Option Pricing and Risk Analysis Using MATLAB

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1 Introduction

In financial mathematics, options are widely used financial derivatives that allow investors to hedge risks or speculate on future price movements of assets. A European call option gives the holder the right (but not the obligation) to purchase an asset at a pre-agreed strike price K on a specific maturity date T. The pricing of such derivatives relies on stochastic models that simulate possible future scenarios for the underlying asset's price.

This report presents the implementation of an option pricing model, which is applied to a real-world dataset containing historical financial information. In addition to pricing, we focus on risk measures such as Value at Risk (VaR) and kurtosis analysis to understand the distribution of potential losses. The study is carried out using MATLAB scripts, and results are visualized through plots and statistical measures.

2 Data and Model

The data used in this project is stored in the file `data.xlsx`, which includes daily information for various financial variables. For the purpose of this analysis, we consider:

- u: Stock price upper movement factor, indicating how much the price can rise.
- d: Stock price lower movement factor, reflecting possible downward movements.

- rf: Risk-free interest rate, often representing the yield on government bonds.
- $S0$: Initial stock price on the first day considered in the model.

The option pricing model implemented in MATLAB is based on the Black-Scholes framework, a popular model for pricing European options. The main parameters for the option are:

- $K = 22$: The strike price, or the price at which the holder of the option can buy the underlying stock.
- $T = 1095$: The maturity, set to three years (1095 days), indicating when the option expires.

The function `call_price.m` calculates the price of the call option based on the input parameters extracted from the dataset. The key feature of this model is the daily recalibration of the option price using updated data for u, d, rf, and $S0$.

3 Price Evolution and Profit/Loss Analysis

A key aspect of this analysis is the evolution of the call option's price over time. Starting from day 365, the option price is recalculated daily using updated market parameters from the dataset. This allows us to observe how the price fluctuates over a period of one year (from day 365 to day 730).

The MATLAB code generates a trend of the option price over this period, which is essential for understanding how the option performs in a dynamic market environment.

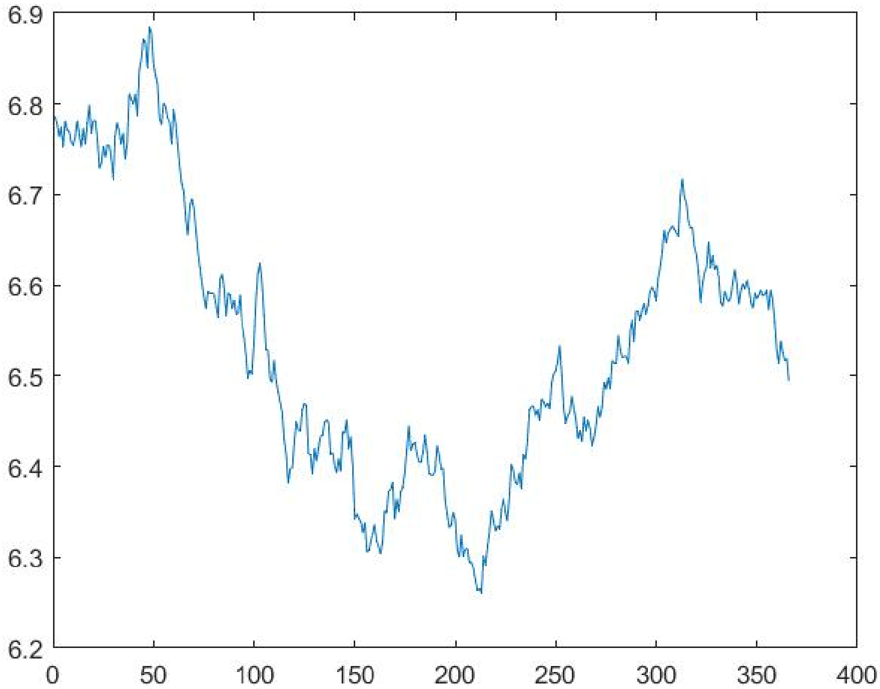

Figure 1: Price Evolution of the Call Option

To further analyze the financial performance of the option, we calculate the daily profits and losses. The daily profit is defined as the difference in the option's price between two consecutive days, while the loss is the negative of the profit. This step is critical for assessing the risk exposure of holding the option over time.

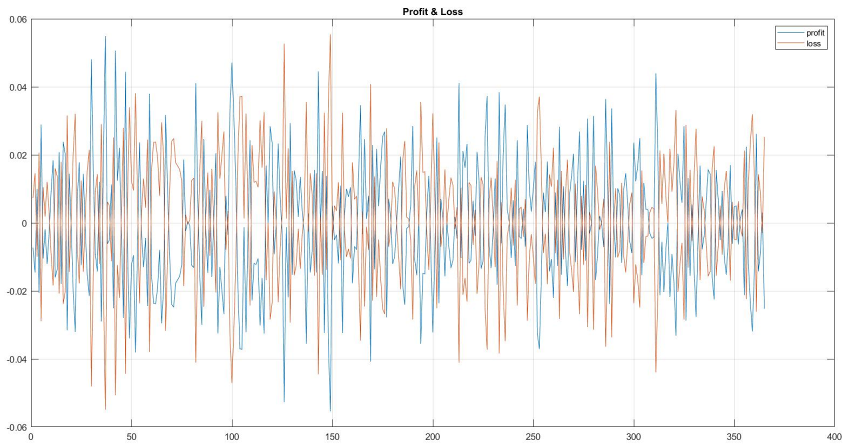

Figure 2: Daily Losses

Additionally, we compute the kurtosis of the losses. Kurtosis is a statistical measure that describes the shape of the distribution of data. Specifically, it tells us whether the distribution of losses is close to a normal (Gaussian) distribution or exhibits fat tails, which would imply a higher likelihood of extreme events. In financial markets, fat-tailed distributions are associated with higher risk and unexpected large losses.

4 Risk Measures: Value at Risk (VaR)

One of the most widely used risk measures in financial markets is Value at Risk (VaR). VaR provides an estimate of the maximum potential loss over a given time period, with a specified confidence level. In this project, we calculate the 5% VaR, which gives us the maximum loss that will not be exceeded with 95% confidence over the time period considered.

The method used for calculating VaR involves simulating price changes based on historical noise (the differences in stock prices between consecutive days) from the first 365 days of data. We add this noise to the values of u, d, rf, and $S0$ observed on day 365, and reprice the option for each simulated scenario. The 95th percentile of the resulting losses is used as the VaR estimate.

Mathematically, if L represents the loss distribution, the VaR at the 95% confidence level is:

$$\mathrm{VaR}_{0.05} = \inf\{x \in R : P(L \leq x) \geq 0.95\}$$

In simpler terms, there is a 5% probability that the loss will exceed this value.

Furthermore, the MATLAB script counts how many times the observed daily losses exceed the calculated VaR. This gives us an indication of the frequency of extreme losses, and thus, of how well the VaR estimate captures the actual risk in the market.

5 Conclusion

In this project, we have implemented a comprehensive model for pricing a European call option and analyzing its risk. The results provide insights into the behavior of the option price over time, as well as the risk of holding the option as measured by profit/loss calculations and Value at Risk.

The model's ability to simulate price movements and compute statistical measures such as kurtosis and VaR is particularly useful for risk management. By understanding how often losses exceed the VaR threshold, we gain a deeper understanding of the risk exposure in different market conditions.

Future extensions of this project could include more sophisticated pricing models, such as the Heston model, or the inclusion of transaction costs and other real-world factors that affect option pricing and portfolio management.