Mathematical Statistics

谢泽健
11810105

Chap1

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第四节

28.

从52张牌中选出25张并进行全排列,5个人的手牌顺序可以改变:

$$\left(\binom{52}{25}25!\right)\Big/(5!)^5 = 297\,686\,658\,367\,751\,290\,178\,415\,114\,240 \tag{1}$$

29.

从剩余的10张黑桃中选出两张,共有

$$\binom{10}{2} = 45 \tag{2}$$

样本总数为

$$\binom{47}{2} = 1081 \tag{3}$$

故概率为

$$\frac{45}{1081} = 0.0416281 \tag{4}$$

1.

a.

$2r$只鞋子中有$2r$种不同的鞋码,则先从n双鞋子中选取$2r$双,再随机选取左脚右脚.

$$P = \left(2^{2r}\binom{n}{2r}\right)\Big/\binom{2n}{2r} \tag{5}$$

b.

从n双鞋子选取一双作为成对的鞋子,而后再按照1中的方式选取剩余$(2r-2)$只鞋

$$P = \left(2\binom{n}{1}\binom{n-1}{2r-2}\right)\Big/\binom{2n}{2r} = \left(2^{2r-2}n\binom{n-1}{2r-2}\right)\Big/\binom{2n}{2r} \tag{6}$$

c.

$$P = \frac{\binom{n}{r}}{\binom{2n}{2r}} \tag{7}$$

2.

将4把钥匙分给四人,共有:

$$4! = 24 \tag{8}$$

恰有一把匹配,有

$$4\times 2 = 8 \tag{9}$$

$$P = \frac{8}{24} = \frac{1}{3} \tag{10}$$

恰有两把匹配:

$$P = \frac{1}{4} \tag{11}$$

恰有四把匹配:

$$P = \frac{1}{24} \tag{12}$$

所以

$$P = (1 + 8 + 6)/24 = \frac{5}{8} \tag{13}$$

第五节

46.

记 H 为事件硬币正面朝上, T 为事件反面朝上, R为事件抽到红球, W 为事件抽到白球.

a.

由全概率公式,

$$P(R) = P(R \mid H)\, P(H) + P(R \mid T)\, P(T) \tag{14}$$

由题可知:

$$P(R \mid H) = \frac{3}{5},\ P(R \mid T) = \frac{2}{7} \tag{15}$$

从而

$$P(R) = \frac{3}{5} \times \frac{1}{2} + \frac{2}{7} \times \frac{1}{2} = \frac{31}{70} \tag{16}$$

b.

由贝叶斯公式,

$$P(H \mid R) = \frac{P(R \mid H)}{P(R \mid T) + P(R \mid H)} = \frac{\frac{3}{5}}{\frac{3}{5} + \frac{2}{7}} = \frac{21}{31} \tag{17}$$

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记 H 为投保人是高风险客户的事件, M 为投保人是中风险客户的事件, L 为投保人是低风险客户的事件, C为投保人获得赔偿的概率.

$$P(H \mid C) = \frac{P(C \mid H)\, P(H)}{P(C \mid H)\, P(H) + P(C \mid M)\, P(M) + P(C \mid L)\, P(L)} = \frac{0.02 \times 0.1}{0.02 \times 0.1 + 0.01 \times 0.2 + 0.0025 \times 0.7} = 0.347826 \tag{18}$$

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a.

由全概率公式

$$P(R_{\text{tomorrow}}) = P(R_{\text{tomorrow}} \mid R_{\text{today}})\, P(R_{\text{today}}) + P\left(R_{\text{tomorrow}} \mid R^c_{\text{today}}\right) P\left(R^c_{\text{today}}\right) = \alpha \text{p} + (1 - \beta)\,(1 - p) = \\ -\beta + \alpha\, p + \beta\, p - p + 1 \tag{19}$$

b.

该气象模型可视为马尔可夫过程,其概率转移矩阵为

$$M = \begin{pmatrix} \alpha & 1 - \alpha \\ 1 - \beta & \beta \end{pmatrix} \tag{20}$$

从而后天下雨的概率为

$$(\,p \quad 1-p\,).M^2 = (\,p \quad 1-p\,).\begin{pmatrix} \alpha^2 + (1-\alpha)(1-\beta) & (1-\alpha)\alpha + (1-\alpha)\beta \\ \alpha(1-\beta) + (1-\beta)\beta & (1-\alpha)(1-\beta) + \beta^2 \end{pmatrix} \tag{21}$$

取第一个分量,即为:

$$P = \alpha + \beta + p(-1+\alpha+\beta)^2 - \beta(\alpha+\beta) \tag{22}$$

c

$$M^n = \begin{pmatrix} \frac{-1+\beta+(-1+\alpha)(-1+\alpha+\beta)^n}{-2+\alpha+\beta} & -\frac{(-1+\alpha)(-1+(-1+\alpha+\beta)^n)}{-2+\alpha+\beta} \\ -\frac{(-1+\beta)(-1+(-1+\alpha+\beta)^n)}{-2+\alpha+\beta} & \frac{-1+\alpha+(-1+\beta)(-1+\alpha+\beta)^n}{-2+\alpha+\beta} \end{pmatrix} \tag{23}$$

所以 n 天后下雨的概率为:

$$\begin{gathered} P(R_n) = p((-1+\beta+(-1+\alpha)(-1+\alpha+\beta)^n)/(-2+\alpha+\beta)) + -(1-p)(((-1+\beta)(-1+(-1+\alpha+\beta)^n))/(-2+\alpha+\beta)) = \\ (-1+\beta+(-1+\alpha+\beta)^n(1-\beta+p(-2+\alpha+\beta)))/(-2+\alpha+\beta) \end{gathered} \tag{24}$$

注意到

$$-1 < -1+\alpha+\beta < 1 \tag{25}$$

这表明

$$\lim_{n\to\infty} (-1+\alpha+\beta)^n = 0 \tag{26}$$

所以当 n 趋近于无穷,下雨的概率为

$$P(R_{n\to\infty}) = (1-\beta)/(2-(\alpha+\beta)) \tag{27}$$

63.

$$P(80 \mid 70) = P\left(80 \cap 70\right) \big/ P(70) = 0.2/0.6 = 1/3 \tag{28}$$

Monty Hall problem.

考虑参赛者一开始选定的门,其后为山羊记为事件G,其后为汽车记为事件C,参赛者最终获奖的事件为 R.

若参赛者选择换门:

$$P(R) = P(R \mid G)\,P(G) + P(R \mid C)\,P(C) = 1 \times \frac{2}{3} + 0 \times \frac{1}{3} = \frac{2}{3} \tag{29}$$

若参赛者不选择换门:

$$P(R) = P(R \mid G)\,P(G) + P(R \mid C)\,P(C) = 0 \times \frac{2}{3} + 1 \times \frac{1}{3} = \frac{1}{3} \tag{30}$$