Homework 3

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Exercise 0.1.

Solution. Suppose the first one's portfolio is $\boldsymbol{\theta} = (\theta_1, \theta_2)$, then

$$\frac{\theta_1 + \theta_2 + 0.5}{2} + \frac{3\theta_1 + \theta_2 + 1.5}{2} - \alpha(q_1\theta_1 + q_2\theta_2)$$

FOC implies that:

$$\begin{cases} 2 = \alpha q_1 \\ 1 = \alpha q_2 \end{cases} \implies q_1 = 2q_2$$

The second should invest $(-\theta_1, -\theta_2)$ and hence

$$\frac{\sqrt{-\theta_1 - \theta_2 + \frac{1}{2}}}{2} + \frac{\sqrt{-3\theta_1 - \theta_2 + \frac{3}{2}}}{2} - \beta(q_1\theta_1 + q_2\theta_2)$$

FOC and $q_1 = 2q_2$ implies

$$\sqrt{-\theta_1 - \theta_2 + \frac{1}{2}} = \sqrt{-3\theta_1 - \theta_2 + \frac{3}{2}}$$

it follows that $\theta_1 = \frac{1}{2}$. Hence, the equilibrium is

$$\theta_1 = \left(\frac{1}{2}, -1\right), \theta_2 = \left(-\frac{1}{2}, 1\right), q = (2\alpha, \alpha)$$