Hackathon

Group 2

April 9, 2025

1 Model problem

We start with a four-dimensional data set for the electric field E and the magnetic field H. They are both complex data of dimension $8 \times 121 \times 111 \times 126$ and can be interpreted as two functions

$$E_i : \mathbb{R}^3 \to \mathbb{C}^3, \quad H_i : \mathbb{R}^3 \to \mathbb{C}^3 \quad \{i = 1, \dots 8\}$$

which are only applied on a grid of grid-size 4mm. Especially the total field strength can be written as

$$E = \sum_{i=1}^{8} E_i, \quad H = \sum_{i=1}^{8} H_i$$

If we adapt the fields at each dipole through an amplitude A_i and phase φ we can write the custom fields as

$$\begin{aligned}\hat{H}_i &= A_i \cdot \exp(\mathrm{i} \cdot \varphi_i) \cdot H_i \\ \hat{E}_i &= A_i \cdot \exp(\mathrm{i} \cdot \varphi_i) \cdot E_i\end{aligned}$$

Now we can define the B_1^+-field as a function which takes a three-dimensional real input x:

$$\begin{aligned}B_1^+ : \mathbb{R}^3 &\to \mathbb{C} \\ x &\mapsto \mathrm{proj}_1(\hat{H}(x)) + \mathrm{i} \cdot \mathrm{proj}_2(\hat{H}(x))\end{aligned}$$

Finally we define the Specific Absorption Rate (SAR) as

$$\mathrm{SAR} = \frac{|E(x)|^2 \cdot \sigma(x)}{\rho(x)}$$

Then we can define two cost functions as follows with D as the human area and $|D|$ as the amount of voxels of the human area:

$$\begin{aligned}
\mathrm{cost}_1(\varphi, A) &= \frac{\mathrm{mean}(|B_1^+|)}{\mathrm{std}(|(B_1^+)|} \\
&\propto \frac{\sum_{x\in D}|B_1^+(x)|}{\sqrt{\sum_{x\in D}\left(|B_1^+(x)| - \frac{\sum_{\hat{x}\in D}|B_1^+(\hat{x})|}{|D|}\right)}} \\
\mathrm{cost}_2(\varphi, A) &= \frac{\mathrm{mean}(|B_1^+|)}{\mathrm{std}(|(B_1^+)|} - \lambda \cdot \max_{x\in D} \frac{|\hat{E}(x)|^2 \cdot \sigma(x)}{\rho(x)} \\
&= \frac{1}{\sqrt{|D|}} \cdot \frac{\sum_{x\in D}|B_1^+(x)|}{\sqrt{\sum_{x\in D}\left(|B_1^+(x)| - \frac{\sum_{\hat{x}\in D}|B_1^+(\hat{x})|}{|D|}\right)}} - \lambda \cdot \max_{x\in D} \frac{|\hat{E}(x)|^2 \cdot \sigma(x)}{\rho(x)}
\end{aligned}$$