Homework 4

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1 Information ratio & GRS statistics

Suppose we manage portfolio A constructed by $\boldsymbol{\omega'}\mathbf{r^e} + \omega_m r_m^e$. By index model:

$$\mathbf{r^e} = \boldsymbol{\alpha} + \boldsymbol{\beta} r_m^e + \boldsymbol{\epsilon}$$

The optimize problem is:

$$\begin{aligned} \min \quad & \frac{1}{2}\sigma^2 = \frac{1}{2}[(\boldsymbol{\omega'}\boldsymbol{\beta})^2\sigma_m^2 + \boldsymbol{\omega'}\mathbf{V}\boldsymbol{\omega} + \omega_m^2\sigma_m^2] \\ s.t. \quad & \boldsymbol{\omega'}(\boldsymbol{\alpha} + \boldsymbol{\beta}\overline{r}_m^e) + \omega_m r_m^e = \overline{r}_p^e \end{aligned}$$

Where $\operatorname{Var}[\epsilon] = \mathbf{V}$. To simplify, let

$$\widetilde{\boldsymbol{\omega}} = \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{\omega_m + \omega'\beta} \end{bmatrix}, \mathbf{r} = \begin{bmatrix} \boldsymbol{\alpha} \\ \overline{r}_m^e \end{bmatrix}$$

and

$$\widetilde{\mathbf{V}} = \begin{bmatrix} \mathbf{V} & \mathbf{0} \\ \mathbf{0} & \sigma_m^2 \end{bmatrix}$$

rewrite above optimize problem:

$$\min_{\widetilde{\boldsymbol{\omega}}, \boldsymbol{\lambda}} \frac{1}{2} \widetilde{\boldsymbol{\omega}}' \widetilde{\mathbf{V}} \widetilde{\boldsymbol{\omega}} + \lambda(\overline{r}_p^e - \widetilde{\boldsymbol{\omega}}' \mathbf{r})$$

FOC with $\widetilde{\boldsymbol{\omega}}$:

$$\widetilde{\mathbf{V}} \widetilde{\boldsymbol{\omega}} - \lambda \mathbf{r} = 0 \implies \widetilde{\boldsymbol{\omega}} = \lambda \widetilde{\mathbf{V}}^{\mathbf{-1}} \mathbf{r}$$

Thus

$$\begin{aligned} \boldsymbol{\omega} &= \lambda \mathbf{V^{-1}} \boldsymbol{\alpha} \\ \omega_m + \boldsymbol{\omega'\beta} &= \lambda \frac{\overline{r}_m^e}{\sigma_m^2} \end{aligned}$$

To get tangency portfolio, impose $\boldsymbol{\omega' e + \omega_m} = 1$, then

$$(\mathbf{e'} - \boldsymbol{\beta'})\boldsymbol{\omega} + \lambda \frac{\overline{r}_m^e}{\sigma_m^2} = 1 \implies \lambda = \frac{1}{(\mathbf{e'} - \boldsymbol{\beta'})\mathbf{V^{-1}}\boldsymbol{\alpha} + \frac{\overline{r}_m^e}{\sigma_m^2}}$$

1.1 1

The squared information ratio of

$$\begin{aligned} IR_A^2 &= \frac{(\boldsymbol{\omega'\alpha})^2}{\boldsymbol{\omega'}\mathbf{V}\boldsymbol{\omega}} \\ &= \frac{\lambda^2 (\boldsymbol{\alpha'}\mathbf{V^{-1}}\boldsymbol{\alpha})^2}{\lambda^2 \boldsymbol{\alpha'}\mathbf{V^{-1}}\boldsymbol{\alpha}} \\ &= \boldsymbol{\alpha'}\mathbf{V^{-1}}\boldsymbol{\alpha} \end{aligned}$$

Thus

$$IR_A^2 = \sum_{i=1}^n \frac{\alpha_i^2}{\sigma_{\epsilon_i}^2}$$

when $\mathbf{V} = diag\left(\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2, \cdots, \sigma_{\epsilon_n}^2\right)$.

1.2 2

The sharpe ratio of portfolio A is given by

$$SR_A^2 = \frac{(\widetilde{\boldsymbol{\omega}}'\mathbf{r})^2}{\widetilde{\boldsymbol{\omega}}'\widetilde{\mathbf{V}}\widetilde{\boldsymbol{\omega}}} = \frac{\lambda^2(\mathbf{r}'\widetilde{\mathbf{V}}^{-1}\mathbf{r})^2}{\lambda^2(\mathbf{r}'\widetilde{\mathbf{V}}^{-1}\mathbf{r})} = \mathbf{r}'\widetilde{\mathbf{V}}^{-1}\mathbf{r}$$

which can be decompositioned by plugging in $\widetilde{\mathbf{V}}$:

$$SR_A^2 = \boldsymbol{\alpha}'\mathbf{V}^{-1}\boldsymbol{\alpha} + \frac{(\overline{r}_m^e)^2}{\sigma_m^2} = \boldsymbol{\alpha}'\mathbf{V}^{-1}\boldsymbol{\alpha} + SR_m^2$$

2 Performance Evaluation

```
pfm <- pfm %>%
  mutate(r1e=yfd-rf,
         r2e=rtxlc-rf,
         rme=market-rf,
         rbe=tbond-rf)

pfm <- pfm %>%
  mutate(mkdum=ifelse(pfm$rme>0,pfm$rme,0))
```

2.1 Annualized average

```
func <- function(vec){
   mean(vec)*12
}

knitr::kable(
    pfm %>% summarise(across(c('r1e','r2e','rme'),func)),
             longtable=TRUE)
```

r1e	r2e	rme
12.75096	9.95927	13.39006

2.2 Annualized standard deviations

```
func <- function(vec){
   sd(vec)*sqrt(12)
}

knitr::kable(
    pfm %>% summarise(across(c('r1e','r2e','rme'),func)),
             longtable=TRUE)
```

r1e	r2e	rme
26.81499	23.27382	30.43868

2.3 Annualized Sharpe ratios

```
func <- function(vec){
    mean(vec)/sd(vec)*sqrt(12)
}

knitr::kable(
    pfm %>% summarise(across(c('r1e','r2e','rme'),func)),
             longtable=TRUE)
```

r1e	r2e	rme
0.475516	0.4279172	0.4399026

2.4 Annualized alpha and t-stats

```
func <- function(vec){
    lms <- lm(vec~pfm$rme+pfm$rbe) %>% summary()
    alpha <- lms$coefficients['(Intercept)','Estimate']
    c(alpha*12,lms$coefficients['(Intercept)','t value'])
}

tbl <- pfm %>% summarise(across(c('r1e','r2e'),func))
rownames(tbl) <- c('alpha','t')

knitr::kable(tbl,
             longtable=TRUE,
             row.names = TRUE)
```

	r1e	r2e
alpha	0.8209598	0.3837345
t	0.2484257	0.1307097

2.5 CAPM beta

```
func <- function(vec){
    lms <- lm(vec~pfm$rme+pfm$rbe) %>% summary()
    b <- lms$coefficients['pfm$rme','Estimate']
    c(b,b*sd(pfm$rme)*sqrt(12))
}

tbl <- pfm %>% summarise(across(c('r1e','r2e'),func))
rownames(tbl) <- c('beta','sys vol')
```

```r
knitr::kable(tbl,
             longtable=TRUE,
             row.names = TRUE)
```

	r1e	r2e
beta	0.7948448	0.6827882
sys vol	24.1940306	20.7831749

2.6 Annualized Treynor ratios

```r
func <- function(vec){
    lms <- lm(vec~pfm$rme+pfm$rbe) %>% summary()
    b <- lms$coefficients['pfm$rme','Estimate']
    mean(vec)*12/b
}

tbl <- pfm %>% summarise(across(c('r1e','r2e'),func))

knitr::kable(tbl,
             longtable=TRUE)
```

r1e	r2e
16.04207	14.58618

2.7 Annualized idiosyncratic volatility

```r
func <- function(vec){
    lms <- lm(vec~pfm$rme+pfm$rbe) %>% summary()
    sd(lms$residuals)*sqrt(12)
}

tbl <- pfm %>% summarise(across(c('r1e','r2e'),func))

knitr::kable(tbl,
             longtable=TRUE)
```

r1e	r2e
11.94453	10.61125

2.8 Goodness of fit

```r
func <- function(vec){
    lms <- lm(vec~pfm$rme+pfm$rbe) %>% summary()
    lms$r.squared
```

```
}

tbl <- pfm %>% summarise(across(c('r1e','r2e'),func))

knitr::kable(tbl,
             longtable=TRUE)
```

r1e	r2e
0.8015812	0.7921272

2.9 Information ratio

```
func <- function(vec){
    lms <- lm(vec~pfm$rme+pfm$rbe) %>% summary()
    alpha <- lms$coefficients['(Intercept)','Estimate']
    alpha/sd(lms$residuals)

}

tbl <- pfm %>% summarise(across(c('r1e','r2e'),func))

knitr::kable(tbl,
             longtable=TRUE)
```

r1e	r2e
0.0198409	0.0104394

2.10 Maximal Drawdown

```
func <- function(vec){
  cumwl <- cumprod(1+vec/100)
  dd <- drawdown(cumwl,FALSE,FALSE)
  max(dd)
}
tbl <- pfm %>% summarise(across(c('yfd','rtxlc'),func))

knitr::kable(tbl,
             longtable=TRUE)
```

yfd	rtxlc
4.653431	3.106981

2.11 Maximal Recovery Period

```
func <- function(vec){
  cumwl <- cumprod(1+vec/100)
  dd <- drawdown(cumwl,FALSE,FALSE)
  rec <- which(dd==0)
  rec <- c(rec %>% diff(),length(vec)-rec[length(rec)])
  max(rec)
}
tbl <- pfm %>% summarise(across(c('yfd','rtxlc'),func))

knitr::kable(tbl,
             longtable=TRUE)
```

yfd	rtxlc
89	90

2.12 Market timing regression test

```
func <- function(vec){
  lms <- lm(vec~pfm$rme+pfm$mkdum+pfm$rbe) %>% summary()
  lms$coefficients['pfm$mkdum',c('Estimate','t value')]
}

tbl <- pfm %>% summarise(across(c('r1e','r2e'),func))
rownames(tbl) <- c('gamma','t')

knitr::kable(tbl,
             longtable=TRUE,
             row.names = TRUE)
```

	r1e	r2e
gamma	-0.0746124	0.0064980
t	-0.8361521	0.0818082

None of them since both gamma are not significant

2.13 Choice

2.13.1 Overall Portfolios

In the case of we are choosing fund as our only and whole portfolio, we should use **Sharpe ratio** as criterion and choose fund 1.

2.13.2 Well diversified

As out portfolio has been well diversified, we only care systematic risk and use **Treynor ratio** as criterion and choose fund 1.

2.13.3 Short allowed

As short sell is allowed for us, the only thing we care is **Information ratio** and choose fund 1. The maximal Sharpe ratio can be attained is given by

$$SR^2 = SR_m^2 + IR_A^2 = 0.434^2 + 0.0198^2 \implies SR = 0.44035$$