihood: -1410.7
AIC: 2837.41
BIC: 2876.67

	Value	StandardError	TStatistic	PValue
Const	3.4854	2.4903	1.3996	0.16164
AR{1}	0.21151	0.86235	0.24527	0.80625
AR{2}	0.57622	0.24962	2.3084	0.020976
AR{3}	-0.26297	0.50262	-0.52319	0.60084
AR{4}	-0.048495	0.24412	-0.19865	0.84254
MA{1}	-0.7695	0.86163	-0.89308	0.37182
MA{2}	-0.010379	0.71592	-0.014498	0.98843
Var	0.98366	0.041368	23.778	5.6281e-125

Estimation and Model Selection

```
[m1AIC m1BIC]=aicbic(m1LLF,sum(any(m1VM)),1000)
[m2AIC m2BIC]=aicbic(m2LLF,sum(any(m2VM)),1000)
[m3AIC m3BIC]=aicbic(m3LLF,sum(any(m3VM)),1000)
[m4AIC m4BIC]=aicbic(m4LLF,sum(any(m4VM)),1000)
[m1AIC m2AIC m3AIC m4AIC]
[m1BIC m2BIC m3BIC m4BIC]
```

AIC

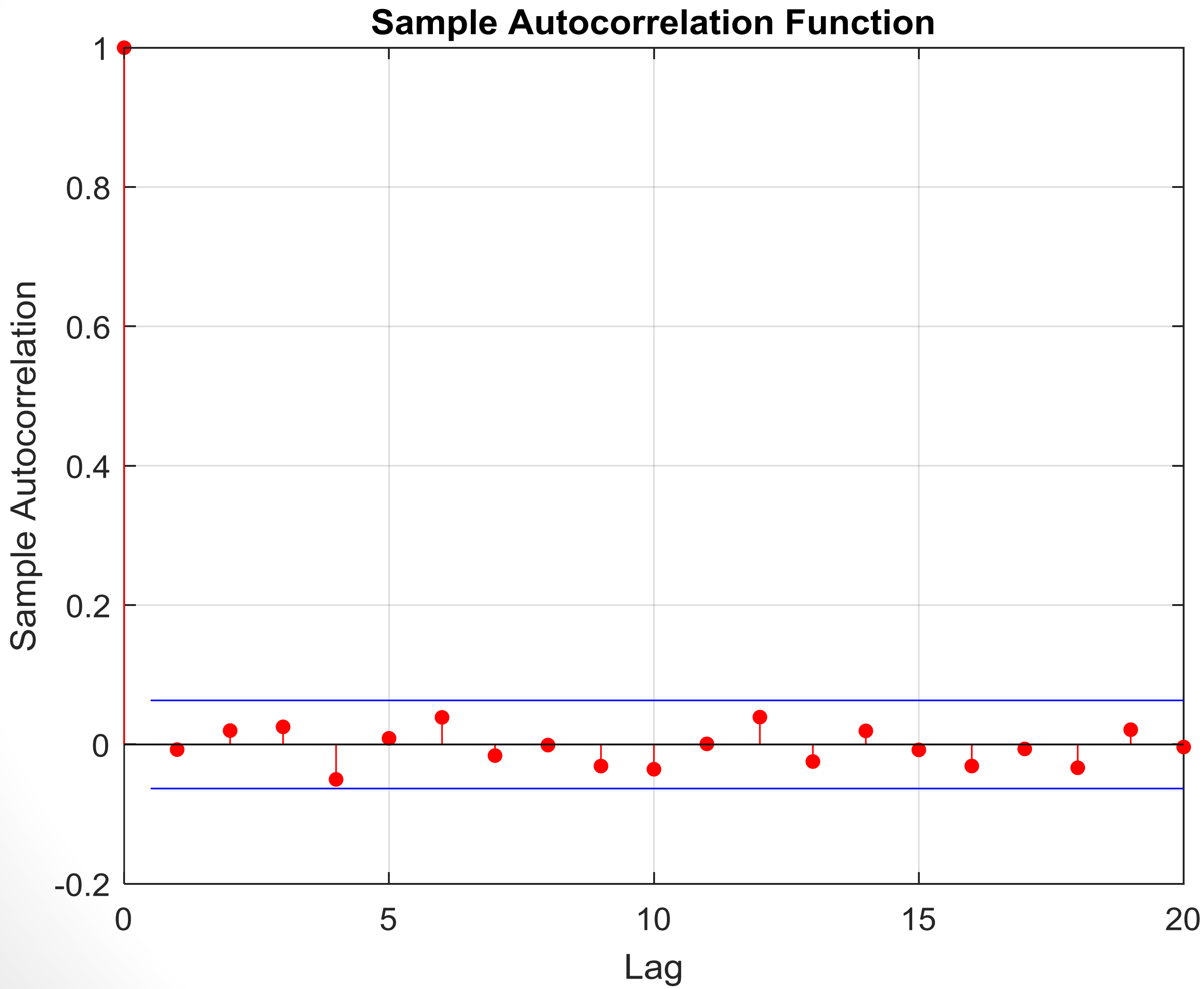
AIC

2.9077e+003	2.8638e+003	2.8348e+003	2.8374e+003

BIC

2.9224e+003	2.8883e+003	2.8642e+003	2.8767e+003

Adequacy Test



Adequacy Test

LBQ Test

[H,p]=lbqtest(Em3)

H = 0

p = 0.8780

Problems

Is the following model invertible?

$$Y_t = e_t - 0.4e_{t-1} - 0.05e_{t-2}$$

Lag (Backshift) operator

$$Be_t = e_{t-1},\ B^2 e_t = e_{t-2},\ \ldots,\ B^M e_t = e_{t-M}$$

$$MA(M): \quad Y_t = \mu + (1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_M B^M)e_t$$

$$Y_t = \mu + \theta(B)e_t$$

$$AR(R): \quad (1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - \cdots - \phi_R B^R)Y_t = c + e_t$$

$$\phi(B)Y_t = c + e_t$$

$$ARMA(R,M): \quad \phi(B)Y_t = c + \theta(B)e_t$$

Invertibility conditions

$$\tilde{Y}_t = Y_t - \mu$$

$$\phi(B)\tilde{Y}_t = \theta(B)e_t$$

$$\theta^{-1}(B)\phi(B)\tilde{Y}_t = e_t$$

$$1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_R B^M = 0$$

Invertibility conditions

All roots of equation (1) *to lie out of the unit circle*

$$1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_M B^M = 0 \qquad (1)$$

or

All roots of equation (2) *to lie in the unit circle*

$$z^M + \theta_1 z^{M-1} + \theta_2 z^{M-2} + \cdots + \theta_M = 0 \qquad (2)$$

$$Y_t = e_t - 0.4e_{t-1} - 0.05e_{t-2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- x^2-0.4x-0.05=0
- D=b^2-4ac=0.16+0.20=0.36
- x1=(0.4+0.6)/2=0.5
- x2=(0.4-0.6)/2=-0.1

What is the value of ρ_1 ?

- If AR(3) model has the following roots 0.8, 0.2 and -0.5

Yule-Walker equations

$$\rho_1 = \phi_1\rho_0 + \phi_2\rho_1 + \cdots + \phi_R\rho_{R-1}$$

$$\cdots$$

$$\rho_R = \phi_1\rho_{R-1} + \phi_2\rho_{R-2} + \cdots + \phi_R\rho_0$$

$$\cdots$$

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} + \cdots + \phi_R\rho_{k-R} \qquad k > R$$

- $(x+0.5)(x-0.8)(x-0.2)=(x^2-0.8x+0.5x-0.4)(x-0.2)$ $=(x^2-0.3x-0.4)(x-0.2)=x^3-0.3x^2-0.4x-$ $0.2x^2+0.06x-0.08=$
- $=x^3-0.5x^2-0.34x+0.08$
- $x=1/B$
- $Y(t)-0.5Y(t-1)-0.34Y(t-2)+0.08Y(t-3)=e(t)$
- $Y(t)=0.5Y(t-1)+0.34Y(t-2)-0.08Y(t-3)+ e(t)$
- $r1=0.5+0.34r1-0.08r2$; $0.66r1=0.5-0.08r2$
- $r2=0.5r1+0.34-0.08r1$;$r2=0.42r1+0.34$
- $0.66r1=0.5-0.08(0.42r1+0.34)=0.5-0.0336r1-0.0272$
- $0.6936r1=0.4728$
- $r1=0.6817$

- If the roots of AR(3) are -0.2, 0.1 and -0.5, calculate ρ_2.

- Answer $\rho_2 = 0.0434$

Calculate the value of ρ_2 for the following model:

$$Y_t = e_t + 0.4e_{t-1} + 0.5e_{t-2} + 0.2e_{t-3}$$

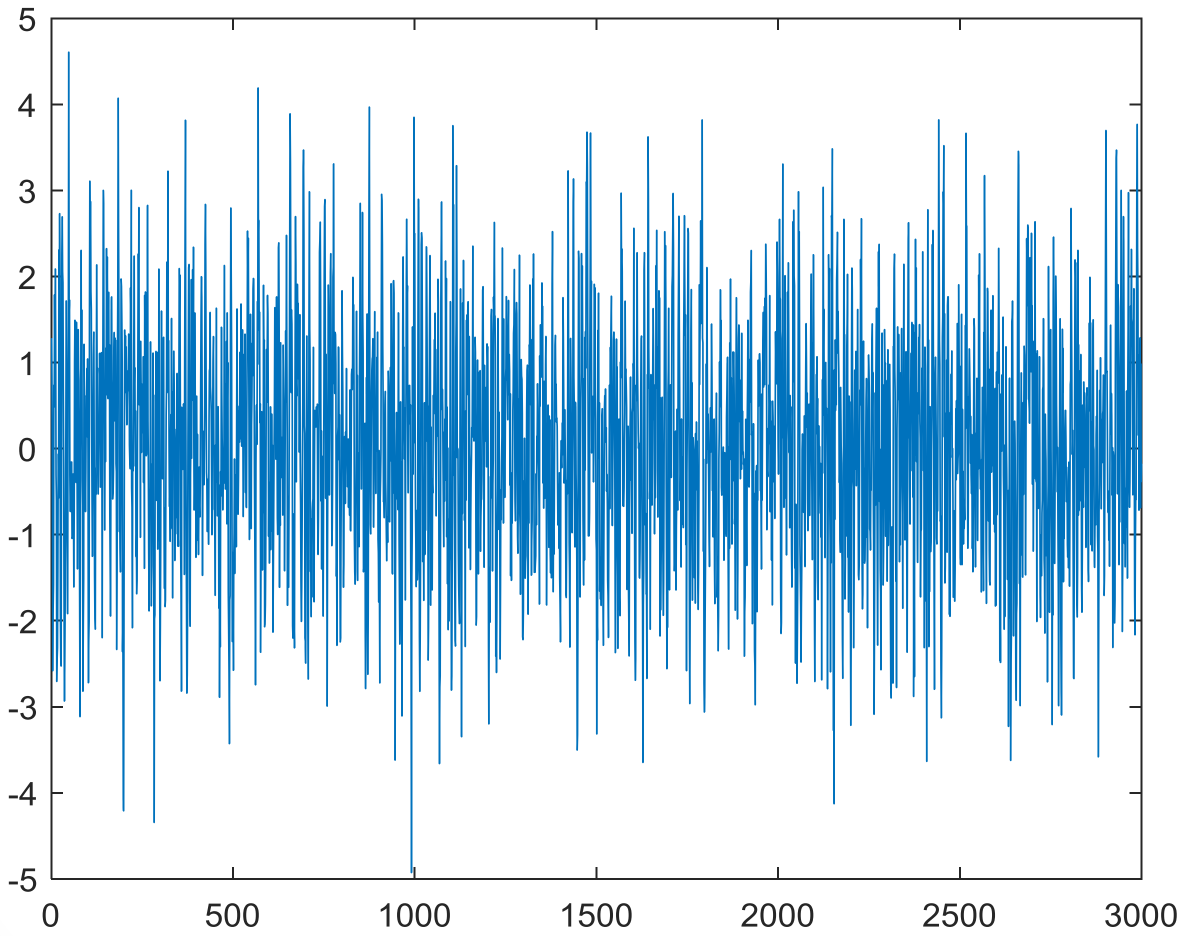
$$Y_t = e_t + 0.4e_{t-1} + 0.5e_{t-2} + 0.2e_{t-3}$$

- Var(Yt)=1+0.16+0.25+0.04=1.45Var(e(t))
- g2=E[Y(t)*Y(t-2)]=
- E[(e(t)+0.4e(t-1)+0.5e(t-2)+0.2e(t-3))…
- (e(t-2)+0.4e(t-3)+0.5e(t-4)+0.2e(t-5))]
- g2=(0.5+0.08)Var(e(t))
- r2=g2/Var(Yt)=0.58/1.45=0.4

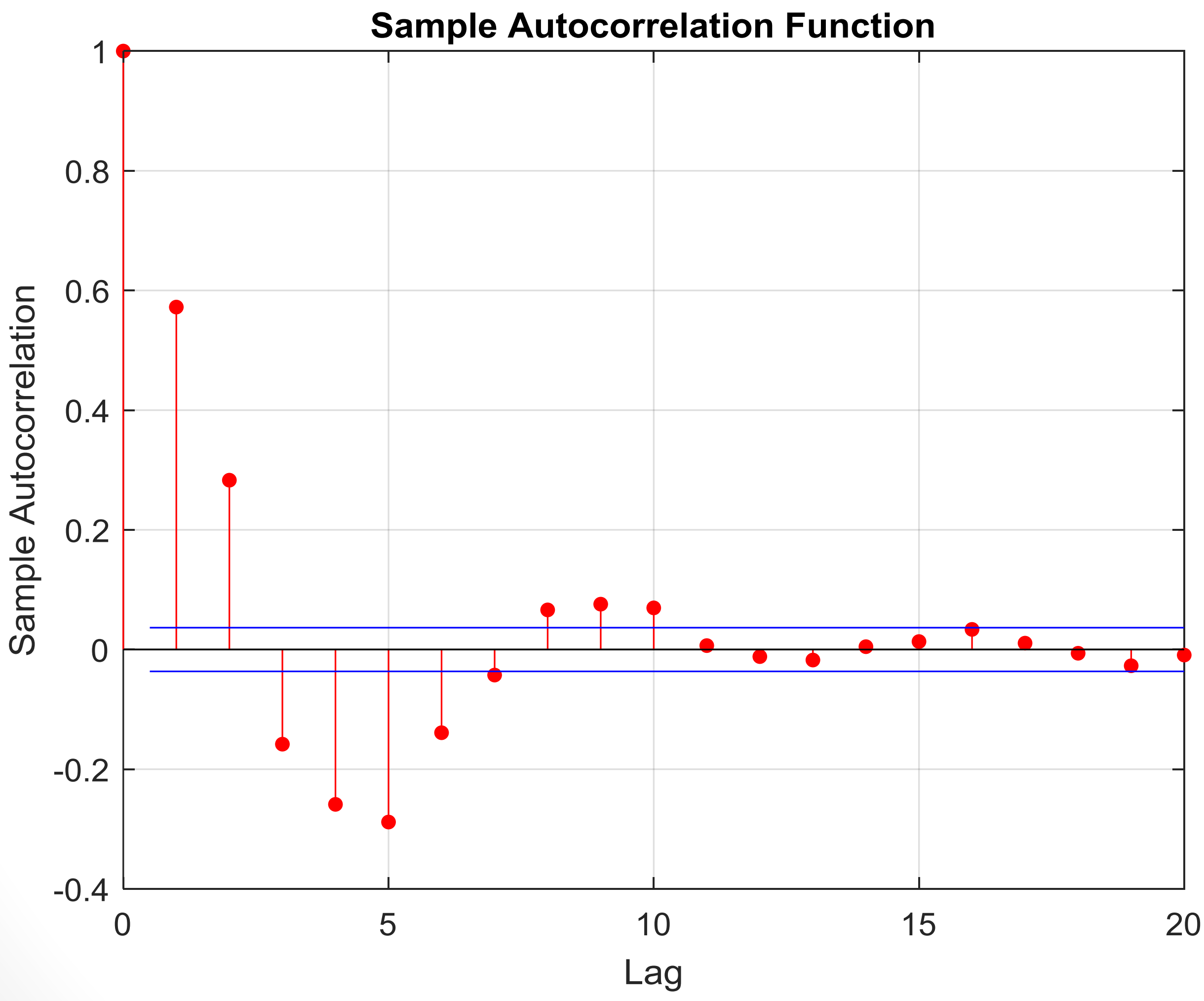
MA(M) model autocorrelations

$$\rho_k = \begin{cases} \dfrac{\theta_k + \theta_1\theta_{k+1} + \cdots + \theta_{M-k}\theta_M}{1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_M^2} & k = 1, \ldots, M \\ 0 & k > M \end{cases}$$

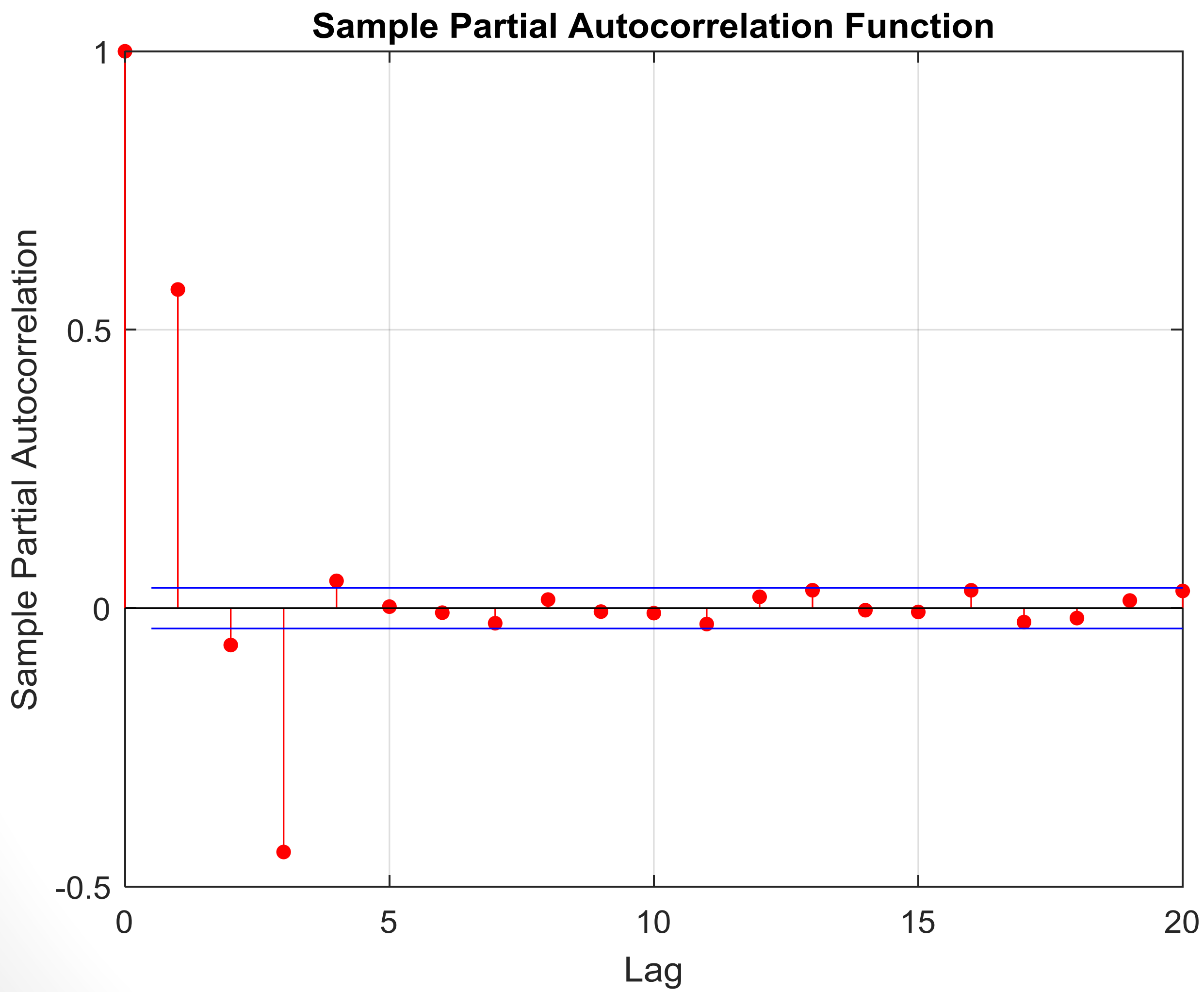
Example 1



Example 1 (n=3000)



Example 1



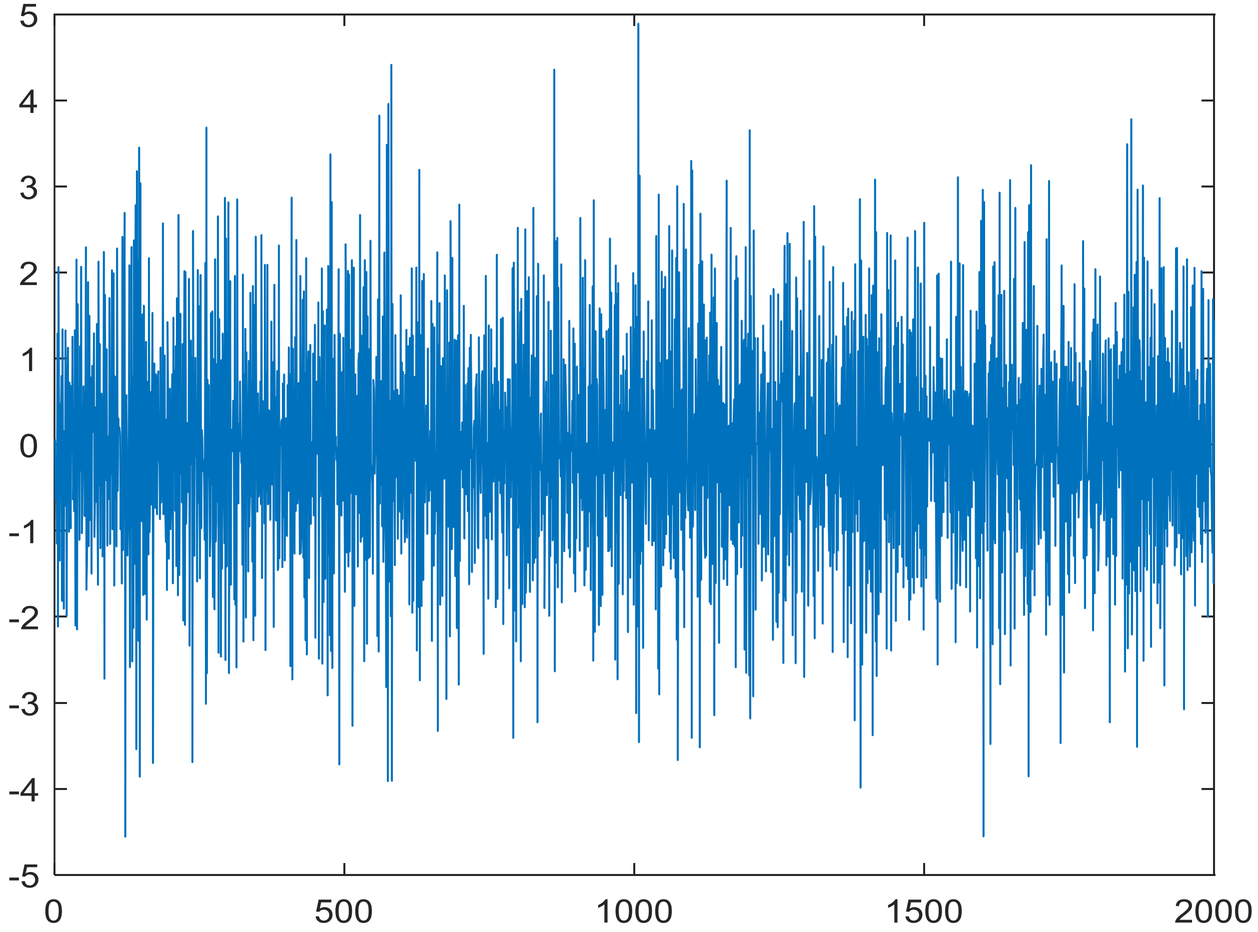
Sample Partial Autocorrelation Function
Sample Partial Autocorrelation
Lag
1
0.5
0
-0.5
0
5
10
15
20

Example 1

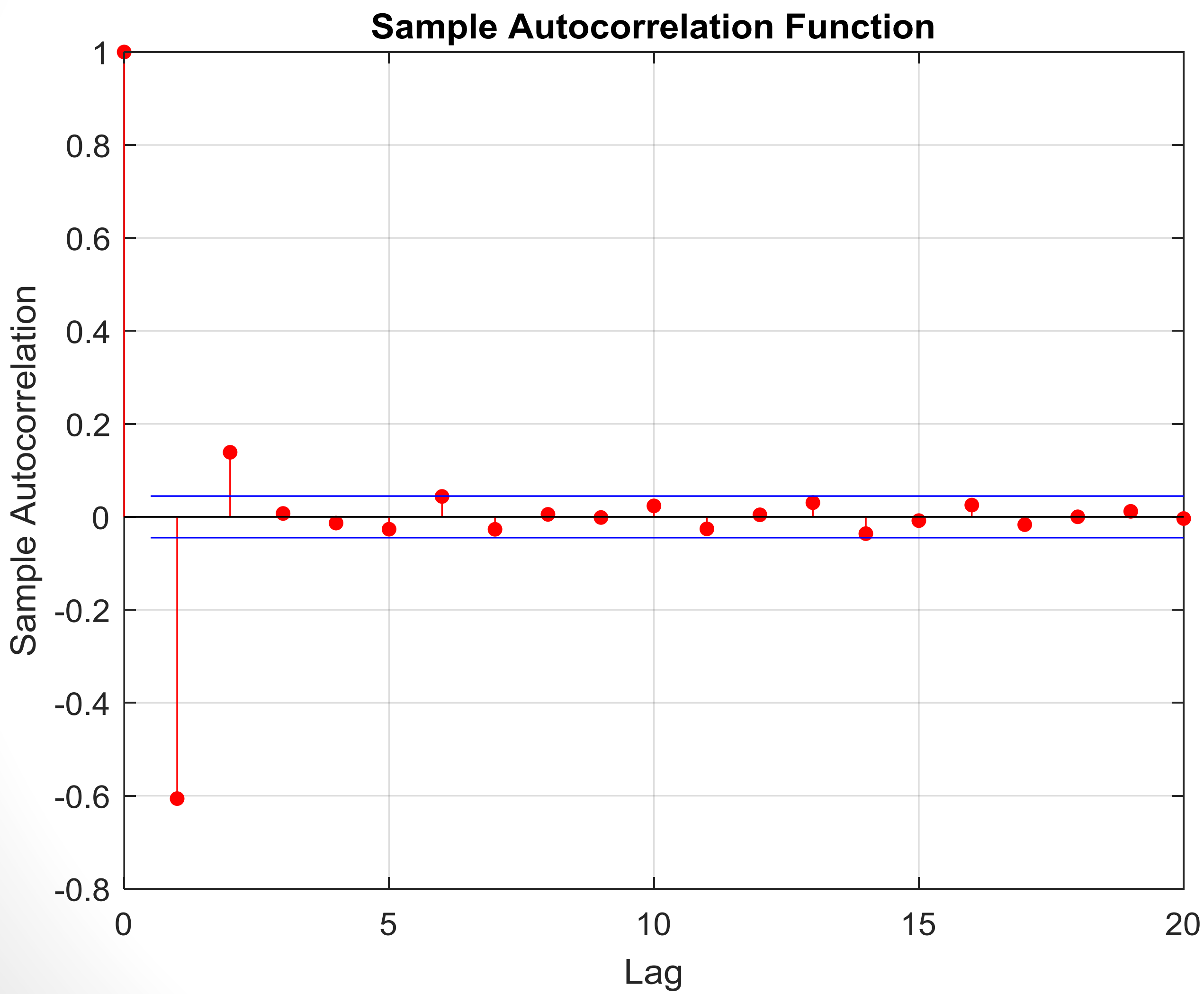
$$y_t = 3 + 0.6y_{t-1} + 0.2y_{t-2} - 0.5y_{t-3} + 0.1y_{t-4} + e_t$$

```
r=roots([1 -0.6 -0.2 .5 -0.1])
  -0.7593 + 0.0000i
   0.5645 + 0.5034i
   0.5645 - 0.5034i
   0.2302 + 0.0000i
abs(r)
    0.7593
    0.7564
    0.7564
    0.2302
```

Example 2

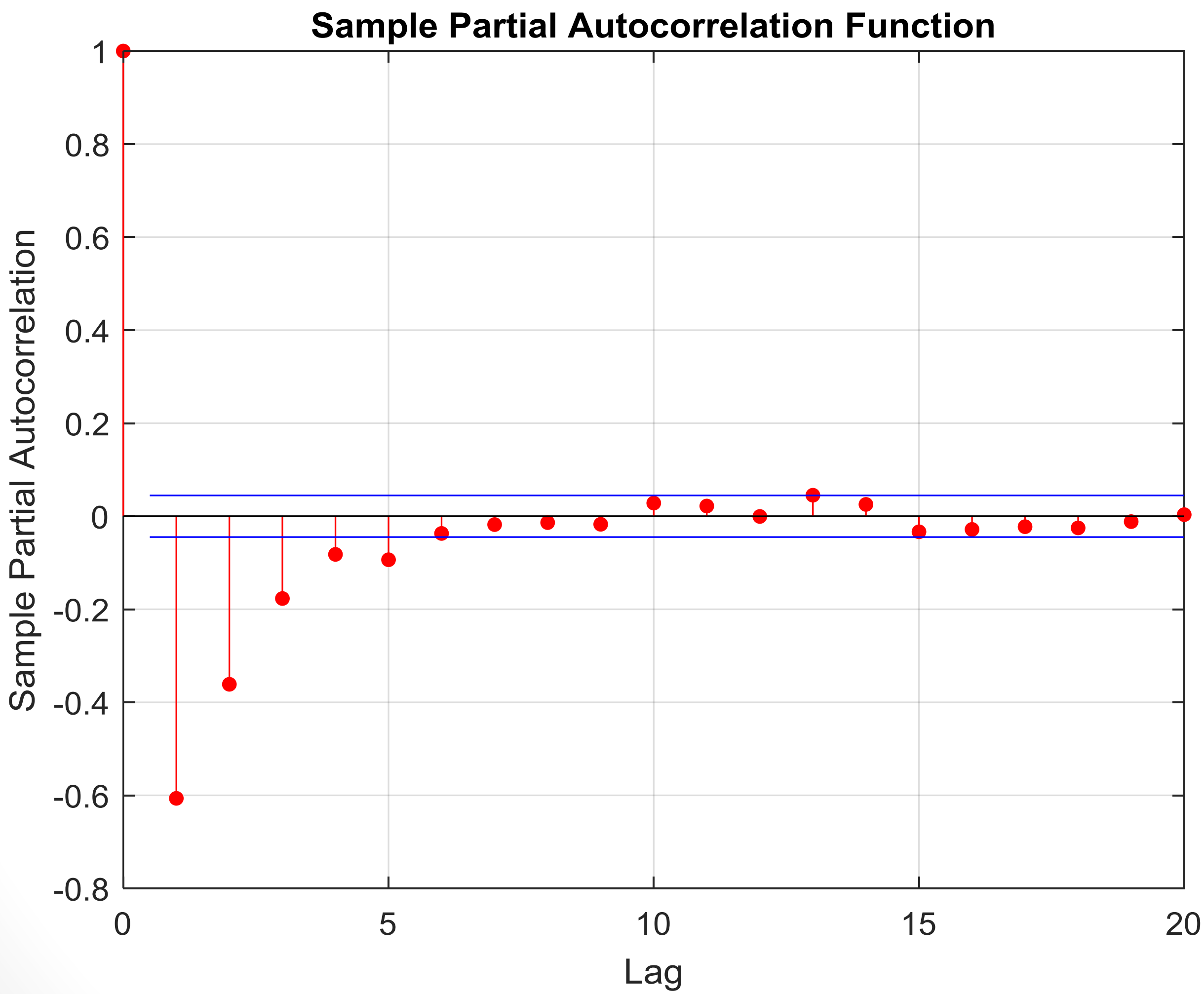


Example 2



Sample Autocorrelation Function
Sample Autocorrelation
Lag
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15
20

Example 2


Sample Partial Autocorrelation Function
Sample Partial Autocorrelation
Lag
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15
20

Example 2

$$y_t = -2 + e_t + 0.9e_{t-1} + 0.23e_{t-2} - 0.001e_{t-3} - 0.0034e_{t-4}$$

```
r=roots(p)
   0.4000 + 0.1000i
   0.4000 - 0.1000i
   0.2000 + 0.0000i
  -0.1000 + 0.0000i
abs(r)
    0.4123
    0.4123
    0.2000
    0.1000
```