Onde piane in un mezzo con perdite I

$$\nabla \times \hat{\mathbf{H}} = \hat{\mathbf{j}} + j\omega\varepsilon\,\overset{\frown}{\mathbf{E}}$$

$$\nabla \times \overset{\frown}{\mathbf{E}} = -\,j\omega\mu\,\hat{\mathbf{H}}$$

Legge di Ohm:

$$\hat{\mathbf{j}} = \sigma\,\overset{\frown}{\mathbf{E}}$$

Onde piane in un mezzo con perdite II

$$\nabla \times \overset{\frown}{\mathbf{E}} = -j\omega\mu\,\hat{\mathbf{H}}$$

$$\nabla \times \hat{\mathbf{H}} = \sigma\,\overset{\frown}{\mathbf{E}} + j\omega\varepsilon\,\overset{\frown}{\mathbf{E}}$$

$$\nabla \times \overset{\frown}{\mathbf{E}} = -j\omega\mu\,\hat{\mathbf{H}}$$

$$\nabla \times \hat{\mathbf{H}} = \left[\sigma + j\omega\varepsilon\right]\overset{\frown}{\mathbf{E}}$$

Onde piane in un mezzo con perdite III

$$\nabla \times \nabla \times \overset{\frown}{\mathbf{E}} = -j\omega\mu\, \nabla \times \hat{\mathbf{H}} = -\, j\omega\mu \left[\sigma + j\omega\varepsilon\right]\overset{\frown}{\mathbf{E}}$$

$$\nabla \times \nabla \times \overset{\frown}{\mathbf{E}} = \nabla\nabla \cdot \overset{\frown}{\mathbf{E}} - \nabla^2 \overset{\frown}{\mathbf{E}}$$

$$\nabla \cdot \overset{\frown}{\mathbf{E}} = 0$$

$$\nabla^2 \overset{\frown}{\mathbf{E}} - \gamma^2 \overset{\frown}{\mathbf{E}} = 0 \qquad \gamma^2 = j\omega\mu\left[\sigma + j\omega\varepsilon\right]$$

Onde piane in un mezzo con perdite IV

$$\gamma = j\omega\sqrt{\varepsilon\mu}\left[1+\frac{\sigma}{j\omega\varepsilon}\right]^{1/2}$$

$$\gamma = j\omega\sqrt{(\varepsilon' - j\varepsilon'')\mu} = j\omega\sqrt{\mu\varepsilon'(1 - j\tan\delta)}$$

γ è la costante di fase

Se assumiamo che l'onda sia uniforme nel piano trasversale alla propagazione:

$$\partial_z^2 \hat{E}_x - \gamma^2 \hat{E}_x = 0$$

$$\hat{E}_x = E_x^+ \exp(-\gamma z) + E_x^- \exp(+\gamma z)$$

Onda piana progressiva con perdite : E

$$\gamma = \alpha + j\beta \qquad \hat{E}_x = E_x^+ \exp(-(\alpha + j\beta)z) =$$

$$\hat{E}_x = E_x^+ \exp\left(-\alpha z\right)\exp\left(-j\beta z\right)$$

$$E_x(z,t) = \operatorname{Re}\left\{\hat{E}_x \exp\left(j\omega t\right)\right\} =$$

$$= E_x^+ \exp\left(-\alpha z\right)\cos\left(\omega t - \beta z\right)$$

Onda piana progressiva con perdite : H

$$\hat{H}_y = \frac{-j}{\omega\mu}\partial_z \hat{E}_x = \frac{-j\gamma}{\omega\mu} E_x^+ \exp\left(-\gamma z\right)$$

$$H_y(z,t) = -\mathrm{Re}\left\{\frac{j(\alpha + j\beta)}{\omega\mu} E_x^+ \exp\left(-(\alpha + j\beta)z\right)\exp\left(j\omega t\right)\right\} =$$

$$H_y(z,t) = -\mathrm{Re}\left\{\frac{j\alpha - \beta}{\omega\mu} E_x^+ \exp\left(-(\alpha + j\beta)z\right)\exp\left(j\omega t\right)\right\} =$$

$$H_y(z,t) = -E_x^+ \frac{\exp(-\alpha z)}{\omega\mu} \operatorname{Re}\left\{ (j\alpha - \beta)\exp\left(j(\omega t - \beta z) \right) \right\} =$$

$$= +E_x^+ \frac{\exp(-\alpha z)}{\omega\mu} \left[\beta \cos(\omega t - \beta z) + \alpha \sin(\omega t - \beta z) \right]$$

Propagazione buon conduttore

σ molto maggiore $\omega\varepsilon$ $\quad \sigma = 5.81 \cdot 10^{7}$ S/m (Cu)

a 1 GHz $\quad \omega\varepsilon = 2\pi \cdot 10^{9} \cdot 8.854 \cdot 10^{-12} = 0.055$ S/m

$$\gamma = j\omega\sqrt{\varepsilon\mu}\left[1 + \frac{\sigma}{j\omega\varepsilon}\right]^{1/2} \approx \sqrt{j\omega\mu\sigma}$$

$$= \frac{1+j}{\sqrt{2}}\sqrt{\omega\mu\sigma} = (1+j)\sqrt{\pi f\mu\sigma}$$

Profondità di penetrazione

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\hat{E}_x = E_x^+ \exp\left(-z/\delta\right)\exp\left(-jz/\delta\right)$$

Profondità di penetrazione di Al, Cu, Au, Ag

$$f = 10 \ \text{GHz}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\delta_{Al} = 8.14 \cdot 10^{-7} \ \text{m}$$

$$\delta_{Au} = 7.86 \cdot 10^{-7} \ \text{m}$$

$$\delta_{Cu} = 6.6 \cdot 10^{-7} \ \text{m}$$

$$\delta_{Ag} = 6.4 \cdot 10^{-7} \ \text{m}$$

A 100 Hz…

$$\delta_{Al} = 8.14 \ \mathrm{mm}$$

$$\delta_{Au} = 7.86 \ \mathrm{mm}$$

$$\delta_{Cu} = 6.6 \ \mathrm{mm}$$

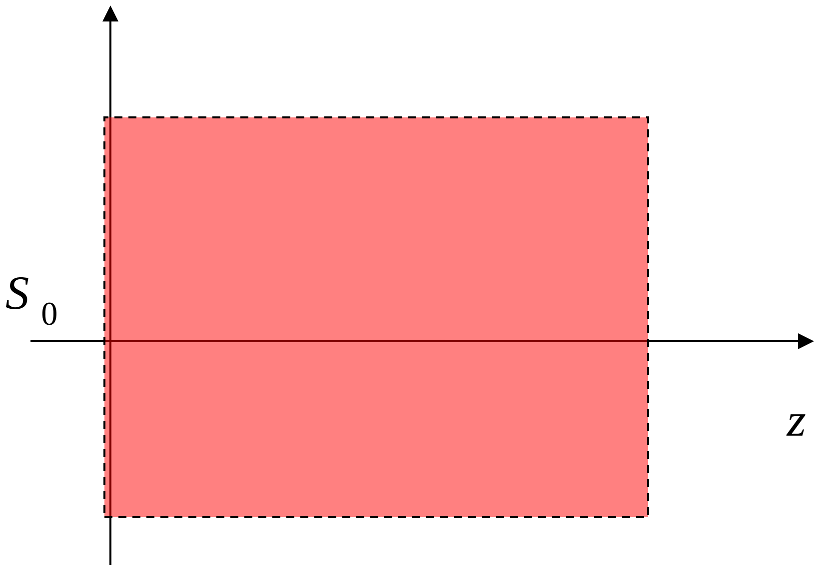
$$\delta_{Ag} = 6.4 \ \mathrm{mm}$$

Impedenza d'onda in un buon conduttore

$$\frac{j\omega\mu_0}{jk} = \frac{j\omega\mu_0}{\gamma} \Rightarrow \eta \approx (1+j)\sqrt{\frac{\omega\mu_0}{2\sigma}} = (1+j)\frac{1}{\sigma\delta}$$

L'angolo di fase dell'impedenza d'onda vale 0° per un Dielettrico senza perdite, mentre è un valore compreso tra –45° e +45 per un mezzo qualsiasi.

Potenza assorbita da un buon conduttore



$$P_{av} = \frac{1}{2}\mathrm{Re}\int_{\partial V} \hat{\mathbf{E}} \times \hat{\mathbf{H}}^{*} \cdot \mathbf{n}\, dS = \frac{1}{2}\mathrm{Re}\int_{S_0} \hat{\mathbf{E}} \times \hat{\mathbf{H}}^{*} \cdot \mathbf{u}_z\, dS =$$

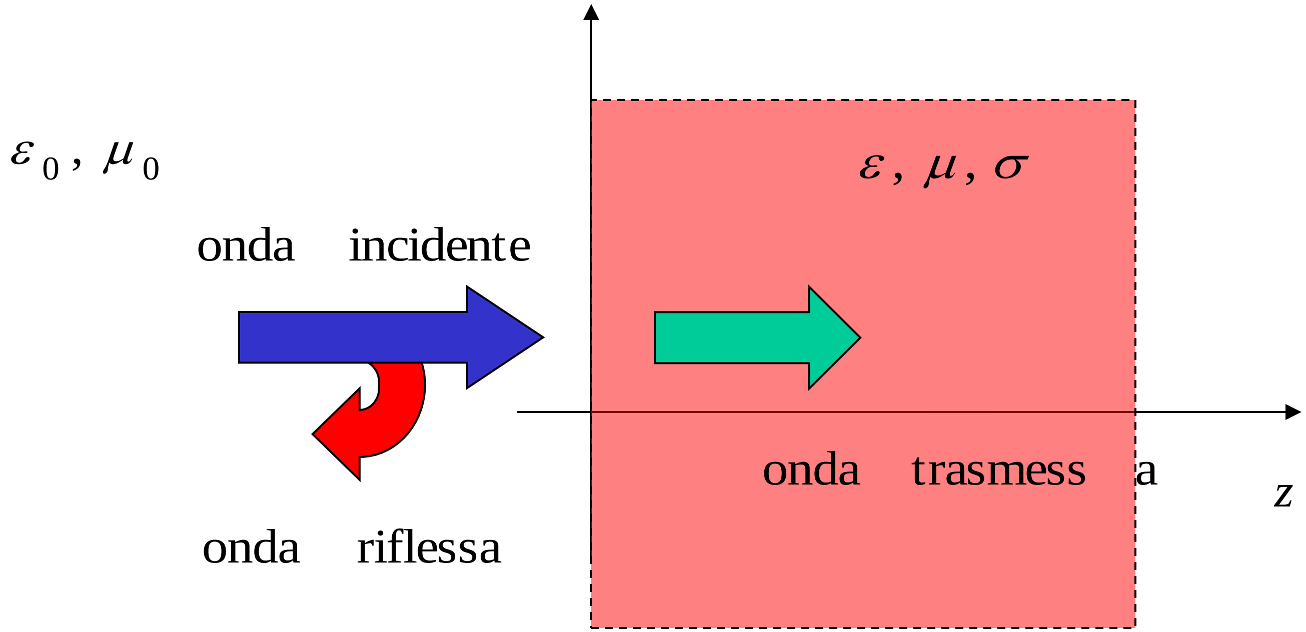
$$P_{av} = \frac{1}{2}\mathrm{Re}\left\{\eta \int_{S_0} \hat{\mathbf{H}} \cdot \hat{\mathbf{H}}^{*} dS\right\} = \frac{1}{2}\mathrm{Re}\{\eta\}\int_{S_0} \left|\hat{\mathbf{H}}\right|^2 dS = \frac{1}{2} R_s \int_{S_0} \left|\hat{\mathbf{H}}\right|^2 dS =$$

$$= \frac{1}{2} R_s \int_{S_0} \left|\hat{\mathbf{J}}_s\right|^2 dS$$

Resistenza superficiale [Ω]

$$R_s = \mathrm{Re}\left\{(1+j)\sqrt{\frac{\omega\mu}{2\sigma}}\right\} = \sqrt{\frac{\omega\mu}{2\sigma}} = \frac{1}{\sigma\delta}$$

Riflessione di un'onda piana



$$\hat{\mathbf{E}}_i = \mathbf{u}_x E_0 \exp\left(-jk_0 z\right) \qquad \hat{\mathbf{E}}_r = \mathbf{u}_x \Gamma E_0 \exp\left(jk_0 z\right)$$

$$\hat{\mathbf{H}}_i = \mathbf{u}_y \frac{E_0}{\eta_0} \exp\left(-jk_0 z\right) \qquad \hat{\mathbf{H}}_r = -\mathbf{u}_y \Gamma \frac{E_0}{\eta_0} \exp\left(jk_0 z\right)$$

Continuità dei campi tangenziali

Onda trasmessa:

$$\hat{\mathbf{E}}_t = \mathbf{u}_x T E_0 \exp(-\gamma z) \qquad \eta = \frac{j\omega\mu}{\gamma}$$

$$\hat{\mathbf{H}}_t = \mathbf{u}_y T \frac{E_0}{\eta} \exp(-\gamma z)$$

$$\gamma = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\sigma/(\omega\varepsilon)}$$

Continuità delle componenti tangenziali:

$$\hat{\mathbf{E}}_i(z=0) + \hat{\mathbf{E}}_r(z=0) = \hat{\mathbf{E}}_t(z=0) \qquad \Leftrightarrow 1 + \Gamma = T$$

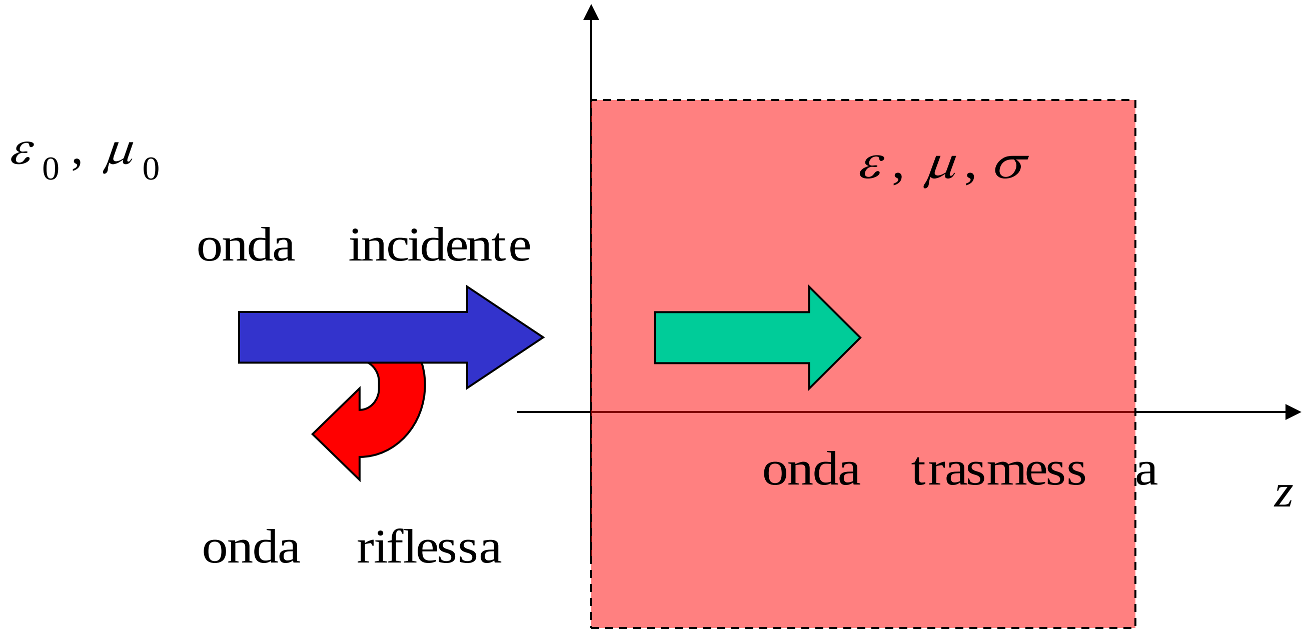
$$\hat{\mathbf{H}}_i(z=0) + \hat{\mathbf{H}}_r(z=0) = \hat{\mathbf{H}}_t(z=0) \qquad \Leftrightarrow \frac{1-\Gamma}{\eta_0} = \frac{T}{\eta}$$

Coefficienti di riflessione e trasmissione

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}$$

$$T = 1 + \Gamma = \frac{2\eta}{\eta + \eta_0}$$

Riflessione di un'onda piana



$$\hat{\mathbf{E}} = \hat{\mathbf{E}}_i + \hat{\mathbf{E}}_r = \mathbf{u}_x E_0 \left[\exp\left(-jk_0 z\right) + \Gamma \exp\left(jk_0 z\right)\right]$$

$$\hat{\mathbf{H}} = \hat{\mathbf{H}}_i + \hat{\mathbf{H}}_r = \mathbf{u}_y \frac{E_0}{\eta_0} \left[\exp\left(-jk_0 z\right) - \Gamma \exp\left(+jk_0 z\right)\right]$$

Vettore di Poynting (z < 0)

$$\hat{\mathbf{E}} \times \hat{\mathbf{H}}^* = \mathbf{u}_x \times \mathbf{u}_y E_0 \left[\exp\left(-jk_0 z\right) + \Gamma \exp\left(jk_0 z\right)\right] \cdot$$

$$\cdot \frac{E_0^*}{\eta_0} \left[\exp\left(jk_0 z\right) - \Gamma^* \exp\left(-jk_0 z\right)\right]$$

$$= \mathbf{u}_z \frac{\left|E_0\right|^2}{\eta_0} \left\{1 - \left|\Gamma\right|^2 + 2j \,\mathrm{Im}\left[\Gamma \exp\left(+2jk_0 z\right)\right]\right\}$$

Vettore di Poynting (z > 0)

$$\hat{\mathbf{E}} \times \hat{\mathbf{H}}^{*} = \mathbf{u}_x \times \mathbf{u}_y E_0 T \exp\left(-\gamma z\right) \cdot \frac{E_0^{*} T^{*}}{\eta^{*}} \exp\left(-\gamma^{*} z\right) =$$

$$= \mathbf{u}_z \frac{\left|E_0\right|^2}{\eta^{*}} \left|T\right|^2 \exp(-2\alpha z)$$

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}$$

$$T = 1 + \Gamma = \frac{2\eta}{\eta + \eta_0}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}; \quad \eta = \frac{j\omega\mu_0}{\gamma} \quad \gamma = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\sigma/(\omega\varepsilon)}$$

Assenza di perdite σ=0, α=0

$$\boxed{\gamma = j\omega\sqrt{\mu\varepsilon}} \qquad \boxed{\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}; \quad \eta = \frac{j\omega\mu}{j\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}}$$

$$\boxed{\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}} \boxed{T = 1 + \Gamma = \frac{2\eta}{\eta + \eta_0}}$$

Entrambi reali

$$(z < 0) \qquad \hat{\mathbf{E}} \times \hat{\mathbf{H}}^* = \mathbf{u}_z \frac{|E_0|^2}{\eta_0} \left\{ 1 - |\Gamma|^2 + 2j\Gamma \sin(2k_0 z) \right\}$$

$$(z > 0) \qquad \hat{\mathbf{E}} \times \hat{\mathbf{H}}^* = \mathbf{u}_z \frac{|E_0|^2}{\eta} |T|^2$$

$$(z > 0)\quad \hat{\mathbf{E}} \times \hat{\mathbf{H}}^{*} = \mathbf{u}_z \frac{\left|E_0\right|^2}{\eta}\left|\frac{2\eta}{\eta + \eta_0}\right|^2 = \mathbf{u}_z \frac{\left|E_0\right|^2}{\left(\eta + \eta_0\right)^2}\, 4\eta$$

$$(z < 0)\quad \operatorname{Re}\left\{\hat{\mathbf{E}} \times \hat{\mathbf{H}}^{*}\right\} = \mathbf{u}_z \frac{\left|E_0\right|^2}{\eta_0}\left(1 - \frac{\left(\eta - \eta_0\right)^2}{\left(\eta + \eta_0\right)^2}\right) =$$

$$= \mathbf{u}_z \frac{\left|E_0\right|^2}{\eta_0}\, \frac{4\eta\eta_0}{\left(\eta + \eta_0\right)^2}$$

Buon conduttore

$$\eta = (1+j)\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{1+j}{\sigma\delta}$$

$$\gamma = \alpha + j\beta = \frac{1+j}{\delta}$$

$$(\mathrm{z} > 0) \qquad \hat{\mathbf{E}} \times \hat{\mathbf{H}}^* = \mathbf{u}_z \frac{\left|E_0\right|^2}{\eta^*} \left|T\right|^2 \exp(-2\alpha z) =$$

$$= \mathbf{u}_z \frac{\left|E_0\right|^2}{2} \sigma\delta\,(1+j)\left|T\right|^2 \exp(-2\alpha z) =$$

$$\left|\Gamma\right|^2 = \left|\frac{(1+j)/(\sigma\delta) - \eta_0}{(1+j)/(\sigma\delta) + \eta_0}\right|^2 = \frac{\left(1/(\sigma\delta) - \eta_0\right)^2 + 1/(\sigma\delta)^2}{\left(1/(\sigma\delta) + \eta_0\right)^2 + 1/(\sigma\delta)^2}$$

$$1 - \left|\Gamma\right|^2 = \frac{\left(1/(\sigma\delta) + \eta_0\right)^2 - \left(1/(\sigma\delta) - \eta_0\right)^2}{\left(1/(\sigma\delta) + \eta_0\right)^2 + 1/(\sigma\delta)^2} =$$

$$= \frac{4\eta_0/(\sigma\delta)}{\left(1/(\sigma\delta) + \eta_0\right)^2 + 1/(\sigma\delta)^2}$$

$$|T|^2 = |1+\Gamma|^2 = \left|1 + \frac{(1+j)/(\sigma\delta) - \eta_0}{(1+j)/(\sigma\delta) + \eta_0}\right|^2 =$$

$$= \left|\frac{(1+j)/(\sigma\delta) + \eta_0 + (1+j)/(\sigma\delta) - \eta_0}{(1+j)/(\sigma\delta) + \eta_0}\right|^2 =$$

$$= \left|\frac{2(1+j)/(\sigma\delta)}{(1+j)/(\sigma\delta) + \eta_0}\right|^2 = \frac{8/(\sigma\delta)^2}{\left(1/(\sigma\delta) + \eta_0\right)^2 + 1/(\sigma\delta)^2}$$

$(z = 0^+)$

$$\hat{\mathbf{E}} \times \hat{\mathbf{H}}^* = \mathbf{u}_z \frac{\left|E_0\right|^2}{2} \sigma\delta\,(1 + j) \frac{8/(\sigma\delta)^2}{\left(1/(\sigma\delta) + \eta_0\right)^2 + 1/(\sigma\delta)^2} =$$

$$\Rightarrow \operatorname{Re}\left(\hat{\mathbf{E}} \times \hat{\mathbf{H}}^*\right) = \mathbf{u}_z \left|E_0\right|^2 \frac{4/(\sigma\delta)}{\left(1/(\sigma\delta) + \eta_0\right)^2 + 1/(\sigma\delta)^2}$$

$(z = 0^-)$

$$\operatorname{Re}\left(\hat{\mathbf{E}} \times \hat{\mathbf{H}}^*\right) = \mathbf{u}_z \frac{\left|E_0\right|^2}{\eta_0} \frac{4\eta_0/(\sigma\delta)}{\left(1/(\sigma\delta) + \eta_0\right)^2 + 1/(\sigma\delta)^2}$$

Onda piana riflessa da un conduttore

Un'onda piana incide normalmente su un semispazio riempito di rame. Se f=1GHz, si calcoli la costante di propagazione, l'impedenza, la profondità di penetrazione. Si calcolino inoltre i coefficienti di riflessione e di trasmissione.

$$\sigma = 5.83 \cdot 10^{7} \text{ S/m} \qquad \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 2.088 \cdot 10^{-6} \text{ m}$$

$$\gamma = \frac{1+j}{\delta} = (4.789 + j4.789) \cdot 10^{5} \text{ m}^{-1}$$

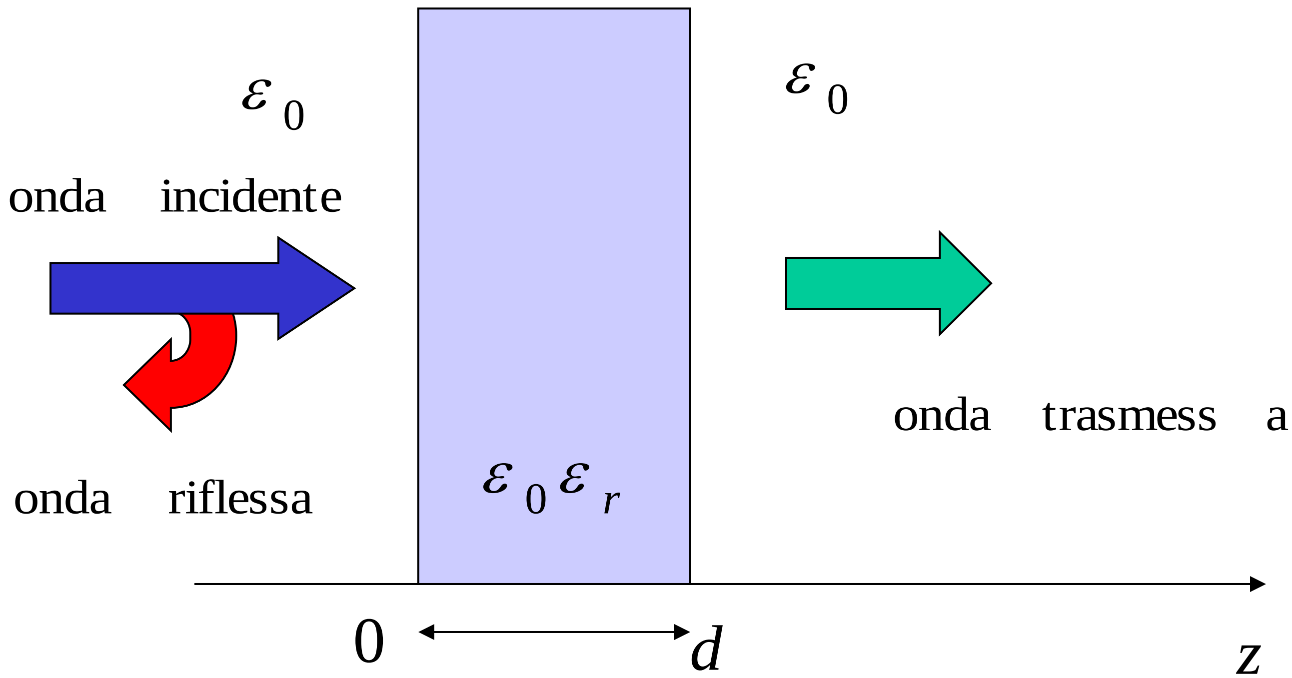
$$\eta = \frac{1+j}{\sigma\delta} = (8.239 + j8.239) \cdot 10^{-3}\ \Omega$$

Onda piana riflessa da un conduttore II

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0} = 1\angle 179.99^\circ$$

$$T = \frac{2\eta}{\eta + \eta_0} = 6.181 \times 10^{-5} \angle 45^\circ$$

Onda piana incidente su uno slab dielettrico



$$d = \frac{\lambda_0}{4\sqrt{\varepsilon_r}}$$