homework 1

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1.1 1

Lemma 1.1. *Suppose* $\mathbf{A}$ *with eigenvalues* λ_i *is symmetric, then*

$$eig\left(\mathbf{I} + c\mathbf{A}\right) = 1 + c\lambda_i$$
$$eig\left(\mathbf{A} - c\mathbf{I}\right) = \lambda_i - c$$

Proof. Note

$$|\mathbf{I} + c\mathbf{A} - \lambda\mathbf{I}| = |\mathbf{I} + c\lambda_i\mathbf{I} - \lambda\mathbf{I}| = 0 \Rightarrow \lambda = 1 + c\lambda_i$$

the other one can be proved similarly.

□

Note

$$\mathbf{A} = \mathbf{I} - \rho(-\mathbf{I} + \mathbf{ee'})$$

Where $\mathbf{e}$ is p one vector. Note $\mathbf{ee'}$ has one eigenvalue of p and $p-1$ eigenvalues of 0, then $\mathbf{A}$ has $p-1$ eigenvalues of $1-\rho$ and one $1+(p-1)\rho$ and thus

$$|A| = (1-\rho)^{p-1}[1+(p-1)\rho]$$

1.2 2

Let $|A| = 0$, we have $(1-\rho) = 0$ or $1+(p-1)\rho = 0$ and thus

$$\rho = \begin{cases} 1 \\ -\frac{1}{p-1} \end{cases}$$

1.3 3

You can't prove a false statement.

2 2

Note eigenvalues of $c\mathbf{A}$ is $c\lambda_i$. Plug $\rho = .5$ and $p = 3$, we find

$$\operatorname{eig}(\mathbf{A}) = 4, 1, 1$$

and the corresponding eigenvectors are:

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

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