Homework 4

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Exercise 0.1.

Solution. As we have

$$1 = A\,\mathbb{E}\,R^{\theta} - B\,\mathrm{Cov}\left[R^{\theta}, e\right]$$

Let $\theta = \theta^0$, we have

$$1 = A\,\mathbb{E}\,R^{0}$$

thus we have

$$\mathbb{E}(R^{\theta} - R^{0}) \propto \mathrm{Cov}\left[R^{\theta}, e\right]$$

From $\left\{R^{\theta} : \theta \in \mathbb{R}^N\right\}$ is a linear space, we have

$$\mathrm{Cov}\left[e - R^{M}, R^{\theta}\right] = 0$$

that implies $\mathrm{Cov}\left[R^{\theta}, e\right] = \mathrm{Cov}\left[R^{\theta}, R^{M}\right]$. Thus

$$\frac{\mathbb{E}\left(R^{\theta} - R^{0}\right)}{\mathbb{E}\left(R^{M} - R^{0}\right)} = \frac{\mathrm{Cov}\left[R^{\theta}, R^{M}\right]}{\mathrm{Var}\,R^{M}} = \beta_{\theta}$$

Exercise 0.2.

Solution. The state price is given by

$$\psi = \mathbf{D^{-1}q} = \frac{1}{1+r}\begin{bmatrix} \frac{1+r-d}{(u-d)} \\ \frac{-1-r+u}{(u-d)} \end{bmatrix}$$

And this option is $C = (uS_0 - K, 0)'$, thus the price is given by

$$\mathbb{E}\,\pi\,\overset{\mathbb{Q}}{\mathbb{E}}\,C = C'\psi = \frac{1}{1+r}\frac{1+r-d}{u-d}(uS_0 - K)$$