# Numeri complessi

Utili per il calcolo di $z_{in}$ e $\Gamma$.

Caso (1):

$z = a + jb$ $\xrightarrow{\text{devo ottenere}}$ $z = M \cdot e^{j\theta}$

$z = a + jb$: numero complesso scritto come parte reale e parte immaginaria

$z = M \cdot e^{j\theta}$: numero complesso scritto come modulo e fase

Ampiezza: $M = |z|$ → modulo

$\angle$ : $\theta = \arg(z)$ → fase

$Re(z) = a$ → parte reale

$Im(z) = b$ → parte immaginaria

$$M = \sqrt{a^2 + b^2}$$

$\theta$ = vedi formule

Caso (2):

$z = M \cdot e^{j\theta}$ $\xrightarrow{\text{devo ottenere}}$ $z = a + jb$

Scrivo $e^{j\theta}$ come:

$$e^{j\theta} = \cos(\theta) + j \operatorname{sen}(\theta),$$

allora:

$a = M \cdot \cos(\theta)$

$b = M \cdot j \operatorname{sen}(\theta)$

# VECTOR DIFFERENTIAL OPERATIONS

Rectangular Coordinates $(x, y, z)$

$$\nabla \Phi = \hat{\mathbf{x}} \frac{\partial \Phi}{\partial x} + \hat{\mathbf{y}} \frac{\partial \Phi}{\partial y} + \hat{\mathbf{z}} \frac{\partial \Phi}{\partial z}$$

$$\nabla \cdot \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla \times \mathbf{H} = \hat{\mathbf{x}} \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) + \hat{\mathbf{y}} \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) + \hat{\mathbf{z}} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right)$$

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

$$\nabla^2 \mathbf{A} = \hat{\mathbf{x}} \nabla^2 A_x + \hat{\mathbf{y}} \nabla^2 A_y + \hat{\mathbf{z}} \nabla^2 A_z$$

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Cylindrical Coordinates $(r, \phi, z)$

$$\nabla \Phi = \hat{\mathbf{r}} \frac{\partial \Phi}{\partial r} + \hat{\boldsymbol{\phi}} \frac{1}{r} \frac{\partial \Phi}{\partial \phi} + \hat{\mathbf{z}} \frac{\partial \Phi}{\partial z}$$

$$\nabla \cdot \mathbf{D} = \frac{1}{r} \frac{\partial}{\partial r} (r D_r) + \frac{1}{r} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla \times \mathbf{H} = \hat{\mathbf{r}} \left[ \frac{1}{r} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right] + \hat{\boldsymbol{\phi}} \left[ \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right] + \hat{\mathbf{z}} \left[ \frac{1}{r} \frac{\partial (r H_\phi)}{\partial r} - \frac{1}{r} \frac{\partial H_r}{\partial \phi} \right]$$

$$\nabla^2 \Phi = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

$$\nabla^2 \mathbf{A} = \hat{\mathbf{r}} \left( \nabla^2 A_r - \frac{2}{r^2} \frac{\partial A_\phi}{\partial \phi} - \frac{A_r}{r^2} \right) + \hat{\boldsymbol{\phi}} \left( \nabla^2 A_\phi + \frac{2}{r^2} \frac{\partial A_r}{\partial \phi} - \frac{A_\phi}{r^2} \right) + \hat{\mathbf{z}} (\nabla^2 A_z)$$

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Spherical Coordinates $(r, \theta, \phi)$

$$\nabla \Phi = \hat{\mathbf{r}} \frac{\partial \Phi}{\partial r} + \hat{\boldsymbol{\theta}} \frac{1}{r} \frac{\partial \Phi}{\partial \theta} + \frac{\hat{\boldsymbol{\phi}}}{r \sin \theta} \frac{\partial \Phi}{\partial \phi}$$

$$\nabla \cdot \mathbf{D} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \, D_\theta) + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{H} = \frac{\hat{\mathbf{r}}}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (H_\phi \sin \theta) - \frac{\partial H_\theta}{\partial \phi} \right] + \frac{\hat{\boldsymbol{\theta}}}{r} \left[ \frac{1}{\sin \theta} \frac{\partial H_r}{\partial \phi} - \frac{\partial}{\partial r} (r H_\phi) \right] + \frac{\hat{\boldsymbol{\phi}}}{r} \left[ \frac{\partial}{\partial r} (r H_\theta) - \frac{\partial H_r}{\partial \theta} \right]$$

$$\nabla^2 \Phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Phi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Phi}{\partial \phi^2}$$

$$\nabla^2 \mathbf{A} = \hat{\mathbf{r}} \left[ \nabla^2 A_r - \frac{2}{r^2} \left( A_r + \cot \theta \, A_\theta + \csc \theta \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_\theta}{\partial \theta} \right) \right] + \hat{\boldsymbol{\theta}} \left[ \nabla^2 A_\theta - \frac{1}{r^2} \left( \csc^2 \theta \, A_\theta - 2 \frac{\partial A_r}{\partial \theta} + 2 \cot \theta \csc \theta \frac{\partial A_\phi}{\partial \phi} \right) \right] + \hat{\boldsymbol{\phi}} \left[ \nabla^2 A_\phi - \frac{1}{r^2} \left( \csc^2 \theta \, A_\phi - 2 \csc \theta \frac{\partial A_r}{\partial \phi} - 2 \cot \theta \csc \theta \frac{\partial A_\theta}{\partial \phi} \right) \right]$$

## 9) Costanti di uso frequente

Costante dielettrica del vuoto : $\epsilon_o = 8.85\ 10^{-12}\ F/m$
Permeabilitá magnetica del vuoto : $\mu_o = 4\pi\ 10^{-7}\ H/m$
Carica dell'elettrone : $e = 1.60\ 10^{-19}\ C$
Massa dell'elettrone : $m_e = 9.1\ 10^{-31}\ kg$
Rapporto e/m dell'elettrone : $e/m = 1.76\ 10^{11}\ C/kg$
Massa del protone : $m_p = 1.67\ 10^{-27}\ kg$
Velocitá delle onde e.m. nel vuoto : $c = 3.0\ 10^{8}\ m/s$
Impedenza del vuoto : $Z_o = 376.7\ \Omega$
Costante di Planck : $h = 6.626\ 10^{-34}\ J \cdot s$
Magnetone di Bohr : $\mu_B = 9.42\ 10^{-24}\ A\ m^2$
Costante gravitazionale : $G = 6.672\ 10^{-11} m^3\ kg{-1}\ s^{-2}$
Numero di Avogadro : $N_A = 6.02252\ 10^{23}\ mol^{-1}$
Costante di Boltzmann : $k = 1.38054\ 10^{-23}\ J\ K^{-1}$
Costante dei gas : $R = 8.314\ J/(mol\ K)$
$= 1.986\ cal/(mol\ K)$
Volume di una mole(STP gas ideale) : $k = 22.414\ 10^{-3}\ m^3 mol^{-1}$
Unitá astronomica : $AU = 1.49598\ 10^{11}\ m$
Raggio(equatoriale)della terra : $R_{\oplus} = 6.378\ 10^{6}\ m$
Massa della terra : $M_{\oplus} = 5.973\ 10^{24}\ kg$
Massa del sole : $M_{\odot} = 1.989\ 10^{30}\ kg$

# Formule di Teoria dei Segnali

L. Verdoliva

## Formule di trigonometria

$$
\begin{aligned}
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \sin\beta\cos\alpha \\
\cos^2\alpha &= \frac{1+\cos 2\alpha}{2} \\
\sin^2\alpha &= \frac{1-\cos 2\alpha}{2} \\
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha \\
\cos\alpha\cos\beta &= \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \\
\sin\alpha\sin\beta &= \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \\
\sin\alpha\cos\beta &= \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]
\end{aligned}
$$

## Formule di Eulero

$$\cos\alpha = \frac{e^{j\alpha}+e^{-j\alpha}}{2} \qquad \sin\alpha = \frac{e^{j\alpha}-e^{-j\alpha}}{2j} \qquad e^{j\alpha} = \cos\alpha + j\sin\alpha$$

## Proprietà $\delta(t)$ e $\delta(n)$

$$\int_{t_1}^{t_2} x(t)\,\delta(t)\,dt = \begin{cases} x(0) & 0 \in (t_1, t_2) \\ 0 & \text{altrimenti} \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$

$$\int_{-\infty}^{+\infty} x(t)\delta(t-t_0)\,dt = x(t_0)$$

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

$$\delta(t) = \delta(-t)$$

$$\int_{-\infty}^{+\infty} x(\alpha)\delta(t-\alpha)\,d\alpha = x(t) * \delta(t) = x(t)$$

$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = \mathrm{u}(t) \quad \leftrightarrow \quad \delta(t) = \frac{d\mathrm{u}(t)}{dt}$$

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{altrimenti} \end{cases}$$

$$\sum_{k=-\infty}^{+\infty} \delta(n-k) = 1$$

$$\sum_{n=-\infty}^{+\infty} x(n)\delta(n-n_0) = x(n_0)$$

$$x(n)\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$$

$$\delta(n) = \delta(-n)$$

$$\sum_{k=-\infty}^{+\infty} x(k)\delta(n-k) = x(n) * \delta(n) = x(n)$$

$$\sum_{k=-\infty}^{n} \delta(k) = \mathrm{u}(n) \quad \leftrightarrow \quad \delta(n) = \mathrm{u}(n) - \mathrm{u}(n-1)$$

# FORMULARIO CEM

## Formule utili per esercizio (1):

$$f_{c\,TE_{10}} = \frac{c}{2a(m)} = \frac{150}{a(mm)} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2}\ [Hz]$$

$$f_{c\,TE_{01}} = \frac{c}{2b(m)} = \frac{150}{b(mm)} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{b}\right)^2}\ [Hz]$$

$$f_{c\,TE_{20}} = \frac{c}{2\pi}\cdot\sqrt{\left(\frac{2\pi}{a}\right)^2}\ [Hz]$$

⋮

$$f_{c\,TE_{nm}} = \frac{c}{2\pi}\cdot\sqrt{\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2}\ [Hz]$$

$c_0 = 3\cdot 10^8\,[m/s]$

$$\boxed{\text{Se } b \leq \frac{a}{2} \Rightarrow f_{c\,TE_{10}} < \text{banda di monomod.} < f_{c\,TE_{20}}}$$

$$Z_0^{TE} = \frac{\omega\mu_0}{\beta}\ [\Omega]\ ,\quad Y_0^{TE} = \frac{1}{Z_0^{TE}} = \frac{\beta}{\omega\mu_0}\ [\Omega^{-1}]$$

$$v_\varphi = \frac{\omega}{\beta}\ [m/s]\ ,\quad v_g = \frac{c_0^2}{v_\varphi}\ [m/s]$$

$$\beta = \sqrt{k_0^2-\left(\frac{\pi}{a}\right)^2}\ [m^{-1}]\ ,\quad k_0 = \omega\sqrt{\mu_0\varepsilon_0}\ [m^{-1}]\ ,\quad \omega = 2\pi f\ [rad/s]$$

$$k_0^2 = \omega^2\mu_0\varepsilon_0\ [m^{-2}]$$

~~Costante di attenuazione~~: $\gamma_{20} = \sqrt{\left(\frac{2\pi}{a}\right)^2-k_0^2}\ [m^{-1}]$ $\quad k_i = \omega\sqrt{\mu_0\varepsilon_0}\,n_i\ [m^{-1}]$

Perdite: $\alpha_c = \left(\frac{R_s}{a^3 b\,\beta k_0\eta}\right)\left(2b\pi^2+a^3k_0^2\right)[Np/m]\ ,\ R_s = \frac{1}{\sigma\cdot\delta}\ [\Omega]$

$$|V^+| = \sqrt{2\cdot Z_0^{TE}\cdot P_{media}}\ [V]$$

$\sigma$ = dipende dal materiale $[S/m]$

$$\delta = \frac{1}{\sqrt{\pi f\mu_0\sigma}}\ [m]$$

## Formule utili per esercizio (2) :

### CASO TE :

$$\Gamma_1^{TE} = \frac{Z_{IN} - Z_1}{Z_{IN} + Z_1} \quad [\text{numero puro}]$$

$$Z_1^{TE} = \eta_1 / \cos(\theta_1) \quad [\Omega]$$

$$Z_{IN}^{TE} = \frac{\eta_2}{\cos(\theta_2)} \cdot \frac{\eta_1/\cos(\theta_1) + j(\eta_2/\cos(\theta_2))\, t_2}{\eta_2/\cos(\theta_2) + j(\eta_1/\cos(\theta_1))\, t_2} \quad [\Omega]$$

### CASO TM :

$$\Gamma_1^{TM} = \frac{Z_{IN} - Z_1}{Z_{IN} + Z_1} \quad [\text{numero puro}]$$

$$Z_1^{TM} = \eta_1 \cos(\theta_1) \quad [\Omega]$$

$$Z_{IN}^{TM} = \eta_2 \cos(\theta_2) \cdot \frac{\eta_1 \cos(\theta_1) + j\eta_2 \cos(\theta_2)\, t_2}{\eta_2 \cos(\theta_2) + j\eta_1 \cos(\theta_1)\, t_2} \quad [\Omega]$$

### Validità generale:

Snell:

posso anche vederla come: $n_1 \operatorname{sen}(\theta_1) = \dots = n_i \operatorname{sen}(\theta_i)$ con $i = 1, \dots, n$

$$n_1 \operatorname{sen}(\theta_1) = n_2 \operatorname{sen}(\theta_2) \Rightarrow \theta_2 = \operatorname{arcsen}\left(\frac{n_1}{n_2} \operatorname{sen}(\theta_1)\right)$$

Risultato deve essere in gradi

$$K_0 = \omega \cdot \sqrt{\mu_0 \varepsilon_0} \quad [m^{-1}] \quad , \quad K_i = \omega\sqrt{\mu_0 \varepsilon_0}\, n_i \quad [m^{-1}]$$

$$n_i = \sqrt{\varepsilon_{r_i}} \quad [\text{numero puro}] \quad , \quad \beta_i = K_i \cos(\theta_i)$$

$$\omega = 2\pi f \quad [\text{rad/s}]$$

$$\lambda = \frac{2\pi}{\omega} \cdot v \quad [m]$$ → spesso $v = c_0$ $\quad c_0 = 3 \cdot 10^8 \, [m/s]$

$$t_2 = \tan(K_2 \cos(\theta_2)\, d) \quad [\text{numero puro}]$$ fare il calcolo in radianti

$T =$ ______

### Angolo di Brewster :

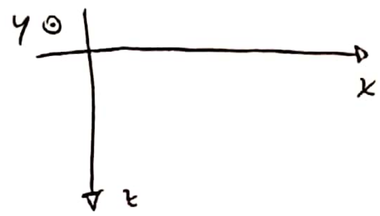
$$\theta_{inc} = \theta_B = \tan\left(\frac{n_2}{n_{inc}}\right) \iff t_2 = 0$$ , cioè se: $K_2 \cos(\theta_2) d = n\pi$

## Continuo:

Potenza trasmessa: $P_T = (1 - |\Gamma|^2) \cdot P_{inc.}$ $[W/m^2]$

Potenza media incidente: $P_{inc} = \frac{1}{2}(Re)\frac{|E_i|^2}{\eta_i}$ $[W/m]$, si calcola con il vettore di Poynting.

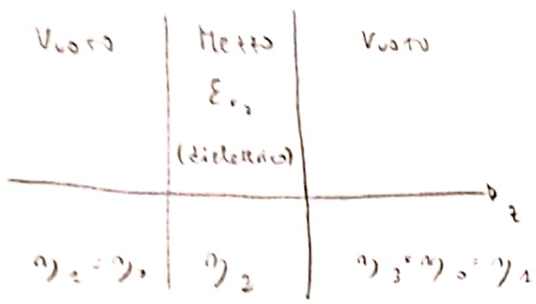
## Caso TE:




Trasverso elettrico: campo $E$ lungo $\hat{y}$

campo $H$ lungo $\hat{x}$ e $\hat{z}$.

### Situazione:




Regione 1: onda incidente e riflessa

Regione 2: dielettrico

Regione 3: onda trasmessa

(Regione 1)

$$\bar{E}_1(x,y,z) = \left[E_{inc}^+ \cdot \left(e^{-jk_1\cos(\theta_1)z} - \Gamma e^{-jk_1\cos(\theta_1)z}\right) e^{-jk_1\sin(\theta_1)x}\right]\hat{y}$$

$$H_{1x} = -\frac{1}{j\omega\mu_0}\nabla\times E_1\Big|_x = \frac{-1}{-j\omega\mu_0}\partial_z E_{y1} = \frac{-\beta_1}{\omega\mu_0}E_{1y}\,\hat{x}$$

$$H_{1z} = \frac{-1}{j\omega\mu_0}\nabla\times E_1\Big|_z = \frac{1}{-j\omega\mu_0}\partial_x E_{y1} = \frac{\beta_1}{\omega\mu_0}E_{1y}\,\hat{z}$$

(Regione 2):

$$\bar{E}_2(x,y,z) = \left[\left(E_2^+ e^{-jk_2\cos(\theta_2)z} + E_2^- e^{-jk_2\cos(\theta_2)z}\right) e^{-jk_1\sin(\theta_1)x}\right]\hat{y}$$

$$H_{2x} = -\frac{1}{j\omega\mu_0}\nabla\times\bar{E}_2\Big|_x = \text{"} \quad \text{"} = -\frac{\beta_2}{\omega\mu_0}E_{2y}\,\hat{x}$$

$$H_{2z} = -\frac{1}{j\omega\mu_0}\nabla\times\bar{E}_2\Big|_z = \text{"} \quad \text{"} = \frac{\beta_2}{\omega\mu_0}E_{2y}\,\hat{z}$$

In $z=0$ (tra 1 e 2)

(i) $E_{y_1} = E_{y_2}$

(ii) $H_{x_1} = H_{x_2}$

In $z=d$, vale:

$$E_2^+ = E_{1\,inc}^+ \left( \frac{1+\Gamma_1}{1+\Gamma_2} \right)$$ , (se mezzo 1 è il vuoto ed il mezzo 2 è il dielettrico), vale:

(iii) $E_2^+ + E_2^- = E_1^+ (1+\Gamma)$ , e

(iv) $\frac{\beta_1}{\omega\mu_0} E_1^+ (1-\Gamma) = \frac{\beta_2}{\omega\mu_0} (E_2^+ - E_2^-)$ (sempre nel caso sopra)

(Regione 3): Solo onda trasmessa

$$\bar{E}_3 = \left[ \left( E_3^+ e^{jk_1\cos(\theta_1)z} e^{-jk_1 \sin(\theta_1)x} \right) \right] \hat{y}$$ , poiché $k_1 \cos(\theta_1) = k_3 \cos(\theta_3)$, è sempre il vuoto.

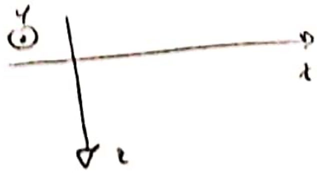
$$H_{3x} = -\frac{1}{j\omega\mu_0} \nabla \times \bar{E}_3 \Big|_x = -\frac{1}{j\omega\mu_0} \cdot -\partial_z E_{3y} \hat{x} = \frac{-\beta_1}{\omega\mu_0} E_{3y} \hat{x}$$

$$H_{3z} = \frac{-1}{j\omega\mu_0} \nabla \times \bar{E}_3 \Big|_z = -\frac{1}{j\omega\mu_0} \partial_z E_{3y} \hat{z} = \frac{\beta_1}{\omega\mu_0} E_{3y} \hat{z}$$

## CASO TM:

Trasverso magnetico : campo E lungo $\hat{x}$ e $\hat{z}$

campo H lungo $\hat{y}$




Situazione precedente

$$E_{i_{x/z}} = \frac{1}{j\omega\varepsilon} \nabla \times H \Big|_{x/z}$$

$$= \frac{\beta_z}{\omega\varepsilon} H_i^+ e \quad \hat{x}$$

$$= \frac{-\beta_x}{\omega\varepsilon} H_i^+ e \quad \hat{z}$$

## Cosa devo scrivere alla fase con $x$?

### Per snell:

$n_1 \operatorname{sen}(\vartheta_1) = n_2 \operatorname{sen}(\vartheta_2)$ , devono essere due quantità uguali

$$K_1 \operatorname{sen}(\vartheta_1) \overset{?}{=} K_2 \operatorname{sen}(\vartheta_2) .$$

Dunque, so che:

$K_1 = K_0 \cdot n_1$ e $K_2 = K_0 \cdot n_2$

Quindi avrei:

$$K_0 \cdot n_1 \cdot \operatorname{sen}(\vartheta_1) = K_0 \cdot n_2 \cdot \operatorname{sen}(\vartheta_2) .$$

A meno del fattore $K_0$, che può essere semplificato, ritrovo la legge di Snell. Pertanto le due espressioni sono uguali!

Posso scrivere:

$e^{-jK_1 \operatorname{sen}(\vartheta_1)x}$ ,

che:

$e^{-jK_2 \operatorname{sen}(\vartheta_2)x}$

# Ricevimento Morini (01/06/2020)

## Polarizzazione TE:

Quando devo trovare le espressioni dei campi, devo: moltiplicare una porzione del campo magnetico ($H_x$ e $H_z$) x tutto il campo elettrico, che si trova lungo y ($E_y$).

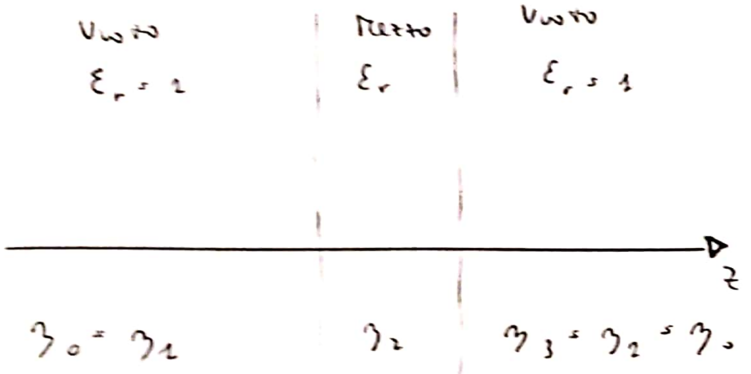
## Polarizzazione TM:

Quando devo trovare le espressioni dei campi, devo: moltiplicare una porzione del campo elettrico ($E_x$ ed $E_z$) x tutto il campo magnetico, che si trova lungo y ($H_y$).

→ Se $b \leq \frac{a}{2}$, che succede?

La banda di monomodalità è tra la freq. di taglio del $TE_{10}$ e quella del $TE_{20}$

Se io ho la situazione:




$$n_1 \operatorname{sen}(\theta_1) = n_2 \operatorname{sen}(\theta_2) = n_3 \operatorname{sen}(\theta_3)$$

Nella regione 3 (vuoto):

$$n_1 \operatorname{sen}(\theta_1) = n_3 \operatorname{sen}(\theta_3) \left(= n_2 \operatorname{sen}(\theta_2)\right)$$

$$\Downarrow$$

$$\theta_3 = \operatorname{arcsen}\left(\frac{n_1}{n_3} \operatorname{sen}(\theta_1)\right) = \theta_1 \quad \Rightarrow \theta_3 = \theta_1$$

$$\operatorname{sen}(\theta_3) = \operatorname{sen}(\theta_1)$$

$$k_3 = k_1,$$

infatti siamo sempre nel vuoto.

Scansionato con CamScanner