Application of Deep Q-Learning techniques on a GridWorld environment

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Overview

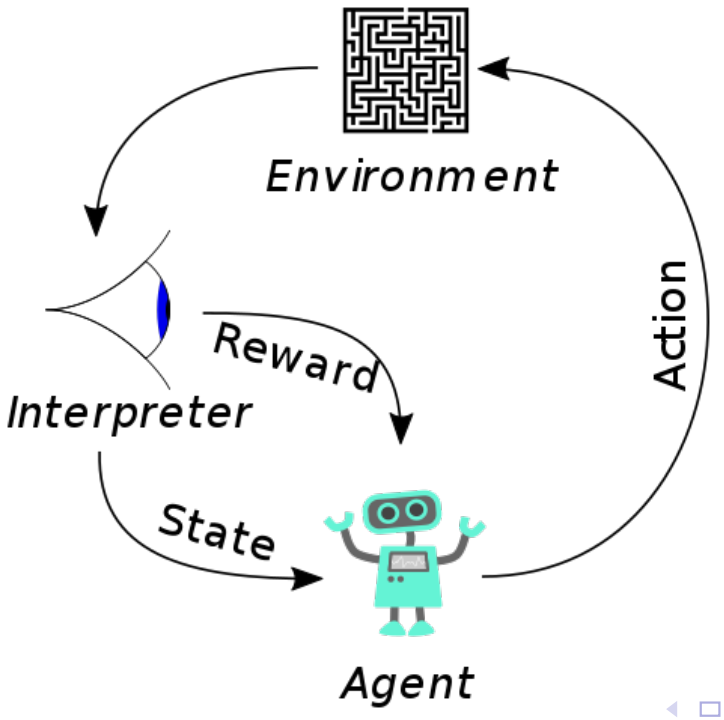
Paradigms of Machine Learning

- **Unsupervised learning**: learning without annotations;
  e.g. K-means, Mean Shift, GMM...
- **Supervised learning**: learning with annotations:
  e.g. classification, regression with SVM, Decision Trees, Neural Networks...
- **Reinforcement learning**: learning *policies for maximizing rewards*:
  - Q-Learning
  - Value iteration
  - Policy iteration

Reinforcement Learning

An environment $\mathcal{E}$ and the interaction with an agent modeled as a dynamical system: $s_{t+1} = f(s_t, a_t, \xi_t)$.
The purpose of the agent is to **maximize** the expected reward.

$$\mathcal{V} = \mathbb{E}_{\xi}\{(\sum_{t=0}^{T-1} r(t, s_t, a_t)) + r(s_T, t)\} \tag{1}$$



Reinforcement Learning

Each of the three strategies is designed to obey a fundamental equation, in reinforcement learning, which is the **Bellman equation of the dynamic programming**.

Theorem (Bellman equation of the dynamic programming at infinite horizon)

$R_t = \alpha^t r(s_t, a_t)$
$\mathcal{V} = \sum_{t=0}^{\infty} R_t$ *is limited if* r *is limited*
$\alpha \in (0, 1)$

$$\mathcal{V}^o(s_t, t) = \max_{a_t} \left\{ r(s_t, a_t) + \alpha \mathcal{V}^o(s_{t+1}, t+1) \right\} \tag{2}$$

$$\gamma^o(s_t, t) = \arg\max_{a_t} \left\{ \mathcal{V}^o(s_t, t) \right\}$$

Q-Learning

We can define the **action-value function** $Q(s, a)$. The Q function is the result of **applying** the action a while in the state s, getting the reward, and adding the discounted **expected reward** from the next state s_{t+1} obtained by applying the optimal policy $\gamma^o(s_{t+1}, t+1)$.

$$Q(s_t, a_t) = r(s_t, a_t) + \alpha \mathcal{V}^o(s_{t+1}, t+1) \tag{3}$$

Theorem ($Q^\star$ satisfies the Bellman equation)

$Q^\star(s_t, a_t) = \max_a Q(s_t, a)$
$\mathcal{V}^o(s_t, t) = Q^\star(s_t, a_t)$ *by definition*
$\implies Q^\star(s_t, a_t) = r(s_t, a_t) + \alpha \max_{a'} Q(s_{t+1}, a_{t+1})$

Therefore, the optimal policy $\gamma^o(s_0, 0)$ is simply defined by **selecting at each time-step** the action with **higher Q-score**.
Issue: **curse of dimensionality**! Complex to solve when state space $\mathbb{X}$ or action space $\mathbb{U}$ are large!

Approximating the Q-function

To counter the course of dimensionality, we can think to **approximate** the Q-function with a Neural Network, which is a **universal function approximator**, parametrized with a set of weights and biases θ.

$$Q(s, a) \approx Q(s, a|\theta) \tag{4}$$

It is trained with a **regression** over the Bellman equation.

$$\begin{aligned} y_i &= r + \alpha \max_{a'} Q(s', a'|\theta_{i-1}) \\ L_i(\theta_i) &= (y_i - Q(s, a|\theta_i))^2 \end{aligned} \tag{5}$$

Note that this method is **model-free**, *i.e.* it has no prior knowledge/estimation of the environment $\mathcal{E}$, but it directly observe its "external" state.

Replay Memory

At each time-step, we collect a tuple from the experience of the agent: $e_t = (s_t, a_t, s_{t+1}, r_t)$. We mantain these experiences in a pool, which we refer as **replay memory**. Then, we are abilitated to sample mini-batches of tuples from this data-structure.
This is efficient for two reasons:

1. it allows to build minibatches of **indipendent samples**, which are mandatory for a correct estimation of the gradient via SGD;
2. it is **computationally-efficient**, since we can leverage parallel-computing hardware and optimized software.

Exploration-exploitation trade-off

During the first stages of training, the parameters are close to being randomly initialized, then the output Q-scores have really few sense, then choosing the *greedy* action hardly is the best option. Before achieving a meaningful approximation, with a probability of ϵ we select a random action ($1 - \epsilon$ selects the greedy action) during the training. As the agent keep accumulating experience and training, ϵ will tend to decrease, letting the agent improving the learned policies. We refer to this mechanism as **exploration-exploitation** trade-off: in my experiments, the value is initialized at 0.9, and linearly decays to 0.1 at the 25% of completion of the training.

This technique makes the Deep Q-Learning **off-policy**, because it does not learn by directly sampling from a policy (a sequence of steps), but rather training on different actions, even randomly sampled.

DQL training algorithm

Algorithm 1 Deep Q-Learning algorithm

Initialize ReplayMemory $\mathcal{D}$ with capacity N
Initialize DQN $Q(x, a|\theta)$ with random weights θ_0
for episode $1, M$ **do**
  Initialize sequence $s_0 = \{x_1\}$
  for $t = 1, T$ **do**
    With probability ϵ_{ep} select the random action a_t
    otherwise select $a_t = \max_a Q(x_t, a|\theta)$
    Execute a_t and observe reward r_t and state change x_{t+1}
    set sequence $s_{t+1} = (x_t, a_t, x_{t+1}, r_t)$
    store sequence s_{t+1} in $\mathcal{D}$
    sample a minibatch of B transitions $s_j = (x_j, a_j, x_{j+1}, r_j)$
    set $y_j = \begin{cases} r_t \text{ if } x_t \text{ is terminal} \\ r_t + \gamma \max_{a'} Q(x_{t+1}, a'|\theta) \text{ otherwise} \end{cases}$
    Compute $L(\theta) = \sum_j^B \frac{1}{B}(y_j - Q(x_j, a_j|\theta))^2$
    Perform a gradient descent update based on $\nabla_\theta L\theta$
  end for
end for

Double Deep Q-Learning

- Sometimes the model learn unrealistically high action values because it includes a maximization step over estimated action values, which tends to prefer overestimated to underestimated values.
- We can decopule the **training network** from the **target values network** y_i (*i.e.* into two network with parameters θ and θ^t). If we expand the equation, then we obtain (in a deterministic environment):

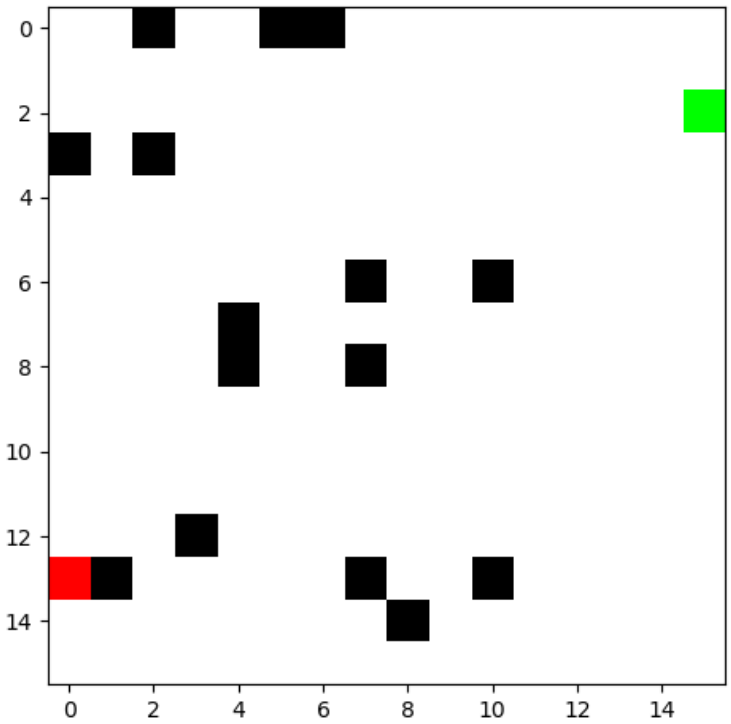
$$\begin{aligned} y_i = r + \alpha \max_{a'} Q(s', a'|\theta) = \\ r + \alpha Q(s', \arg\max_{a} Q(s', a|\theta)|\theta) \end{aligned} \tag{6}$$

The Eq. 6 becomes

$$r + \alpha Q(s', \arg\max_{a} Q(s, a|\theta)|\theta^t) \tag{7}$$

During the training, the roles of the two Q-network will be switched after a number of steps.

The *GridWorld* game



- The state is a tensor of dimension $16 \times 16 \times 3$;
- The agent can move the green square at each step the green square in one of the four direction (up, right, down, left);
- The goal is to avoid the obstacles (black cells) and reach the arrival point (red square).
- There are 16×16 reachable position for the agent, multiplied by all the possible start and finish combinations which are more than ~ 60. The number of possible states then is over 15360.

Environment reward system

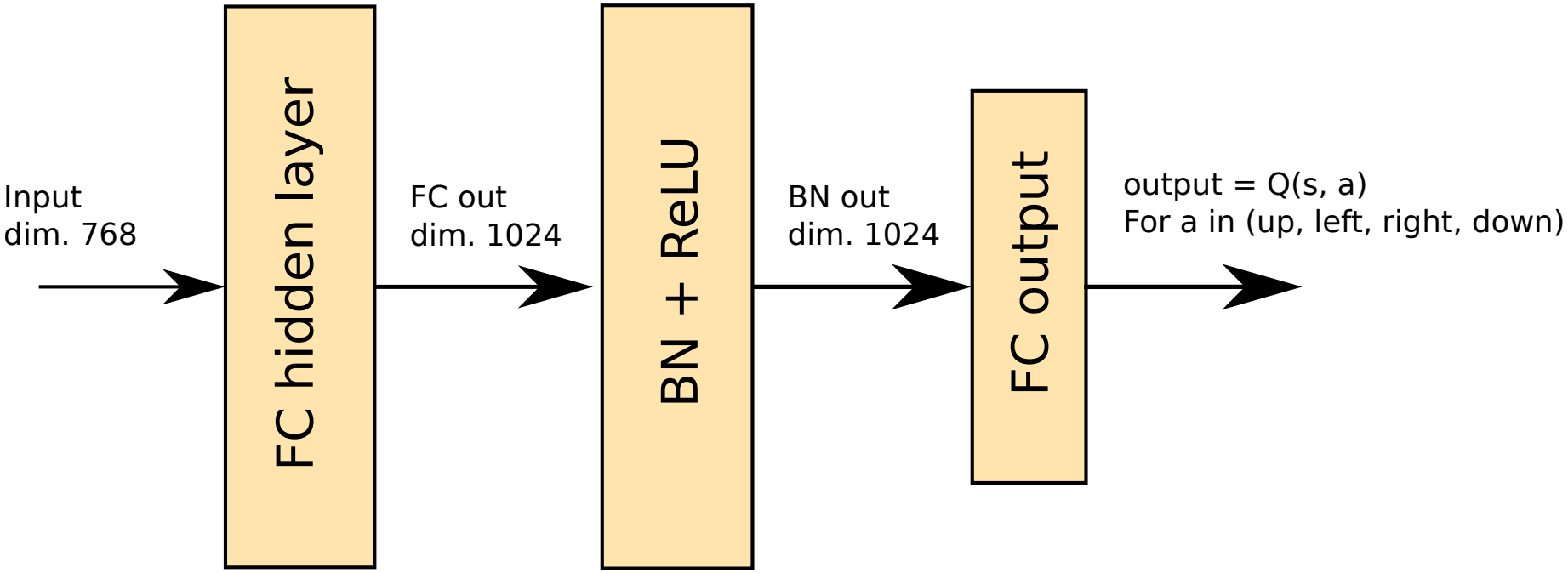
The reward system works as follows:

- $r_t = 1$ If the agent gets the green closer to the ending square but in a white cell;
- $r_t = -1$ If the agent moves the green square upon an obstacle;
- $r_t = -1$ If the agent enters again in a cell already visited (for discouraging loops) **or** tries a move towards a wall;
- $r_t = 2$ If the agent can reach the red square;
- $r_t = 0$ If the agent gets farther from the objective, but in a white cell not visited yet.

The game terminates when the agent reach the green square; otherwise we adopted an early exit strategy, meaning when the game cumulative reward reaches a negative score of -500 the game is also terminated.

Network architecture

We used a MLP architecture with 1 hidden layer and *Batch Normalization*;



Training hyper-parameters

- The network is trained for 500 episodes;
- Each episode, the network is updated 1000 times;
- The samples drawn from the Replay Memory in mini-batches of 32
- The discount factor is fixed at $\alpha = 0.85$.
- The exploration rate ϵ starts at 0.9, linearly decaying until the quarter-way episode until it reaches value $\epsilon = 0.1$.
- The employed SGD optimizer is Adam, with *learning rate* $\eta = 10^{-4}$ and *weight decay* (L2 regularization) $= 10^{-6}$.

  Before computing the action-value function, the state is processed as follows: it is flattened and normalized to fit in the range $[-1, 1]$ with the processing $s_{normalized} = (s - 0.5) * 2$.
- When using the Double-Q Learning, the network are swapped every 5 episodes.

Evaluating the model

After the agent plays 1,000 matches of the game, we evaluate our agent in terms of:

- The number of matches terminated with a **positive rate**;
- The number of matches terminated on the arrival square (**Arrivals rate**);
- The **average total reward** and **average positive reward** of all the matches. Additionally, this results are sided by the result reached by an "oracle agent", computed as $d(start, finish) + 1$, where $d(*, *)$ is the *Manhattan* distance between two cells.

Method	Pos. rate (%)	Arr. rate (%)	Avg. final reward	Avg. oracle reward
DQL	96.0	99.8	18.06	24.0 ± 0.05
Double DQL	94.5	99.5	14.65	24.0 ± 0.05

Reward per episode curves

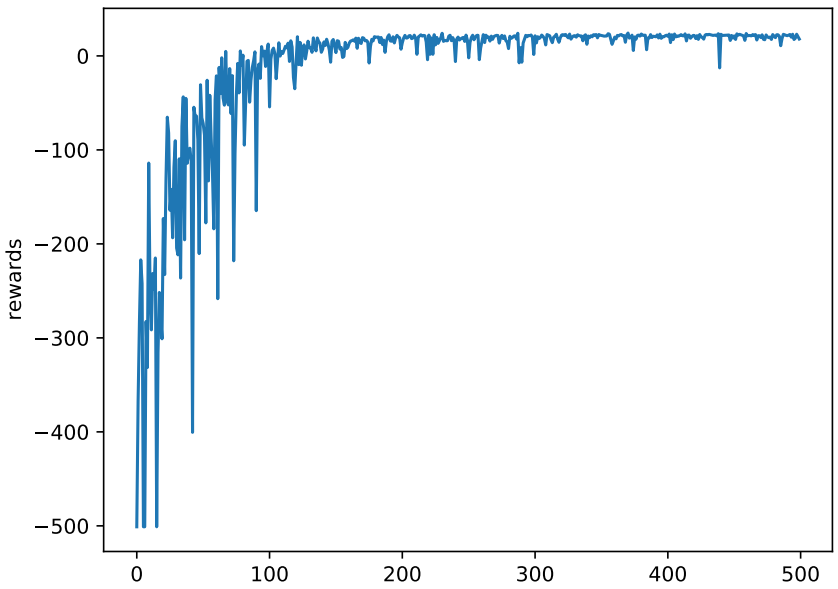


Figure: Reward curve of DQL algorithm.

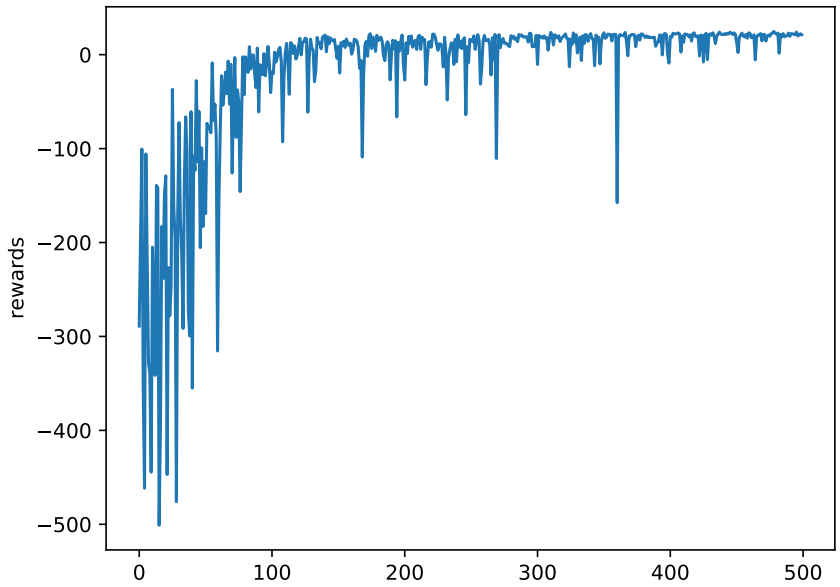


Figure: Reward curve of DoubleDQL algorithm.

Loss curves

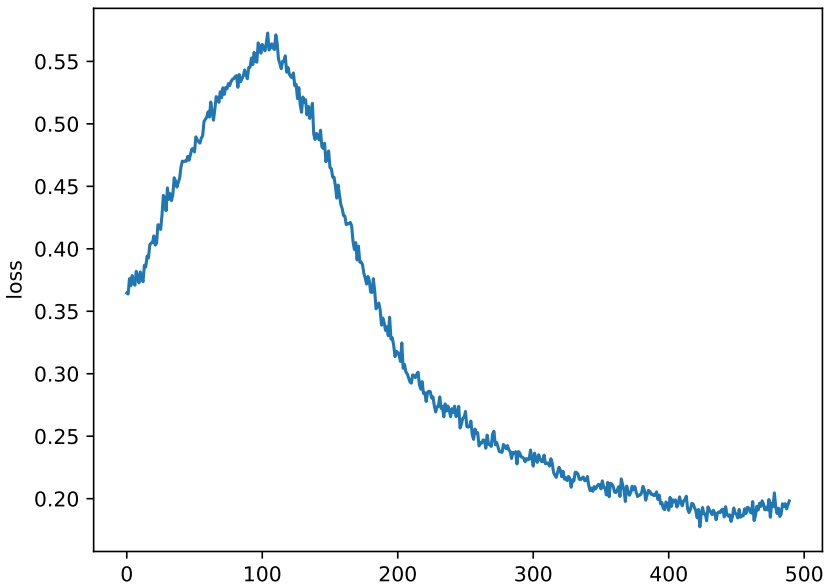


Figure: Loss of DQL algorithm.

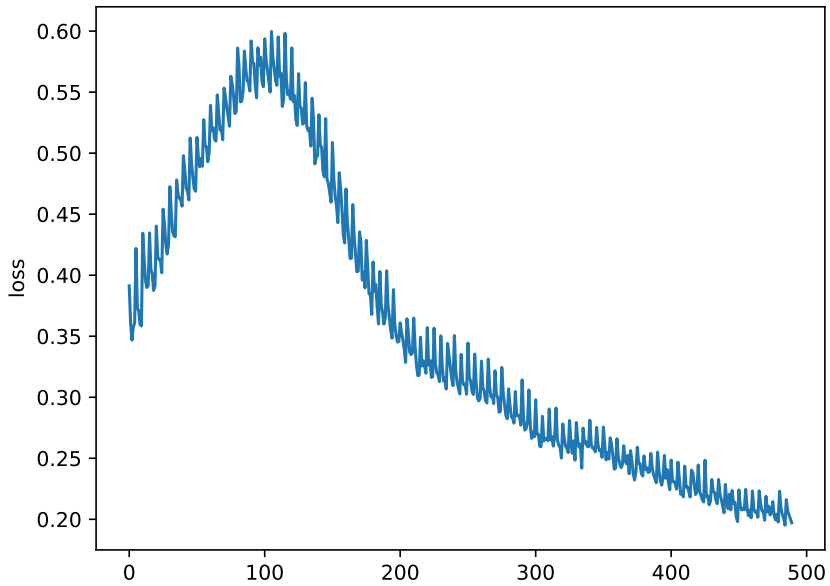


Figure: Loss curve of DoubleDQL algorithm.

References

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Thanks for listening!