Introduction to Time Series Analysis

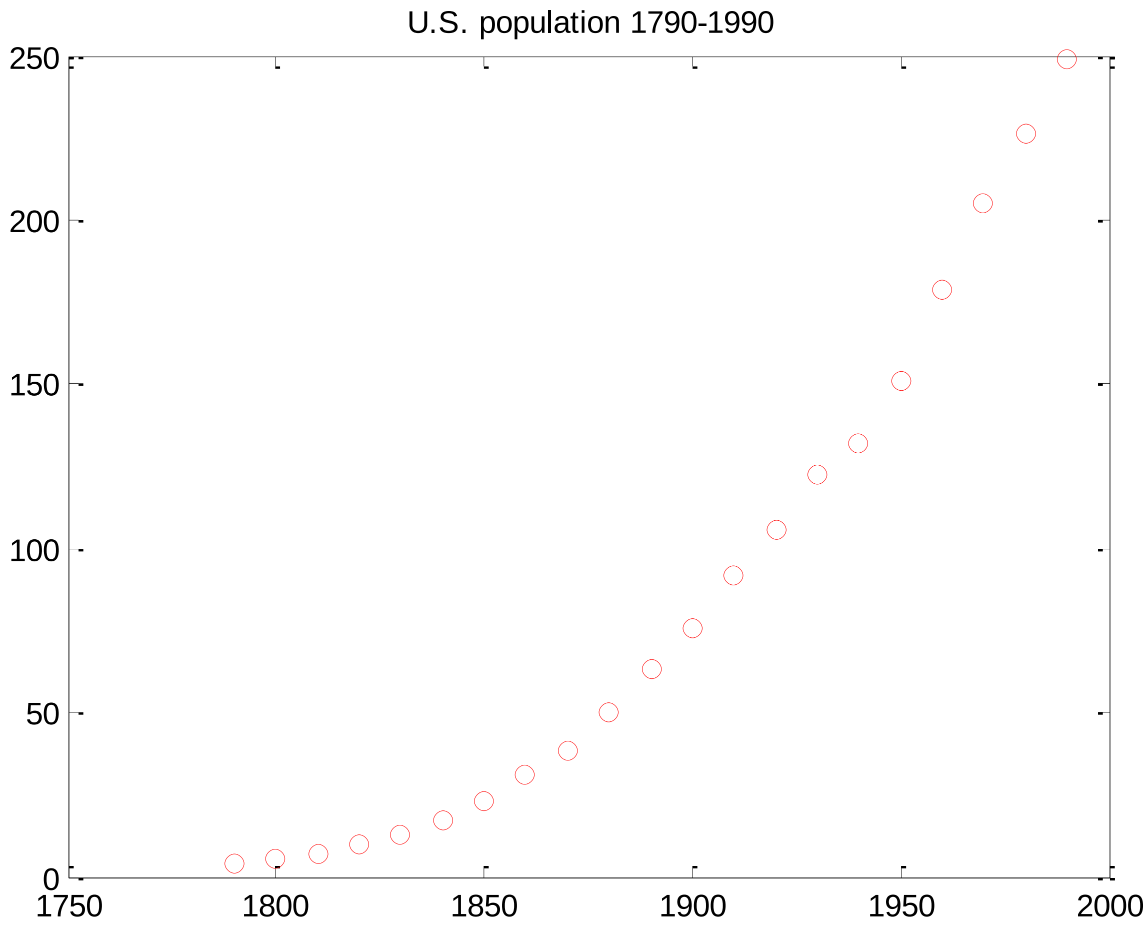
Time Series Definition

- A **time series** is a set of observations *Yt* , each one being recorded at a specific time *t*
- A *discrete-time time series* is one in which the set of times at which observations are made is a discrete set and observations are made at fixed time intervals
- *Continuous time time series* are obtained when observations are recorded continuously over some time interval

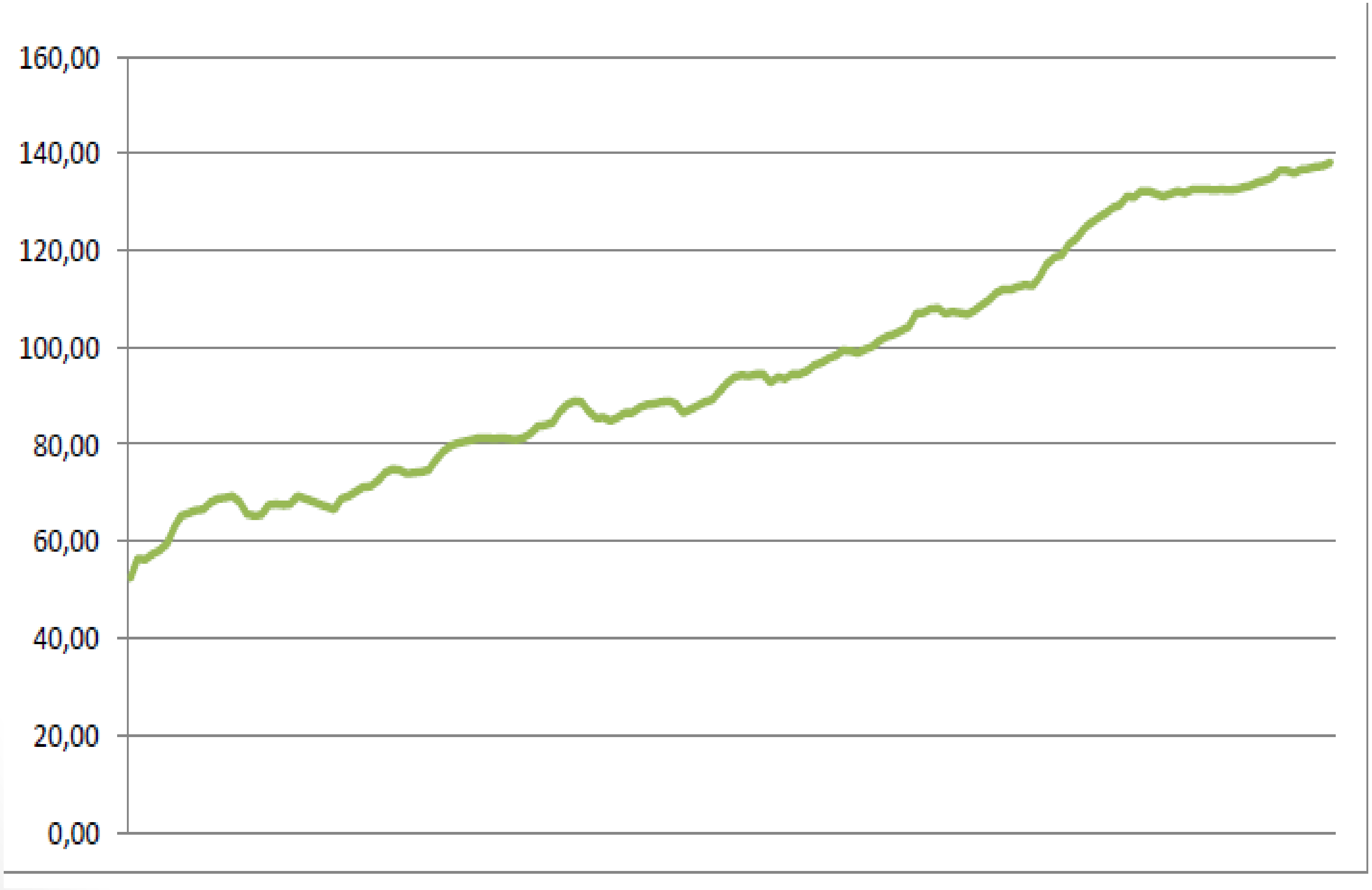
Examples of Time Series

- Example 1 - Population of the U.S.A., 1790–1990
- Example 2 - Bulgarian Harmonized CPI 1997-2010
- Example 3 - SOFIX index 2004-2010
- Example 4 - Bad and Restructured Consumer Loans 2004-2009

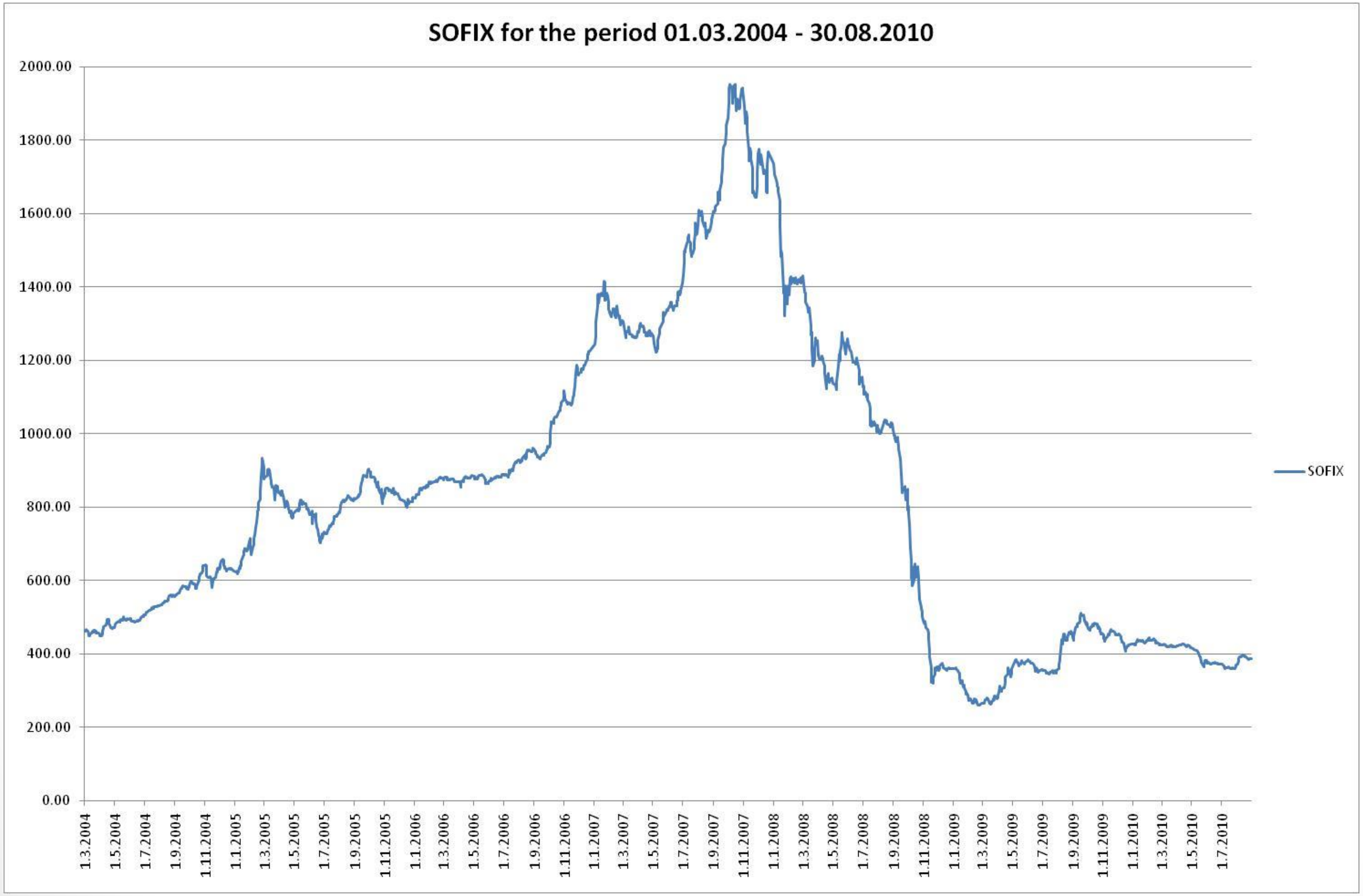
Example 1 - Population of the U.S.A., 1790–1990



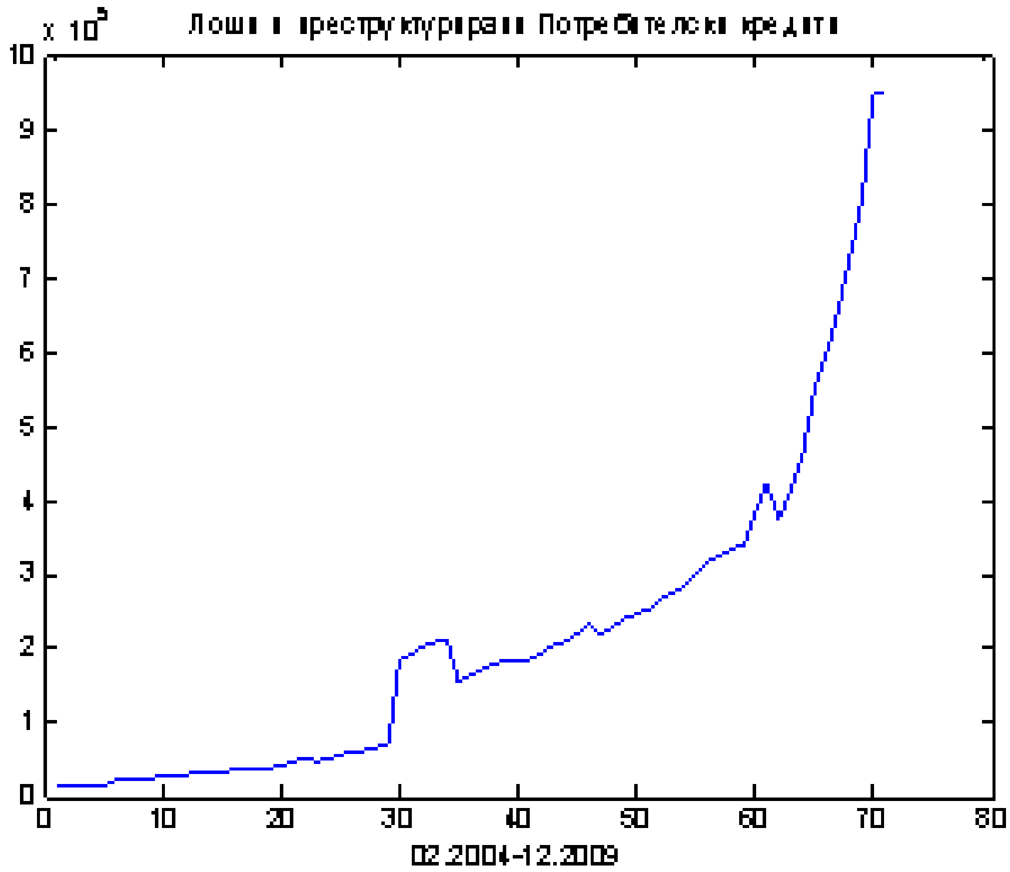
Bulgarian Harmonized CPI 1997-2010



SOFIX index 2004-2010



Example 4 - Bad and Restructured Consumer Loans 2004-2009



A General Approach to Time Series Modeling

- Plot the series and examine the main features of the graph:
  - Trend
  - Seasonal component
  - Apparent sharp changes in behavior
  - Outlying observations

A General Approach to Time Series Modeling

- Remove the trend and seasonal components to get *stationary* residuals
- To achieve this goal it may sometimes be necessary to apply a preliminary transformation to the data
- For example, if the magnitude of the fluctuations appears to grow roughly linearly with the level of the series, then the transformed series $\{\ln X_1, \ldots, \ln X_n\}$ will have fluctuations of more constant magnitude

A General Approach to Time Series Modeling

- There are several ways in which trend and seasonality can be removed
- The first approach is to estimate the components and subtract them from the data
- The second method depend on *differencing* the data, i.e., replacing the original series $\{X_t\}$ by $\{Y_t : X_t - X_{t-d}\}$ for some positive integer d
- The aim is to produce a stationary series

A General Approach to Time Series Modeling

- Choose a model to fit the transformed time series, starting from large initial set of "candidates"
- Check whether the selected model is adequate in the sense of satisfactory explaining the series peculiarities
- Generate forecasts for the transformed time series and then invert the transformations to arrive at forecasts of the original series

Simple Time Series Models

- Moving Average
- Exponential Smoothing
- Holt-Winters Algorithms

Moving Average

$$L_t = \tfrac{1}{n}(Y_t + Y_{t-1} + Y_{t-2} + \cdots + Y_{t-n+1})$$

$$\hat{Y}_t(h) = L_t,\ h = 1,2,3,\ldots$$

Exponential Smoothing

$$L_t = \alpha Y_t + \alpha(1-\alpha)Y_{t-1} + \alpha(1-\alpha)^2 Y_{t-2} + \cdots, \quad \alpha \in (0,1)$$

$$L_t = \alpha Y_t + (1-\alpha)L_{t-1}, \quad \alpha \in (0,1)$$

$$L_1 = Y_1$$
$$\hat{Y}_t(h) = L_t$$

Matlab function “expsm.m”

```
function e=expsm(a,X)
%Calculates forecasting error  for one step forecast
n=length(X);
L=zeros(n,1);
Xf=zeros(n,1);
L(1)=X(1);
for i=2:n,
   L(i)=a*X(i)+(1-a)*L(i-1);
end
e=mean((X(2:n)-L(1:n-1)).^2);
t=[1:n]';
plot(t,X,t(2:n),L(1:n-1),'m*')
```

Holt-Winters with local trend

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1}) \quad 0 < \alpha < 1$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \quad 0 < \beta < 1$$

$$L_2 = Y_2, \quad T_2 = Y_2 - Y_1$$

$$\hat{Y}_t(h) = L_t + hT_t$$

$$e_{t+h} = \hat{Y}_t(h) - Y_{t+h}$$

Function “holt.m”

```
function [e,L,T]=holt(a,X)
n=length(X); L=zeros(n,1); T=zeros(n,1); Xf=zeros(n,1);

L(2)=X(2);
T(2)=X(2)-X(1);
for i=3:n,
   L(i)=a(1)*X(i)+(1-a(1))*(L(i-1)+T(i-1));
   T(i)=a(2)*(L(i)-L(i-1))+(1-a(2))*T(i-1);
end
for i=3:n
   Xf(i)=L(i-1)+T(i-1);
end
e=mean((X(3:n)-Xf(3:n)).^2);
t=[1:n]';
plot(t,X,t(3:n),Xf(3:n),'m*')
```

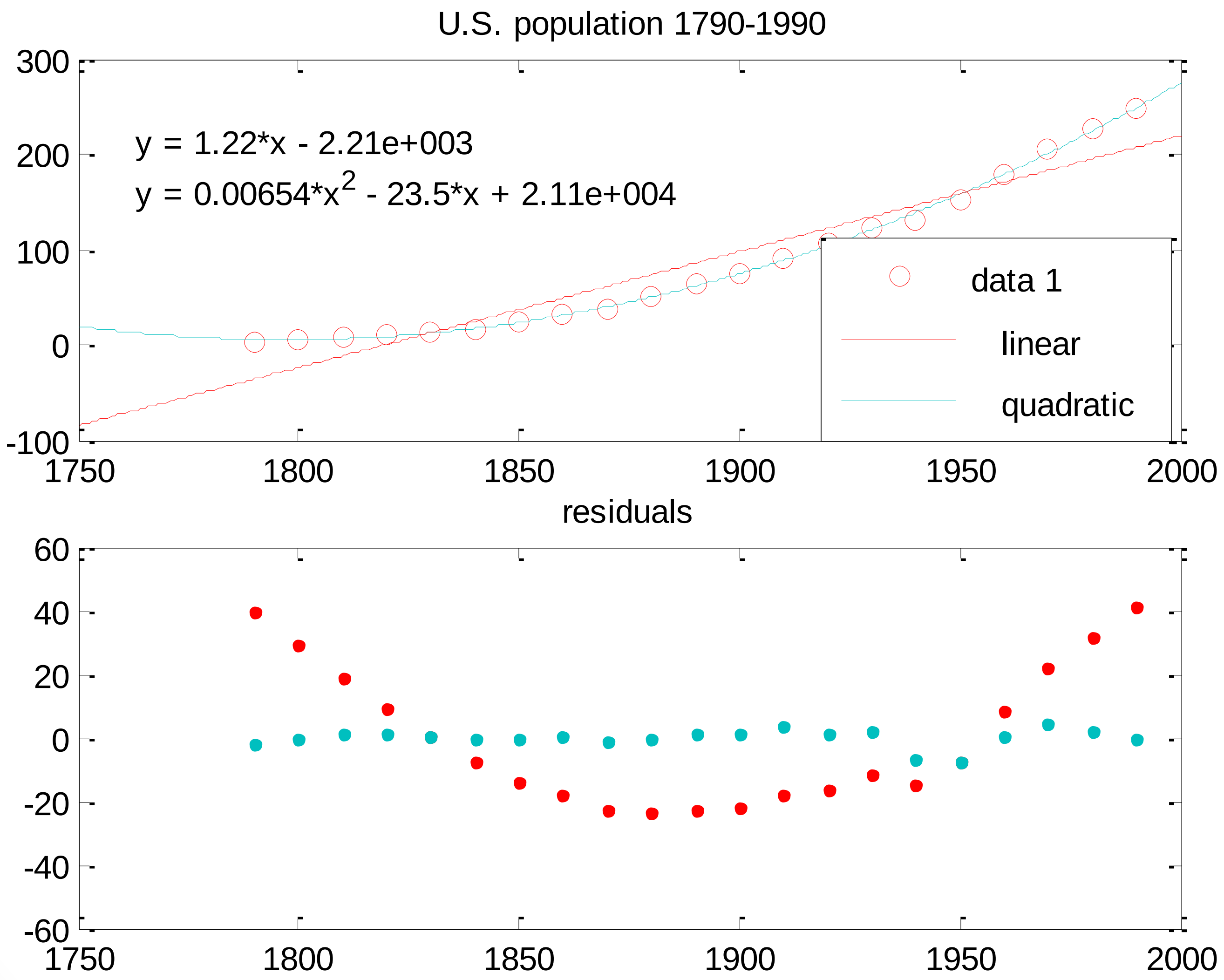
Trend modeling

$$T_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \cdots + \beta_n t^n$$

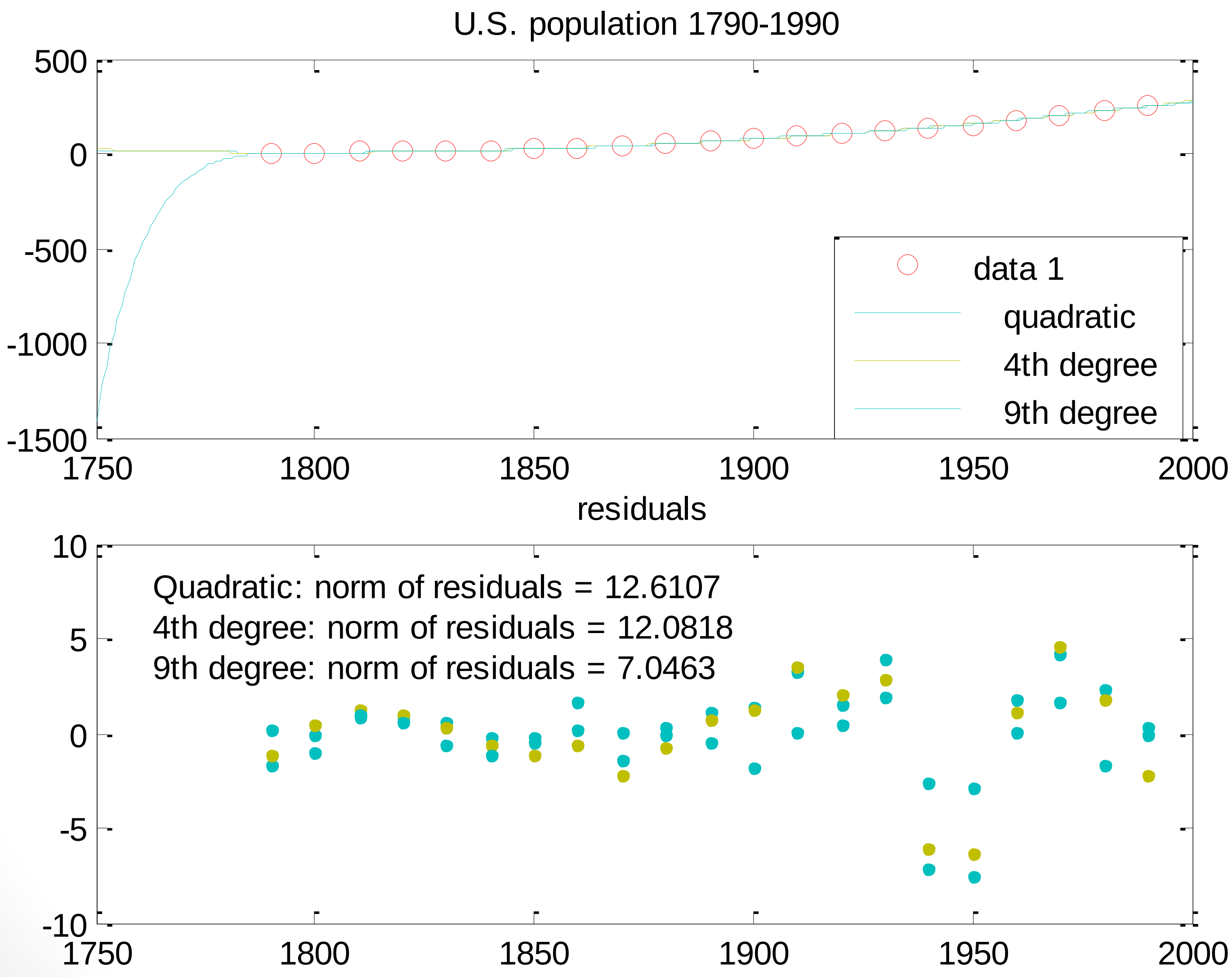
$$Y_t = T_t + I_t$$

$$\hat{Y}_t(h) = T_{t+h}$$

Trend modeling



Trend modeling



Other local trend models

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1}T_{t-1}) \quad 0 < \alpha < 1$$

$$T_t = \frac{\beta L_t}{L_{t-1}} + (1-\beta)T_{t-1} \quad 0 < \beta < 1$$

$$L_2 = Y_2, \quad T_2 = 1$$

$$\hat{Y}_t(h) = L_t T_t^{\,h}$$

Other local trend models- dampening growth

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + \phi T_{t-1}) \quad 0 < \alpha < 1, 0 < \phi \leq 1$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)\phi T_{t-1} \quad 0 < \beta < 1$$

$$\hat{Y}_t(h) = L_t + \sum_{i=1}^{h} \phi^i T_t$$

Holt-Winters with seasonality

$$L_t = \alpha(Y_t - F_{t-s}) + (1-\alpha)(L_{t-1} + T_{t-1}) \quad 0 < \alpha < 1$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \quad 0 < \beta < 1$$

$$F_t = \gamma(Y_t - L_t) + (1-\gamma)F_{t-s}$$

$$\hat{Y}_t(h) = L_t + hT_t + F_{t+h-s} \quad h = 1,2,\ldots,s$$

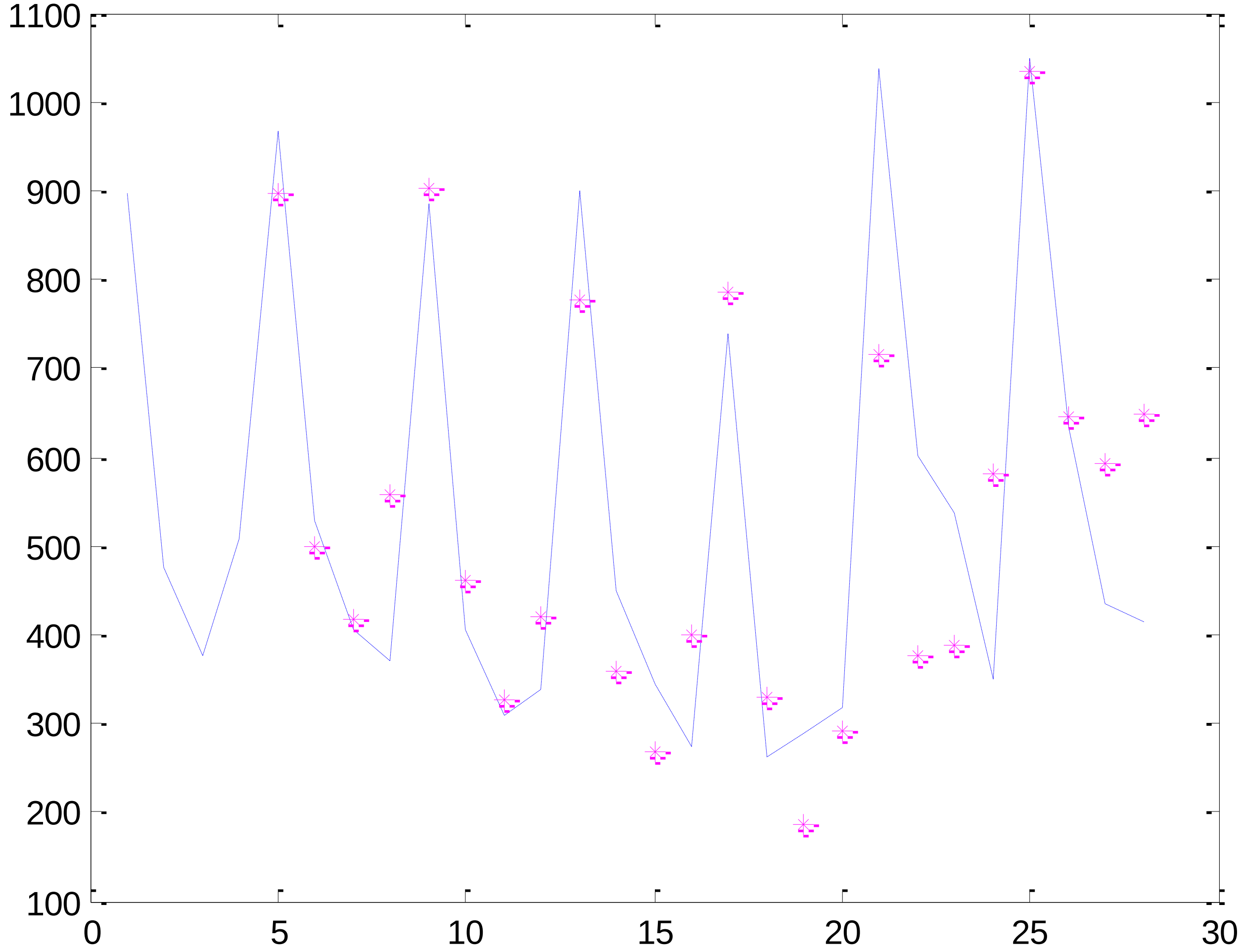
Holt-Winters with seasonality

$$L_4 = \frac{Y_1 + Y_2 + Y_3 + Y_4}{4}$$

$$T_4 = 0$$

$$F_i = Y_i - L_4, \quad i = 1,2,3,4$$

Example: Monthly sales



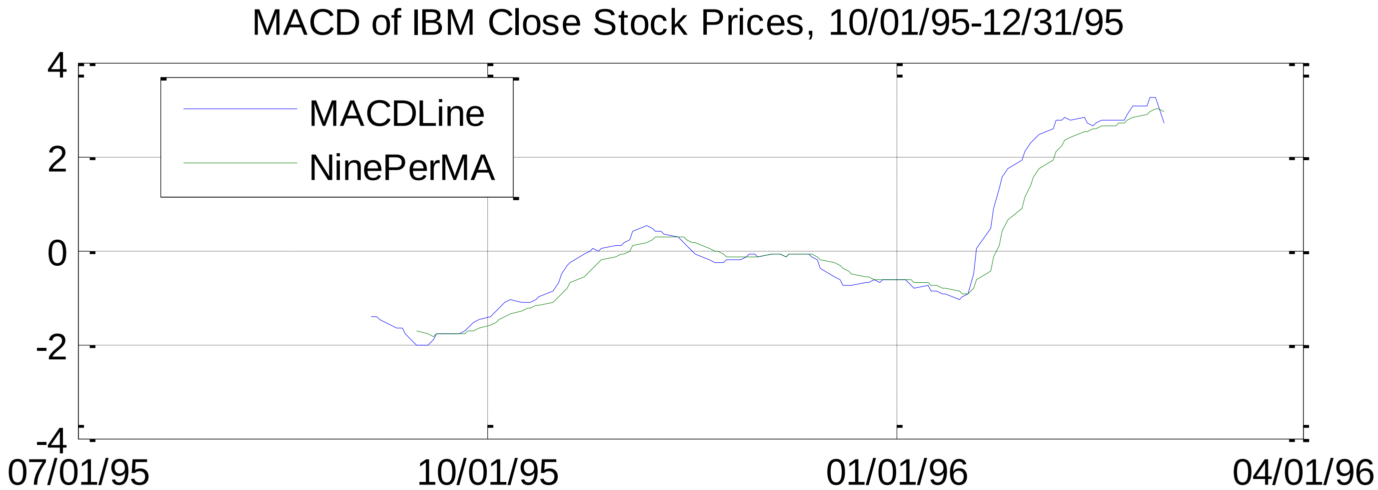
Moving Average Convergence – Divergence (MACD) indicator

- The MACD is calculated by subtracting from 12-day moving average of the security's price 26-day moving average of its price
- MACD oscillates above and below zero
- When the MACD is above zero, it means that the 12-day moving average is higher than the 26-day moving average
- This means, that the market is "bullish"

Moving Average Convergence – Divergence (MACD) indicator

- When MACD is below zero we have “bearish” market
- Usually 9-day moving average of MACD is also calculated and used as a “signal” line
- “Buy” signal is generated when MACD rise above its signal line
- “Sell” signal is generated in the opposite case

MACD


MACD of IBM Close Stock Prices, 10/01/95-12/31/95
MACDLine
NinePerMA
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2
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-2
-4
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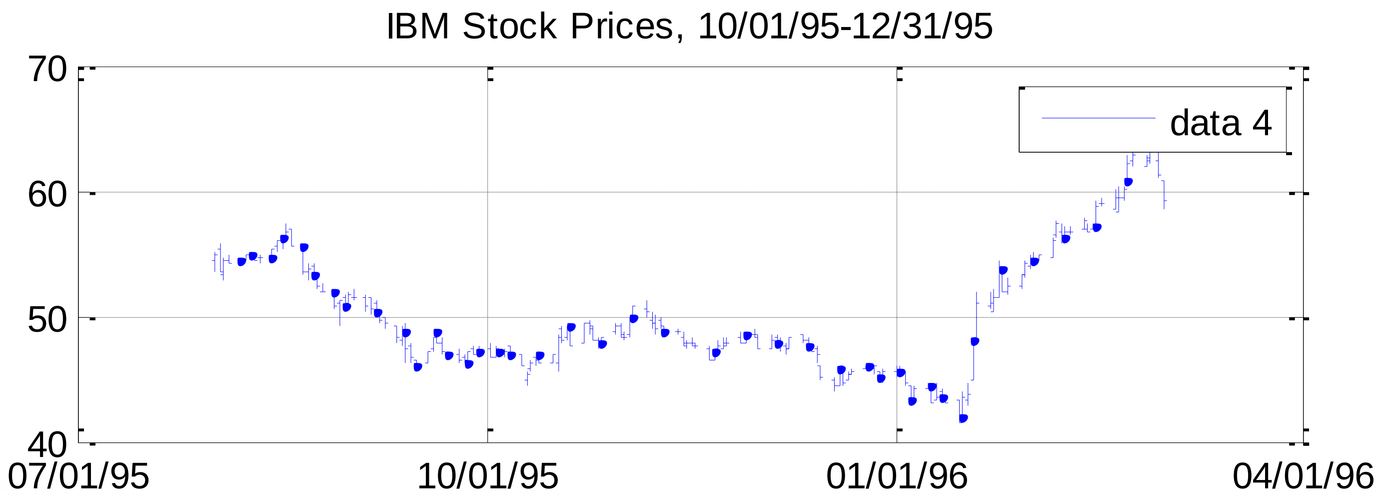
04/01/96


IBM Stock Prices, 10/01/95-12/31/95
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