Practice Quiz 2: Time Value of Money (Solutions)

1. What is more valuable: $1,000 today or $1,000 in one year? Why?

Ans. $1,000 today is more valuable. Because it can earn interest over the year, the $1,000 taken today can grow into more than $1,000 in one year.

2. Using an interest rate of 7%, calculate the present value of $300 to be received in (a) one year, (b) five years, and (c) ten years.

Ans. By discounting using a rate of 7%:

(a) $PV = \dfrac{F}{(1+r)^T} = \dfrac{\$300}{(1.07)^1} = \$283.37$

(b) $PV = \dfrac{F}{(1+r)^T} = \dfrac{\$300}{(1.07)^5} = \$213.90$

(c) $PV = \dfrac{F}{(1+r)^T} = \dfrac{\$300}{(1.07)^{10}} = \$152.50$

3. John plans to purchase a one-year Treasury Bill (T-Bill). The T-Bill will pay the holder $1,000 in one year. If John requires a return of at least 2% on such an investment, what is the most he will be willing to pay for the T-Bill?

Ans. If John requires a return of 2%, he should discount the $1,000 by that 2% to find the present value:

$$PV = \frac{F}{\left(1+r\right)^T} = \frac{\$1{,}000}{\left(1.02\right)^1} = \$980.40$$

This is the most he can pay for the T-Bill and earn a return of 2%. If he pays more, his realized return will be less than 2%. If he pays less, he will realize a return of greater than 2%.

4. Mike is considering quitting his job to start a bakery, his dream work. To do so, he would need to make an investment of $80,000 today. He estimates that the bakery would generate revenues of $90,000 over the next five years and would require $20,000 in expenses. At his current job he earns $50,000. Therefore, Mike estimates that the incremental cash flows from opening the bakery would be $20,000 per year for the next five years. Calculate the NPV of the business using a discount rate of 15%. Should Mike quit his job and start the bakery?

Ans. Using a financial calculator, the NPV of this project can easily be found:

Cash Flow Worksheet	
CF_0	-$80,000
C01	$20,000
F01	5
I	15%
NPV=	-$12,956.90

Because the NPV of starting the bakery is negative, Mike should not quit his job and start the bakery.

Note that the NPV can also be found using the annuity formula:

$$NPV = -\$80,000 + \$20,000\left(\frac{1}{0.15} - \frac{1}{0.15*1.15^5}\right) = -\$12,956.90$$

Or by discounting each cash flow:

$$NPV = -\$80,000 + \frac{\$20,000}{1.15} + \frac{\$20,000}{1.15^2} + \frac{\$20,000}{1.15^3} + \frac{\$20,000}{1.15^4} + \frac{\$20,000}{1.15^5}$$
$$= -\$12,956.90$$

5. Calculate the IRR for the project described in problem 4. If Mike requires a return of 15% on the business, should he start the bakery?

Ans. Using a financial calculator:

Cash Flow Worksheet	
CF_0	-$80,000
C01	$20,000
F01	5
IRR=	7.93%

Because this is less than the 15% required return, Mike should not start the bakery. This is consistent with the conclusion reached in problem 4.

6. If Mike instead invests an additional $5,000 in new equipment and upgrades for the bakery each year, the bakery will remain operational and generate net cash flows of $15,000 into perpetuity. Given the same initial investment of $80,000 and discount rate of 15%, calculate the NPV of opening the bakery. Should Mike quit his job and start the bakery?

Ans. The NPV can be found using the perpetuity formula:

$$NPV = -\$80{,}000 + \frac{\$15{,}000}{0.15} = \$20{,}000$$

In this case the NPV is positive, so Mike should open the bakery.

Note: Although Mike will not be able to operate the bakery into perpetuity, if the business is arranged such that it can continue to operate, he can sell the business at retirement and realize the present value of the perpetual cash flows. Therefore, valuing the business as a perpetuity is acceptable.

7. Bobby buys a scratch-off ticket every day and today he hits it big with a $100,000 winning ticket. But when he turns in his ticket, he's informed of the fine print that states the $100,000 is payable in annual installments of $10,000 per year over the next 10 years. If he wants a lump sum today, he will only get $85,000. If the interest rate is 5%, is it better for Bobby to take the ten $10,000 annual installments or the $85,000 lump sum?

Ans. To compare the two options, find the present value of the annual installments:

Time Value of Money	
P/Y	1
PMT	$10,000
N	10
FV	$0
I/Y	5%
PV=	-$77,217.35

So the present value of ten annual payments of $10,000 is $77,217. Because this is less than the $85,000, it would be better for Bobby to take the $85,000. If he wants the money later, he can invest it at 5% and withdraw more than $10,000 a year for 10 years.

This present value can be found with the following keystrokes on a _TI BA II Plus_:

To	Press	Display
Enter annual payment	10000, [PMT]	PMT = 10,000.00
Set the number of payments	10, [N]	N = 10.00
Set the final value to zero	0, [FV]	FV = 0.00
Enter the interest rate	5, [I/Y]	I/Y = 5.00
Compute the present value	[CPT], [PV]	PV = -77,217.35

Note: The payments per year (P/Y) should be set to 1. If not, you must set it.

8. Jordan is considering starting a Widget manufacturing company. The initial investment in tools will cost $1,500,000. Jordan estimates that he can earn profits of $50,000 a year for the first 2 years while he establishes the business, $100,000 a year for the next 3 years, and then $200,000 a year for 10 more years before the tools wear out. The project is risky, because Jordan is not certain he will earn these profits. To compensate for this risk, Jordan requires a high return of 25% on such an investment. What is the net present value (NPV) of this business project? Should Jordan undertake the investment?

Ans. Even though the $2,400,000 will earn from the business is enough to make back his $1,500,000 investment, comparing the undiscounted cash flows is incorrect because it ignores the time value of money. The NPV must be computed to evaluate this project. The NPV can be calculated using the Cash Flow Worksheet:

Cash Flow Worksheet	
CF_0	-$1,500,000
C01	$50,000
F01	2
C02	$100,000
F02	3
C03	$200,000
F03	10
I	25%
NPV=	-$1,069,076

Because the NPV of the project is negative, Jordan should not undertake this investment. It's like throwing away $1,000,000!

The keystrokes to calculate the NPV with a *TI BA II Plus* are:

To	Press	Display
Open cash flow worksheet	[CF]	CF0 = 0.00
Clear cash flows	[2nd], [CLR WORK]	CF0 = 0.00
Enter today's cash flow	1500000, [+|-], [ENTER]	CF0 = -1,500,000.00
Enter first cash flow	[DOWN], 50000, [ENTER]	C01 = 50,000.00
Enter first cash flow frequency	[DOWN], 2, [ENTER]	F01 = 2.00
Enter second cash flow	[DOWN], 100000, [ENTER]	C02 = 100,000.00
Enter second cash flow frequency	[DOWN], 3, [ENTER]	F02 = 3.00
Enter third cash flow	[DOWN], 200000, [ENTER]	C03 = 200,000.00
Enter third cash flow frequency	[DOWN], 10, [ENTER]	F03 = 10.00
Open NPV worksheet	[NPV]	I = 0.00
Enter interest rate	25, [ENTER]	I = 25.00
Move to NPV	[DOWN]	NPV = 0.00
Compute NPV	[CPT]	NPV = -1,069,076.00

9. Alice is a young professional and is considering a certification program for her field. The certification program is a self-study program that will take three years. The cost is $1,000 per year. Further, Alice estimates that it will take 200 hours of study time per year, and since she values her time at her wage of $30 per hour, she considers the $6,000 of lost time per year a cost of the program. On average, professionals with the certification make $5,000 more per year. Assuming a cost of $7,000 per year for the next three years and an increase in income of $5,000 for the following 30 years, what is the return on this certification? If Alice requires a return of 10%, should she pursue the certification?

Ans. The IRR of the certification is:

Cash Flow Worksheet	
CF$_0$	$0
C01	-$7,000
F01	3
C02	$5,000
F02	30
IRR=	19.60%

Because the 19.6% implied return on the certification exceeds Alice's benchmark rate of 10%, she should pursue the certification.

The keystrokes on a *TI BA Plus* are:

To	Press	Display
Open cash flow worksheet	[CF]	CF0 = 0.00
Clear cash flows	[2nd], [CLR WORK]	CF0 = 0.00
Enter first cash flow	[DOWN], 7000, [+|-], [ENTER]	C01 = -7,000.00
Enter first cash flow frequency	[DOWN], 3, [ENTER]	F01 = 3.00
Enter second cash flow	[DOWN], 5000, [ENTER]	C02 = 5,000.00
Enter second cash flow frequency	[DOWN], 30, [ENTER]	F02 = 30.00
Open IRR worksheet	[IRR]	IRR = 0.00
Compute IRR	[CPT]	IRR = 19.60