Mean-variance portfolio notes

Xie Zejian

Contents

denoted $\delta = \mathbf{e}'\mathbf{V}^{-}\mathbf{e}, \alpha = \overline{\mathbf{r}}'\mathbf{V}^{-}\mathbf{e}, \xi = \overline{\mathbf{r}}'\mathbf{V}^{-}\overline{\mathbf{r}}$ where $\delta, \xi > 0$ since $\mathbf{V}$ is positive define. Thus we have a linear equations:

$$\begin{bmatrix} \xi & \alpha \\ \alpha & \delta \end{bmatrix} \begin{bmatrix} \lambda \\ \gamma \end{bmatrix} = \begin{bmatrix} \overline{r}_p \\ 1 \end{bmatrix}$$

Note $\delta\xi - \alpha^2 > 0$ since $(\alpha\overline{\mathbf{r}} - \xi\mathbf{e})'\mathbf{V}^{-}(\alpha\overline{\mathbf{r}} - \xi\mathbf{e}) = \xi(\delta\xi - \alpha^2) > 0$ and thus such equations is consistent.

solve and get

$$\lambda = \frac{\delta\overline{r}_p - \alpha}{\delta\xi - \alpha^2}, \gamma = \frac{\xi - \alpha\overline{r}_p}{\delta\xi - \alpha^2}$$