Portfolio Performance Evaluation

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1 Portfolio performance

So far, we have following evaluation methods:

- Sharpe ratio: $SR = \frac{r^e}{\sigma}$
- Treynor ratio: $T = \frac{r^e}{\beta}$
- Jensen's alpha: $\alpha = r^e - \beta r_m^e$
- Information ratio: $IR = \frac{\alpha}{\sigma}$

However the sharpe ratio is hard interpret, so we may need M^2:

$$M^2 = (S - S_m)\sigma_m = r_{p_\sigma}^e - r_m^e$$

With interpretation that the return difference of the portfolio to market with control σ fixed.

Similarly,

$$T^2 = (T - T_m) = r_{p_\beta}^e - r_m^e$$

Describe difference between the portfolio and market share the same β.

Plug in α, we have

$$T^3 = \frac{\alpha}{\beta}$$

As hedge fund can short, they prefer information ratio. Otherwise, i.e., short is not allowed, we should consider Treynor ratio.

1.1 Other performance measure

As variance can be good or bad, there are various alternative measures of the trade-off between return and **downside risks**:

1.1.1 Sortino ratio

Suppose we have a most conservative goal for return, this is **Minimum Acceptable Return**(MAR).

Suppose is given by:

$$SOR = \frac{\mathrm{E}\left[r_t - MAR\right]}{\sqrt{\frac{\sum_{t=1}^{T}(r_t - MAR)^2 \chi_{\{r_t < MAR\}}}{T}}}$$

r_f is a default choice of MAR.

2 Market timing

Market timing is that we change the weight on market based on our expectation to market. If we may increase ω when market does well and decrease ω as market does badly, the plot of the return against market return look like figure 1

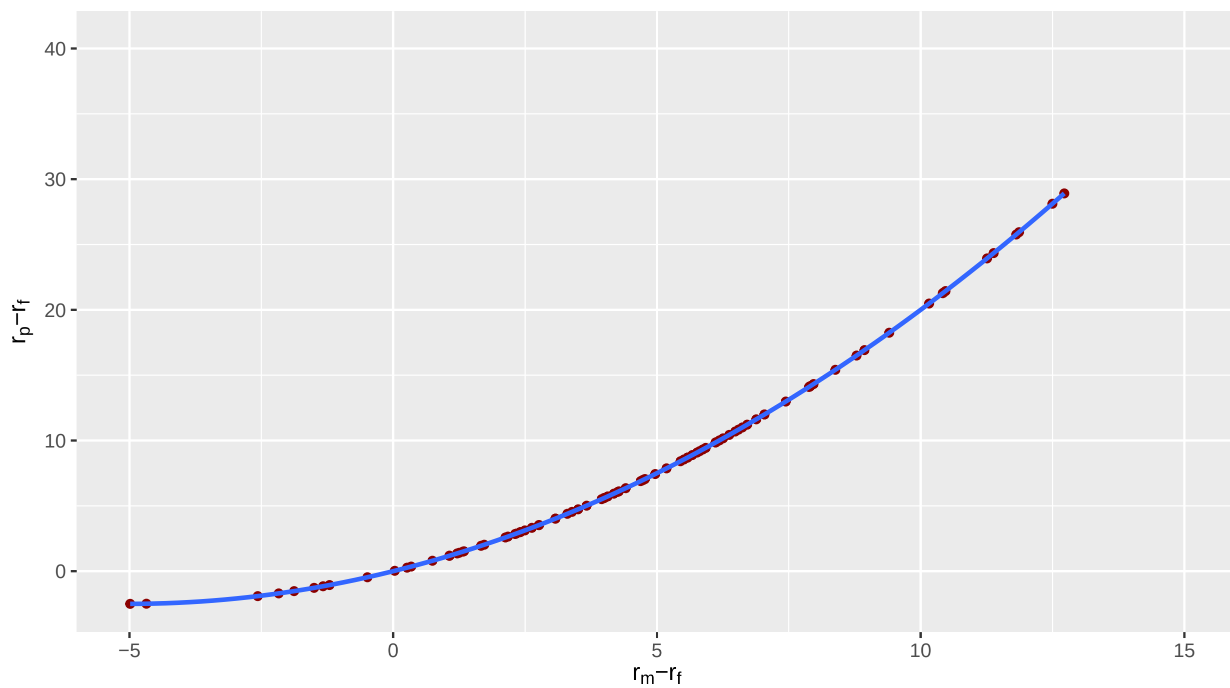

Figure 1: Market timing behavior

Thus we may estimate

$$r_i^e = \alpha + b r_m^e + c {r_m^e}^2 + e_i$$

to determine whether there is a quadratic term or estimate

$$r_i^e = \alpha + br_m^e + cr_m^e \chi_{r_m^e > 0} + e_i$$

to determine whether the slope is different with the sign of $r_m - r_f$, that is

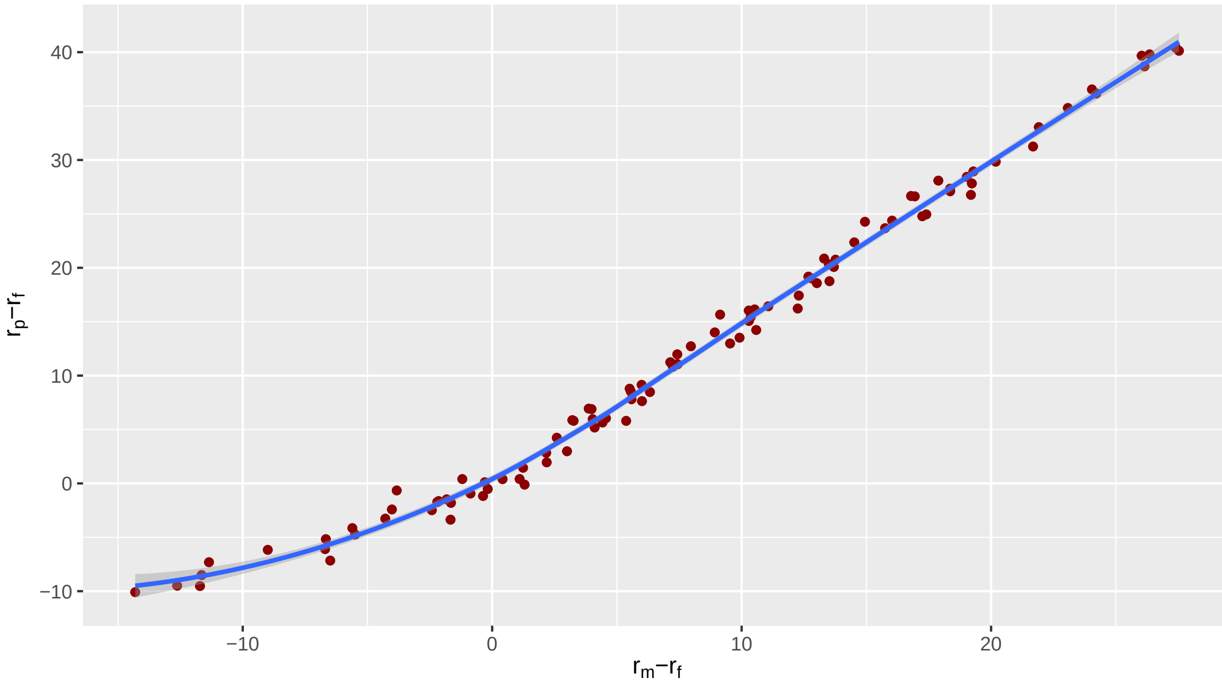

Figure 2: piecewise market timing behavior

Given initial wealth S_0, the value of perfect market timing is equivalent to holding an European call option on market with strike $(1 + r_f)\, S_0$.

Recall the BSM model $C\,(S_0, K, r, T, \sigma)$

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

Where $d_1 = \frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

- C is increasing in σ
- C is increasing in T
- C is increasing in r

As $K = (1 + r_f)^T\, S_0 = e^{rT} S_0$ and set $S_0 = 1$, then

$$d_1 = \frac{-rT + rT + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} = \frac{1}{2}\sigma\sqrt{T} = -d_2$$

Thus

$$C = N(\frac{1}{2}\sigma\sqrt{T}) - N(-\frac{1}{2}\sigma\sqrt{T}) = 2N(\frac{1}{2}\sigma\sqrt{T}) - 1$$

Which is now independent to r.

As on one can predict perfectly, recall the confusion matrix:

$$\begin{bmatrix} TP & FP \\ FN & TN \end{bmatrix}$$

One way to measure timing ability is used **informedness**, which is defined as

$$J = \frac{TP}{TP + FN} + \frac{TN}{TN + FP} - 1$$

And the value of imperfect timer is given by $J \cdot C$. Suppose the confidence of such timer is ω, that is, s(he) only shifts ω between bills and market, then the value is ωJC.

Suppose the fund ask F for your invest I, in competitive market, is the same as you buy the timing ability with F and thus:

$$F = (I - F)C$$

3 Performance Attribution

3.1 Brinson's method

Performance attribution identify the sources of the active return. Recall

$$\begin{aligned} r &= \boldsymbol{\omega}'\mathbf{r} \\ b &= \boldsymbol{\omega}'_{\mathbf{m}}\mathbf{r}_{\mathbf{m}} \end{aligned}$$

Now we may classify active return sources into

- Market timing: Different weight allocation between ω and ω_m:
$$(\boldsymbol{\omega}' - \boldsymbol{\omega}'_{\mathbf{m}})\mathbf{r}_{\mathbf{m}}$$
- Security selection: Different assets selection ($\mathbf{r}$ and $\mathbf{r}_{\mathbf{m}}$):
$$\boldsymbol{\omega}'_{\mathbf{m}}(\mathbf{r} - \mathbf{r}_{\mathbf{m}})$$
- Interaction between of them.

$$r - b - (\boldsymbol{\omega}' - \boldsymbol{\omega}'_{\mathbf{m}})\mathbf{r}_{\mathbf{m}} - \boldsymbol{\omega}'_{\mathbf{m}}(\mathbf{r} - \mathbf{r}_{\mathbf{m}}) = \boldsymbol{(\omega' - \omega'_{\mathbf{m}})(\mathbf{r} - \mathbf{r}_{\mathbf{m}})}$$