Mathematical Statistics

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Chap1

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4.

当 $n = 2$:

$$P\left(\bigcup_{i=1}^{n} A_i\right) = P(A_1) + P(A_2) - P\left(A_1 \cap A_2\right) \le \sum_{i=1}^{n} P(A_i) \tag{1}$$

假设对于 $n = k$,

$$P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i) \tag{2}$$

则当 $n = k + 1$:

$$P\left(\bigcup_{i=1}^{k+1} A_i\right) = P\left(\bigcup_{i=1}^{k} A_i \cup A_{k+1}\right) \le P\left(\bigcup_{i=1}^{n} A_i\right) + P(A_{k+1}) \le \sum_{i=1}^{n} P(A_i) + P(A_{k+1}) \le \sum_{i=1}^{k+1} P(A_i) \tag{3}$$

又对于 $n = 1$ 显然成立,故由数学归纳法:

$$\forall\, n \ge 1,\ P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i) \tag{4}$$

7.

只需证:

$$1 \ge P(A) + P(B) - P\left(A \cap B\right) \tag{5}$$

注意到:

$$P(A) + P(B) - P\left(A \cap B\right) = P\left(A \cup B\right) \tag{6}$$

对于一个事件,显然有:

$$P\left(A \cup B\right) \le 1 \tag{7}$$

以上每步可逆,故而

$$P\left(A \cap B\right) \ge P(A) + P(B) - 1 \tag{8}$$