

# homework 1

Xie zejian

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## **1**

### **1.1**

**Lemma 1.1.** Suppose  $\mathbf{A}$  with eigenvalues  $\lambda_i$  is symmetric, then

$$\begin{aligned} \text{eig}(\mathbf{I} - c\mathbf{A}) &= 1 + c\lambda_i \\ \text{eig}(\mathbf{A} - c\mathbf{I}) &= \lambda_i - c \end{aligned}$$

Note

$$\mathbf{A} = \mathbf{I} - \rho(-\mathbf{I} + \mathbf{e}\mathbf{e}')$$

Where  $\mathbf{e}$  is  $p$  one vector. Note  $\mathbf{e}\mathbf{e}'$  has one eigenvalue of  $p$  and  $p - 1$  eigenvalues of 0, then  $\mathbf{A}$  has  $p - 1$  eigenvalues of  $1 - \rho$  and one  $1 + (p - 1)\rho$  and thus

$$|A| = (1 - \rho)^{p-1}[1 + (p - 1)\rho]$$

### **1.2**

Let  $|A| = 0$ , we have  $(1 - \rho) = 0$  or  $1 + (p - 1)\rho = 0$  and thus

$$\rho = \begin{cases} 1 \\ -\frac{1}{p-1} \end{cases}$$

### **1.3**

This statement is false and can't be proved.