

Models for Conditional Heteroscedastisity

Models with conditional heteroscedastisity-GARCH(P,Q)

$$\sigma_t^2 = K + \sum_{i=1}^P G_i \sigma_{t-1}^2 + \sum_{j=1}^Q A_j e_{t-j}^2$$

$$\sum_{i=1}^P G_i + \sum_{j=1}^Q A_j < 1$$

$$e_t \sim \text{Gaussian}$$

$$z_t = \frac{e_t}{\sigma_t} \text{ i.i.d}$$

$$K > 0, \quad G_i \geq 0, \quad A_j \geq 0$$

Basic characteristics of GARCH(P,Q)

- Fat tails
- Volatility clustering

Basic GARCH model

$$Y_t = C + e_t$$

$$\sigma_t^2 = K + G_1 \sigma_{t-1}^2 + A_1 e_{t-1}^2$$

$$e_t \sim \textit{Gaussian}$$

$$z_t = \frac{e_t}{\sigma_t} \textit{i.i.d}$$

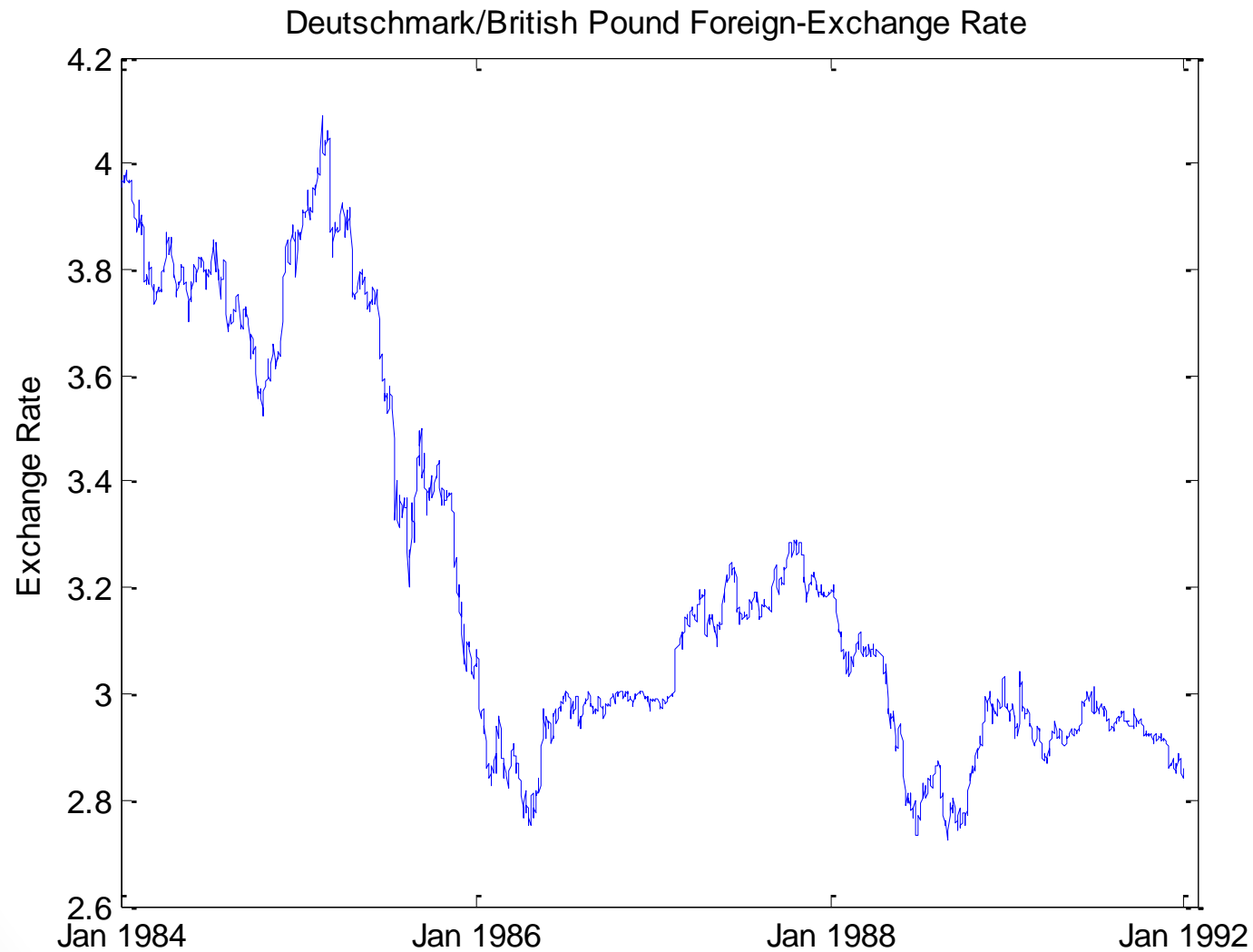
ARIMA-GARCH

Case 1

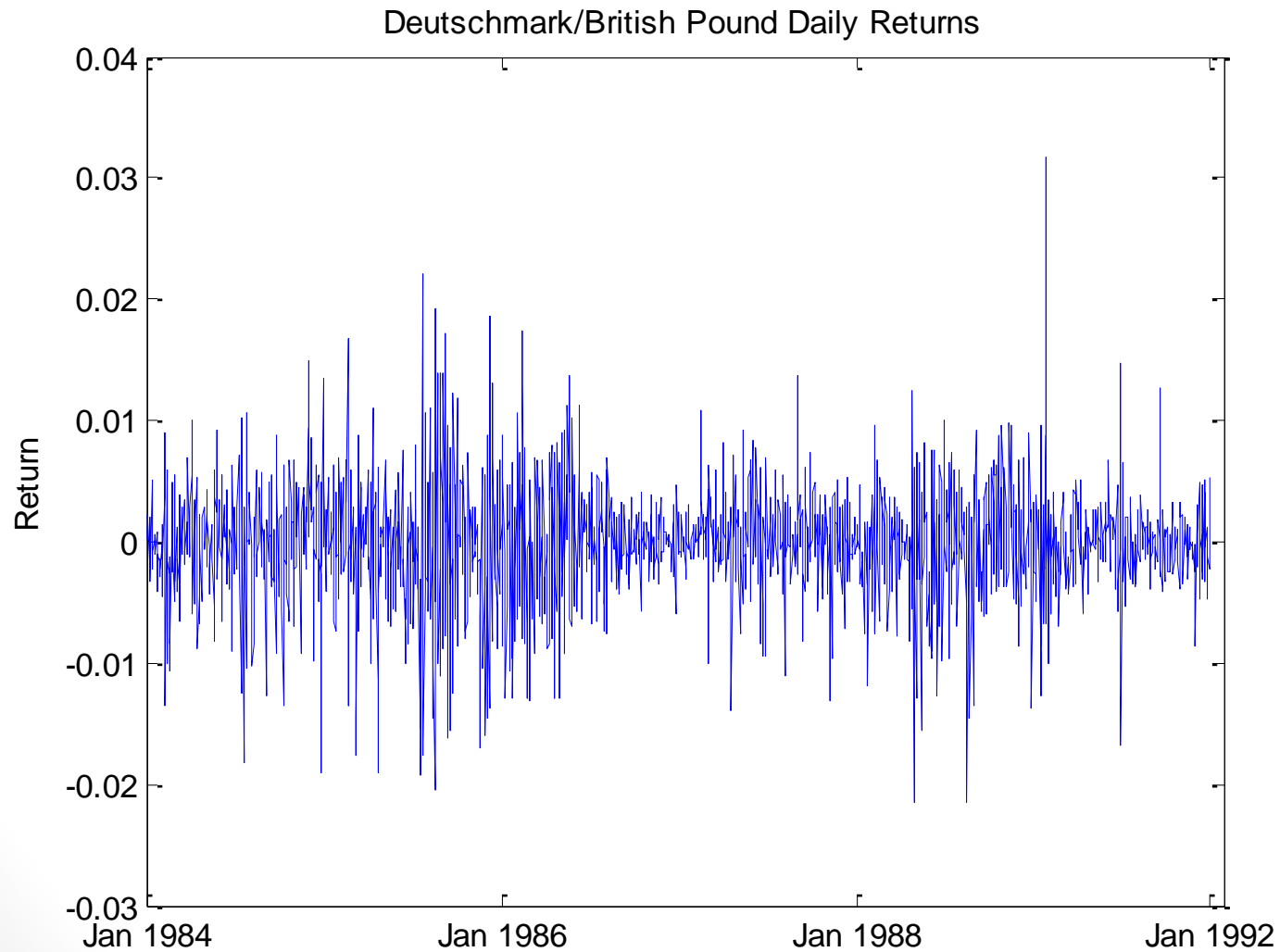
Case 1 – DEM/GBP

- Deutschmark/British Pound , 1984-1991

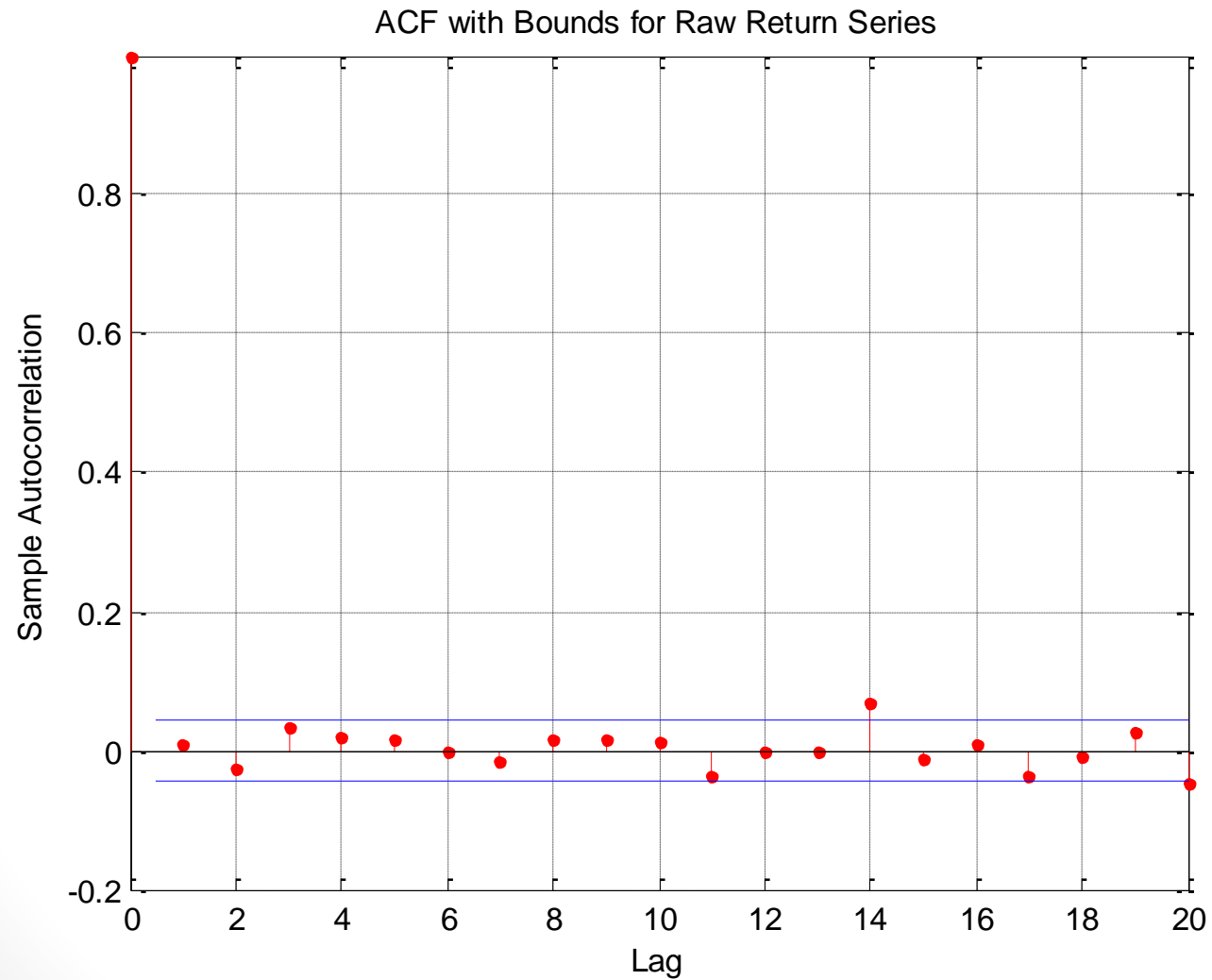
Case 1



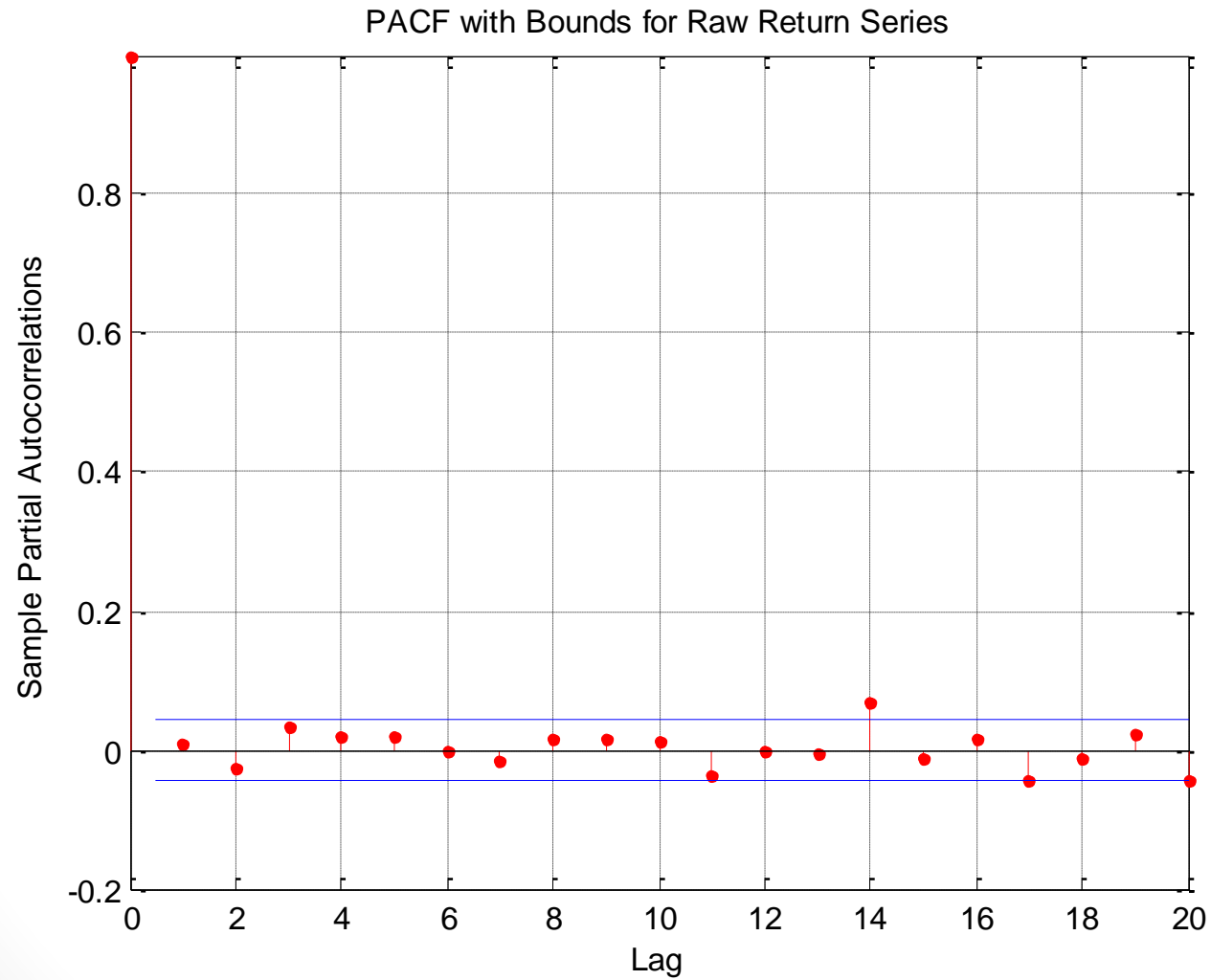
Case 1



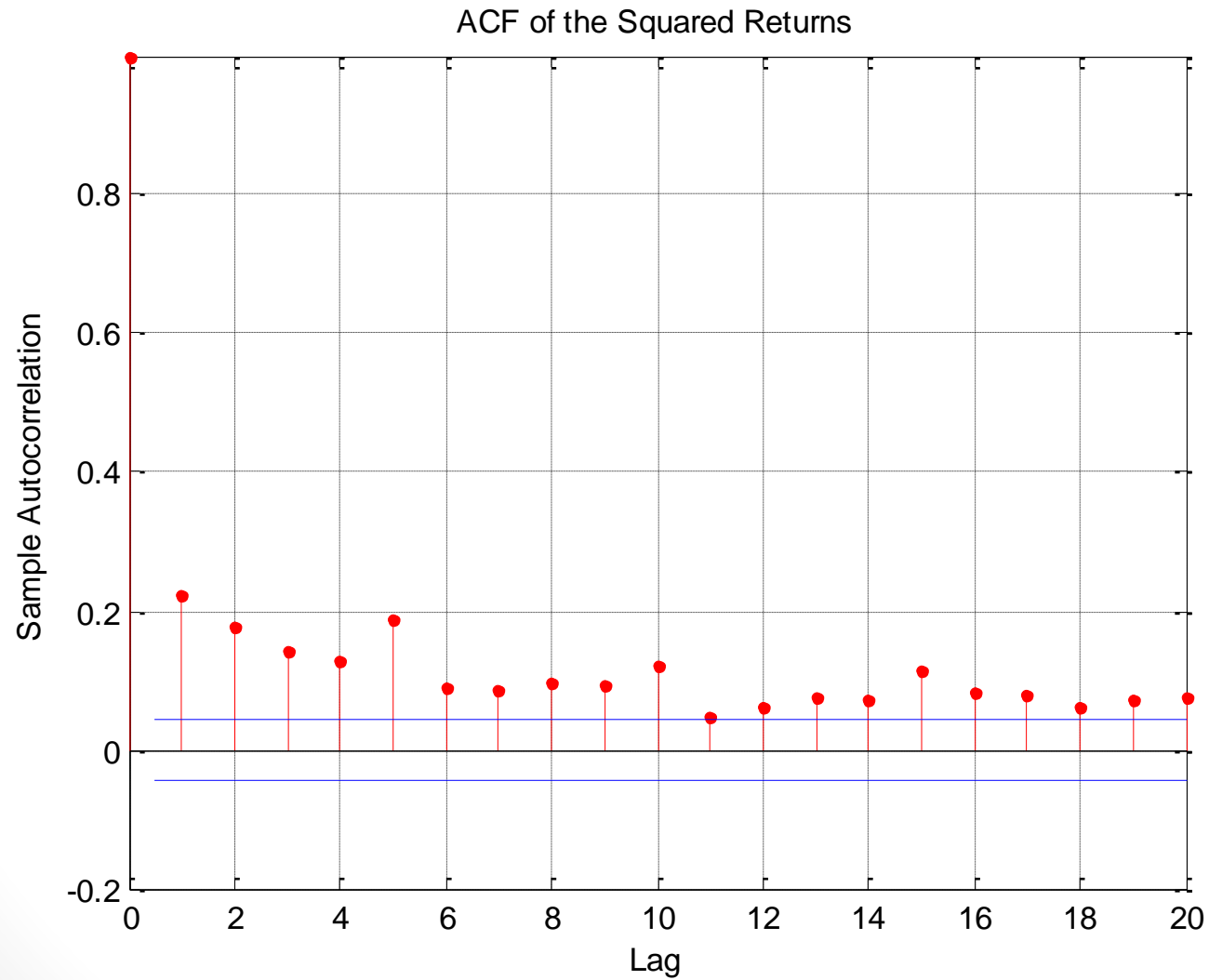
Case 1



Case 1



Case 1



Case 1

```
[H,pValue,Stat,CriticalValue] = ...  
    lbqtest(markpound-  
mean(markpound),[10 15 20]',0.05);  
[H pValue Stat CriticalValue]
```

```
ans =
```

0	0.7278	6.9747	18.3070
0	0.2109	19.0628	24.9958
0	0.1131	27.8445	31.4104

Case 1

```
[H,pValue,Stat,CriticalValue] = ...  
    lbqtest((markpound-  
mean(markpound)).^2,[10 15 20]',0.05);  
[H pValue Stat CriticalValue]
```

```
ans =
```

1.0000	0	392.9790	18.3070
1.0000	0	452.8923	24.9958
1.0000	0	507.5858	31.4104

Case 1

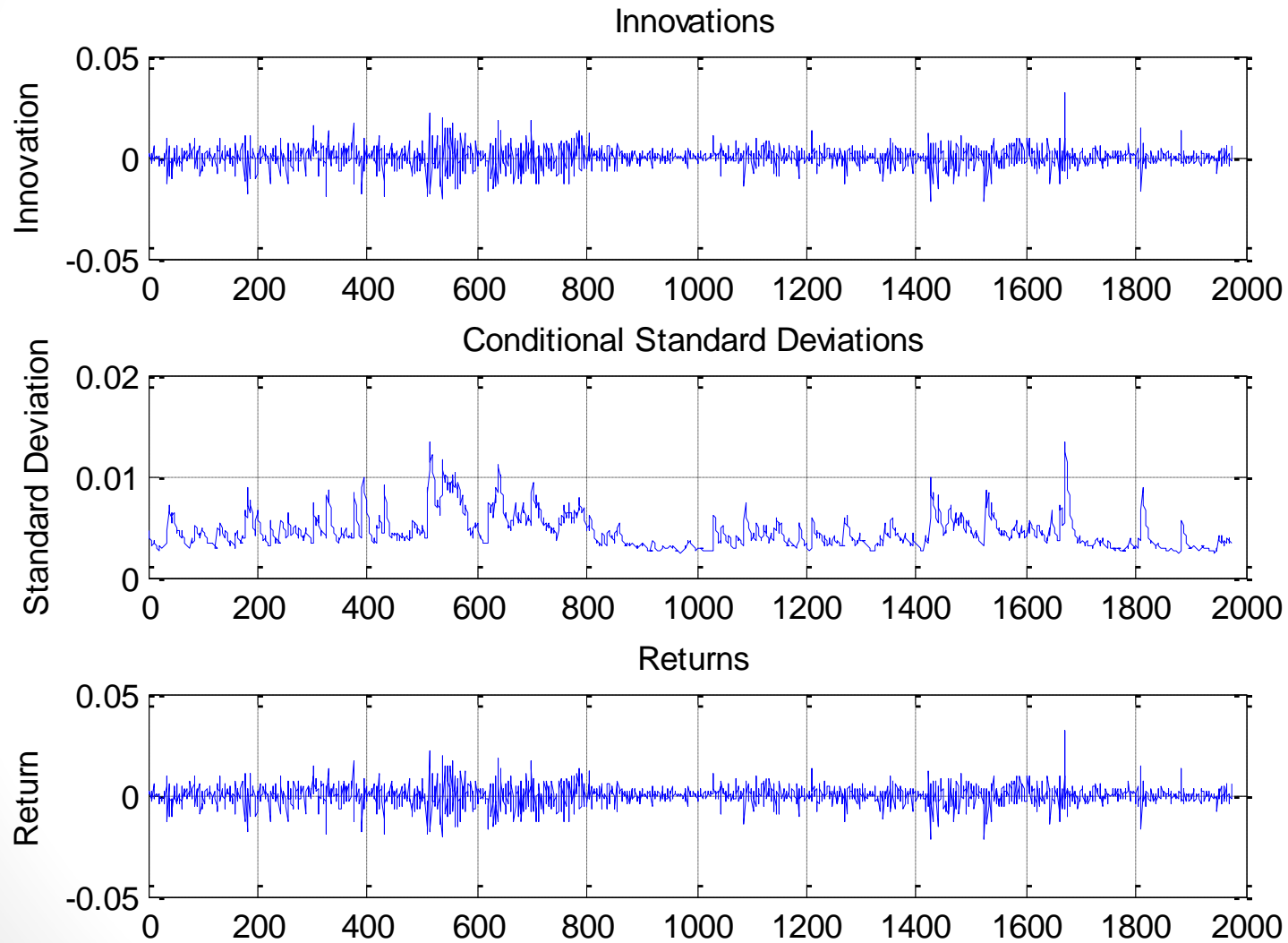
Mean: ARIMA(0,0,0); Variance: GARCH(1,1)

Conditional Probability Distribution: Gaussian

Number of Model Parameters Estimated: 4

Parameter	Value	Standard Error	T Statistic
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C	-6.373e-005	8.3788e-005	-0.7606
K	9.9717e-007	1.2328e-007	8.0890
GARCH(1)	0.81458	0.015727	51.7953
ARCH(1)	0.14721	0.013285	11.0813

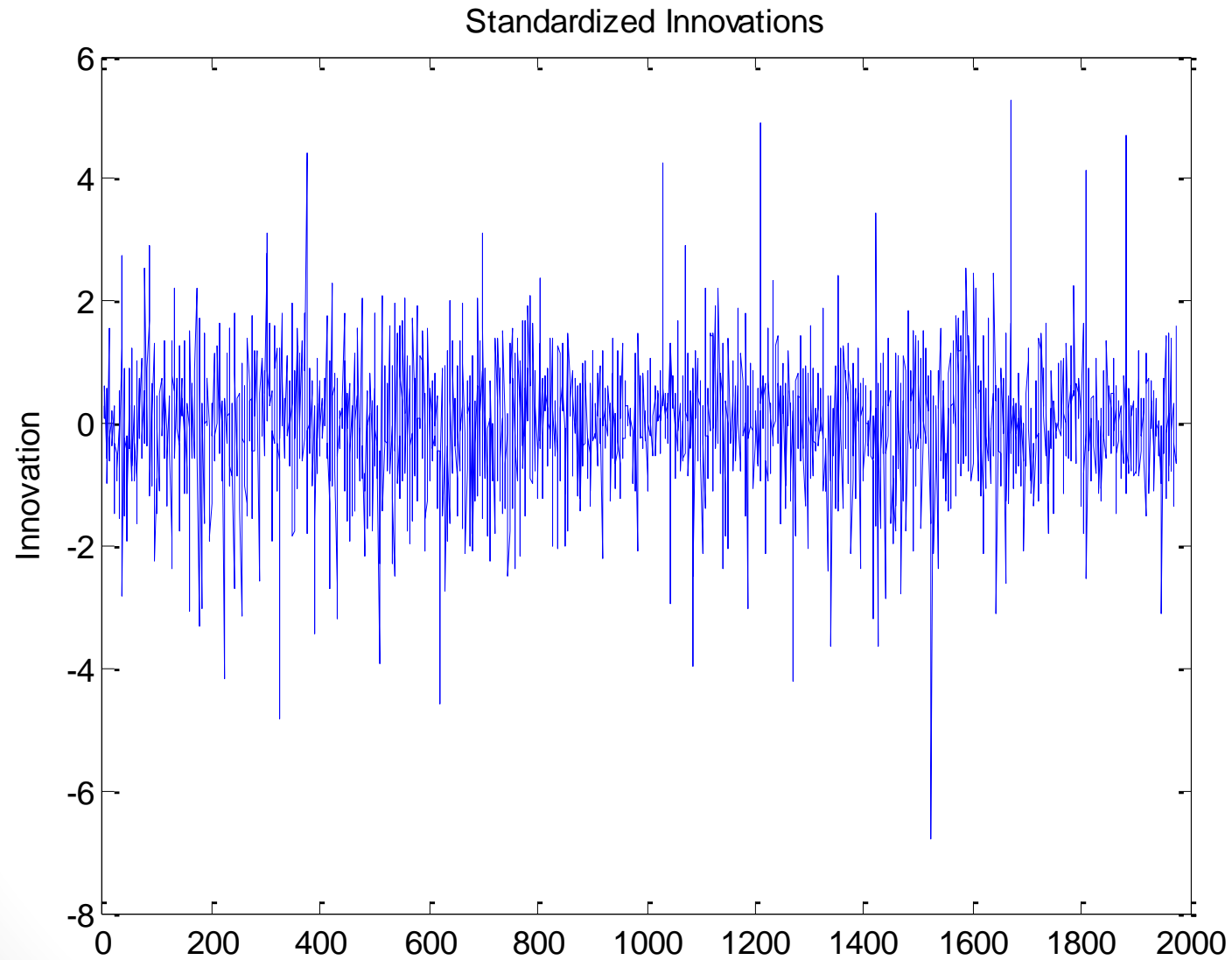
Case 1



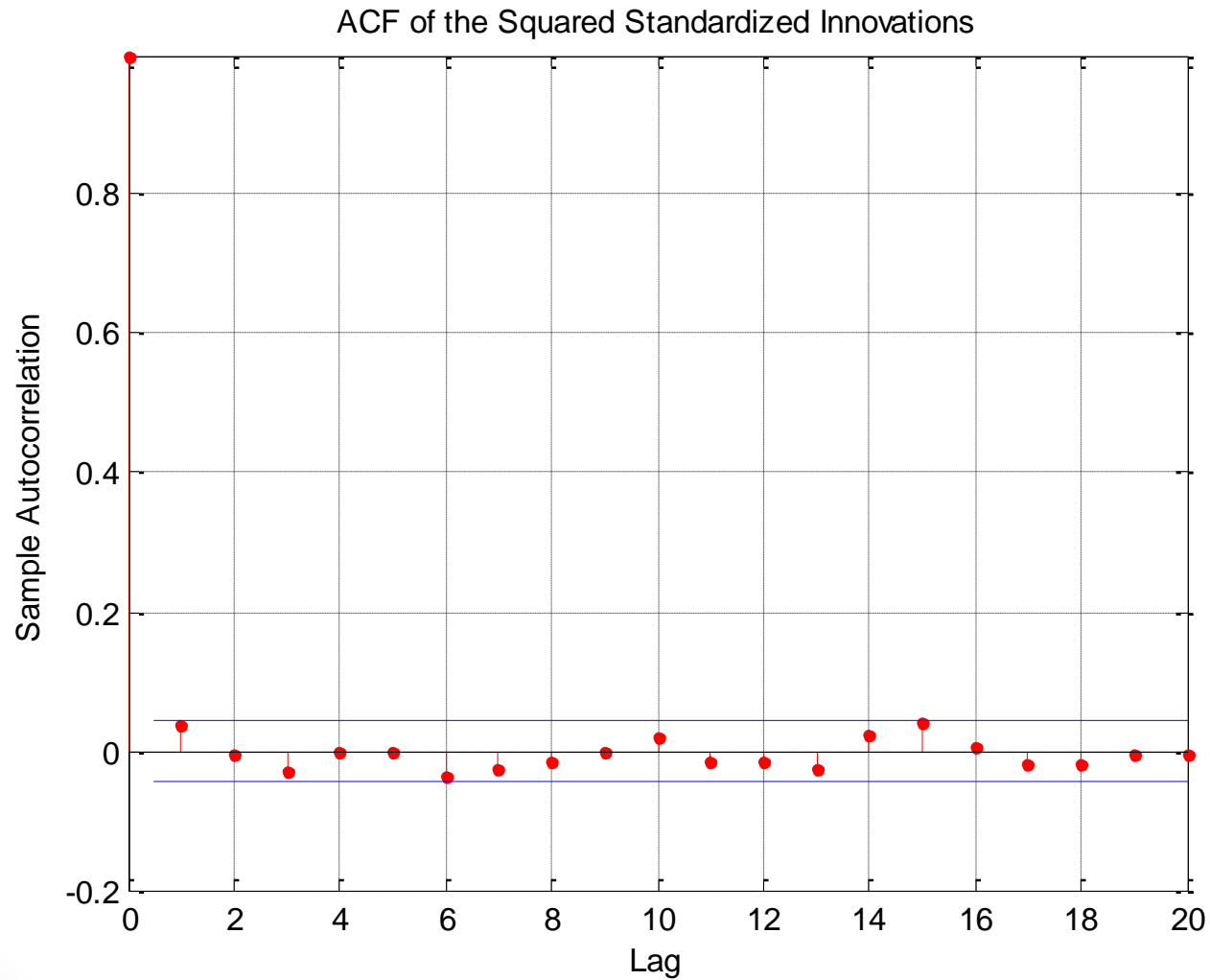
Case 1

```
plot(innovations./sigmas)  
ylabel('Innovation')  
title('Standardized Innovations')
```


Case 1



Case 1



Case 1

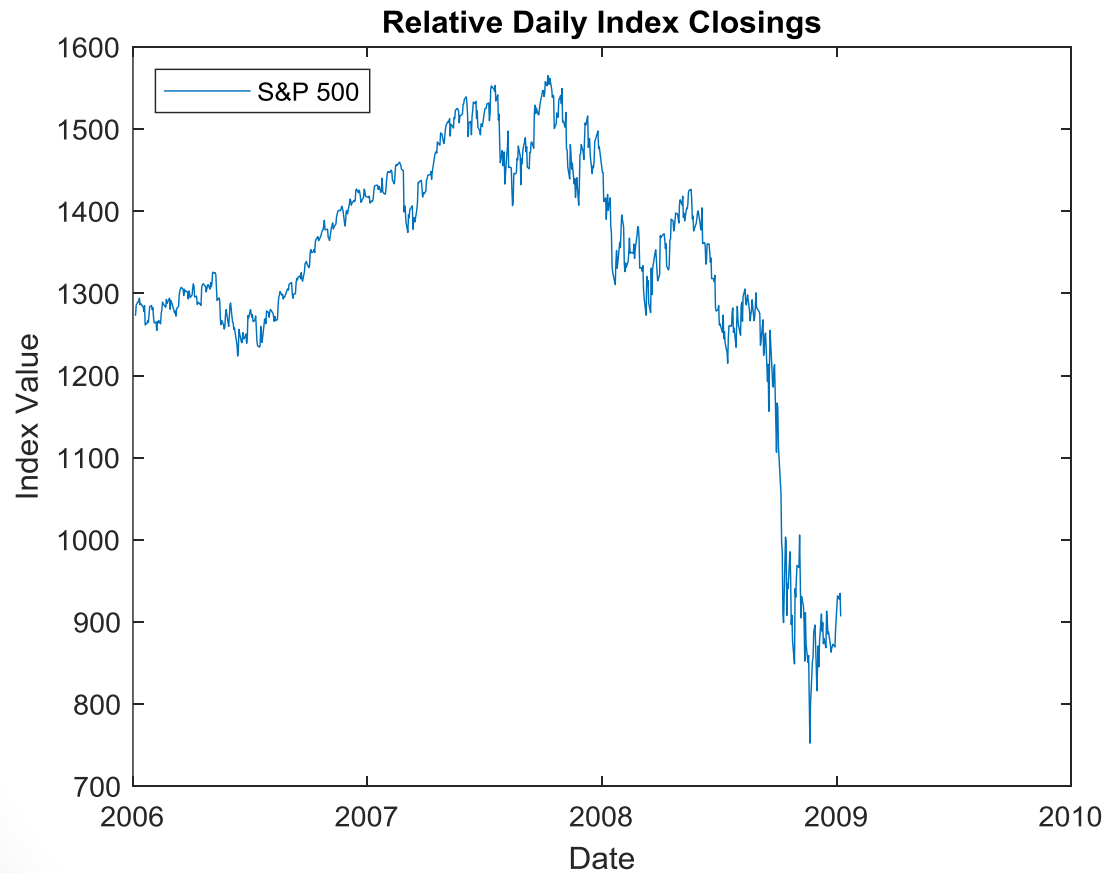
```
[H, pValue, Stat, CriticalValue] = ...  
    lbqtest((innovations./sigmas).^2,[10 15  
20]',0.05);  
[H pValue Stat CriticalValue]
```

ans =

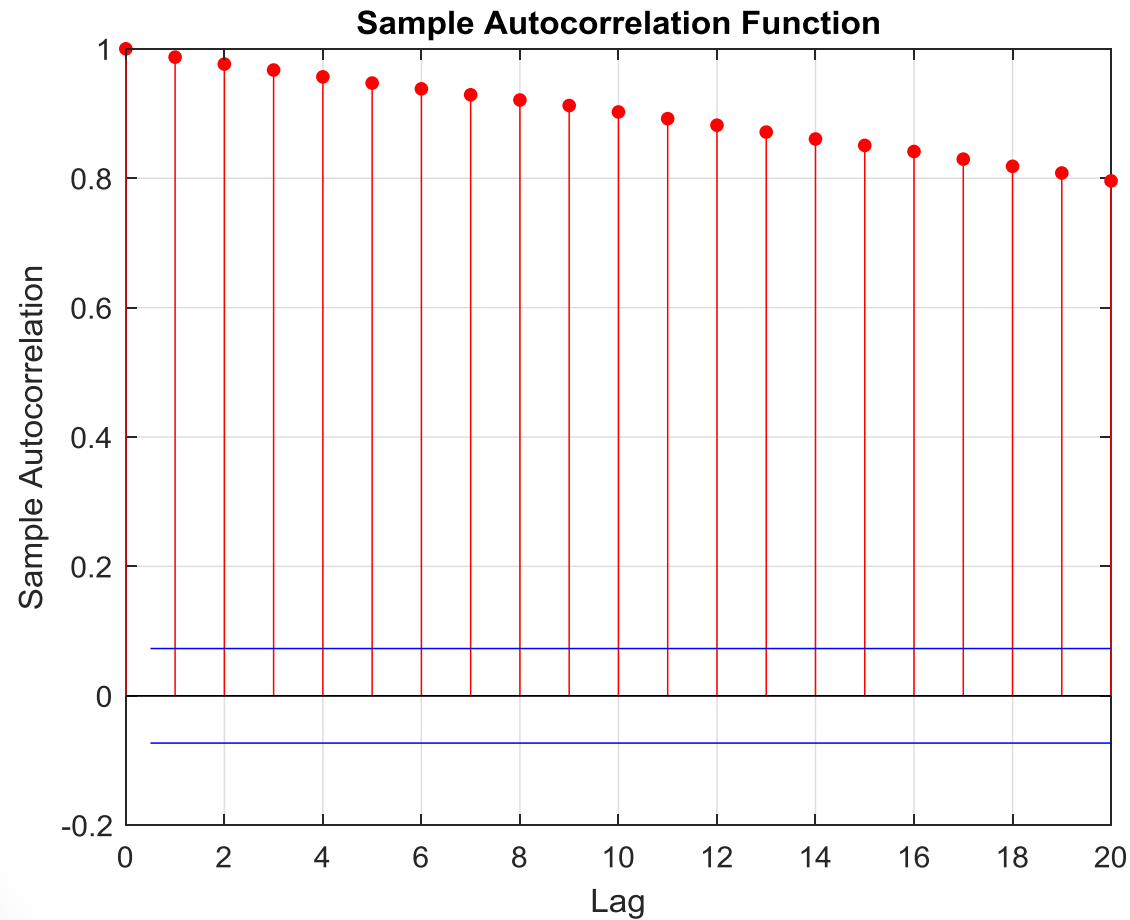
0	0.5014	9.3271	18.3070
0	0.3674	16.2221	24.9958
0	0.6019	17.7793	31.4104

ARIMA-GARCH Case 2

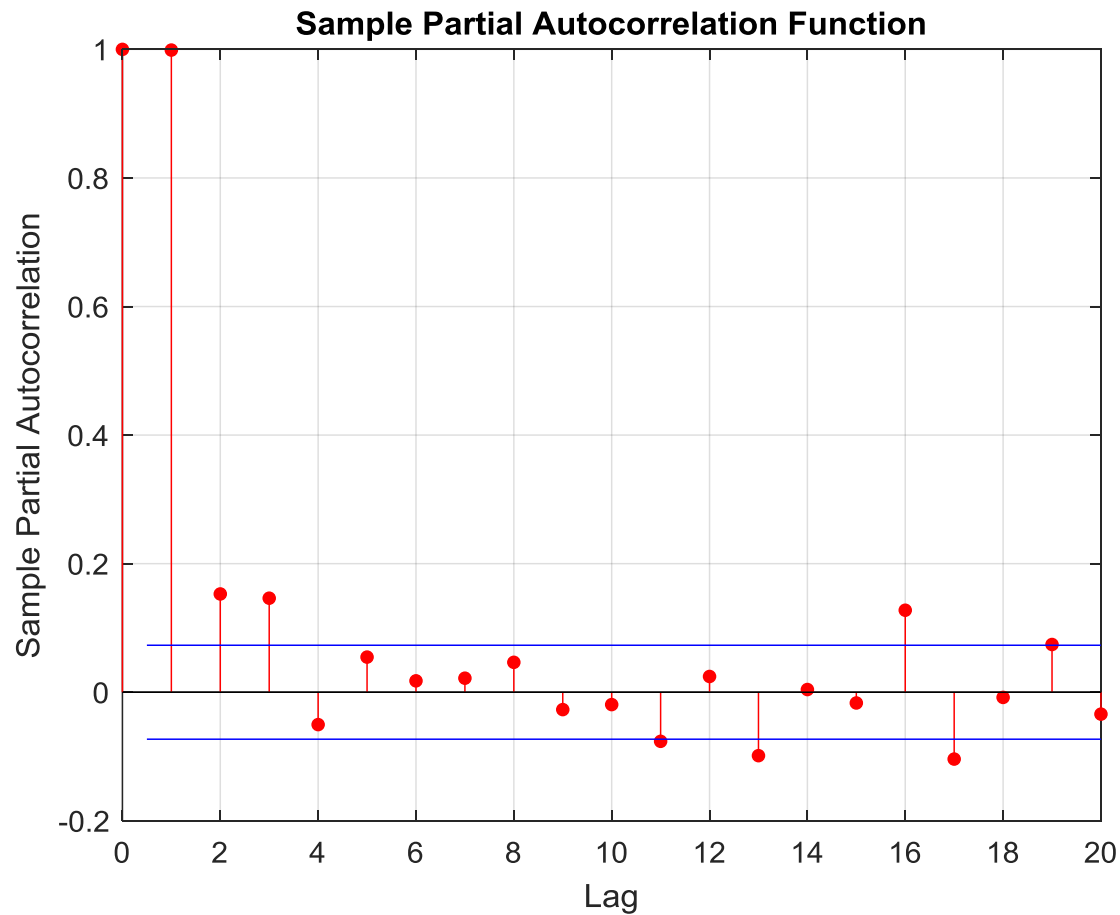
Data – S&P 500



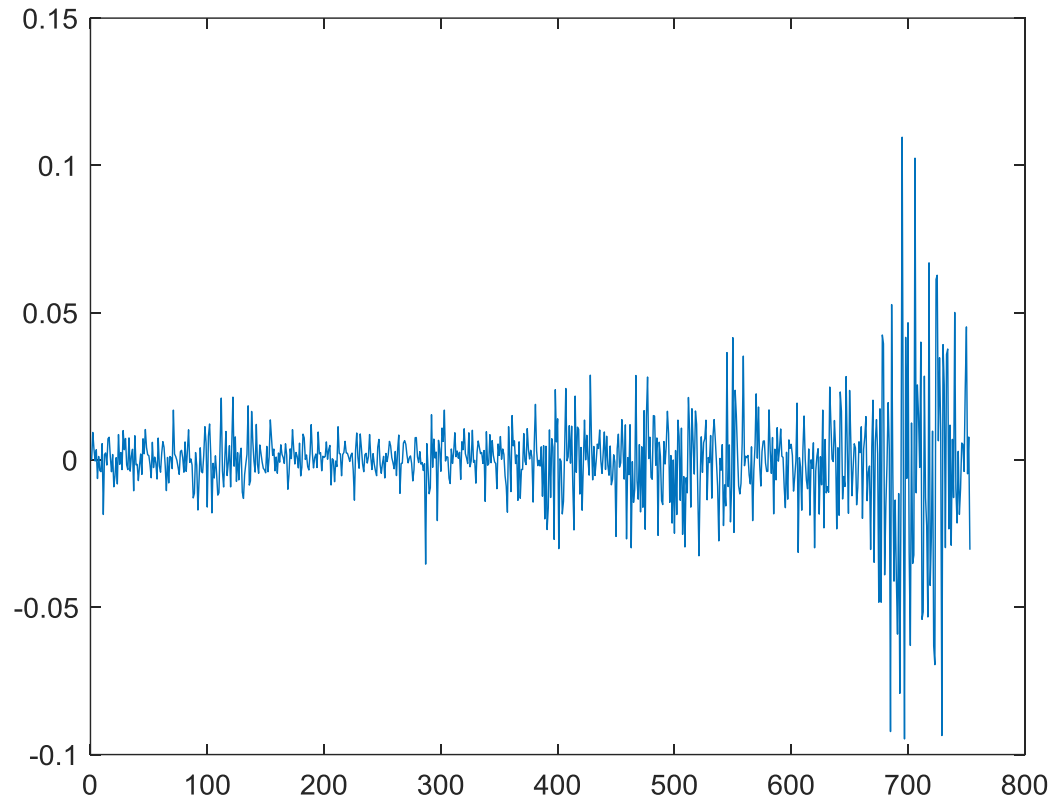
Data – S&P 500



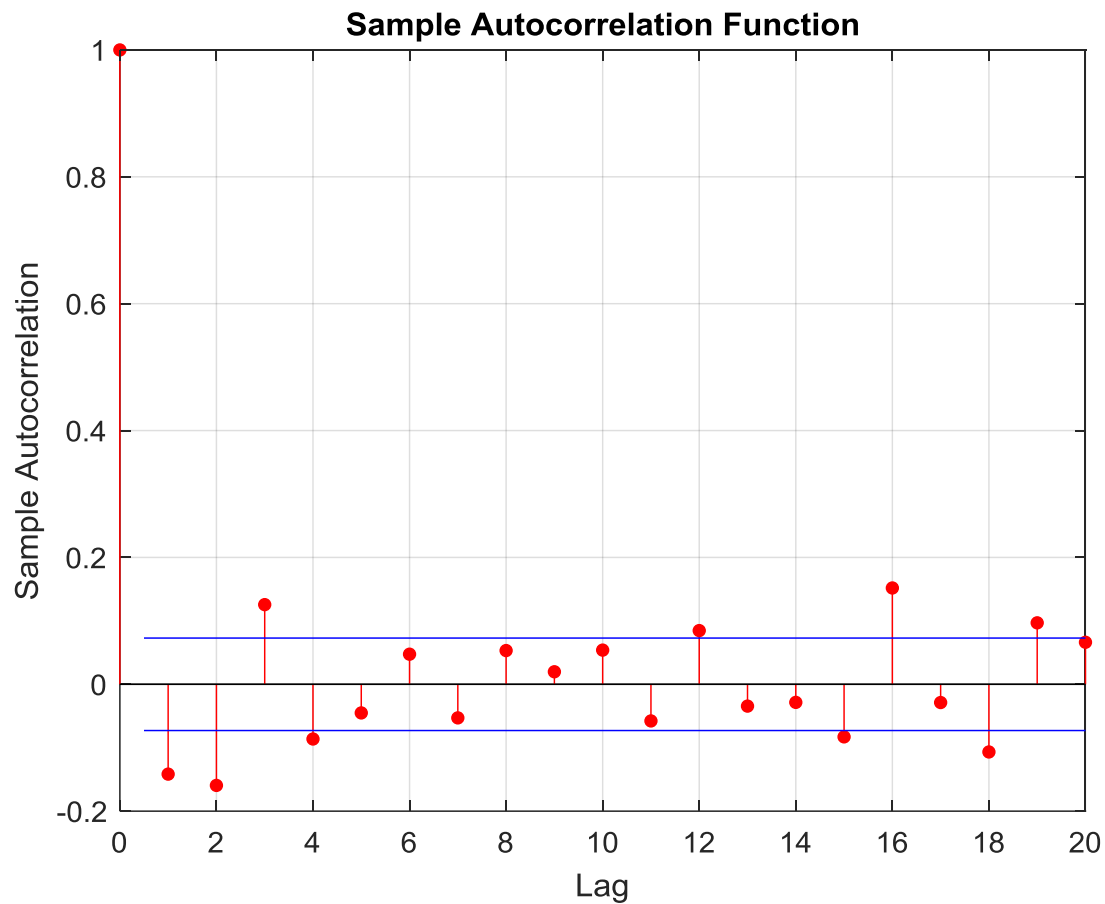
Data – S&P 500



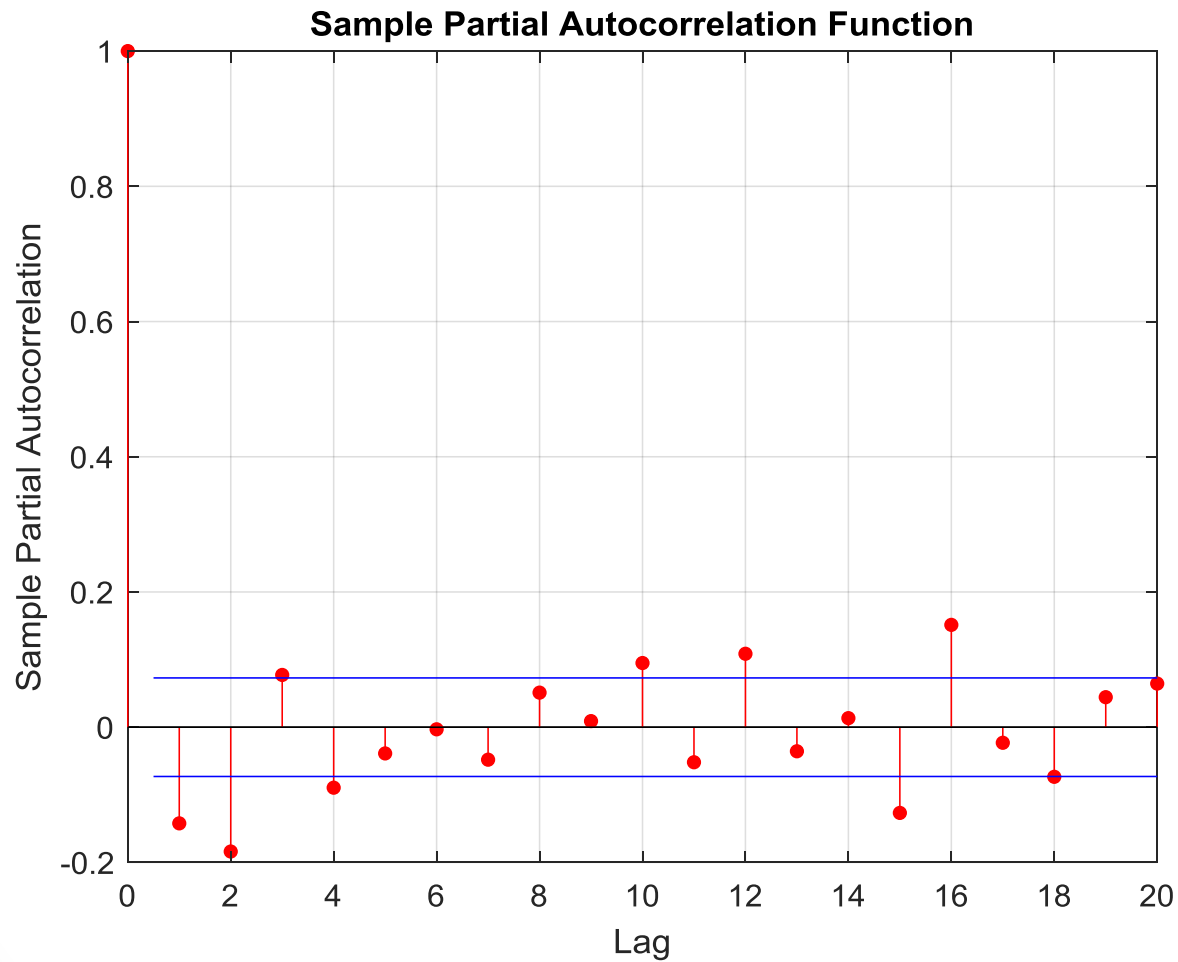
Logarithmic Returns



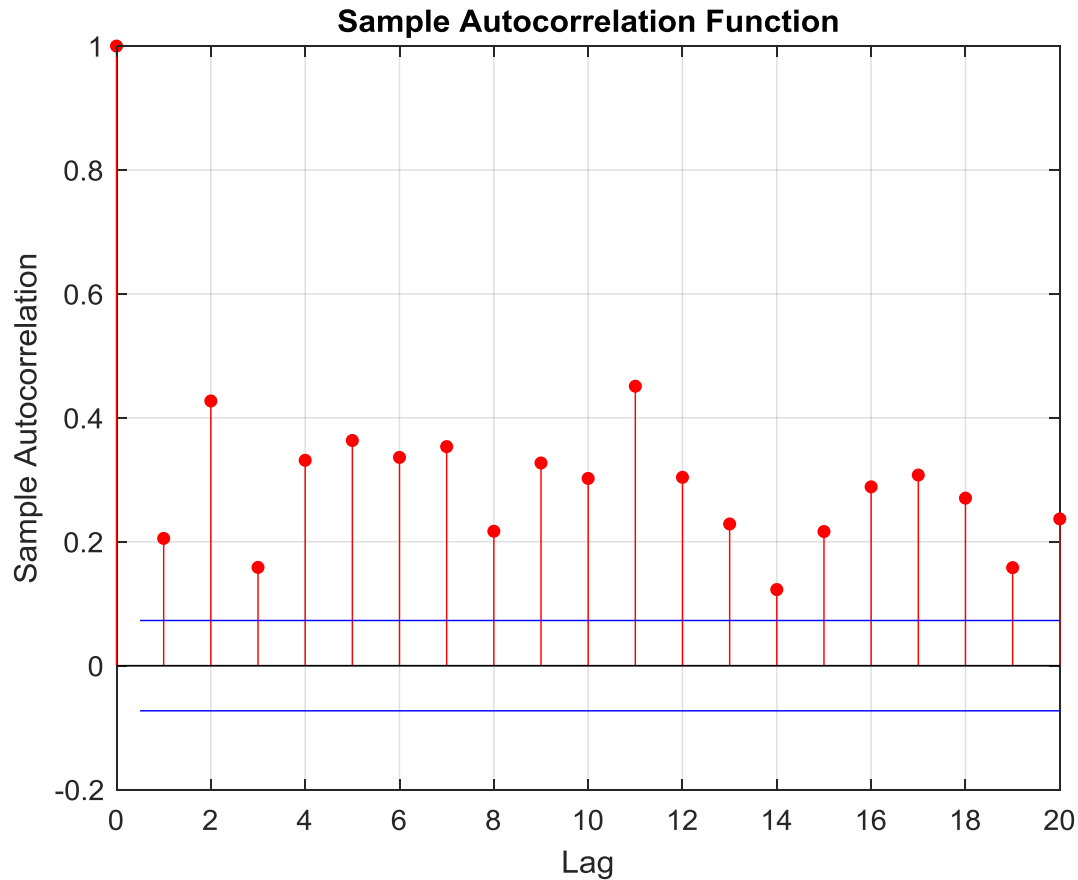
Logarithmic Returns



Logarithmic Returns



Variance of Logarithmic Returns



ARIMA(1,0,0) – Model 1 (Gaussian Distribution)

	Value	StandardError	TStatistic	PValue
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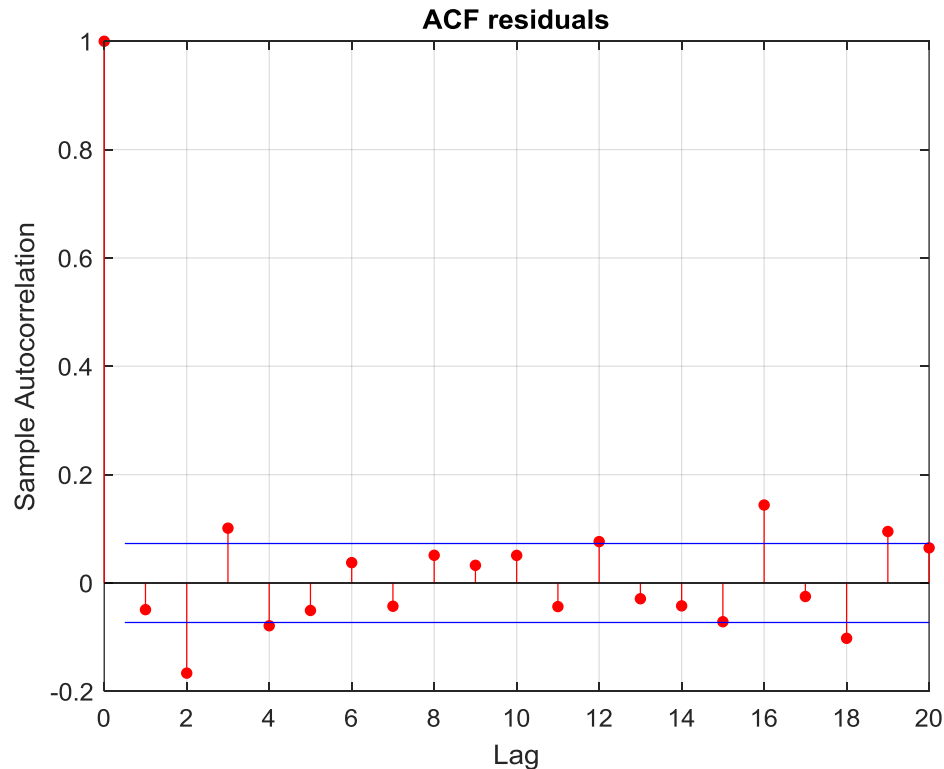
Constant	0.0004201	0.0003359	1.2507	0.21105
AR{1}	-0.1138	0.046183	-2.464	0.013739

GARCH(1,1) Conditional Variance Model (Gaussian Distribution):

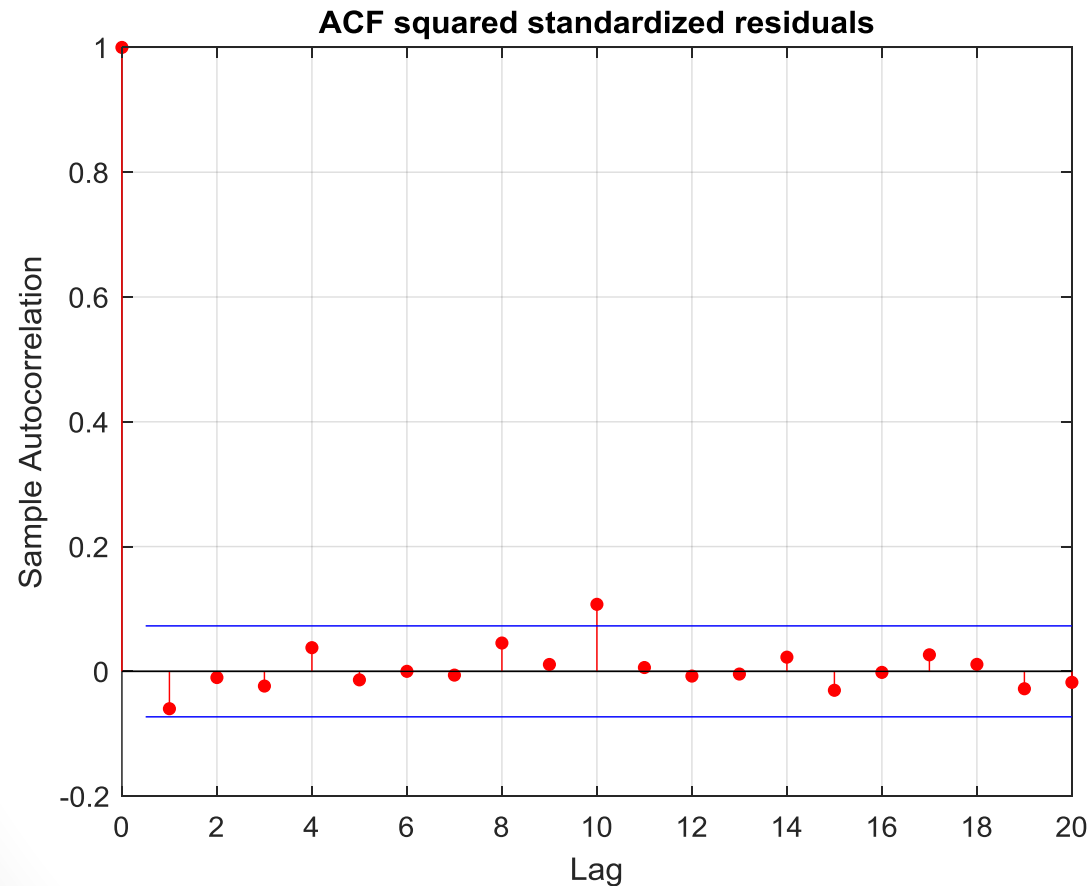
	Value	StandardError	TStatistic	PValue
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Constant	1.6928e-06	9.8177e-07	1.7242	0.084669
GARCH{1}	0.894	0.018359	48.695	0
ARCH{1}	0.099863	0.01634	6.1115	9.8672e-10

ARIMA(1,0,0) – Model 1 (Gaussian Distribution)



ARIMA(1,0,0) – Model 1 (Gaussian Distribution)



GJR Model

$$Y_t = C + e_t$$

$$\sigma_t^2 = K + G_1 \sigma_{t-1}^2 + A_1 e_{t-1}^2 + \Psi[e_{t-1} < 0] e_{t-1}^2$$

$$e_t \sim t(\nu)$$

$$z_t = \frac{e_t}{\sigma_t} \text{ i.i.d}$$

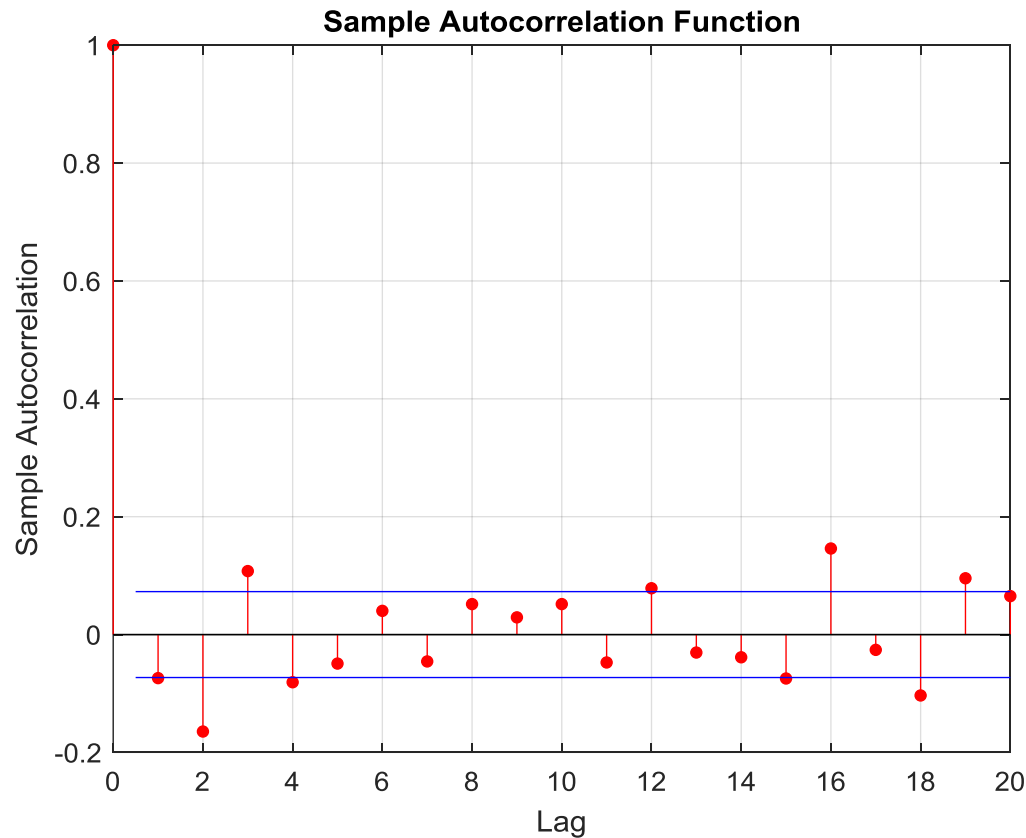
ARIMA(1,0,0) – Model 2 (t Distribution)

	Value	StandardError	TStatistic	PValue
Constant	0.000467	0.0002702	1.7284	0.083925
AR{1}	-0.083614	0.040809	-2.0489	0.040471
DoF	5.1251	1.0437	4.9107	9.0753e-07

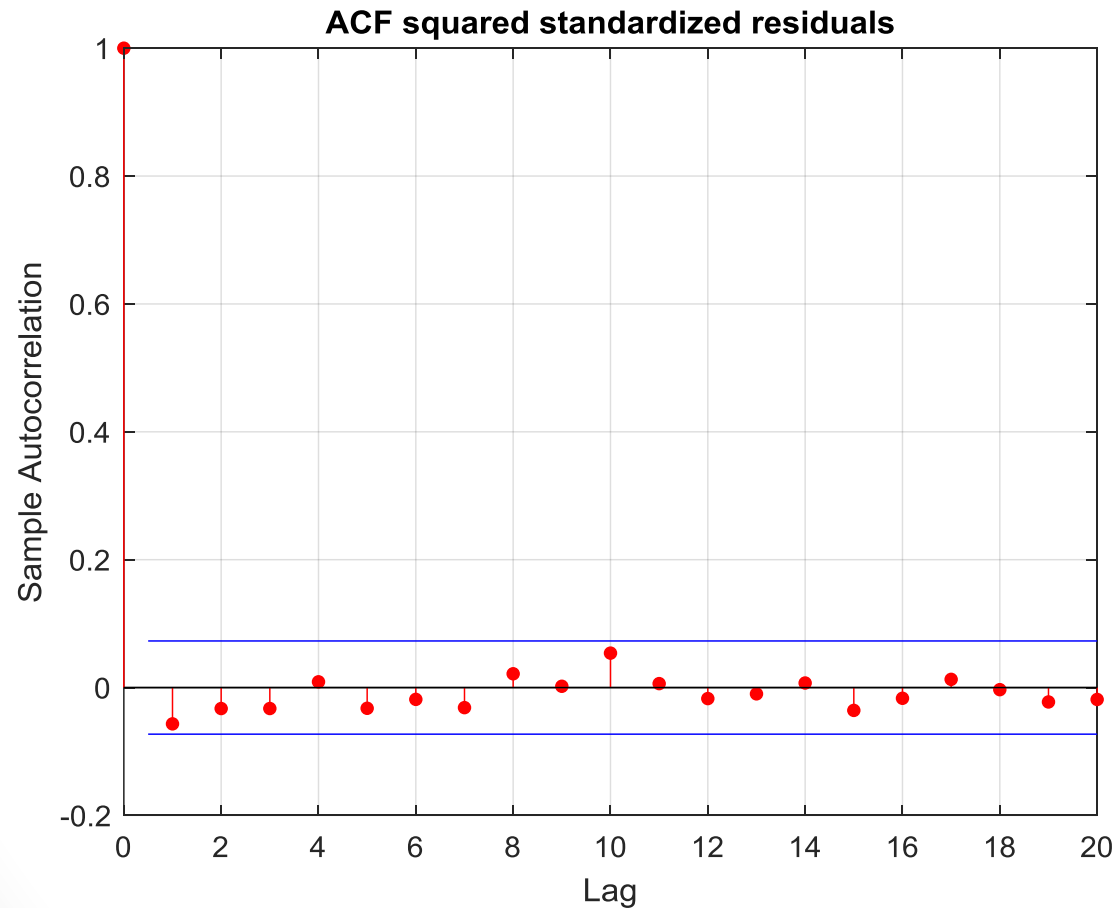
GJR(1,1) Conditional Variance Model (t Distribution):

	Value	StandardError	TStatistic	PValue
Constant	1.0749e-06	9.1462e-07	1.1753	0.23989
GARCH{1}	0.90483	0.022744	39.782	0
ARCH{1}	2e-12	0.036817	5.4323e-11	1
Leverage{1}	0.17998	0.048115	3.7406	0.00018358
DoF	5.1251	1.0437	4.9107	9.0753e-07

ARIMA(1,0,0) – Model 2 (t Distribution)



ARIMA(1,0,0) – Model 2 (t Distribution)



ARIMA(4,0,0) - Model 3 (t Distribution)

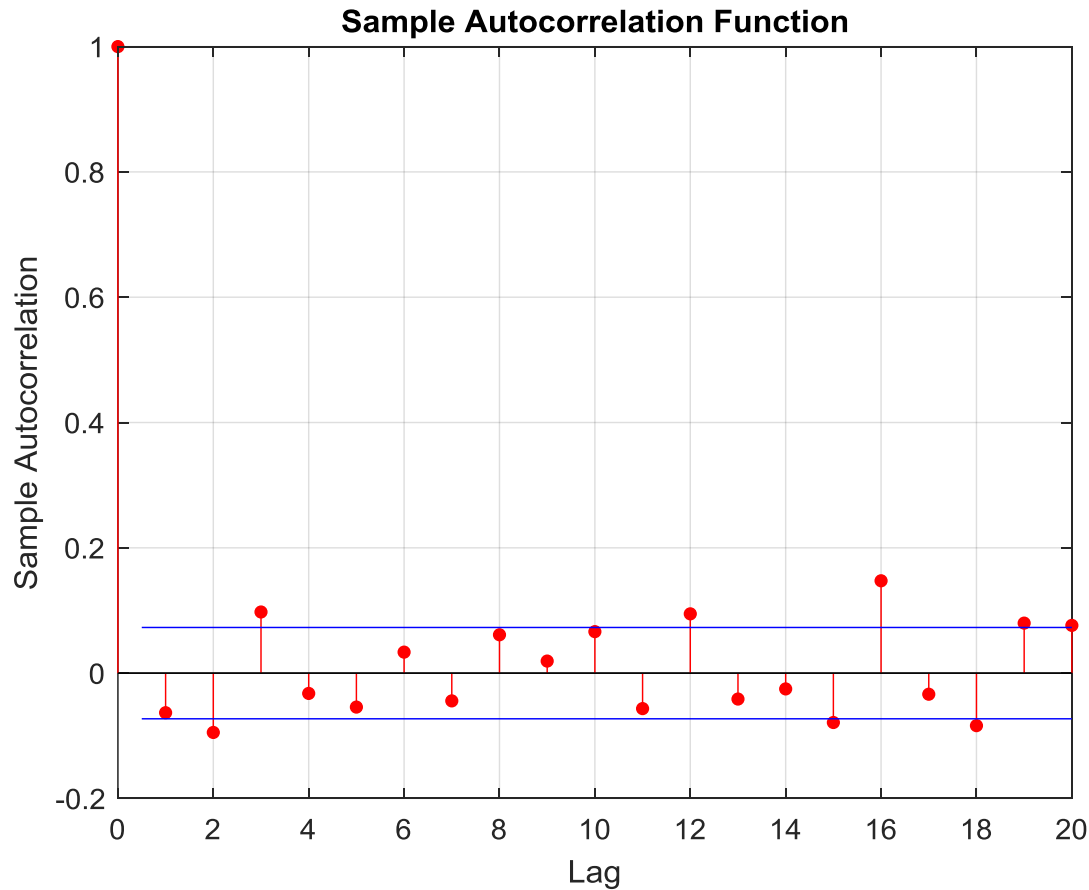
	Value	StandardError	TStatistic	PValue
Constant	0.000649	0.00027089	2.3958	0.016585
AR{1}	-0.091428	0.041264	-2.2157	0.026715
AR{2}	-0.085777	0.037517	-2.2863	0.022236
AR{3}	-0.012399	0.037015	-0.33496	0.73765
AR{4}	-0.0598	0.035297	-1.6942	0.090227
DoF	5.0979	1.0171	5.0125	5.374e-07

ARIMA(4,0,0) - Model 3 (t Distribution)

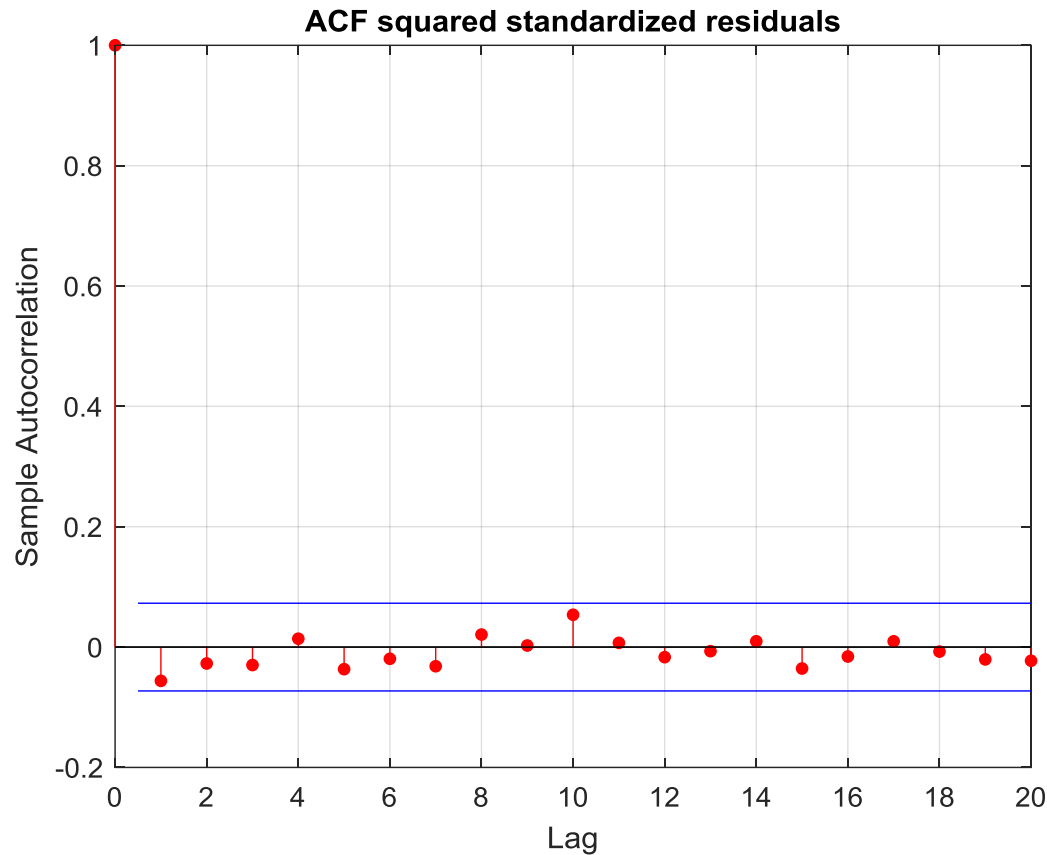
GJR(1,1) Conditional Variance Model (t Distribution):

	Value	StandardError	TStatistic	PValue
Constant	1.1163e-06	8.9992e-07	1.2405	0.2148
GARCH{1}	0.90482	0.023198	39.005	0
ARCH{1}	2e-12	0.038542	5.1892e-11	1
Leverage{1}	0.17202	0.045561	3.7755	0.00015
DoF	5.0979	1.0171	5.0125	5.374e-07

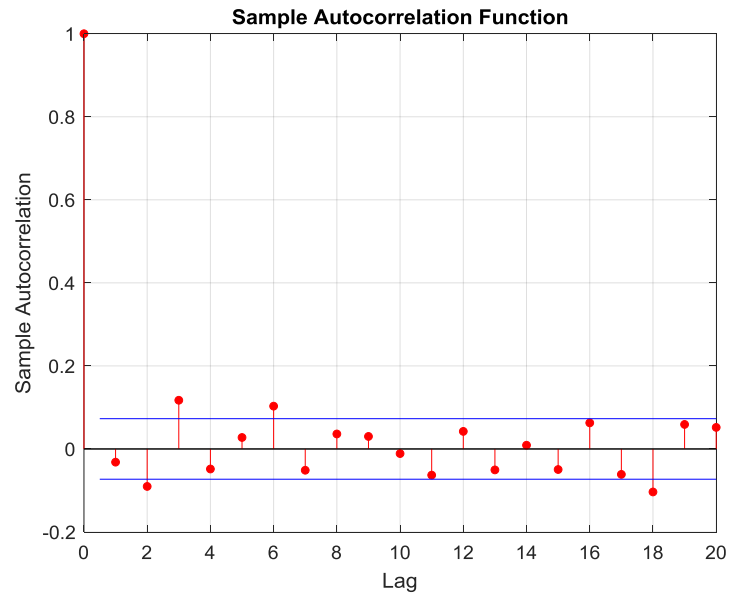
ARIMA(4,0,0) – Model 3 (t Distribution)



ARIMA(4,0,0) – Model 3 (t Distribution)



ARIMA(2,0,16) – Model 4 (t Distribution)



AIC-BIC

	m1	m2	m3	m4	
AIC:	-4.6764	-4.7537	-4.7555	-4.7564	*10 ³
BIC:	-4.6518	-4.7193	-4.7064	-4.6386	*10 ³