Belief Change
From 1985 to present days

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Abstract

The 1985 paper by Carlos Alchourrón (1931–1996), Peter Gärdenfors, and David Makinson (AGM), “On the Logic of Theory Change: Partial Meet Contraction and Revision Functions” was the starting-point of a large and rapidly growing literature that employs formal models in the investigation of changes in belief states and databases.

Abstract

In this lecture, the first 35 years of this development are summarized. The topics covered include equivalent characterizations of AGM operations, extended representations of the belief states, change operators not included in the original framework, iterated change, applications of the model, its connections with other formal frameworks, computatibility of AGM operations, and criticism of the model.

The Problem

Belief Revision: An example

(Gärdenfors & Rott 1995)

Beliefs:

- The bird caught in the trap is a swan
- The bird caught in the trap comes from Sweden
- Sweden is part of Europe
- All European swans are white

Consequences:

- The bird caught in the trap is white

New information:

- The bird caught in the trap is black

Which sentence(s) would you give up?

Belief Revision: An example

Some Conclusions:

- Consistency
- Minimal Change
- Logic is not enough to make a decision

Problem arises in several areas …

Databases: New entry can be inconsistent with the database. Minimal ways to erase data.

Robotics: Sensor information can be inconsistent with plans.

Diagnosis: Device behaviour inconsistent with device description

Problem arises in several areas …

Argumentation: The construction of argument may implies changes of beliefs in agents

Ontologies: an ontology will be expected to evolve, either as domain information is corrected and refined, or in response to a change in the underlying domain.

History

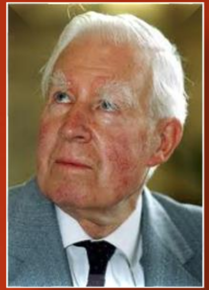
The Beginning

PHILOSOPHY

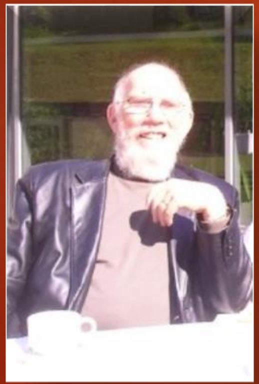
Philosophy


Jaakko Hintikka

- 1962 “Knowledge and Belief: An Introduction to the Logic of the Two Notions”


Henrik von Wright

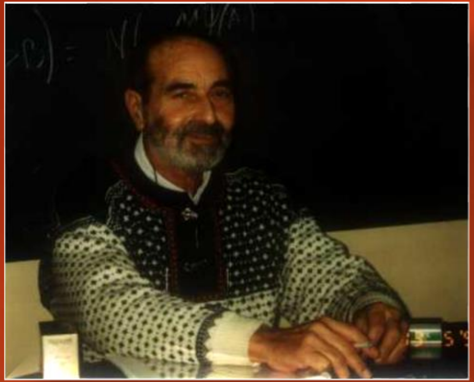
- 1971, “Explanation and understanding”


Isaac Levi

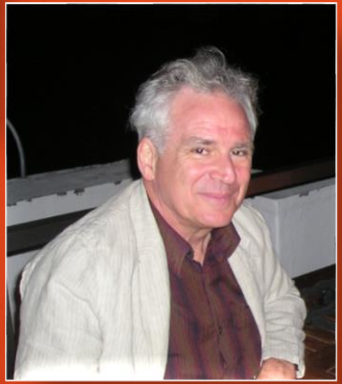
- 1977, “Subjunctives, Dispositions and Chances”
- 1977 “Rational Conceptual Change”


William Harper

Philosophy



- Deontic Logic
- Philosophy of law

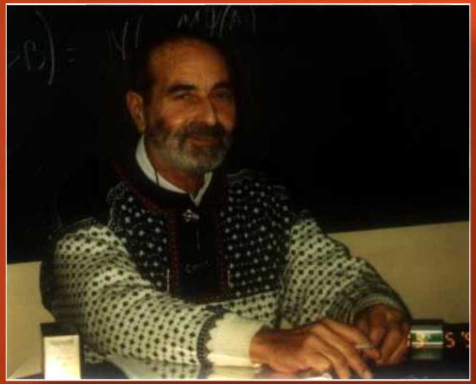


- Epistemology
- Cognitive Science

- Modal Logic
- Deontic Logic

Philosophy: Alchourrón and Makinson

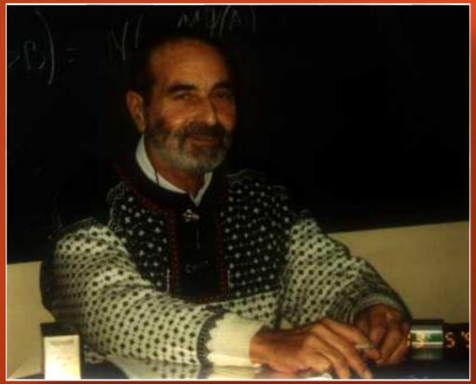
- To analyse the logical structure of the derogation procedure of a norm contained in a legal code.
- To find the general principles that any derogation should satisfy.
- To define a family of all the possible derogations.



“Hierarchies of Regulations and their Logic” 1981

Philosophy: Alchourrón and Makinson

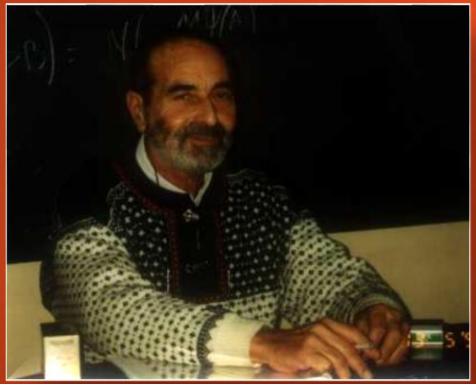
- Given a code A, create a partial order between the norms of A and induce an order on the set of parts of A.
  The maximal sets of A that did not involve the standard were called "remainders"



"Hierarchies of Regulations and their Logic" 1981

Philosophy: Alchourrón and Makinson

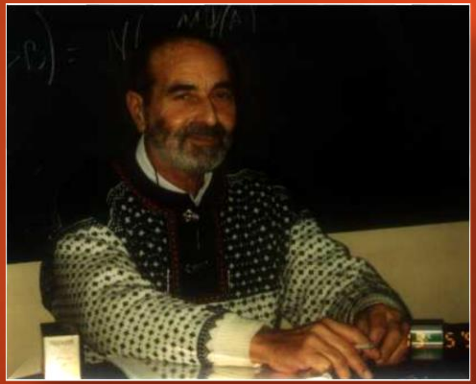
- The problem was not limited only to a set of norms.
  - The set A might be an arbitrary set of formulae and the problem now was how to eliminate one of the formulae or one of the consequence of the set.



"On the Logic of Theory Change:
Contraction functions and their associated Revision functions", 1982

Philosophy: Alchourrón and Makinson

- In the paper two different ways to contract a theory by means of remainder sets was analyzed.
- Maxichoice and Full Meet

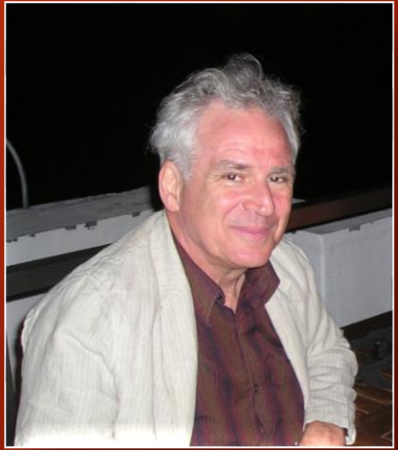


"On the Logic of Theory Change:
Contraction functions and their associated Revision functions", 1982

Philosophy: Gärdenfors

- He was looking for a model for Explanations
- Gärdenfors thought that Explanations can be expressed as different types of conditional sentences
- Gärdenfors receives an important influence from the philosophers Levi and Harper, leading him to make a thorough study of epistemic conditionals.

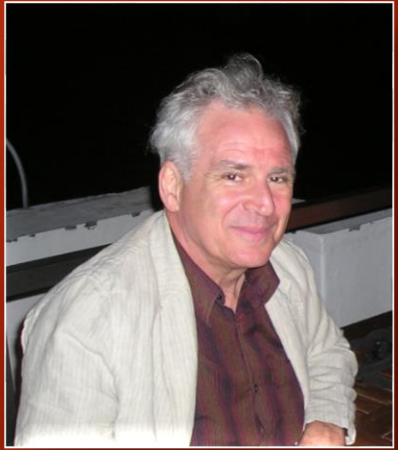
"Conditional and Change of Belief" , 1978
"A pragmatic approach to explanations", 1980

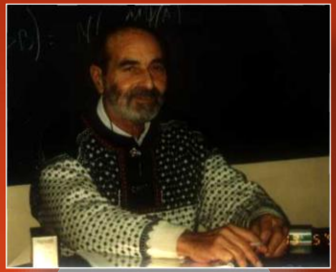


Philosophy: Gärdenfors

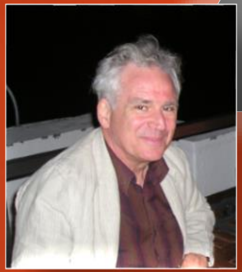
- He looking for a semantic for the epistemic conditionals.
- This semantic must be based on belief states and belief changes.

"An epistemic approach to conditionals", 1981
"Rules for rational changes of beliefs" 1982





AGM



1985

"On the Logic of Theory Change:
Partial Meet Contractions and Revision Functions",
1985 – Journal of Symbolic Logic

The Beginning

BELIEF REVISION IN

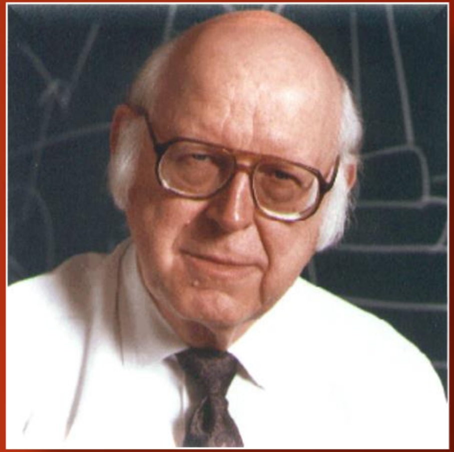
ARTIFICIAL INTELLIGENCE

The AI crisis in the ´80

- Allen Newell pointed out three indicators:

"A first indicator comes from our continually giving to representation a somewhat magical role.

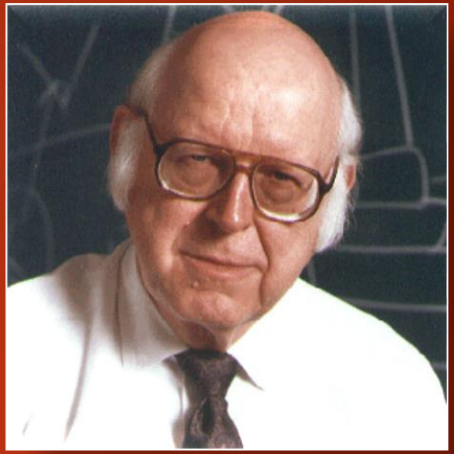
What is indicative of underlying difficulties is our inclination to treat representation like a homunculus, as the locus of real intelligence".



"The Knowledge Level" 1981

The AI crisis in the ´80

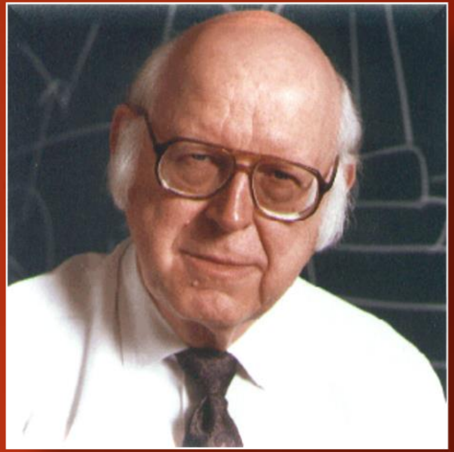
"A second indicator is the great theorem-proving controversy of the late sixties and early seventies. Everyone in AI has some knowledge of it, no doubt, for its residue is still very much with us. It needs only brief recounting."



"The Knowledge Level" 1981

The AI crisis in the ´80

The results of a questionnaire promoted in 79/80 by Brachman & Smith which was sent to the AI community



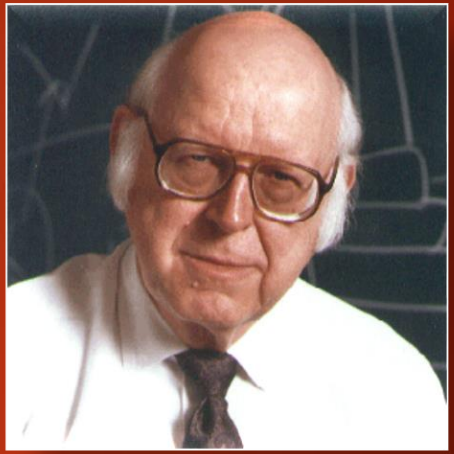
"The main result was overwhelming diversity -a *veritable jungle of opinions. There is no consensus on any question of substance."*

"As one [of the respondents] said, "Standard practice of representation of knowledge is the scandal of AI."

"The Knowledge Level" 1981

The AI crisis in the ´80

- *Knowledge Representation and Reasoning* (KRR) must be a priority in the AI agenda.
- He postulates the existence of a “Knowledge Level”



“…there exists a distinct computer system level, lying immediately above the symbol level, which is characterized by knowledge as the medium and the principle of rationality as the law of behavior.”

“The Knowledge Level” 1981

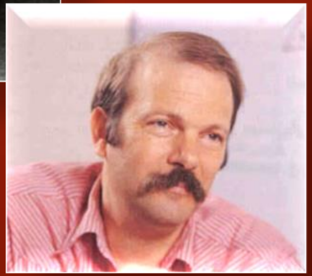
KRR

- Newell's work had an enormous influence on AI researchers: Brachman, Levesque, Moore, Halpern, Moses, Lifschitz,Vardi, Fagin, Ullman, Shapiro, Borgida, Winslett, etc.

Looking for a model in AI

- *"On the semantics of updates in databases"2nd ACM SIGACT-SIGMOD symposium on Principles of database systems Georgia   Março 21 - 23, 1983. Fagin, Ullman, Vardi*

"The ability of the database user to modify the content of the database, the so-called update operation, is fundamental to all database management systems".

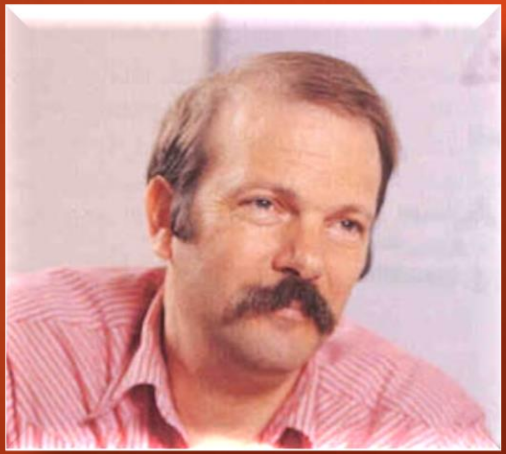


"First we consider the problem of updating arbitrary theories by inserting into them or deleting from them arbitrary sentences"

"when replacing an old theory by a new one we wish to minimize the change in the theory"

TARK (Theoretical Aspects of Rationality and Knowledge) – 1986 - ....

- Originally planned as a little workshop.
- Attended 40 researchers and other 250 integrate the *email list*.



"…included computer scientists, mathematicians, philosophers and linguists" […] "given the evident interest in the area by groups so diverse, it seemed appropriate a conference, particularly one that could increase the knowledge of the workers of one field about the work developed in other fields"

Moshe Vardi

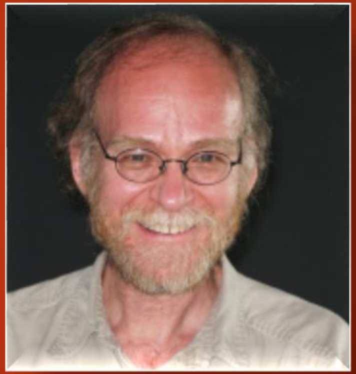
Looking for a model in AI

Reasoning About Knowledge: An Overview
Joseph Halpern

TARK 86 (March 19-22 California)

Keynote

"Most of the work discussed above has implicitly or explicitly assumed that the messages received are consistent. The situation gets much more complicated if messages may be inconsistent."



"This quickly leads into a whole complex of issues involving ***belief revision*** *and reasoning in the presence of inconsistency.*

Although ***I won't attempt to open this can of worms here****, these are issues that must eventually be considered in designing a knowledge base"*

http://www.tark.org/proceedings/tark_mar19_86/proceedings.html

- In 1988, Gärdenfors e Makinson presented the AGM model on TARK 88

- The can was opened …

… let’s go to see inside.

The AGM model

AGM Model

Belief Set: Set of sentences closed under logical consequence Cn.

Cn satisfies:

inclusion ($X \subseteq Cn(X)$)
idempotence ($Cn(Cn(X)) = Cn(X)$)
monotony ($Cn(X) \subseteq Cn(Y)$ if $X \subseteq Y$)

as well as supraclassicality, deduction and compactness.

Consequently for every theory K we have that: $Cn(K) = K$.

AGM Model

Belief Base: Set of sentences.

Finite Belief Base: Finite set of sentences. We identify the base K with a formula φ which is the conjunction of the formulae of K.

Three basic epistemic attitudes are assumed:

If $K \vdash \alpha$, α is accepted.

If $K \vdash \neg\alpha$, α is rejected.

If neither $K \vdash \alpha$ nor $K \vdash \neg\alpha$, α is undetermined.

AGM Model

The three basic operations of the AGM model, that correspond with a change of epistemic attitude towards the input sentence α, are the following:

Expansion: This operation is in charge of incorporating sentences in the original set, without eliminating any sentence from it. It allows the passage from an epistemic state in which a belief is undetermined to another epistemic state in which the belief is accepted or rejected.

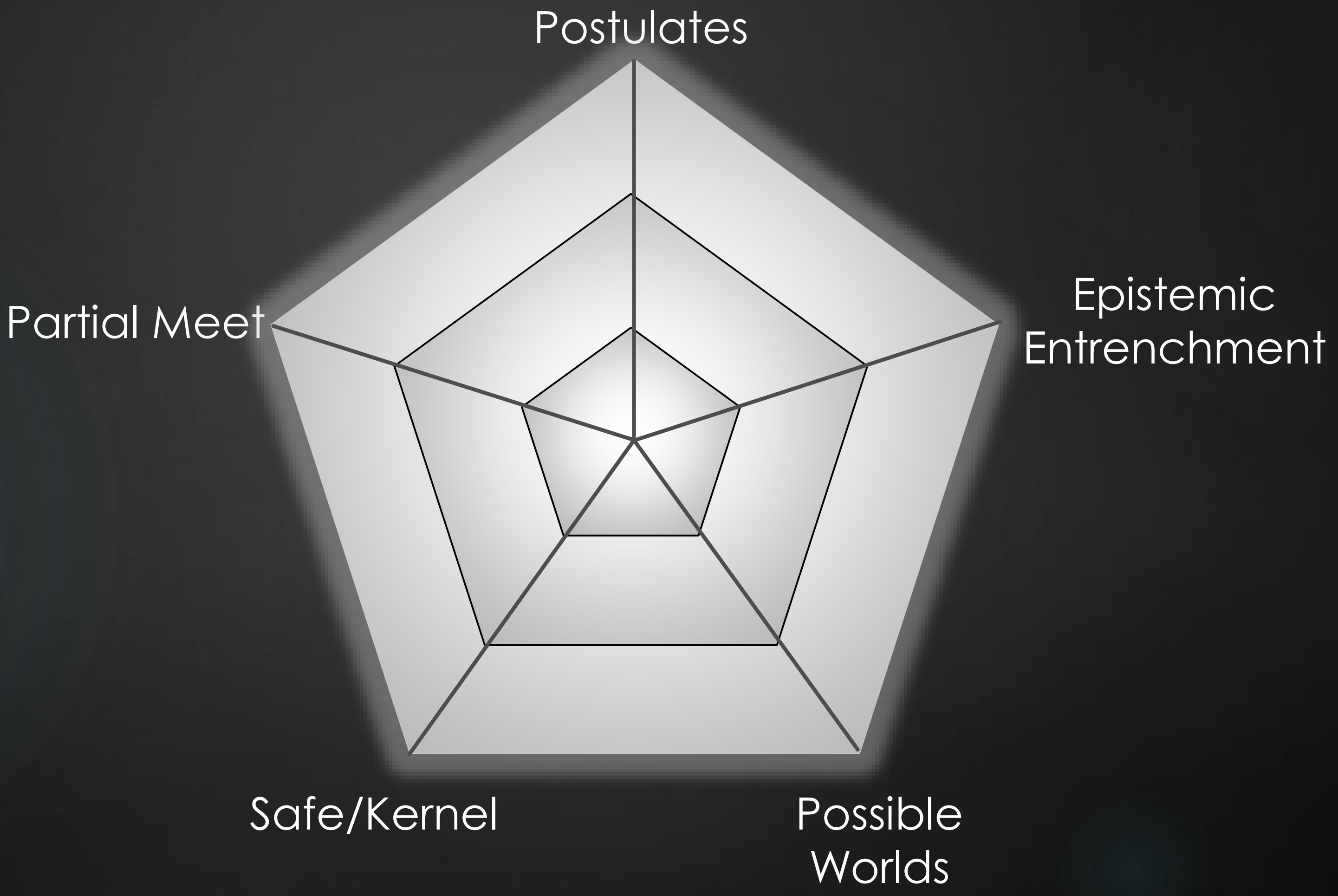
Expansion is defined as $K{+}\alpha = Cn(K \cup \{\alpha\})$

Contraction: This operation eliminates sentences from the original set without incorporating any new ones. It allows the passage from an epistemic state in which a belief is accepted or rejected to another epistemic state in which the belief is undetermined.

Revision: This operation incorporates a sentence in the original set, but it can eliminate some beliefs in order to preserve consistency of the revised set. It allows the passage from an epistemic state in which a belief is accepted (rejected) to another state in which the belief is rejected (accepted).

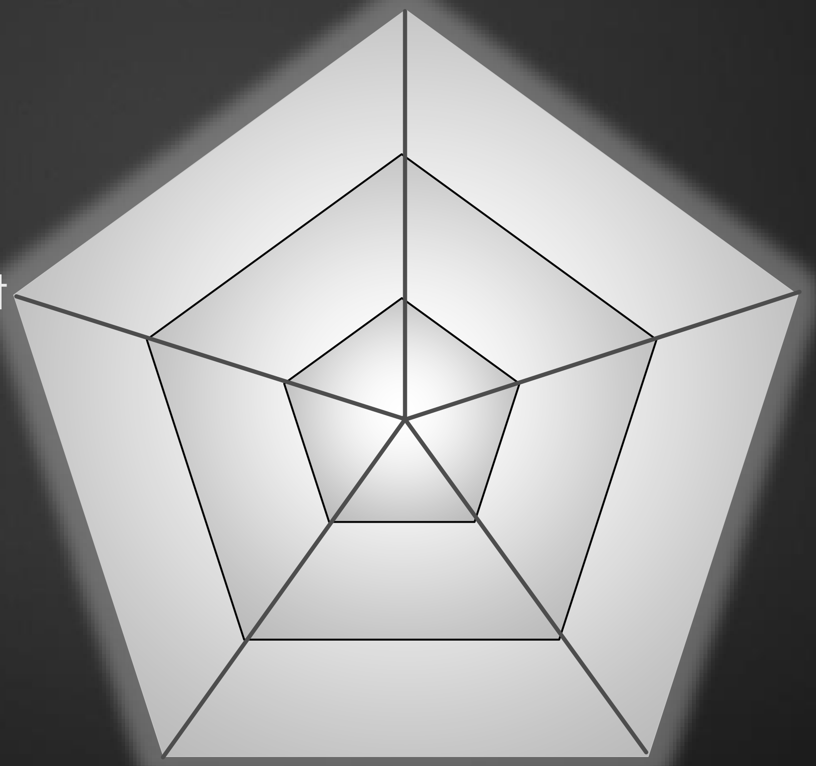
AGM

5 different equivalent presentations



AGM

5 different equivalent presentations



Expansion

$$K+\alpha = Cn\left(K \cup \{\alpha\}\right)$$

Postulates for Contraction

Closure $K{-}\alpha$ is a belief set whenever K is a belief set.

Success If $\nvdash \alpha$, then $K{-}\alpha \nvdash \alpha$.

Inclusion $K{-}\alpha \subseteq K$.

Vacuity If $K \nvdash \alpha$, then $K \subseteq K{-}\alpha$.

Extensionality If $\vdash \alpha \leftrightarrow \beta$ then $K{-}\alpha = K{-}\beta$.

Recovery $K \subseteq (K{-}\alpha) + \alpha$.

Conjunctive factoring $K{-}(\alpha \wedge \beta) = \begin{cases} K{-}\alpha, \text{ or} \\ K{-}\beta, \text{ or} \\ K{-}\alpha \cap K{-}\beta \end{cases}$

Postulates for Revision

Closure: $K * \alpha$ is a belief set whenever K is a belief set.

Success: $\alpha \in K * \alpha$.

Inclusion: $K * \alpha \subseteq K + \alpha$.

Vacuity: If $K \nvdash \neg\alpha$ then $K * \alpha = K + \alpha$.

Consistency: $\nvdash \neg\alpha$ then $K * \alpha \neq \perp$.

Extensionality: If $\vdash \alpha \leftrightarrow \beta$ then $K * \alpha = K * \beta$.

Disjunctive factoring $K * (\alpha \vee \beta) = \begin{cases} K * \alpha, \text{ or} \\ K * \beta, \text{ or} \\ K * \alpha \cap K * \beta \end{cases}$

Contraction vs. Revision

Levi's Identity: $K * \alpha = (K - \neg\alpha) + \alpha$.

Harper's Identity: $K - \alpha = K \cap K * \neg\alpha$.

K&M - Alternative formulation

Assuming *finite representability*, K must be expressed by a single sentence φ, Katsuno and Mendelzon reformulate the AGM postulates for such case.

(R1) $\varphi \circ \alpha \vdash \alpha$

(R2) If $\varphi \wedge \alpha \nvdash \perp$ then $\varphi \circ \alpha \equiv \varphi \wedge \alpha$

(R3) If $\alpha \nvdash \perp$ then $\varphi \circ \alpha \nvdash \perp$

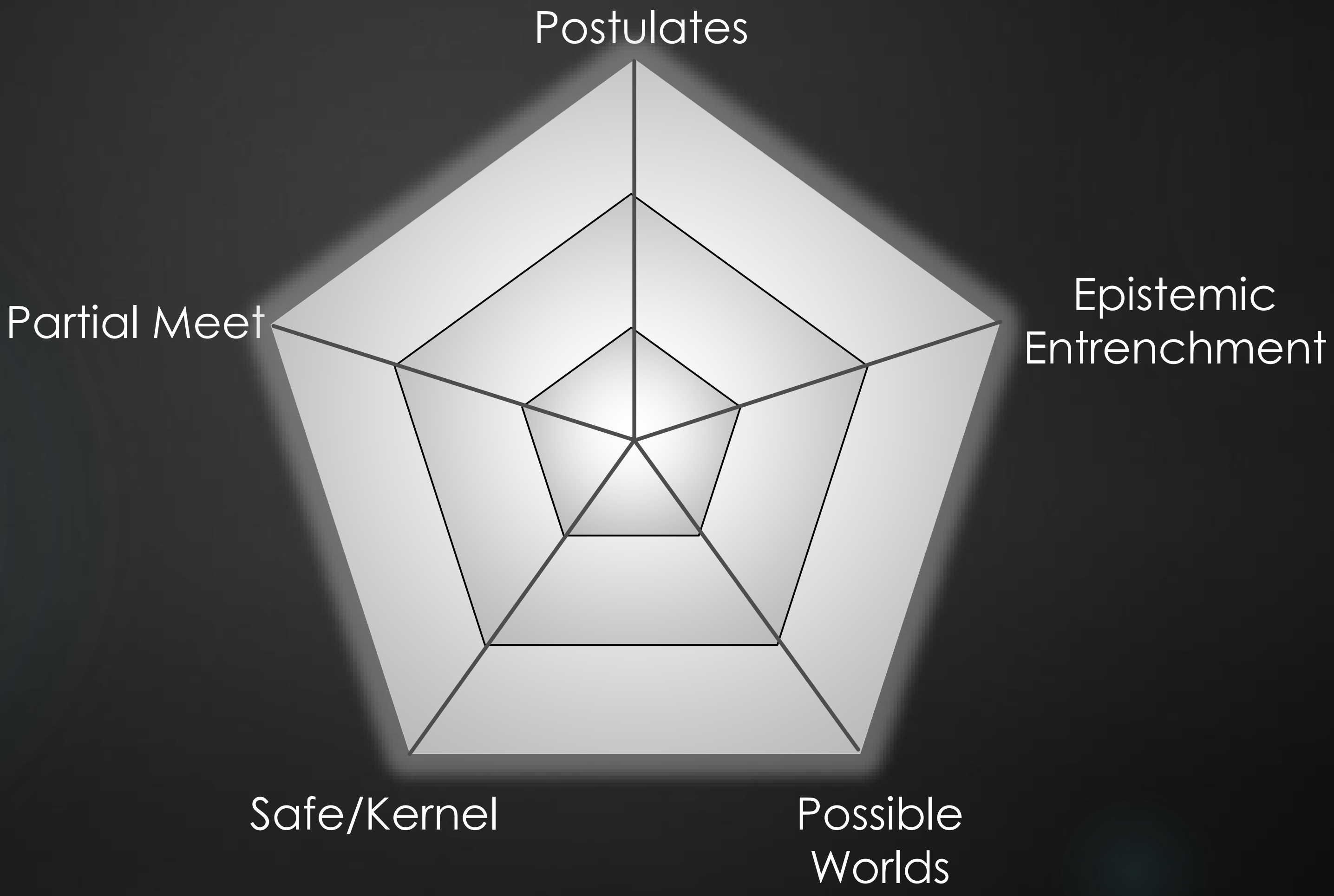
(R4) If $\varphi_1 \equiv \varphi_2$ and $\alpha_1 \equiv \alpha_2$ then $\varphi_1 \circ \alpha_1 \equiv \varphi_2 \circ \alpha_2$

(R5) $(\varphi \circ \alpha) \wedge \psi \vdash \varphi \circ (\alpha \wedge \psi)$

(R6) If $(\varphi \circ \alpha) \wedge \psi \nvdash \perp$ then $\varphi \circ (\alpha \wedge \psi) \vdash (\varphi \circ \alpha) \wedge \psi$

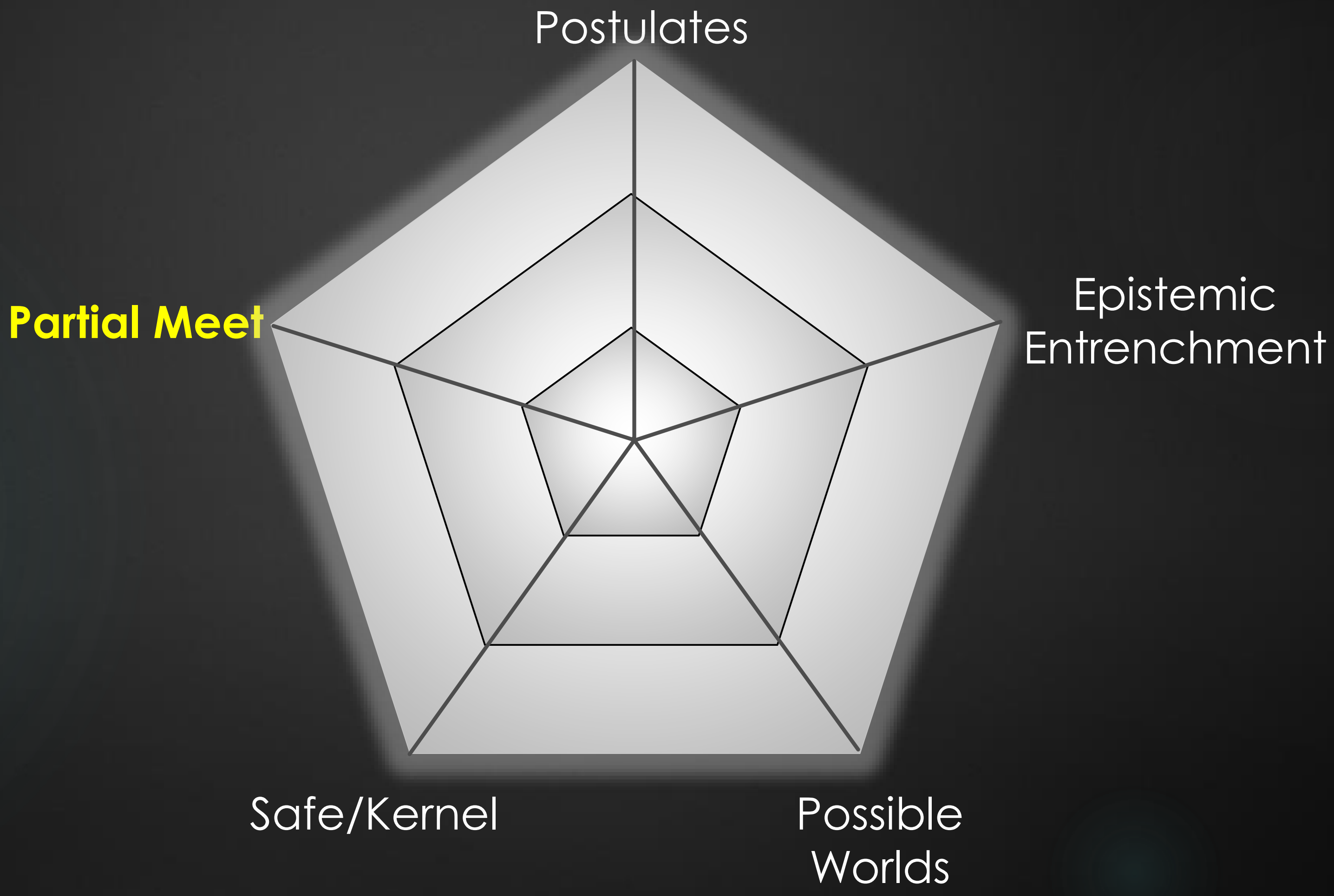
AGM

5 different equivalent presentations



AGM

5 different equivalent presentations



Partial Meet Contraction

Based on the inclusion-maximal subsets of K that do not imply α.

$K \perp \alpha$ (*K remainder α*)

$$H \in K \perp \alpha \text{ if and only if: } \begin{cases} H \subseteq K \\ H \nvdash \alpha \\ \text{There is no set } H' \text{ such that} \\ \qquad H \subset H' \subseteq K \text{ and } H' \nvdash \alpha \end{cases}$$

Partial Meet Contraction: Special Cases

There are two limiting cases: 1) By selecting just one element (Maxichoice) and 2) By choosing what all the members of the set have in common (Full meet contraction).

If – is a Maxichoice, then it satisfies

Saturability If $\alpha \in K$, then for any $\beta \in \mathcal{L}$, either $\alpha \vee \beta \in K-\alpha$ or $\alpha \vee \neg\beta \in K-\alpha$

If – is a full meet contraction then

$$K-\alpha = K \cap Cn(\neg\alpha)$$

Partial Meet Contraction

γ is a *selection function* for K if and only if:

If $K \perp \alpha \neq \varnothing$, then $\varnothing \subset \gamma(K \perp \alpha) \subseteq K \perp \alpha$.
If $K \perp \alpha = \varnothing$, then $\gamma(K \perp \alpha) = K$.

γ is relational if and only if there is a relation $\sqsubseteq$ such that for all sentences α, if $K \perp \alpha$ is non-empty, then

$$\gamma(K \perp \alpha) = \{B \in K \perp \alpha \mid C \sqsubseteq B \text{ for all } C \in K \perp \alpha\}$$

γ is transitively relational if and only if this hold for some transitive relation $\sqsubseteq$.

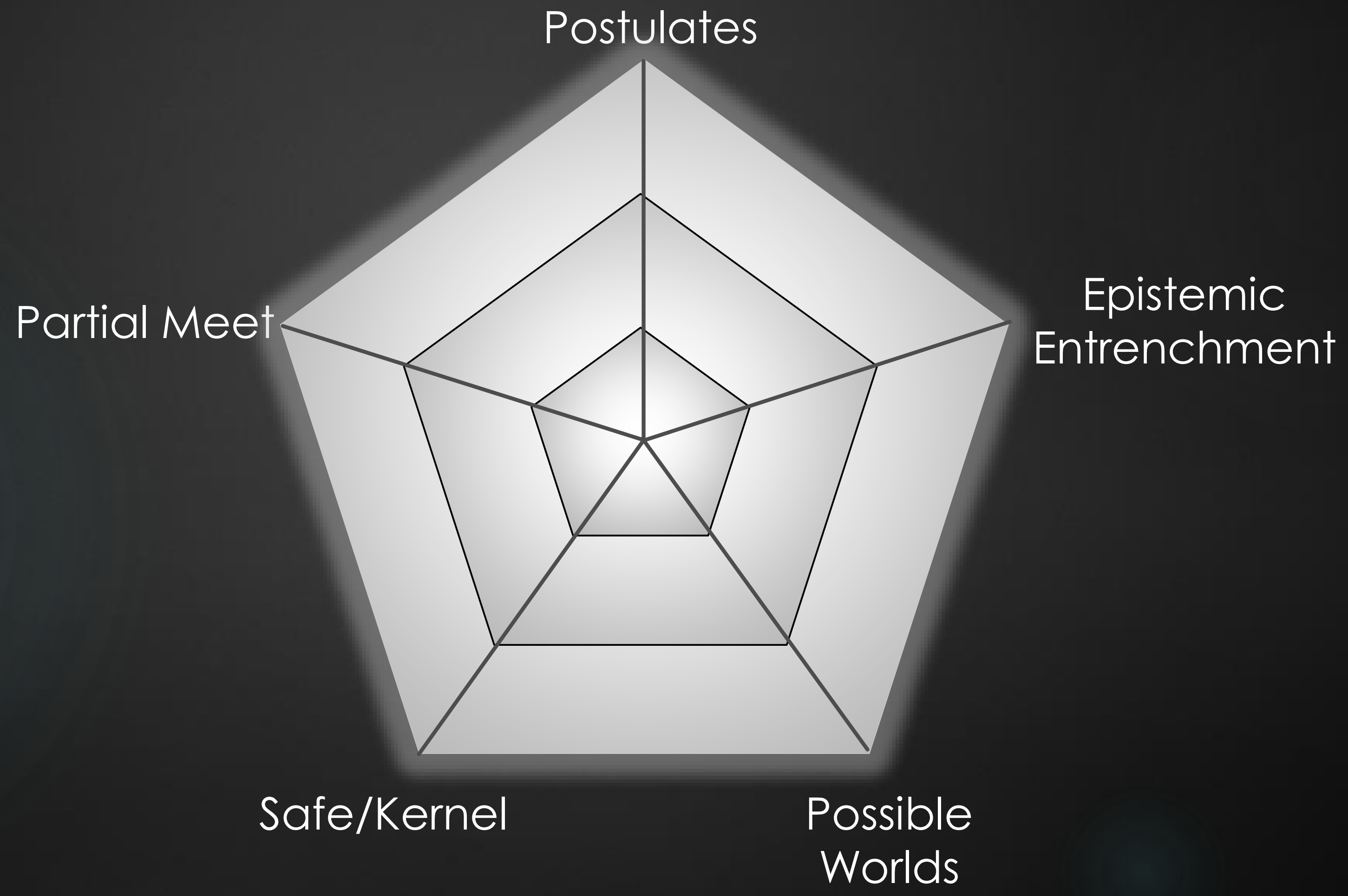
Partial Meet Functions

$$K \sim_{\gamma} \alpha = \cap\gamma(K \perp \alpha)$$

$$K * \alpha = (K \sim_{\gamma} \neg\alpha) + \alpha$$

AGM

5 different equivalent presentations



AGM

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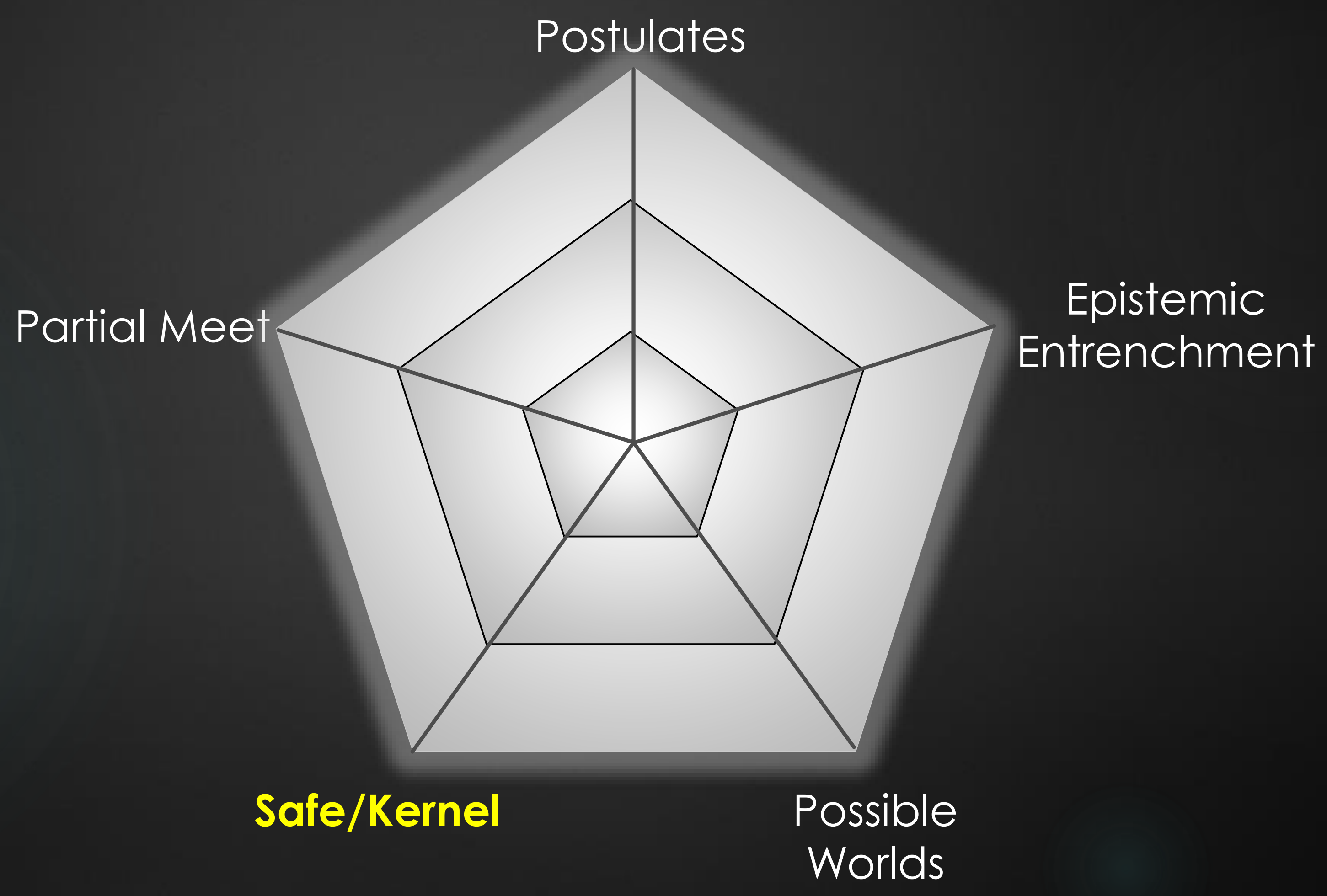


Kernel / Safe Contraction

Based on the inclusion-minimal subsets of K that imply α

$K \perp\!\!\!\perp \alpha$ (kernel set of K with respect to α)

$$A \in K \perp\!\!\!\perp \alpha \textit{ if and only if: } \begin{cases} A \subseteq K \\ A \vdash \alpha \\ \textit{If } B \subset A \textit{ then } B \nvdash \alpha \end{cases}$$

Kernel Contraction

An incision function σ *for K is a function such that for all sentences* α*:*

$$\begin{cases} \sigma(K \perp\!\!\!\perp \alpha) \subseteq \bigcup(K \perp\!\!\!\perp \alpha) \\ \textit{If } \varnothing \neq A \in K \perp\!\!\!\perp \alpha, \textit{ then } A \cap \sigma(K \perp\!\!\!\perp \alpha) \neq \varnothing \end{cases}$$

Kernel Contraction

$$K -_{\sigma} \alpha = Cn(K \setminus \sigma(K \perp\!\!\!\perp \alpha))$$

$$K * \alpha = Cn(K \setminus \sigma(K \perp\!\!\!\perp \neg\alpha)) + \alpha$$

Safe Contraction

K is ordered according to a relation $\angle$

$\beta \angle \delta$ means that δ should be retained rather that β if we have to give up one of them.

$\angle$ must be an acyclic, irreflexive and asymmetric relation.

$\angle$ is virtually connected if and only if for all $\alpha, \beta, \delta \in K$:
if $\alpha \angle \beta$ *then either* $\alpha \angle \delta$ *or* $\delta \angle \beta$.

$\angle$ is regular if and only if it satisfies:
continuing-up (*If* $\alpha \angle \beta$ and $\beta \vdash \delta$, *then* $\alpha \angle \delta$).
continuing-down (*If* $\alpha \vdash \beta$ and $\beta \angle \delta$, *then* $\alpha \angle \delta$).

Safe Contraction

Any sentence β *in a belief set* K *is safe with respect to* α *if and only if* β *is not minimal under* $\angle$ *with respect to the elements of any* $A \in K \perp\!\!\!\perp \alpha$. *The set of all* safe *sentences of* K *respect to* α *is denoted by* K/α.

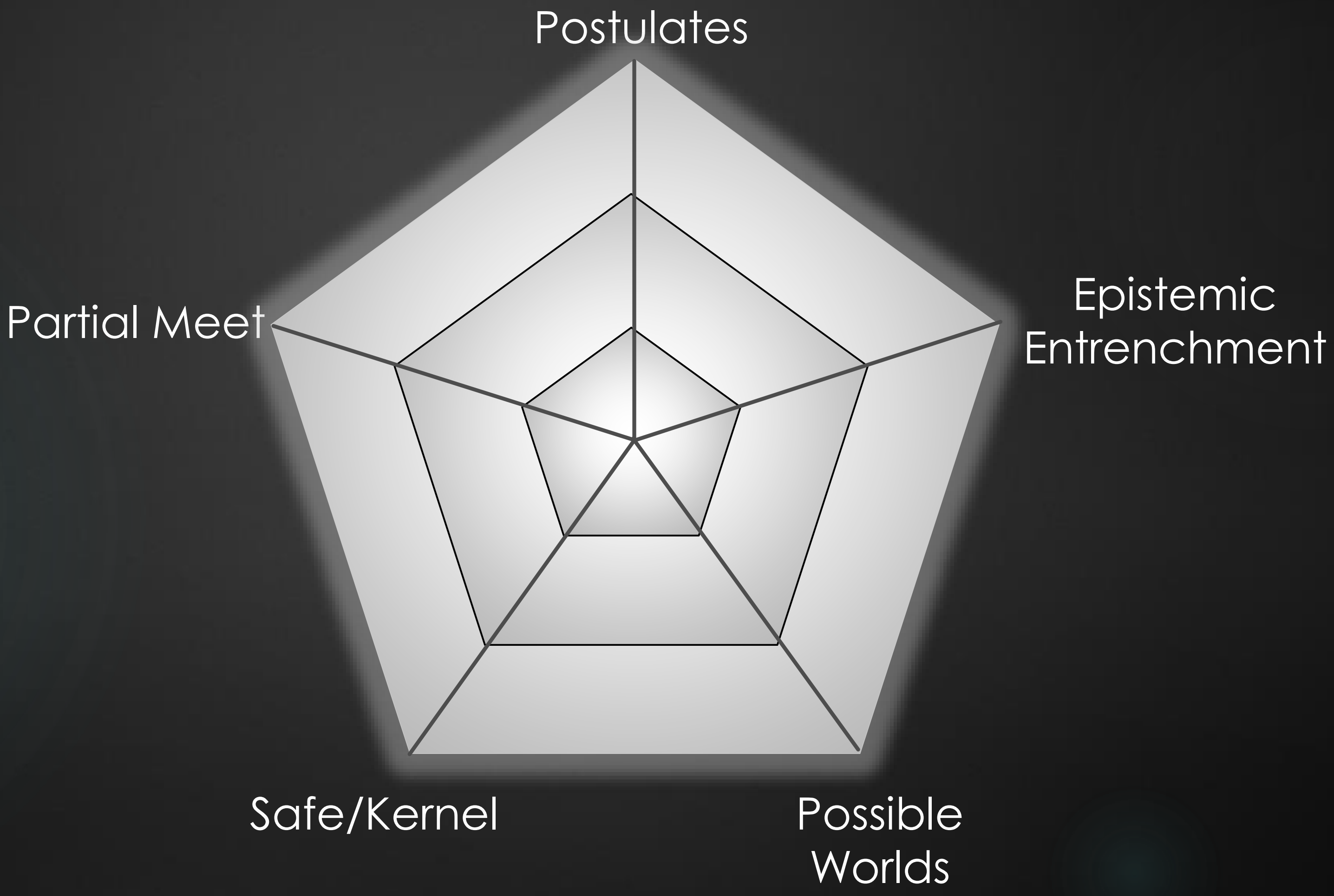
Definition

$K \sim \alpha$ *is a safe contraction, based on a regular and virtually connected hierarchy* $\angle$, *if and only if:*

$$K \sim \alpha = Cn(K/\alpha)$$

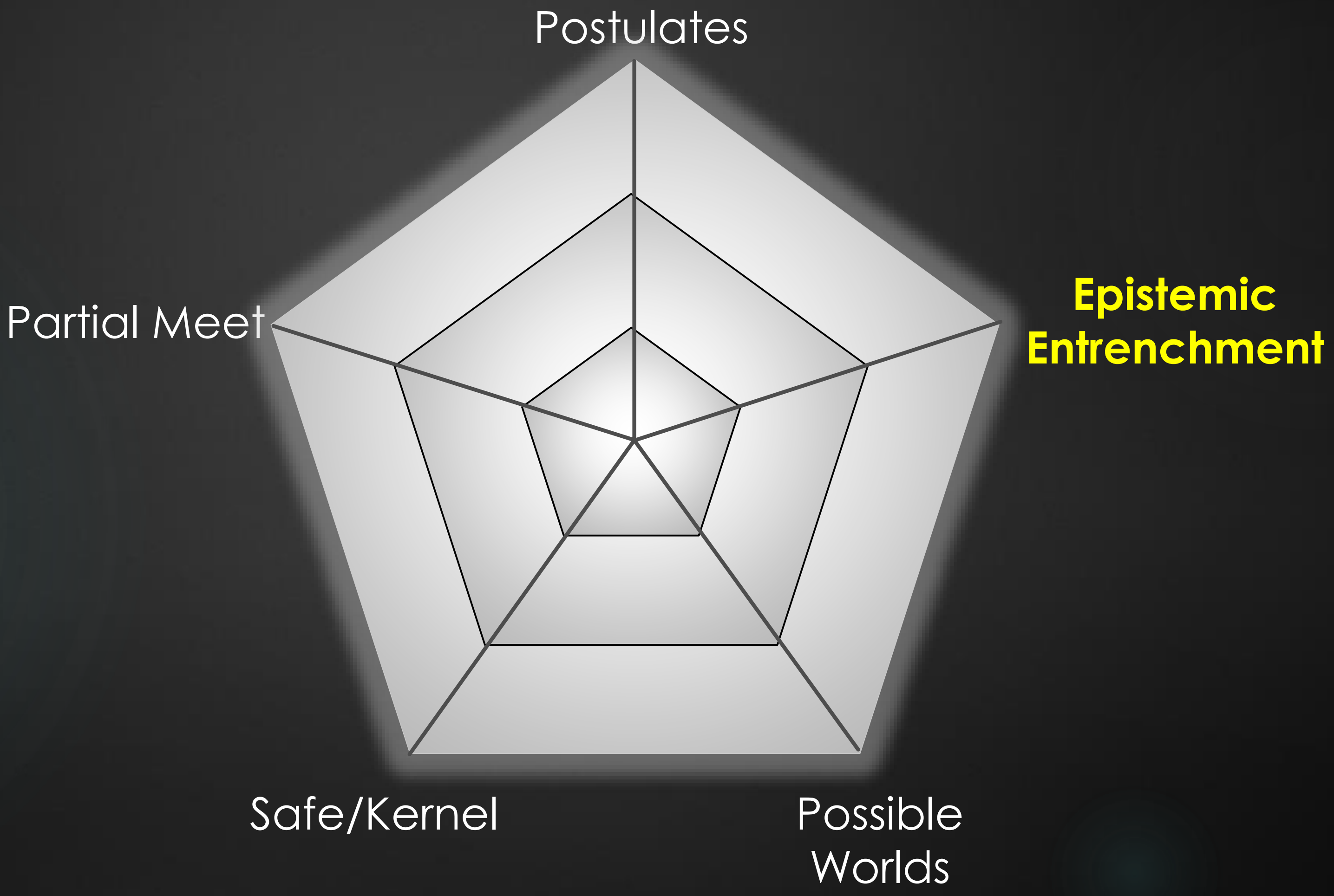
AGM

5 different equivalent presentations



AGM

5 different equivalent presentations



Epistemic Entrenchment

Epistemic entrenchment is a binary relation $\leq$ on the sentences in the belief set K such that in contraction, giving up beliefs with lower entrenchment is preferred to giving up those with higher entrenchment.

Epistemic Entrenchment

(EE1) Transitivity If $\alpha \leq_K \beta$ and $\beta \leq_K \delta$, then $\alpha \leq_K \delta$.

(EE2) Dominance If $\alpha \vdash \beta$, then $\alpha \leq_K \beta$.

(EE3) Conjunctiveness $\alpha \leq_K (\alpha \wedge \beta)$ or $\beta \leq_K (\alpha \wedge \beta)$.

(EE4) Minimality If $K \nvdash \perp$, then $\alpha \notin K$ if and only if $\alpha \leq_K \beta$ for all β.

(EE5) Maximality If $\beta \leq_K \alpha$ for all β, then $\vdash \alpha$.

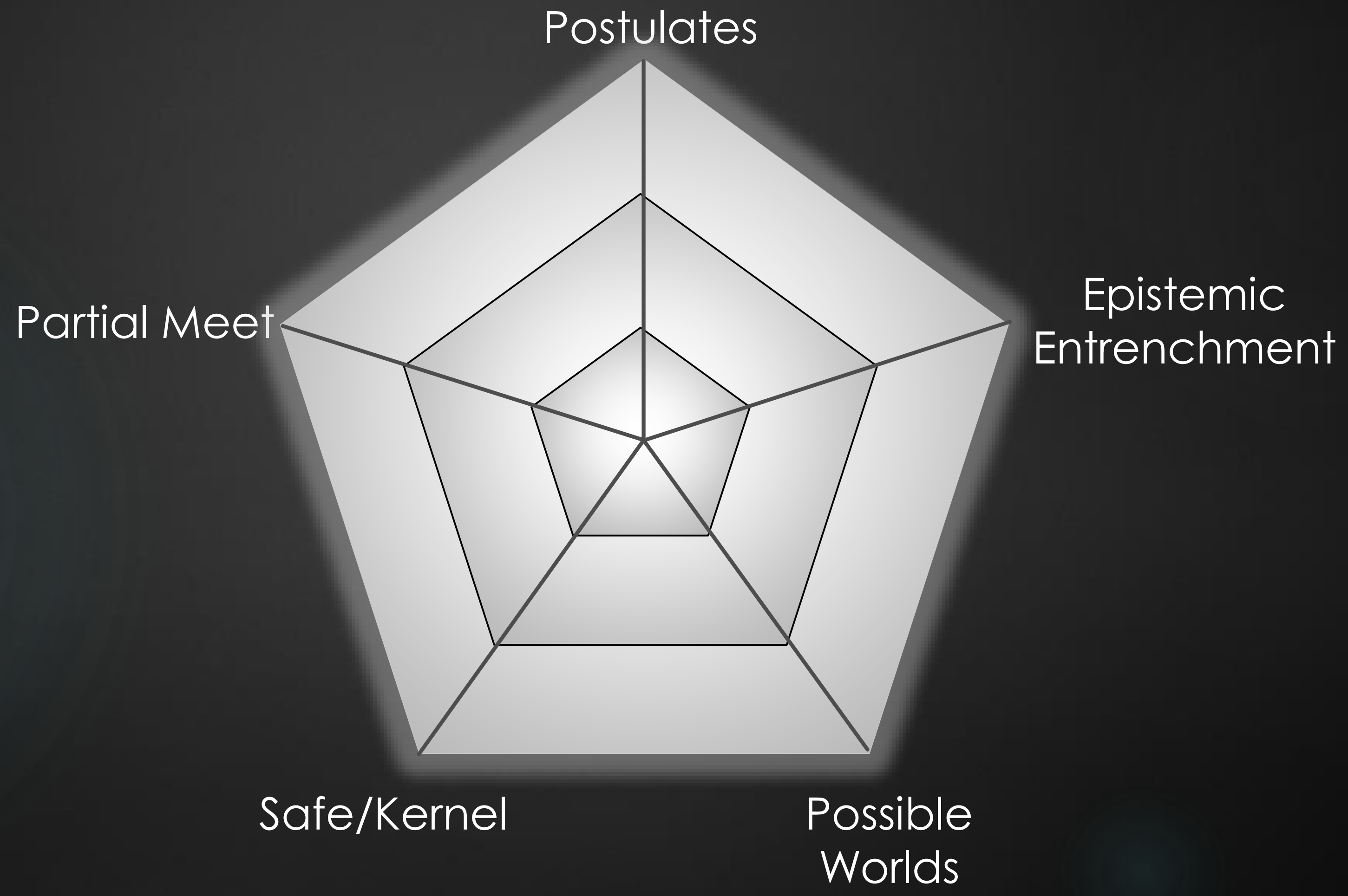
Epistemic Entrenchment

$(C \leq)$	$\alpha \leq_K \beta$ if and only if $\alpha \notin K-(\alpha \wedge \beta)$ or $\vdash (\alpha \wedge \beta)$.
$(-_G)$	$\beta \in K-\alpha$ if and only if $\beta \in K$ and either $\vdash \alpha$ or $\alpha <_K (\alpha \vee \beta)$.

$(C \leq_*)$	$\alpha \leq_K \beta$ if and only if: If $\alpha \in K*\neg(\alpha \wedge \beta)$ then $\beta \in K*\neg(\alpha \wedge \beta)$.
$(*_{EBR})$	$\beta \in K*\alpha$ if and only if either $(\alpha \rightarrow \neg\beta) <_K (\alpha \rightarrow \beta)$ or $\alpha \vdash \bot$.

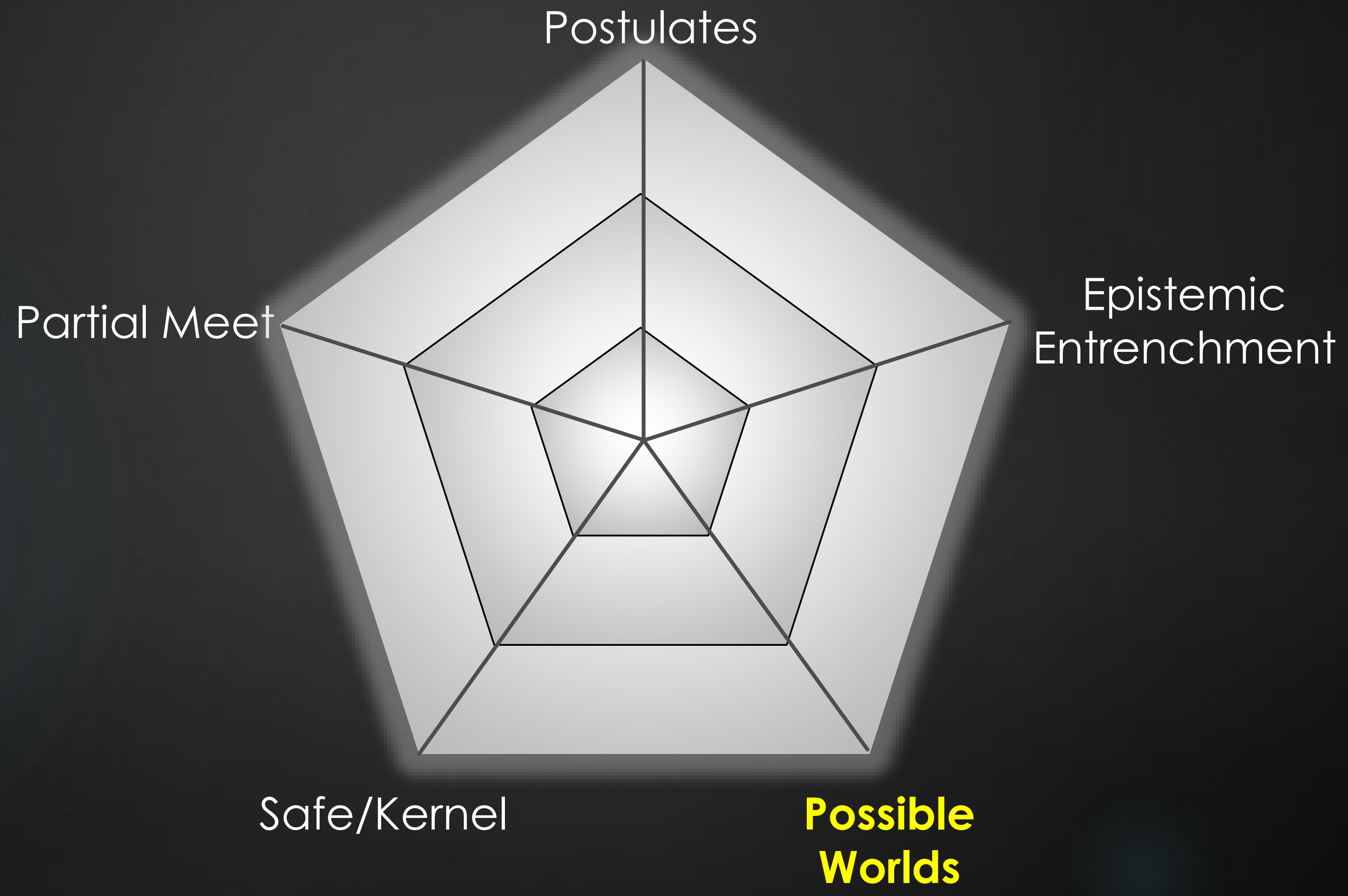
AGM

5 different equivalent presentations



AGM

5 different equivalent presentations



Possible Worlds

$$\|K\| = \{X \mid K \subseteq X \in \mathcal{L}\perp\perp\}$$

$$\|\alpha\| = \|Cn(\{\alpha\})\|$$

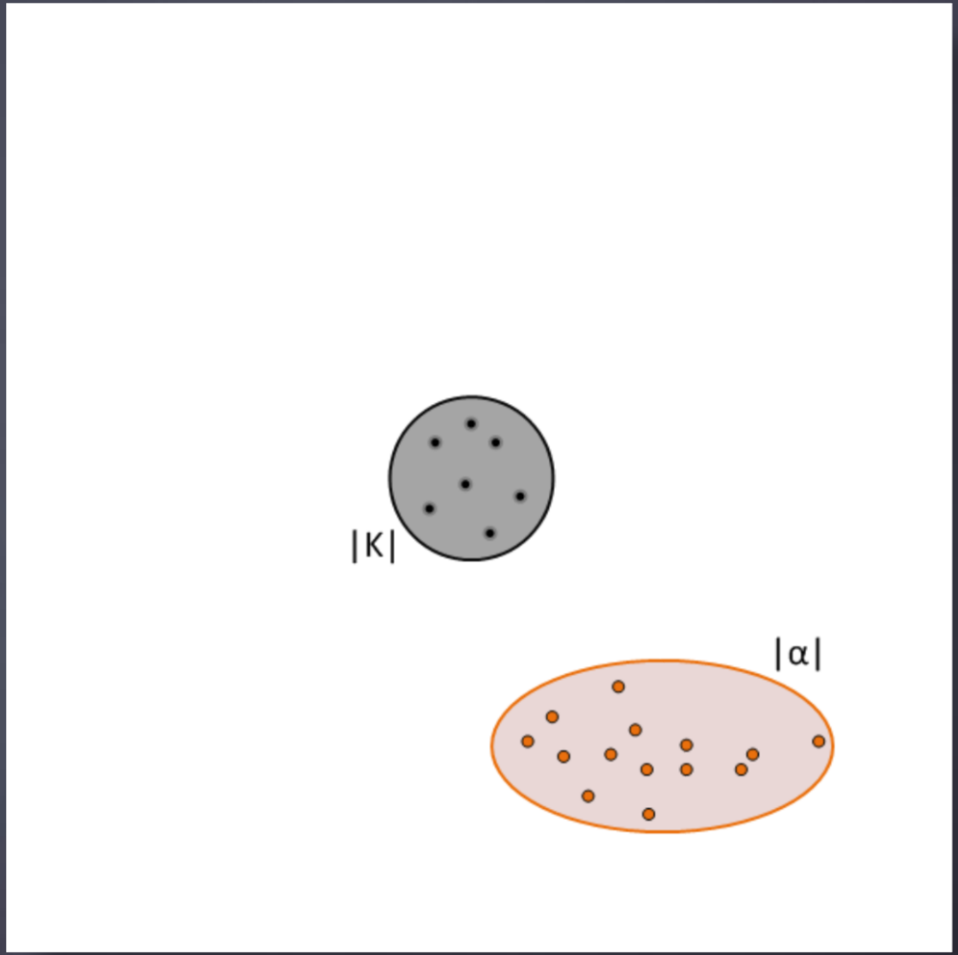
$$\|K + \alpha\| = \|K\| \cap \|\alpha\|$$

$$\|K - \alpha\| = \|K\| \cup f(\|\neg\alpha\|)$$

$$\|K * \alpha\| = f(\|\alpha\|)$$

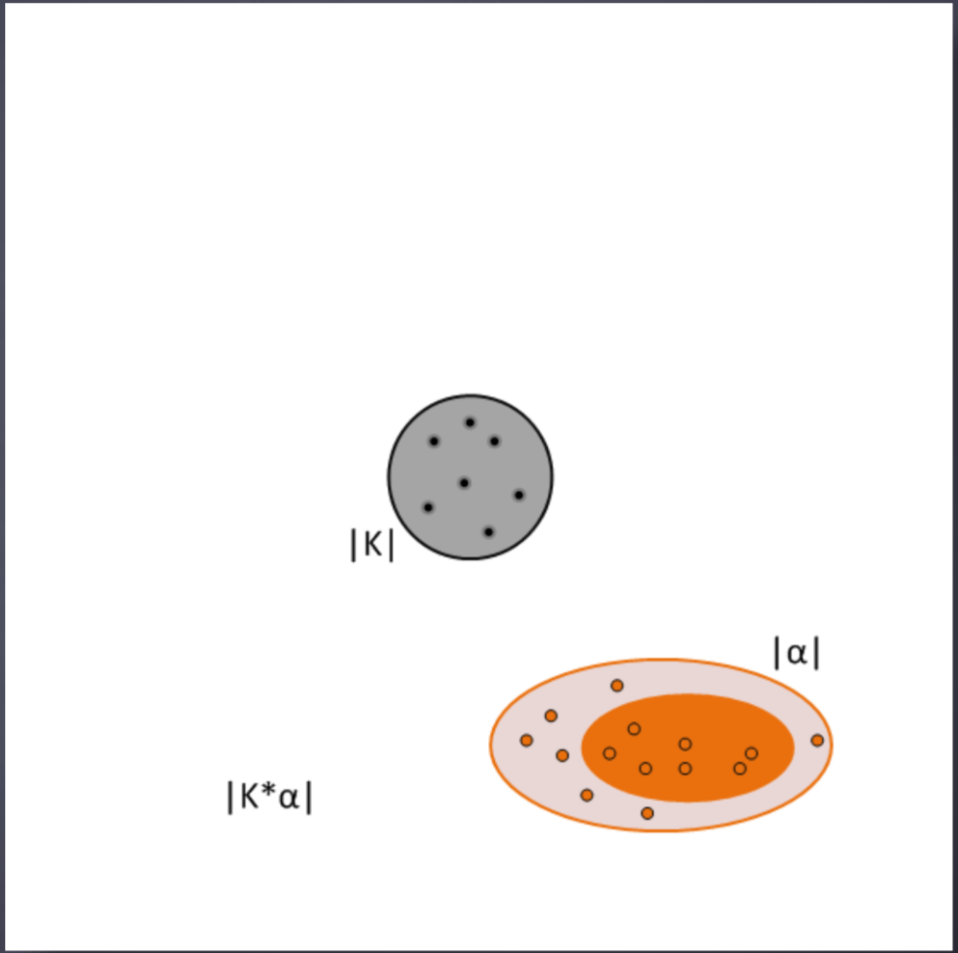
where $f(\|\alpha\|)$ selects a subset of $\|\alpha\|$.

Possible Worlds



The graphics style used is due to Sebastien Konieczny.
Thanks Sebastien!

Possible Worlds



The graphics style used is due to Sebastien Konieczny.
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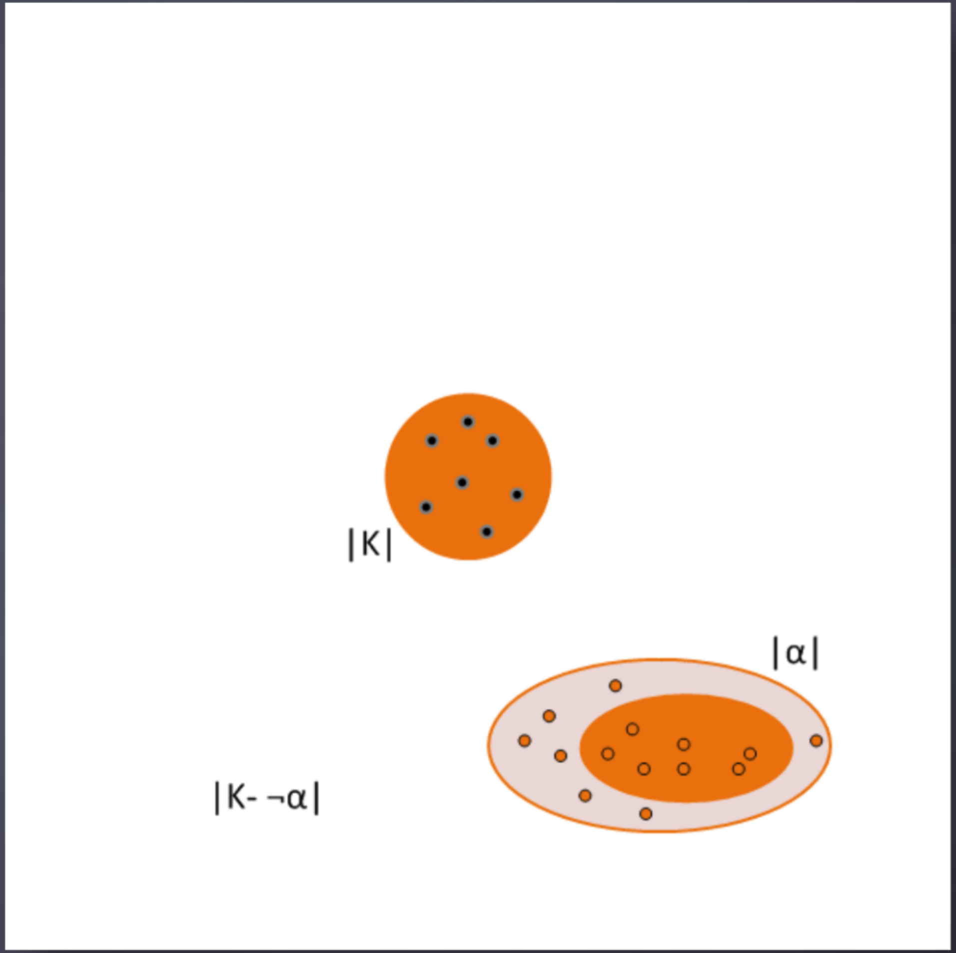
Possible Worlds



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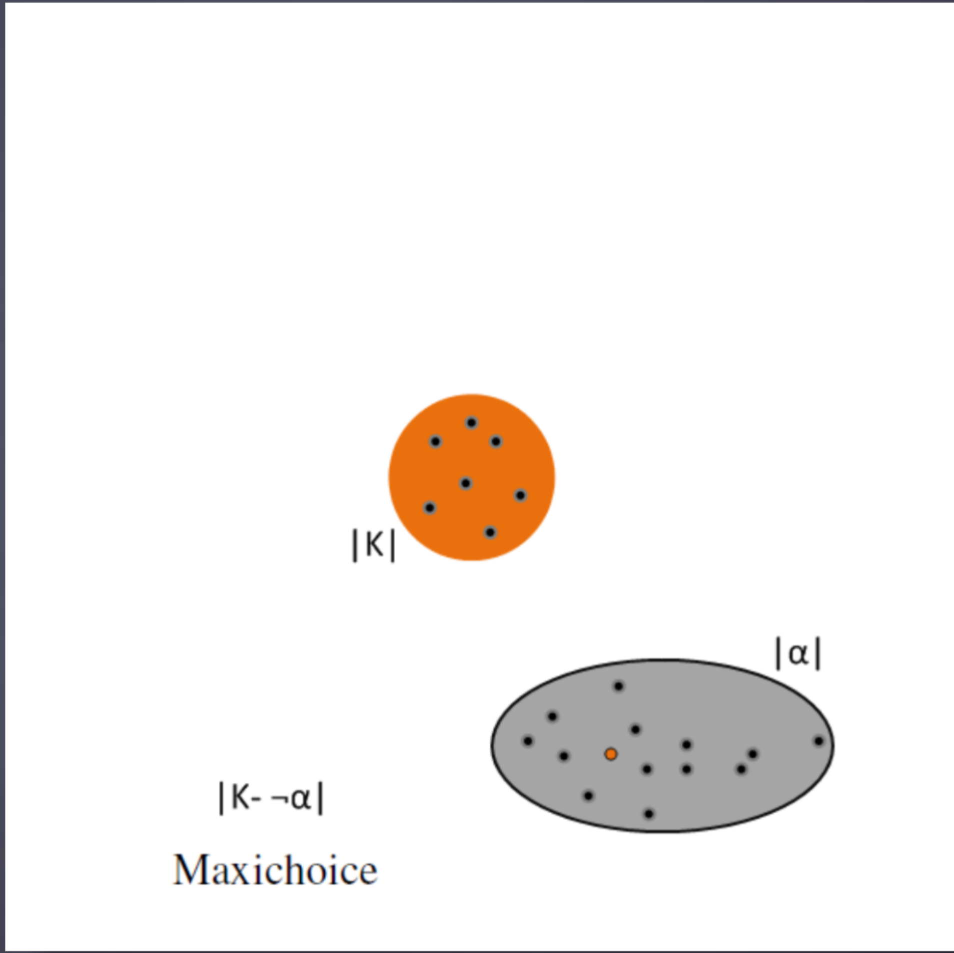
Possible Worlds



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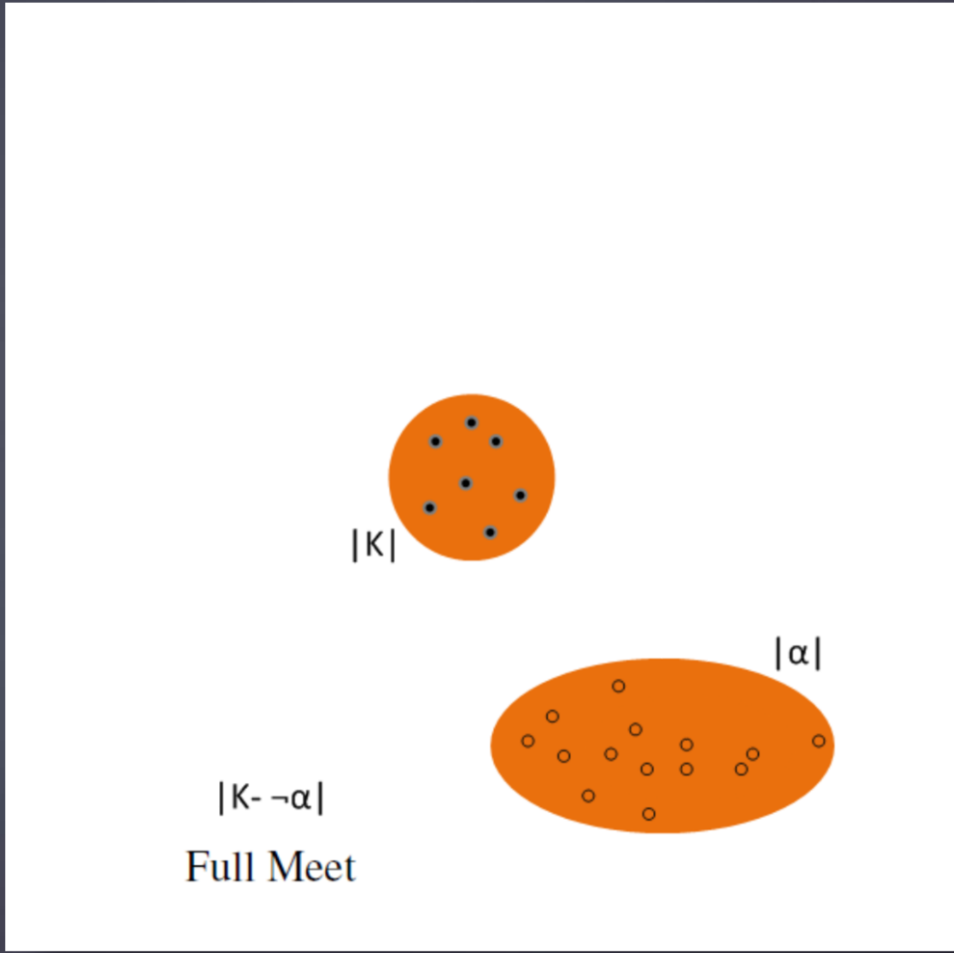

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Spheres System

The *Grove's sphere-system* makes use of a system of concentric spheres around the proposition.

Intuitively, each sphere represents a degree of closeness or similarity to $\|K\|$.

In contraction by α, the closest $\neg\alpha$-worlds are added to $\|K\|$.

In revision by α, the closest α-worlds are the worlds for $\|K * \alpha\|$.

Faithful Assignment

Alternatively a sphere system can be characterized by a total preorder $\leq$ between worlds, i.e. a reflexive, transitive and total relation on $\mathcal{W}$.

A faithful assignment is a function mapping each base φ to a pre-order $\leq_\varphi$ such that:

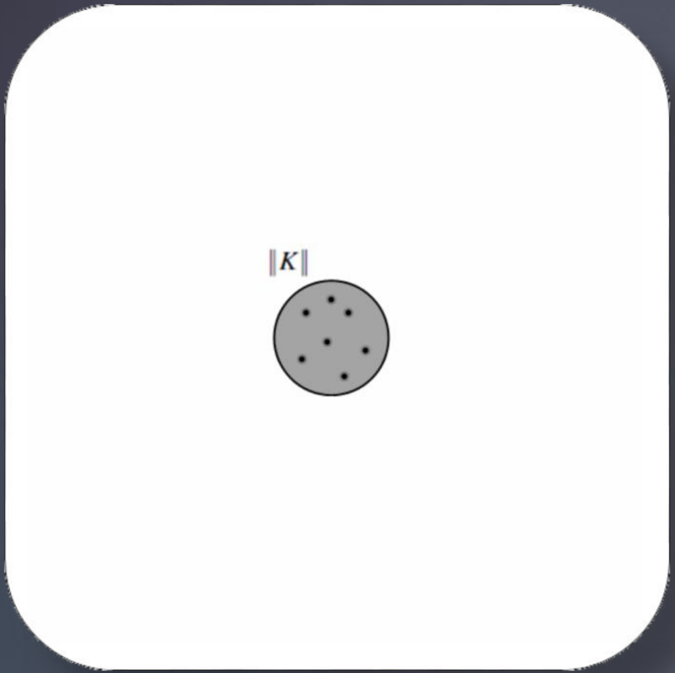
1. If $\omega \vDash \varphi$ and $\omega' \vDash \varphi$, then $\omega \simeq_\varphi \omega'$
2. If $\omega \vDash \varphi$ and $\omega' \nvDash \varphi$, then $\omega <_\varphi \omega'$
3. If $\varphi \equiv \varphi'$, then $\leq_\varphi = \leq_{\varphi'}$

Faithful Assignment

An operator $*$ is a revision operator that satisfies (R1)-(R6) if and only if there exists a faithful assignment that maps each base φ to a total pre-order $\leq_\varphi$ such that

$$\mathrm{mod}\ \varphi * \alpha = \min(\mathrm{mod}(\alpha), \leq_\varphi)$$

Spheres System / Faithful Assignment

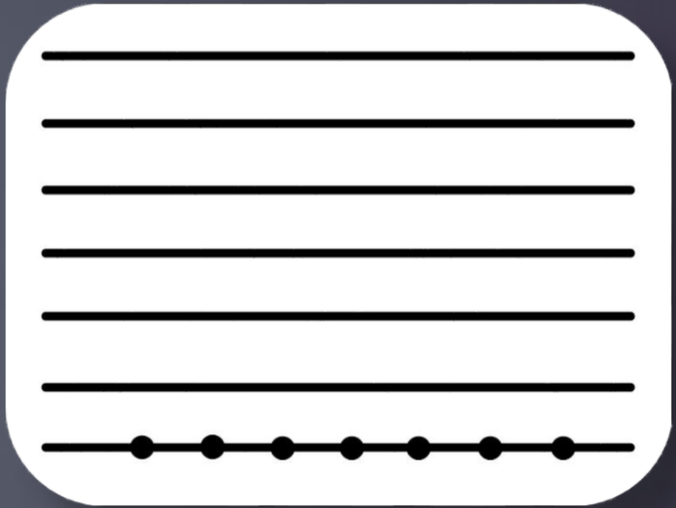


Grove's
Notation

Konieczny and Pino
Perez's Notation

Spheres System / Faithful Assignment

Grove's
Notation



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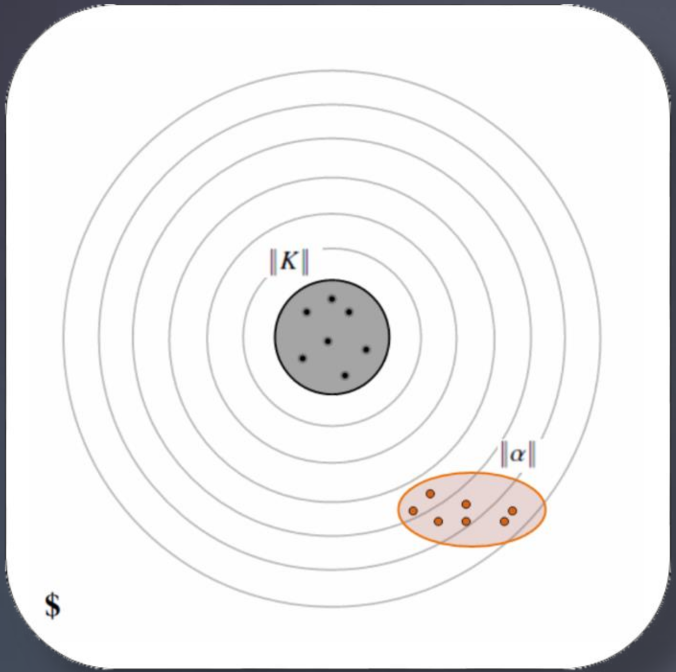

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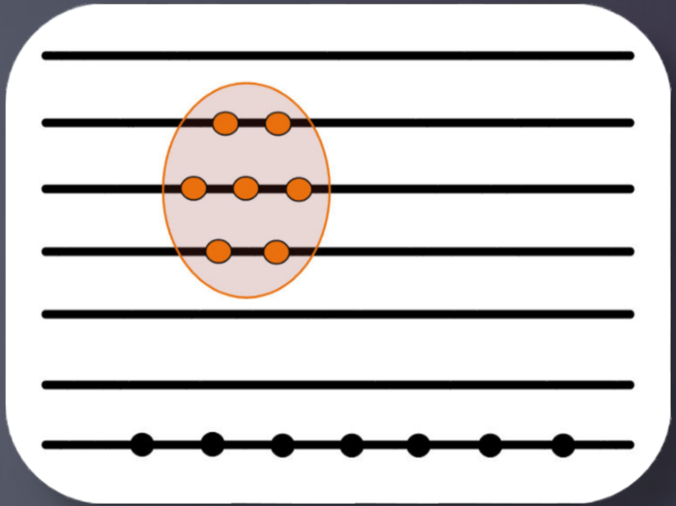

Konieczny and Pino Perez's Notation

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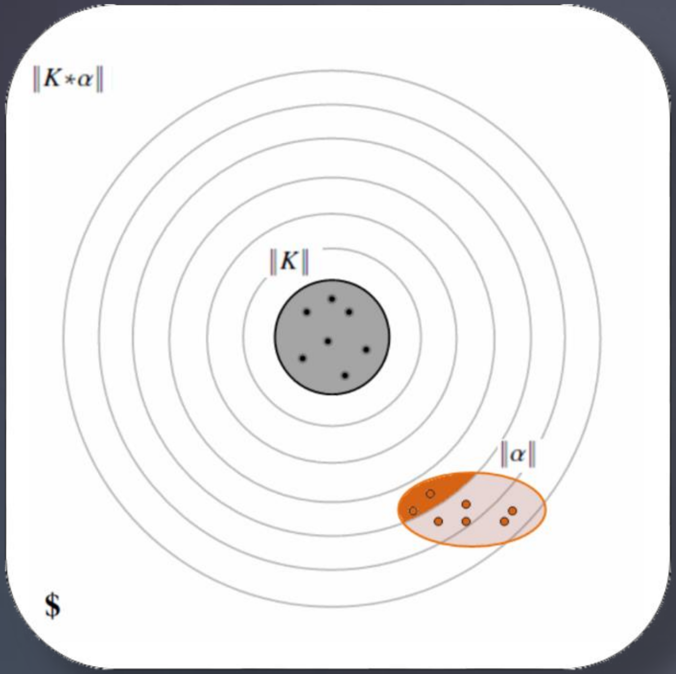

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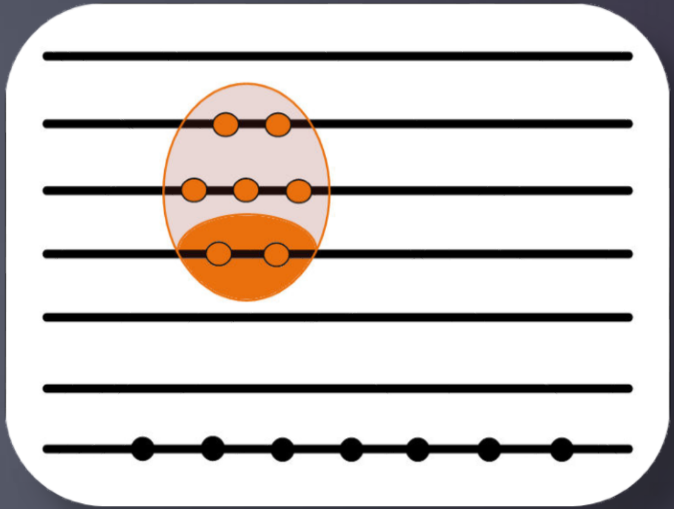

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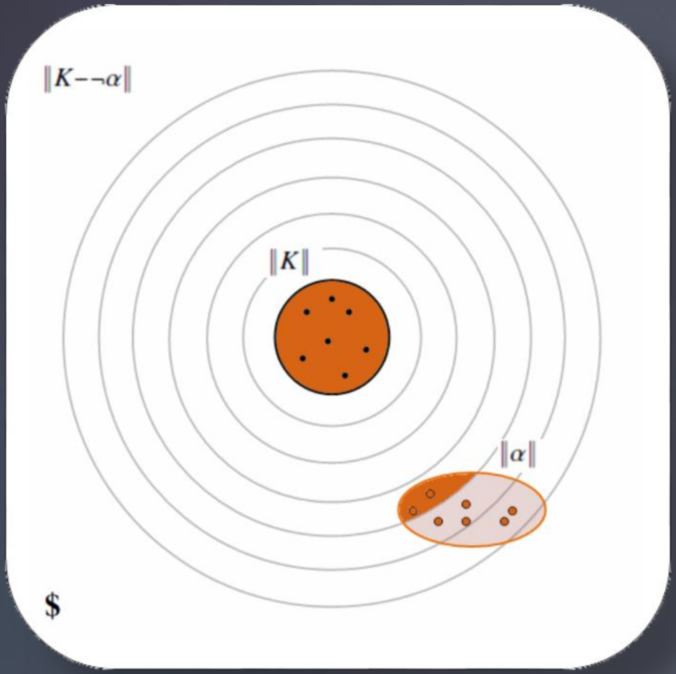

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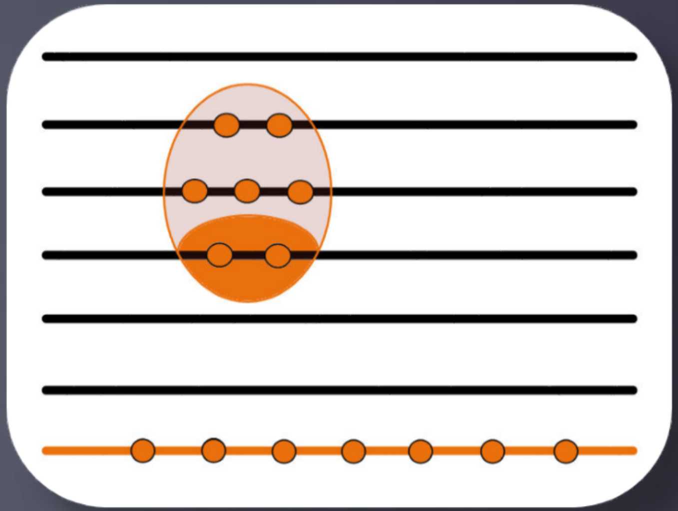
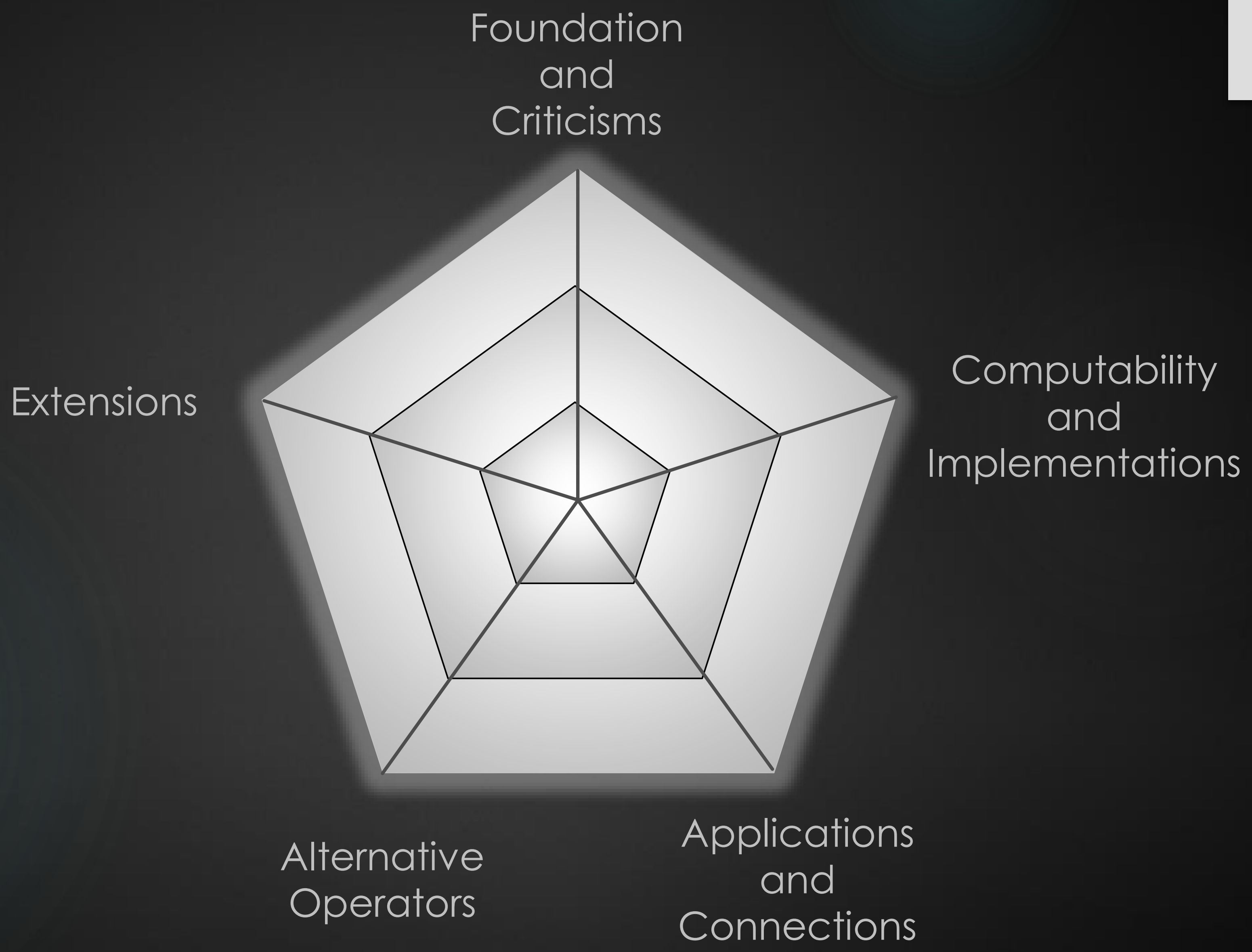


Konieczny and Pino Perez's Notation

35 years of AGM

AGM
35 years

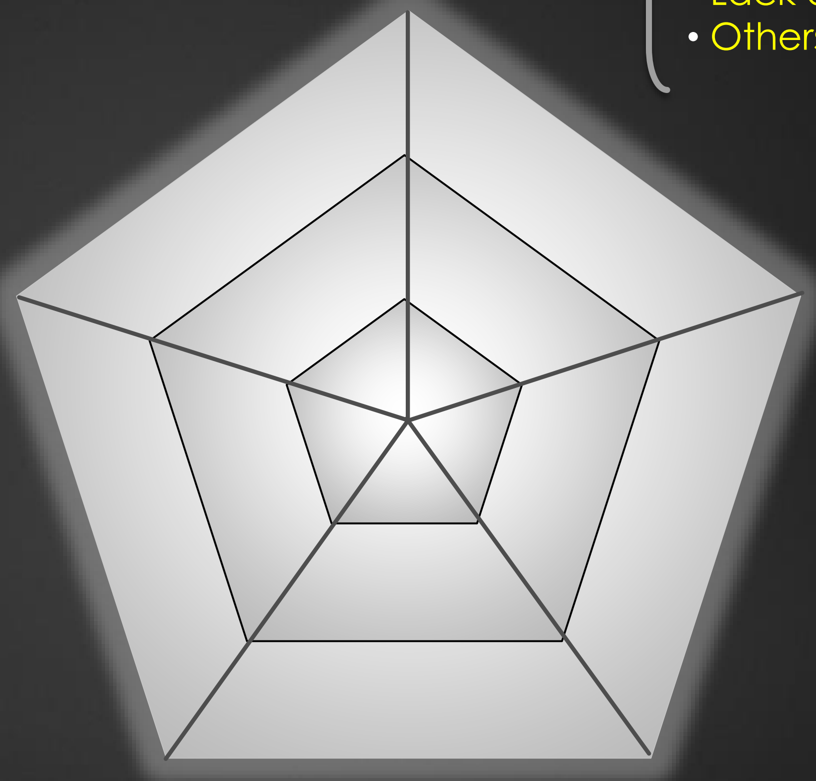

Foundation
and
Criticisms
Computability
and
Implementations
Applications
and
Connections
Alternative
Operators
Extensions

AGM

35 years

Foundation and Criticisms

- Foundations
- Recovery
- Success
- The use of Belief Sets
- Lack of Information in the belief sets
- Others



AGM

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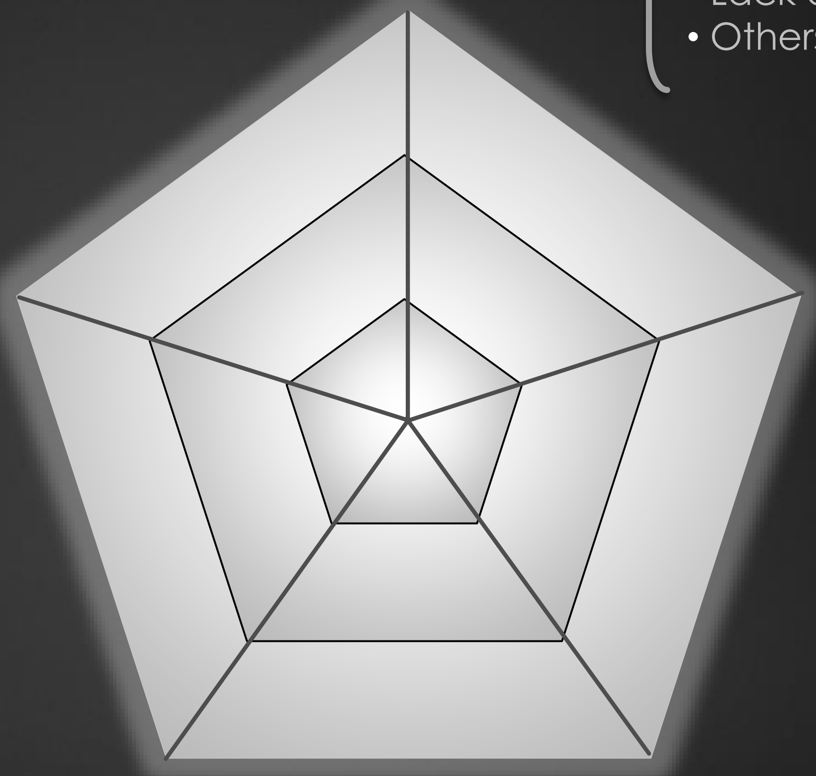
- Belief Bases
- Iteration
- Multiple Change
- Probability and possibility
- Ranking
- Extension on the language
- Others

Extensions



Computability and Implementations

Alternative Operators

Applications and Connections

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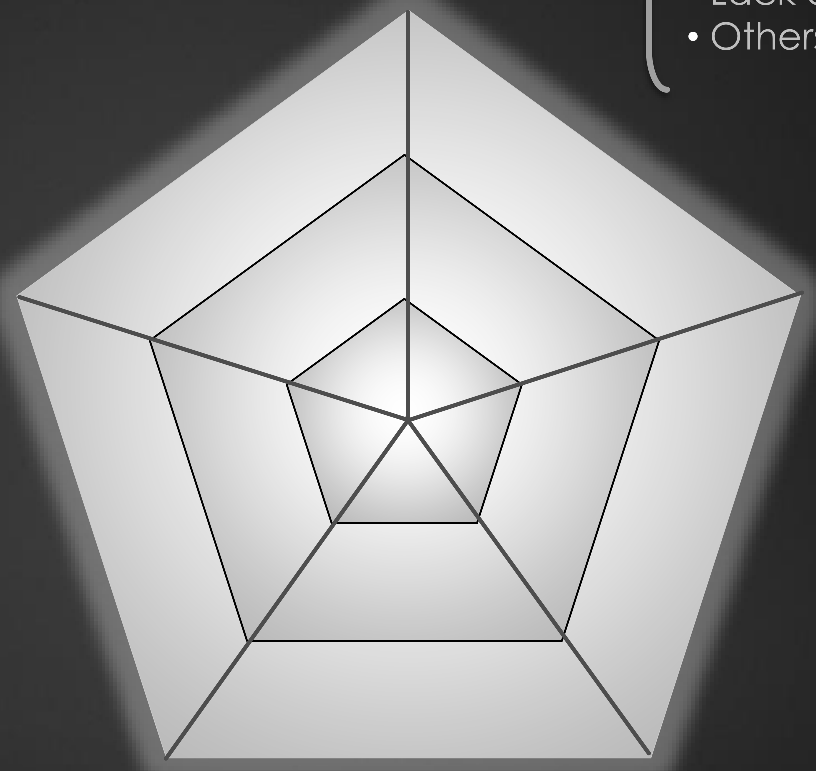

Computability and Implementations

Alternative Operators

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- Non-Priorityzed Change
- Withdrawal Functions
- Change in the strenght of belief
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- Merging
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Applications and Connections

AGM

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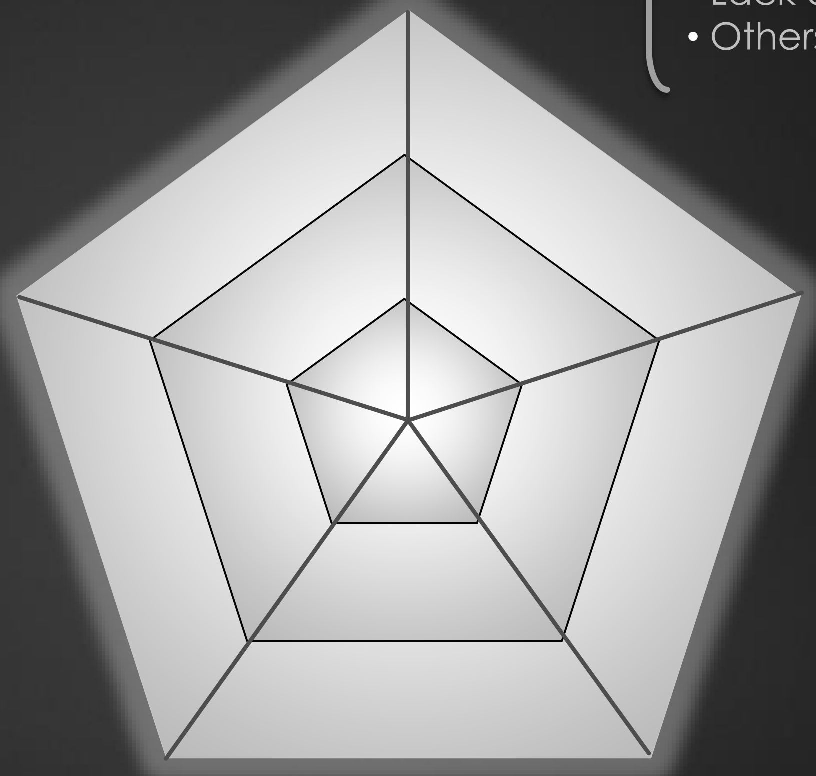

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AGM

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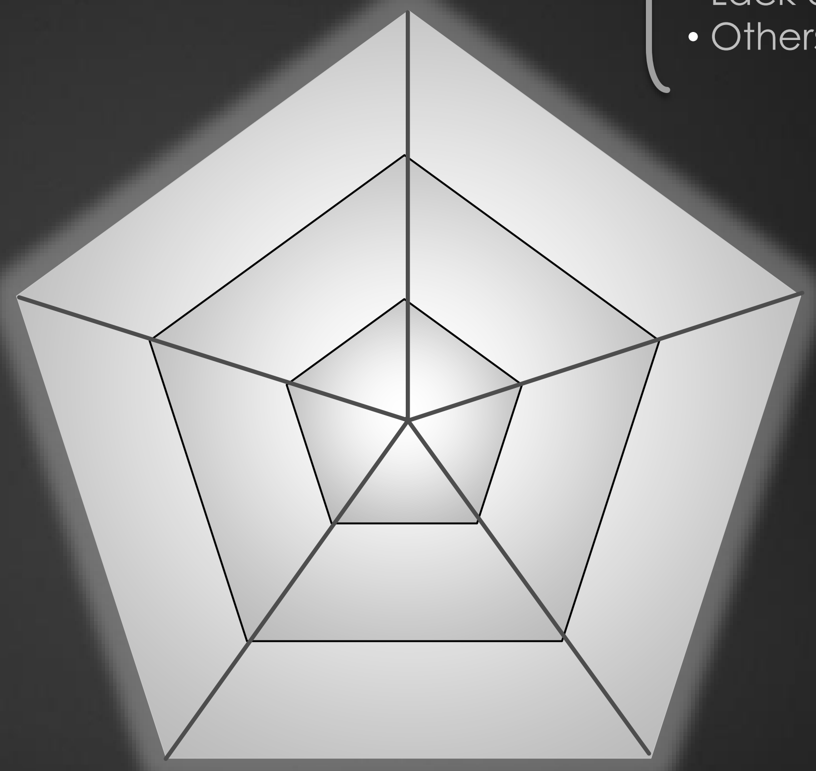

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BELIEF BASES

Belief Bases

"in real life, when we perform a contraction or derogation, we never do it to the theory itself (in the sense of a set of propositions closed under consequence) but rather on some finite or recursive or at least recursively enumerable base for the theory" Makinson

Belief Bases

Belief Base:

A set A of formulas (usually finite).

Epistemic attitudes:

- $\alpha \in Cn(A)$: α (implicitly) believed.
- $\alpha \in A$: α explicitly believed.
- $\alpha \in Cn(A) \setminus A$: α merely derived.

Two Traditions in Belief Bases

à la Dalal: This case is associated with a coherentist epistemic representation in which the corpus of beliefs is considered as a whole and none of the parts has a structural feature that differentiates it from the others. Thus belief bases as a mere expressive resource,and the belief change operation use the whole theory to perform the change, i.e.; if $Cn(H_1) = Cn(H_2)$, then $Cn(H_1 - \alpha) = Cn(H_2 - \alpha)$. This principle is known as *irrelevance of syntax*.

Two Traditions in Belief Bases

à la Hansson: This case is associated with a foundacionist epistemic representation. In this case, the belief change is performed in the belief base.

Example:

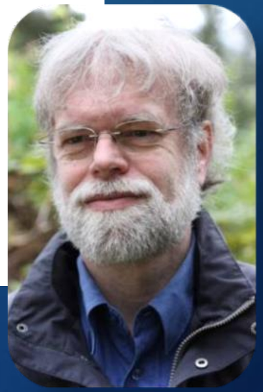
α: Paris is the capital of France.

β: There is milk in the fridge.

$\alpha, \beta \in B \Rightarrow \alpha \leftrightarrow \beta \in Cn(B)$

Contracting by β must give up β and $\alpha \leftrightarrow \beta$ (*"Disbelief Propagation"*).

In the rest of this part, we will assume the Hansson's approach.



Filtering Condition

"If β has been retracted from a base B in order to bar derivations of α from B, then the contraction of Cn(B) by α should not contain any sentences which were in Cn(B) "just because" β was in Cn(B)."

(Fuhrmann)

Why belief bases?

Expressivity $A_1 = \{\alpha, \beta\}$, $A_2 = \{\alpha, \alpha \leftrightarrow \beta\}$.

$Cn(A_1) = Cn(A_2)$

$A_1 * \neg\alpha = \{\neg\alpha, \beta\}$

$A_2 * \neg\alpha = \{\neg\alpha, \alpha \leftrightarrow \beta\}$

$\beta \in Cn(A_1 * \neg\alpha)$, but $\beta \notin Cn(A_2 * \neg\alpha)$.

Inconsistency Tolerance $A_1 = \{p. \neg p, q_1, q_2, q_3\}$

$A_2 = \{p, \neg p, \neg q_1, \neg q_2, \neg q_3\}$

$Cn(A_1) = Cn(A_2)$, but $Cn(A_1 \dot{-} p) \neq Cn(A_2 \dot{-} p)$

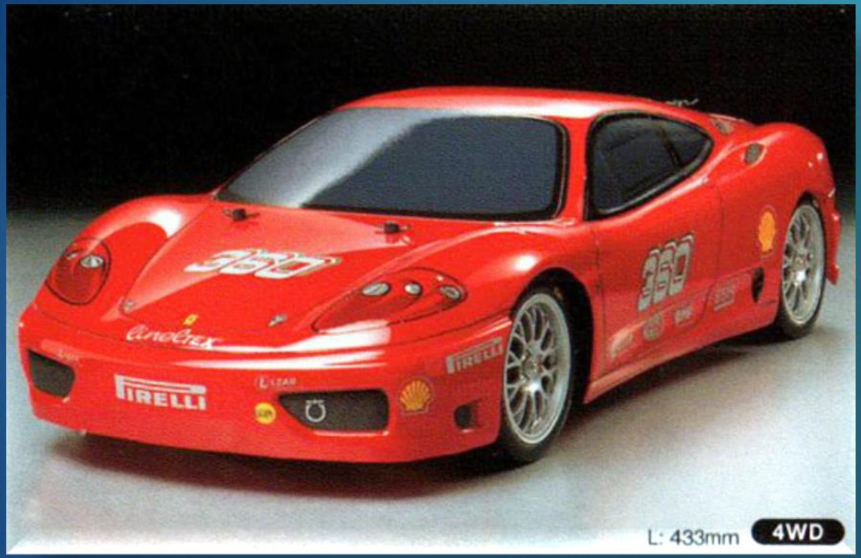
Why belief bases?

Implementations should use belief bases.

Define models with belief bases reduces the implementation gap.

The implementation Gap

- The theoretical model



- The implementation

Belief Bases (à la Hansson) Expansion and Revision

Expansion:

$$A + \alpha = A \cup \{\alpha\}.$$

Revision:

$$A * \alpha = (A - \neg\alpha) \cup \{\alpha\} \text{ (via Levi identity).}$$

Partial Meet Base Contraction Construction

- $A \perp \alpha$: maximal subsets of A that fail to imply α
- γ: function that selects some elements of $A \perp \alpha$
- $A -_{\gamma} \alpha = \bigcap \gamma(A \perp \alpha)$

Partial Meet Base Contraction

If $\alpha \notin Cn(\varnothing)$, then $\alpha \notin Cn(A - \alpha)$ **(success)**

$A - \alpha \subseteq A$ **(inclusion)**

If $\beta \in A \setminus (A - \alpha)$, then there is some A' such that $A - \alpha \subseteq A' \subseteq A$, $\alpha \notin Cn(A')$ and $\alpha \in Cn(A' \cup \{\beta\})$

(relevance)

If for all subsets A' of A, $\alpha \in Cn(A')$ if and only if $\beta \in Cn(A')$, then $A - \alpha = A - \beta$

(uniformity)

Kernel Base Contraction

- $A \perp\!\!\!\perp \alpha$: minimal subsets of A that imply α
- σ: function that selects at list one element of each set in $A \perp\!\!\!\perp \alpha$
- $A -_{\sigma} \alpha = A \setminus \sigma(A \perp\!\!\!\perp \alpha)$

Kernel Base Contraction

If $\alpha \notin Cn(\varnothing)$, then $\alpha \notin Cn(A - \alpha)$ **(success)**

$A - \alpha \subseteq A$ **(inclusion)**

If $\beta \in A \setminus A - \alpha$, then there is some $A' \subseteq A$ such that $\alpha \notin Cn(A')$ and $\alpha \in Cn(A' \cup \{\beta\})$

(core-retainment)

If for all subsets A' of A, $\alpha \in Cn(A')$ if and only if $\beta \in Cn(A')$, then $A - \alpha = A - \beta$

(uniformity)

Smooth Kernel Base Contraction

σ satisfies

- if it holds for all subsets A' of A that if $A' \vdash \beta$ and $\beta \in \sigma(A \perp\!\!\!\perp \alpha)$ then $A' \cap \sigma(A \perp\!\!\!\perp \alpha) \neq \varnothing$.

if and only if – satisfies

$A \cap Cn(A - \alpha) \subseteq A - \alpha.$ **(relative closure)**

Basic Related AGM Contraction

$$A - \alpha = (Cn(A) \div \alpha) \cap A$$

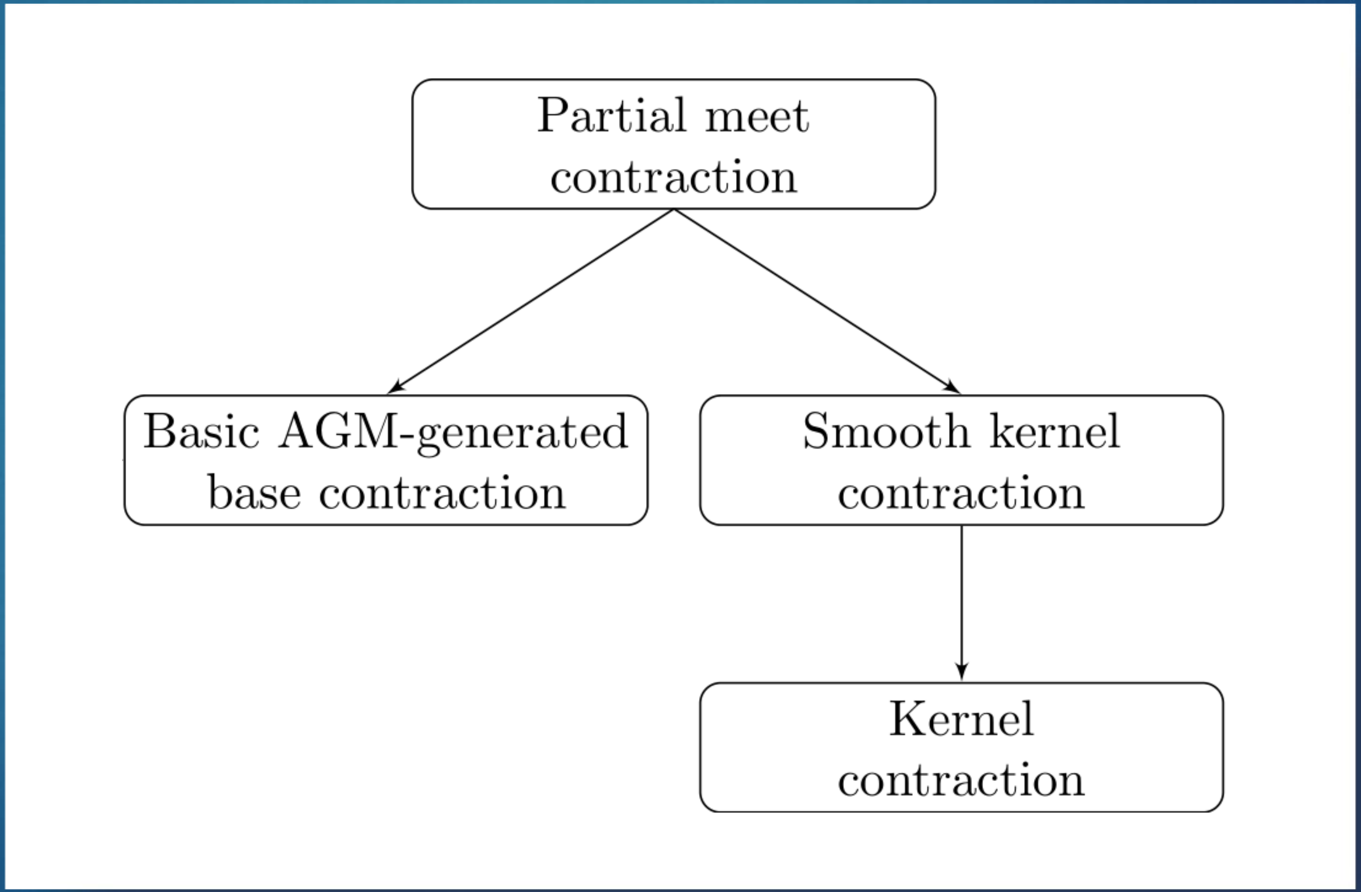
If $\alpha \notin Cn(\varnothing)$, then $\alpha \notin Cn(A - \alpha)$ **(success)**

$A - \alpha \subseteq A$ **(inclusion)**

If $\beta \in A$ and $\beta \notin A - \alpha$ then $A - \alpha \nvdash \alpha \vee \beta$. **(disjunctive elimination)**

If $\vdash \alpha \leftrightarrow \beta$, then $A - \alpha = A - \beta$ **(extensionality)**

Relation between the different base contraction functions

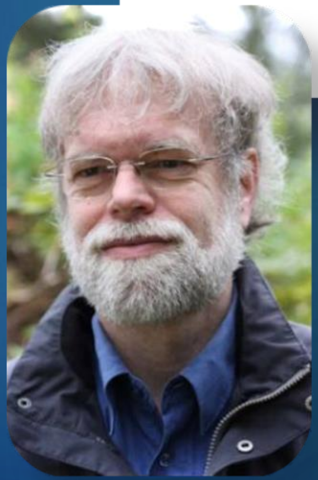


Local Change

Main idea: Not all of the agent's belief are relevant.

Example: I believe it is always raining in Holland.
I see it is a sunny day in Amsterdam $\Rightarrow$ revision

The revision can be done without checking consistency with irrelevant beliefs.



Some Challenges for Belief Bases

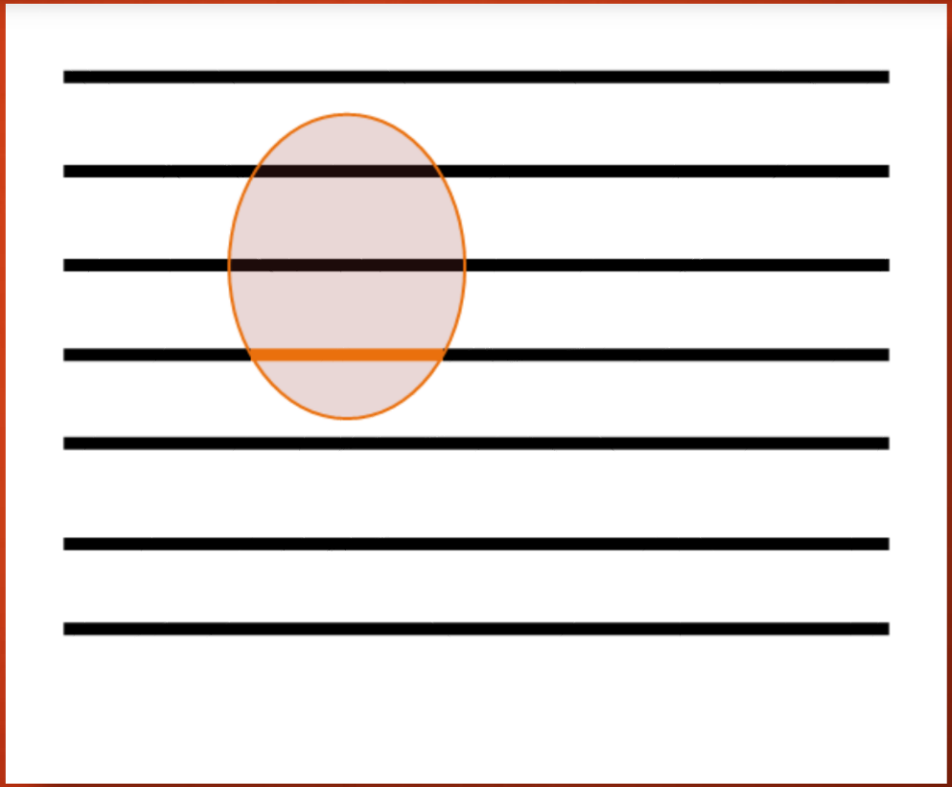
- Complete the theoretical models
- Understand better the concept of minimal change.
- Understand how the model works at the supplementary level.
- Implement (outside the toys problems) belief base change for resource-bounded agents.

Extensions

ITERATED MODELS

Iteration

An AGM contraction or revision takes us from a belief set to a new belief set.



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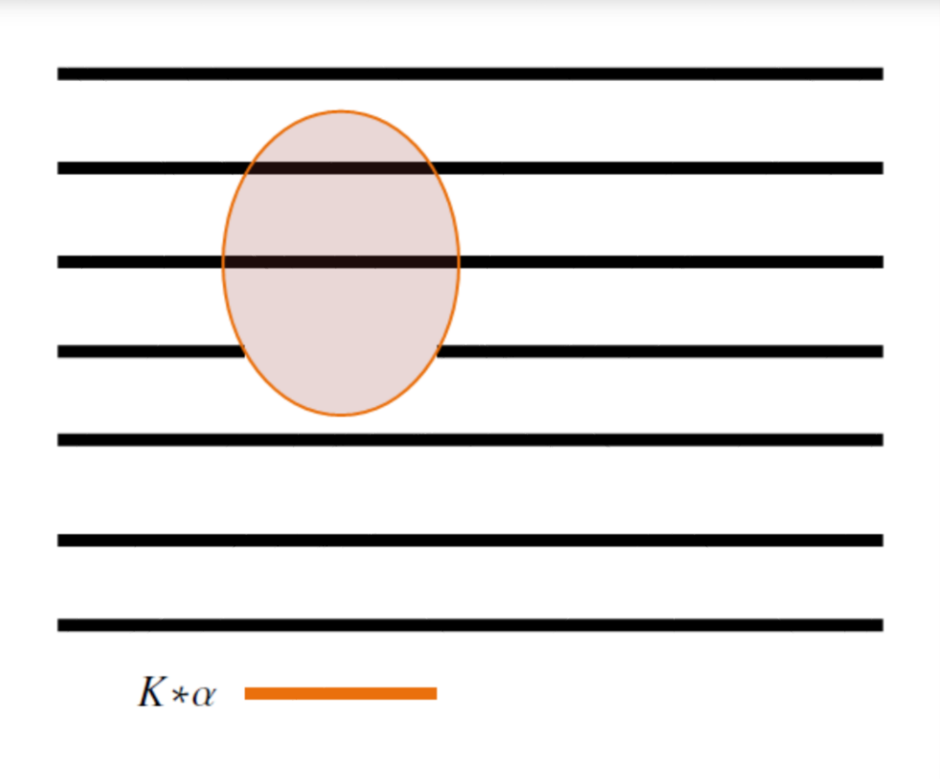
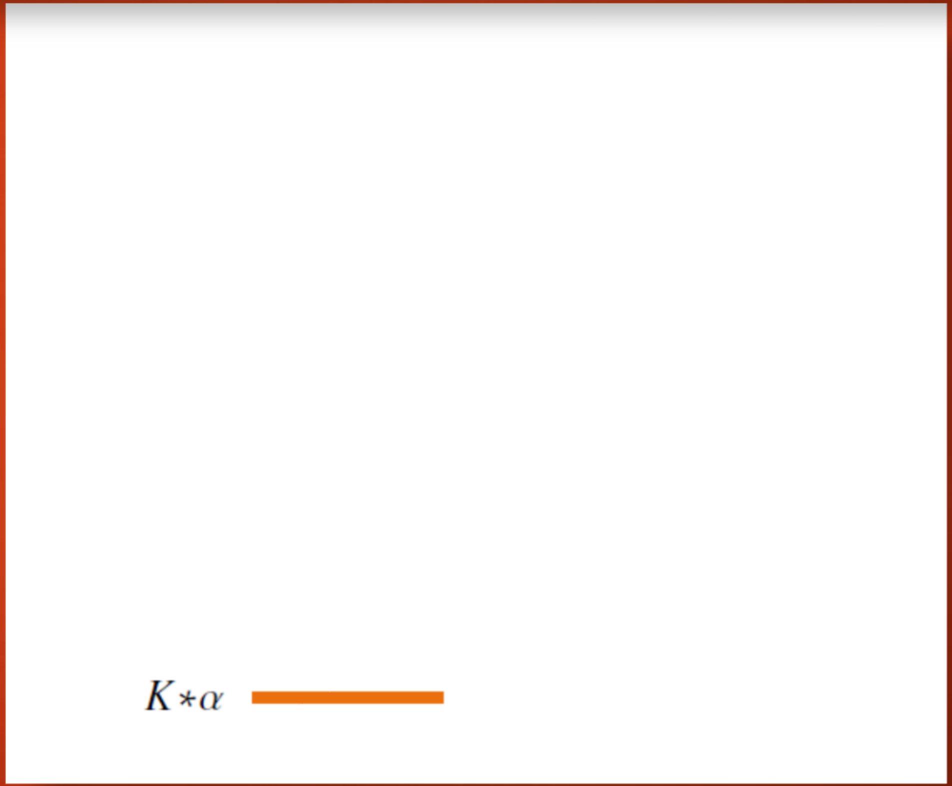


Iteration

However, it does not provide a new selection mechanism to be used for further changes of the new belief set.



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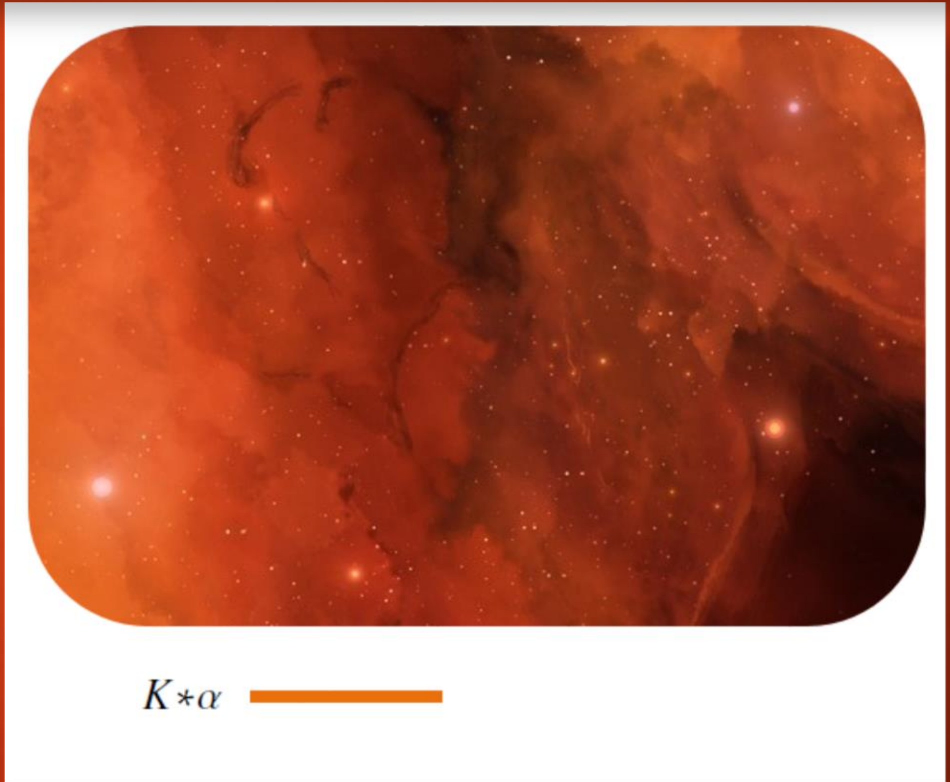


Iteration

The problem of constructing models that allow for iterated change is probably the most studied problem in the literature on belief change.

Major Classes of Iterated Operators

Hans Rott has identified at least Twenty-Seven Iterated Theory Change Operators.

Belief States

Furthermore, the operation of change has to yield a complete such belief state representation as its outcome, not merely a new belief set.

There are several ways to represent such an extended epistemic state. The most common of these is a preorder on the set of possible worlds, or equivalently a complete sphere system.

The belief set can be inferred from this preorder; it is simply the intersection of the worlds in the highest equivalence class (innermost sphere).

An operation of change gives rise to a new preorder (sphere system), from which the new belief set can be inferred, and which can in its turn be subject to further changes.

Major Classes of Iterated Operators

We can divide iterable operators into three classes according to their ability to remember and to take the revision history into account :

Operators with full memory: In this case the full history of changes is conserved, so that rollbacks of previous changes are possible.

Major Classes of Iterated Operators

We can divide iterable operators into three classes according to their ability to remember and to take the revision history into account :

Operators without memory: In this case, each belief set is revised in a predetermined way, independently of how it was obtained:

If $\Psi \circ \alpha$ and $\Upsilon \circ \alpha$ have the same belief set, then so have $\Psi \circ \alpha \circ \beta$ and $\Upsilon \circ \alpha \circ \beta$.

Major Classes of Iterated Operators

We can divide iterable operators into three classes according to their ability to remember and to take the revision history into account :

Operators with partial memory: In this case it makes a difference for future revisions how a belief set was arrived at, but the information remembered is not sufficient to identify the previous states.
Most of the proposed iterable revision operators are of this type.

AGM Revision Postulates for Belief States

Closure $B(\Psi - \alpha) = Cn(\Psi - \alpha)$

Success If $\nvdash \alpha$, then $B(\Psi - \alpha) \nvdash \alpha$.

Inclusion $B(\Psi - \alpha) \subseteq B(\Psi)$.

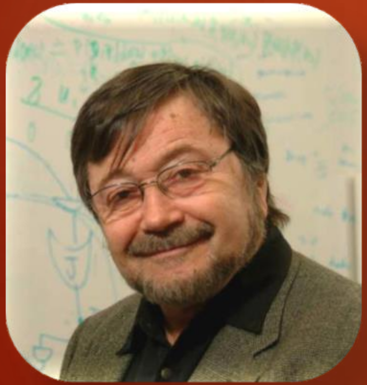
Vacuity If $B(\Psi) \nvdash \alpha$, then $B(\Psi) \subseteq B(\Psi - \alpha)$.

Extensionality If $\vdash \alpha \leftrightarrow \beta$ then $B(\Psi - \alpha) = B(\Psi - \beta)$.

Recovery $B(\Psi) \subseteq (B(\Psi - \alpha)) + \alpha$.

Conjunctive factoring $B(\Psi - (\alpha \wedge \beta)) = \begin{cases} B(\Psi - \alpha), \text{ or} \\ B(\Psi - \beta), \text{ or} \\ B(\Psi - \alpha) \cap B(\Psi - \beta) \end{cases}$

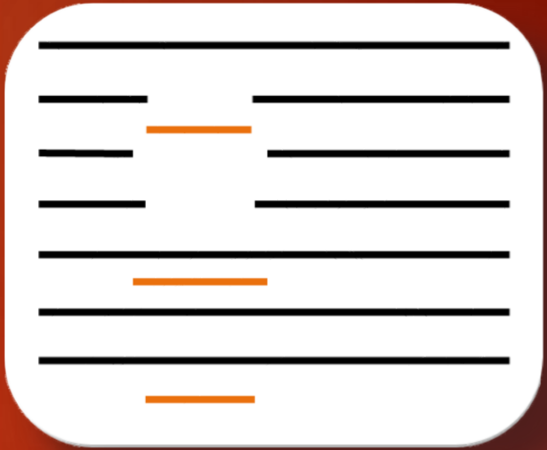
DP Postulates



Darwiche and Pearl proposed the following conditions for iteration

(DP1*) If $\beta \vdash \alpha$, then $(\Psi * \alpha) * \beta = \Psi * \beta$.
(DP2*) If $\beta \vdash \neg\alpha$, then $(\Psi * \alpha) * \beta = \Psi * \beta$.
(DP3*) If $B(\Psi * \beta) \vdash \alpha$, then $(\Psi * \alpha) * \beta \vdash \alpha$.
(DP4*) If $B(\Psi * \beta) \nvdash \neg\alpha$, then $(\Psi * \alpha) * \beta \nvdash \neg\alpha$

(DPR1*) If $\alpha \in \omega_1$ and $\alpha \in \omega_2$, then $\omega_1 \leq_\Psi \omega_2$ if and only if $\omega_1 \leq_{\Psi * \alpha} \omega_2$.
(DPR2*) If $\neg\alpha \in \omega_1$ and $\neg\alpha \in \omega_2$, then $\omega_1 \leq_\Psi \omega_2$ if and only if $\omega_1 \leq_{\Psi * \alpha} \omega_2$.
(DPR3*) If $\alpha \in \omega_1$, $\neg\alpha \in \omega_2$ and $\omega_1 <_\Psi \omega_2$, then $\omega_1 <_{\Psi * \alpha} \omega_2$.
(DPR4*) If $\alpha \in \omega_1$, $\neg\alpha \in \omega_2$ and $\omega_1 \leq_\Psi \omega_2$, then $\omega_1 \leq_{\Psi * \alpha} \omega_2$.

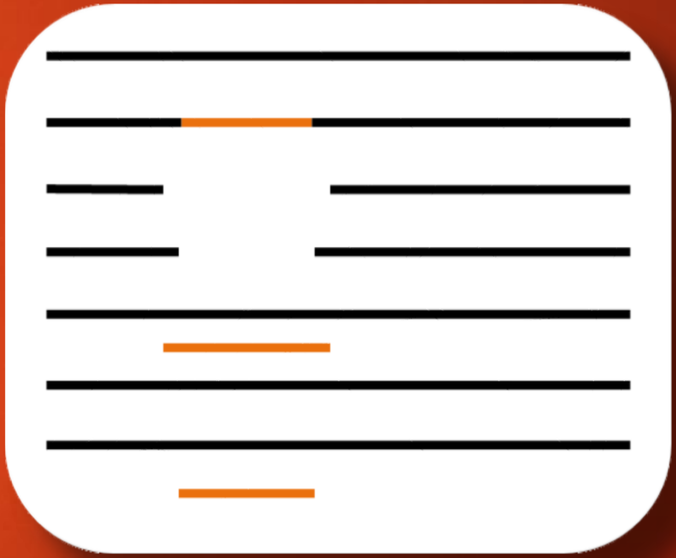


DP Postulates

Jin & Thielscher and Booth & Meyer have pointed out that these postulates are too permissive.
They proposed the following additional condition:

(P*) If $B(\Psi * \alpha) \nvdash \neg\beta$ then $B((\Psi * \beta) * \alpha) \vdash \beta$.

(PR*) If $\alpha \in \omega_1$, $\neg\alpha \in \omega_2$ and $\omega_1 \leq_\Psi \omega_2$, then $\omega_1 <_{\Psi * \alpha} \omega_2$.



Satisfies (DP1*) - (DP4*) but not (P*)

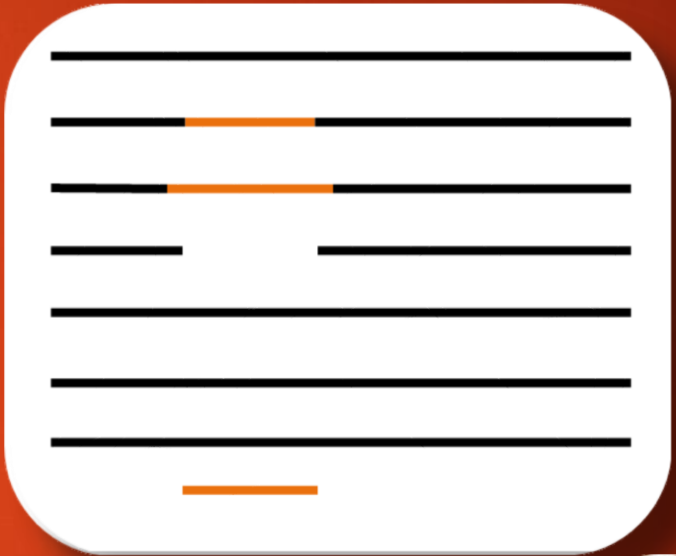
Three Classes of Operators with Partial Memory

Conservative revision, originally called natural revision, has been studied by Boutilier. This operation is conservative in the sense that it only makes the minimal changes of the preorder that are needed to accept the input.

In revision by α, the maximal α-worlds are moved to the top of the preorder which is otherwise left unchanged.

(Nat*) If $B(\Psi * \alpha) \vdash \neg\beta$, then $B((\Psi * \alpha) * \beta) = B(\Psi * \beta)$.

(NatR*) If $B(\Psi * \alpha) \not\vDash \omega_1$ and $B(\Psi * \alpha) \not\vDash \omega_2$, then $\omega_1 \leq_\Psi \omega_2$ iff $\omega_1 \leq_{\Psi * \alpha} \omega_2$

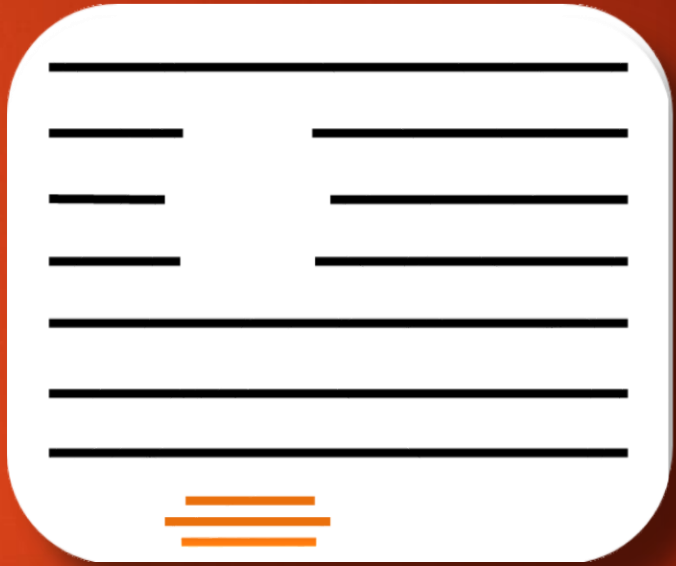


Three Classes of Operators with Partial Memory

Moderate revision, also called lexicographic revision, was originally studied by Nayak. When revising by α it rearranges the preorder by putting the α-worlds at top (but conserving their relative order) and the $\neg\alpha$-worlds at bottom (but conserving their relative order).

(Lex*) If $\beta \nvdash \neg\alpha$, then $B((\Psi * \alpha) * \beta) \vdash \alpha$.

(LexR*) If $\alpha \in \omega_1$ and $\neg\alpha \in \omega_2$, then $\omega_1 <_{\Psi * \alpha} \omega_2$



Three Classes of Operators with Partial Memory

Radical revision Proposed by Segerberg. Is similar to moderate revision, but it differs in making the new belief irrevocable, i.e., impossible to remove.

In radical revision by α, the relative order of the α-worlds is retained whereas the $\neg\alpha$-worlds are removed from the preorder, thus becoming inaccessible.

(Irr*) $B((\Psi * \alpha) * \neg\alpha) \vdash \bot$.

(CRIrr*) For all $\omega_i \in \mathfrak{W}$, then $\omega_i \subset B((\Psi * \alpha) * \neg\alpha)$.

AGM Contraction Postulates for Belief States

Closure: $B(\Psi * \alpha) = Cn(\Psi * \alpha)$
Success: $\alpha \in B(\Psi * \alpha)$.
Inclusion: $B(\Psi * \alpha) \subseteq B(\Psi + \alpha)$.
Vacuity: If $\neg\alpha \notin \Psi$, then $B(\Psi + \alpha) \subseteq B(\Psi * \alpha)$.
Consistency: If $\nvdash \neg\alpha$, then $B(\Psi * \alpha) \nvdash \bot$.
Extensionality: If $\vdash \alpha \leftrightarrow \beta$, then $B(\Psi * \alpha) = B(\Psi * \beta)$.

Disjunctive factoring $B(\Psi * (\alpha \wedge \beta)) = \begin{cases} B(\Psi * \alpha), \text{ or} \\ B(\Psi * \beta), \text{ or} \\ B(\Psi * \alpha) \cap B(\Psi * \beta) \end{cases}$

DP Contraction Postulates (v1)

(DP1–) If $\beta \vdash \neg\alpha$, then $(\Psi - \alpha) * \beta = \Psi * \beta$.
(DP2–) If $\beta \vdash \alpha$, then $(\Psi - \alpha) * \beta = \Psi * \beta$.
(DP3–) If $B(\Psi * \beta) \vdash \neg\alpha$, then $B((\Psi - \alpha) * \beta) \vdash \neg\alpha$.
(DP4–) If $B(\Psi * \beta) \nvdash \alpha$, then $B((\Psi - \alpha) * \beta) \nvdash \alpha$

(DPR1–) If $\neg\alpha \in \omega_1$ and $\neg\alpha \in \omega_2$, then $\omega_1 \leq_\Psi \omega_2$ if and only if $\omega_1 \leq_{\Psi-\alpha} \omega_2$.
(DPR2–) If $\alpha \in \omega_1$ and $\alpha \in \omega_2$, then $\omega_1 \leq_\Psi \omega_2$ if and only if $\omega_1 \leq_{\Psi-\alpha} \omega_2$.
(DPR3–) If $\neg\alpha \in \omega_1$, $\alpha \in \omega_2$ and $\omega_1 <_\Psi \omega_2$, then $\omega_1 <_{\Psi-\alpha} \omega_2$.
(DPR4–) If $\neg\alpha \in \omega_1$, $\alpha \in \omega_2$ and $\omega_1 \leq_\Psi \omega_2$, then $\omega_1 \leq_{\Psi-\alpha} \omega_2$.




DP Contraction Postulates (v2)

(C8) If $\neg\alpha \vdash \gamma$ then $B(\Psi - (\alpha \vee \beta)) \vdash B(\Psi - \alpha)$
$\Leftrightarrow B(\Psi - \gamma - (\alpha \vee \beta)) \vdash B(\Psi - \gamma - \alpha)$

(C9) If $\gamma \vdash \alpha$ then $B(\Psi - (\alpha \vee \beta)) \vdash B(\Psi - \alpha)$
$\Leftrightarrow B(\Psi - \gamma - (\alpha \vee \beta)) \vdash B(\Psi - \gamma - \alpha)$

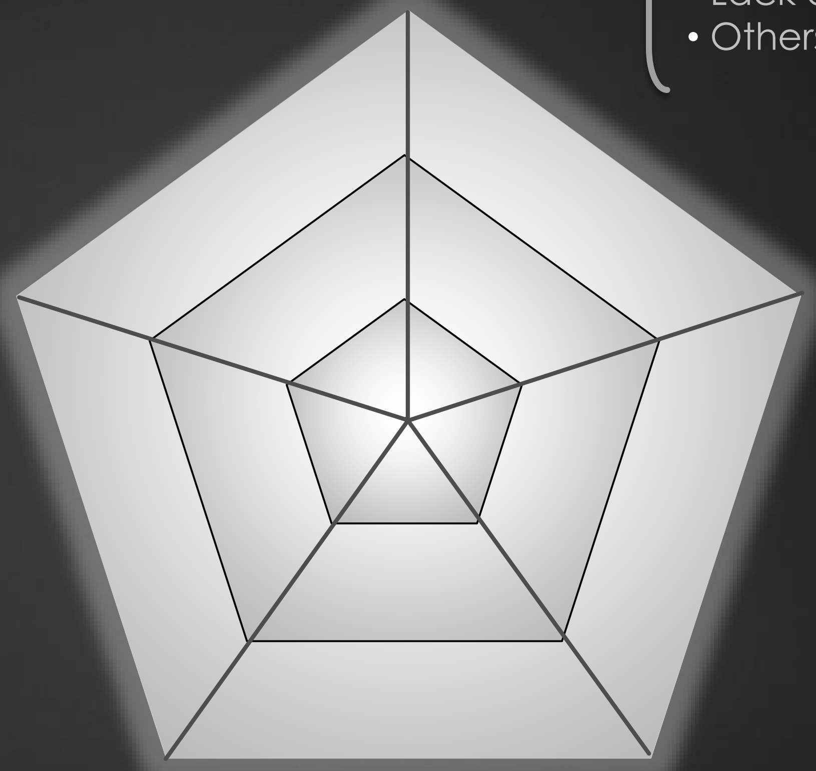
(C10) If $\neg\beta \vdash \gamma$ then $B(\Psi - \gamma - (\alpha \vee \beta)) \vdash B(\Psi - \gamma - \alpha)$
$\Rightarrow B(\Psi - (\alpha \vee \beta)) \vdash B(\Psi - \alpha)$

(C11) If $\gamma \vdash \beta$ then $B(\Psi - \gamma - (\alpha \vee \beta)) \vdash B(\Psi - \gamma - \alpha)$
$\Rightarrow B(\Psi - (\alpha \vee \beta)) \vdash B(\Psi - \alpha)$

If $\omega, \omega' \in [[\gamma]]$ *then* $\omega \leq_\Psi \omega' \Leftrightarrow \omega \leq_{\Psi - \gamma} \omega'$
If $\omega, \omega' \in [[\neg\gamma]]$ *then* $\omega \leq_\Psi \omega' \Leftrightarrow \omega \leq_{\Psi - \gamma} \omega'$
If $\omega \in [[\neg\gamma]]$ *and* $\omega' \in [[\gamma]]$ *then* $\omega <_\Psi \omega' \Rightarrow \omega <_{\Psi - \gamma} \omega'$
If $\omega \in [[\neg\gamma]]$ *and* $\omega' \in [[\gamma]]$ *then* $\omega \leq_\Psi \omega' \Rightarrow \omega \leq_{\Psi - \gamma} \omega'$

AGM

30 years



Foundation and Criticisms

- Foundations
- Recovery
- Success
- The use of Belief Sets
- Lack of Information in the belief sets
- Others

Extensions

- Belief Bases
- Iteration
- Multiple Change
- Probability and possibility
- Ranking
- Extension on the language
- Others

Computability and Implementations

Alternative Operators

- Update
- Non-Priorityzed Change
- Withdrawal Functions
- Change in the strenght of belief
- Abductive Models
- Merging
- Others

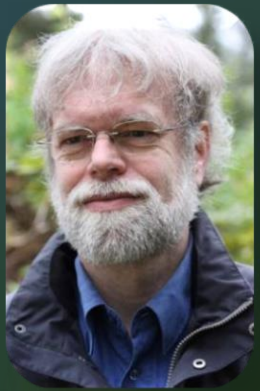
Applications and Connections

Alternative Models

Example: Selective Revision

DEFINITION: Let K be a belief set, $*$ a *partial meet revision* for K and f a function from $\mathcal{L}$ to $\mathcal{L}$. The *selective revision* $\circ$, based on $*$ and f, is the operation such that for all sentences α:

$$K \circ \alpha = K * f(\alpha)$$



Example: Selective Revision

Plausible properties for f:

implication $\vdash \alpha \rightarrow f(\alpha)$.

idempotence $\vdash f(f(\alpha)) \leftrightarrow f(\alpha)$.

extensionality If $\vdash \alpha \leftrightarrow \beta$ then $\vdash f(\alpha) \leftrightarrow f(\beta)$.

consistency preservation If $\nvdash \neg\alpha$, then $\nvdash \neg f(\alpha)$.

maximality $\vdash f(\alpha) \leftrightarrow \alpha$.

weak maximality If $K \nvdash \neg\alpha$, then $\vdash f(\alpha) \leftrightarrow \alpha$.

Example: Selective Revision

Weak success If $K \nvdash \neg\alpha$, then $K \circ \alpha \vdash \alpha$.

Proxy success There is a sentence β, such that $K \circ \alpha \vdash \beta$, $\vdash \alpha \rightarrow \beta$, and $K \circ \alpha = K \circ \beta$

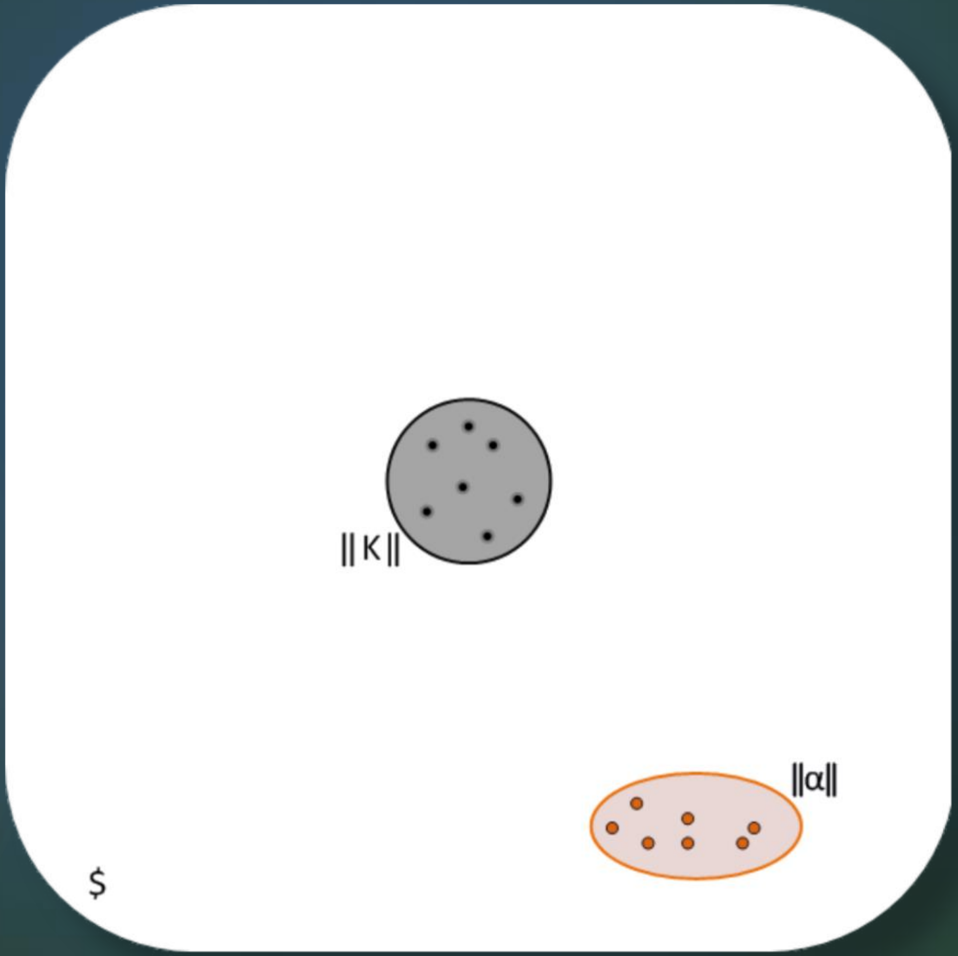
Weak proxy success There is a sentence β, such that $K \circ \alpha \vdash \beta$ and $K \circ \alpha = K \circ \beta$.

Consistent expansion If $K \nsubseteq K \circ \alpha$ then $K \cup (K \circ \alpha) \vdash \perp$.

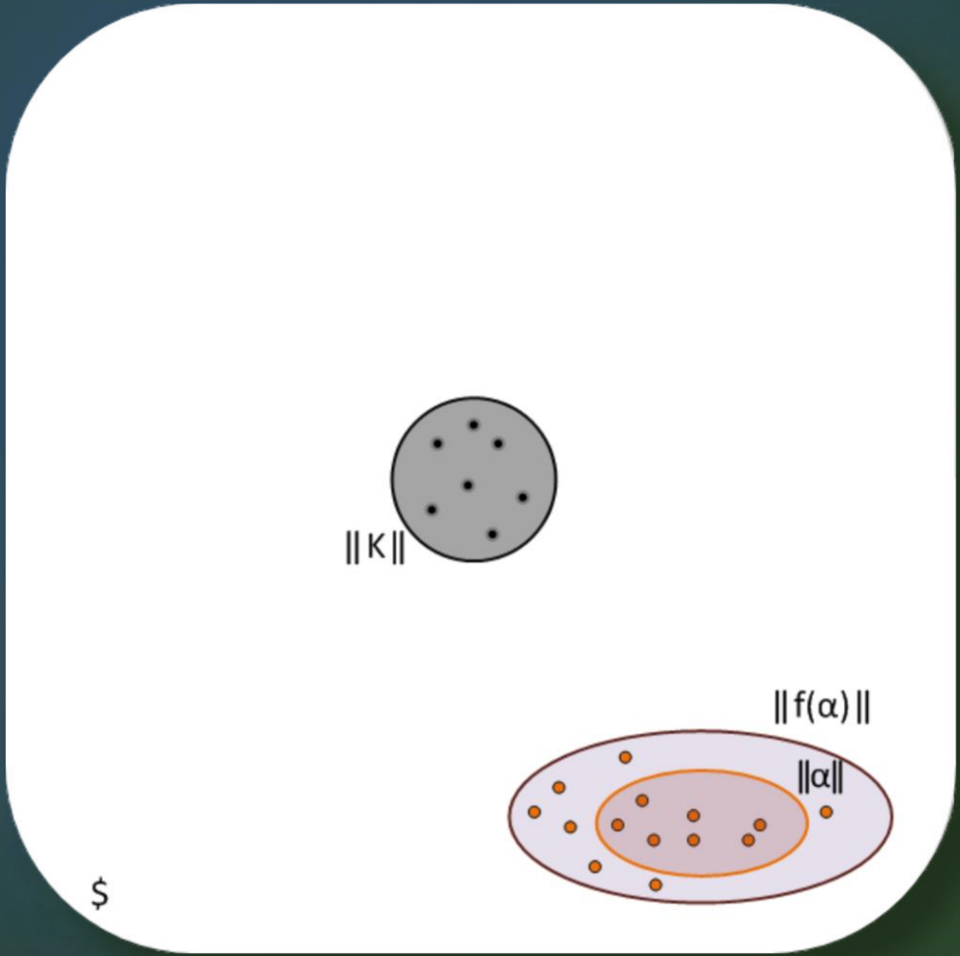
Example: Selective Revision

$$\begin{array}{lcl} \circ & & f \\ \left.\begin{array}{l} \text{Closure} \\ \text{Inclusion} \\ \text{Vacuity} \\ \text{Consistency} \\ \text{Extensionality} \\ \text{C. Expansion} \\ \text{P. Success} \end{array}\right\} & \begin{array}{c} K \circ \alpha = K * f(\alpha) \\ \Longleftrightarrow \end{array} & \left\{\begin{array}{l} \text{Extensionality} \\ \text{C. Preservation} \\ \text{W. Maximality} \\ \text{Idempotence} \\ \text{Implication} \end{array}\right. \end{array}$$

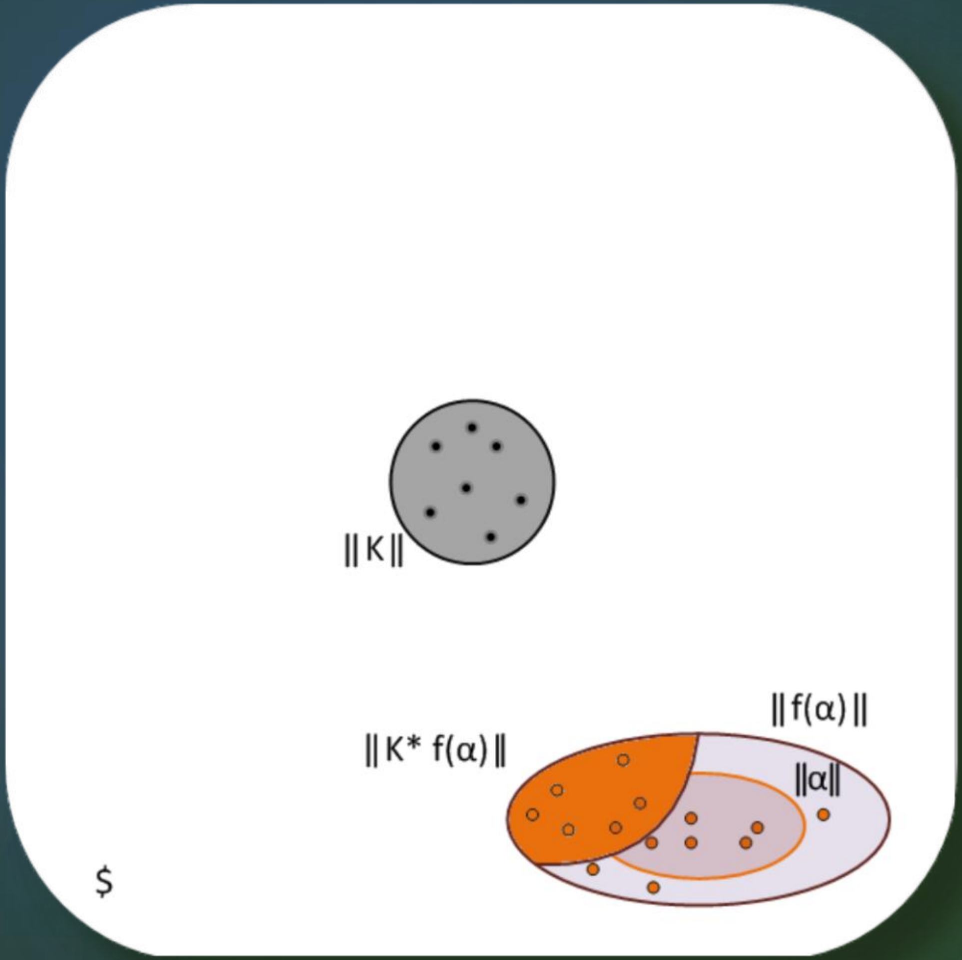
Example: Selective Revision



Example: Selective Revision



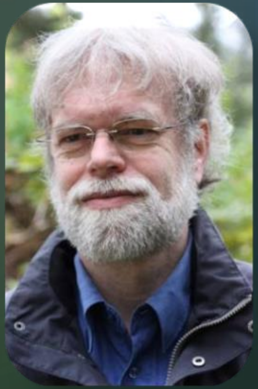
Example: Selective Revision



Example: Credibility-Limited Revision

DEFINITION: Let K be a belief set. The operation $\circ$ on K is a credibility-limited revision on K if and only if there is an AGM revision $*$ on K and a set $\mathcal{C}$ of sentences such that for all sentences α:

$$K \circ \alpha = \begin{cases} K * \alpha & \text{if } \alpha \in \mathcal{C} \\ K & \text{otherwise} \end{cases}$$




Example: Credibility-Limited Revision

Plausible properties for $\mathcal{C}$

Closure under Logical Equivalence If $\vdash \alpha \leftrightarrow \beta$ and $\alpha \in \mathcal{C}$, then $\beta \in \mathcal{C}$.

Single sentence closure If $\alpha \in \mathcal{C}$, then $Cn(\{\alpha\}) \subseteq \mathcal{C}$.

Disjunctive completeness If $\alpha \vee \beta \in \mathcal{C}$, then either $\alpha \in \mathcal{C}$ or $\beta \in \mathcal{C}$.

Negation completeness $\alpha \in \mathcal{C}$ or $\neg\alpha \in \mathcal{C}$.

Element consistency If $\alpha \in \mathcal{C}$, then $\alpha \nvdash \perp$.

Expansive credibility If $K \nvdash \alpha$, then $\neg\alpha \in \mathcal{C}$.

Example: Credibility-Limited Revision

Relative success $\alpha \in K \circ \alpha$ or $K \circ \alpha = K$.

Disjunctive success $\alpha \in K \circ \alpha$ or $\neg \alpha \in K \circ \alpha$.

Strict improvement If $\alpha \in K \circ \alpha$ and $\alpha \to \beta$, then $\beta \in K \circ \beta$.

Regularity If $\beta \in K \circ \alpha$ then $\beta \in K \circ \beta$.

Strong regularity If $\neg \beta \notin K \circ \alpha$ then $\beta \in K \circ \beta$.

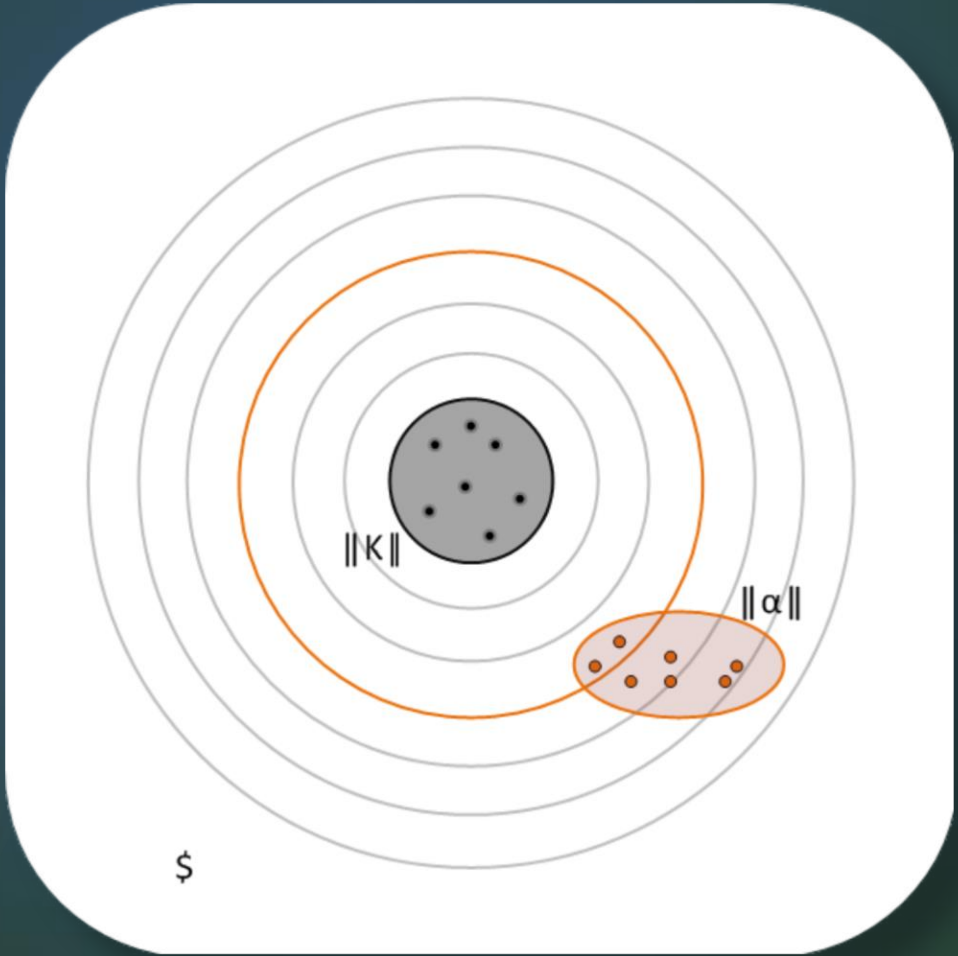
Strong consistency $K \circ \alpha \neq K_{\perp}$.

Consistency preservation If $K \neq K_{\perp}$ then $K \circ \alpha \neq K_{\perp}$.

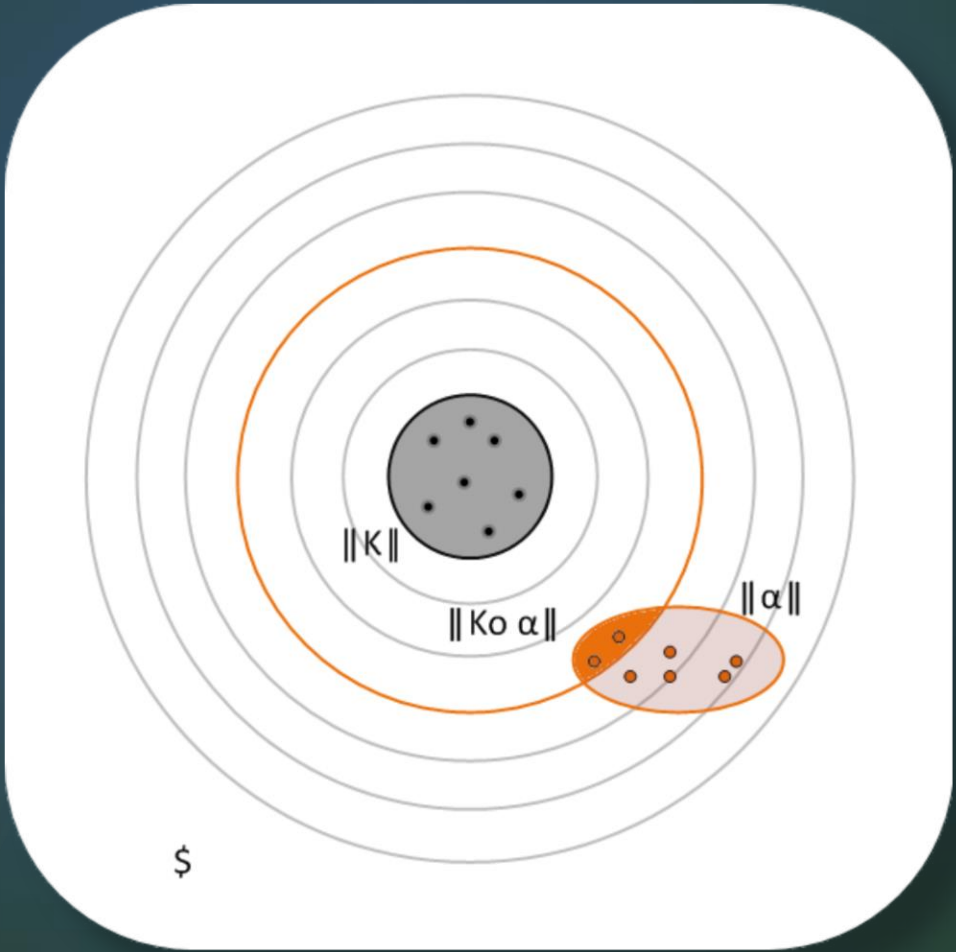
Disjunctive constancy If $K \circ \alpha = K \circ \beta = K$ then $K \circ (\alpha \vee \beta) = K$.

Consistent expansion If $K \nsubseteq K \circ \alpha$ then $K \cup (K \circ \alpha) \vdash \perp$.

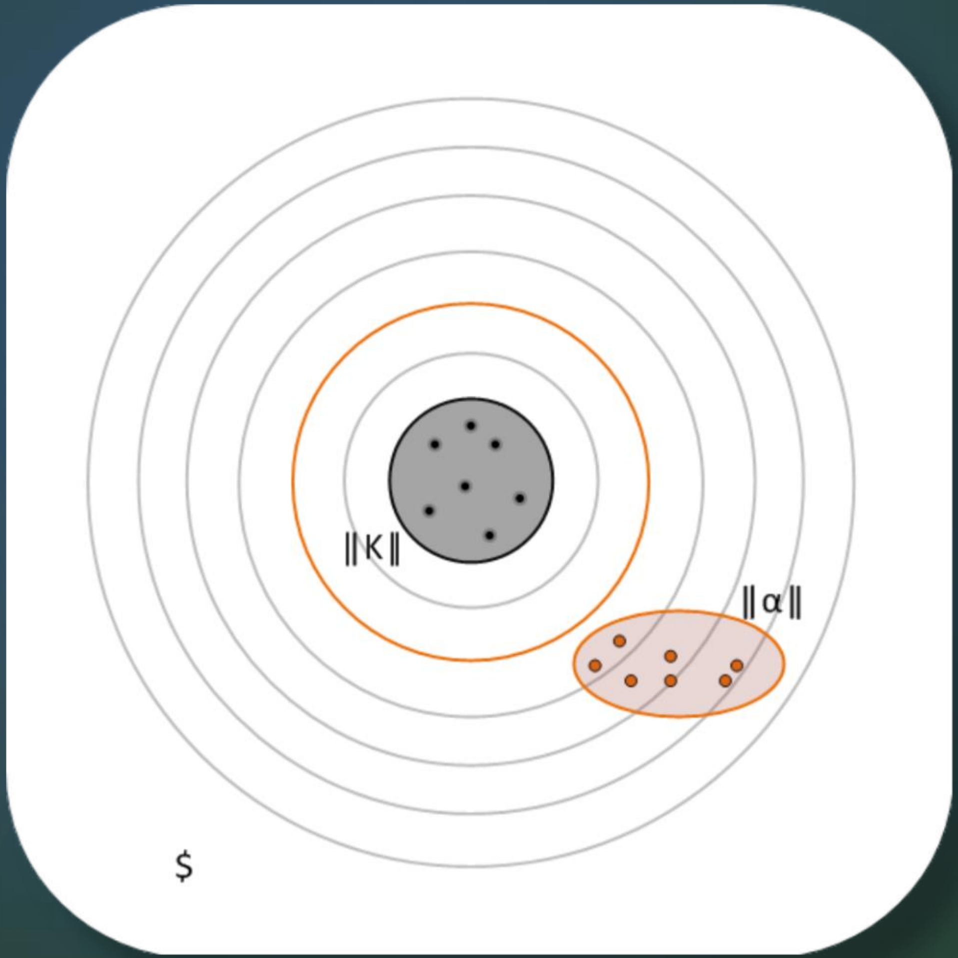
Example: Credibility-Limited Revision



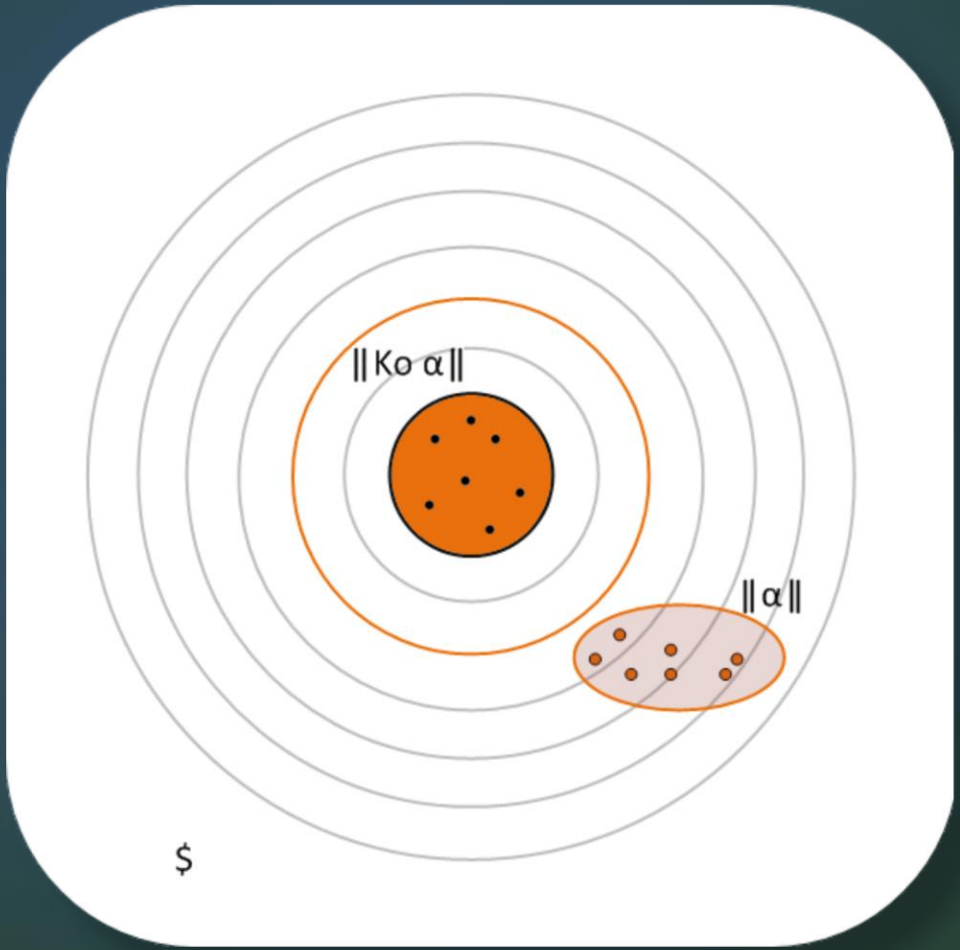
Example: Credibility-Limited Revision



Example: Credibility-Limited Revision



Example: Credibility-Limited Revision



Improvement

Also it was proposed by Konieczny and Pino Perez

- Weak primacy of update.
- The plausibility of the new information must be increased after the improvement
- This can facilitate the acceptance of α in later, additional operations, so that we can have $\varphi \circ \alpha \nvdash \alpha$, but $\varphi \circ \alpha \circ \alpha \vdash \alpha$,
- *(Imp):* Let $\mu \models \alpha$ and $\phi \models \neg\alpha$. If $\phi \ll_{\Psi} \mu$, then $\mu \leq_{\Psi \circ \alpha} \phi$.

Improvement

(I1) There exists n such that $\Psi \circ^n \alpha \vdash \alpha$
(I2) If $\Psi \wedge \alpha \nvdash \bot$, then $\Psi \star \alpha \equiv \Psi \wedge \alpha$
(I3) If $\alpha \nvdash \bot$, then $\Psi \circ \alpha \nvdash \bot$
(I4) For any positive integer n if $\alpha_i \equiv \beta_i$ for all $i \leq n$ then

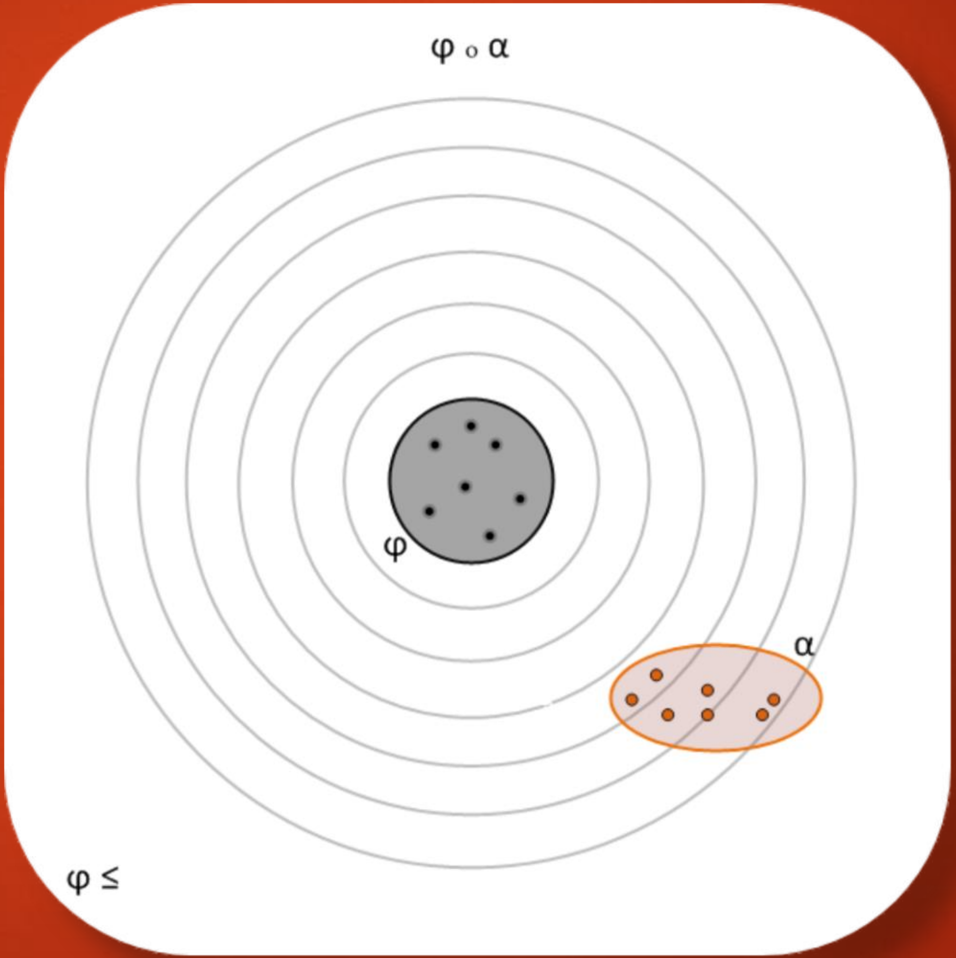
$$\Psi \circ \alpha_1 \circ \cdots \circ \alpha_n \equiv \Psi \circ \beta_1 \circ \cdots \circ \beta_n$$

(I5) $(\Psi \star \alpha) \wedge \beta \vdash \Psi \star (\alpha \wedge \beta)$
(I6) If $(\Psi \star \alpha) \wedge \beta \nvdash \bot$, then $\Psi \star (\alpha \wedge \beta) \vdash (\Psi \star \alpha) \wedge \beta$

(I7) If $\alpha \vdash \beta$ then $(\Psi \circ \beta) \star \alpha \equiv \Psi \star \alpha$
(I8) If $\alpha \vdash \neg\beta$ then $(\Psi \circ \beta) \star \alpha \equiv \Psi \star \alpha$
(I9) If $\Psi \star \alpha \nvdash \neg\beta$ then $(\Psi \circ \beta \star \alpha) \vdash \beta$
(I10) If $\Psi \star \alpha \vdash \neg\beta$ then $(\Psi \circ \beta \star \alpha) \nvdash \beta$
(I11) If $\Psi \star \alpha \vdash \neg\beta$, $\alpha \wedge \beta \nvdash \bot$ and $\alpha \lll_\Psi \alpha \wedge \beta$
then $(\Psi \circ \beta) \star \alpha \nvdash \neg\beta$

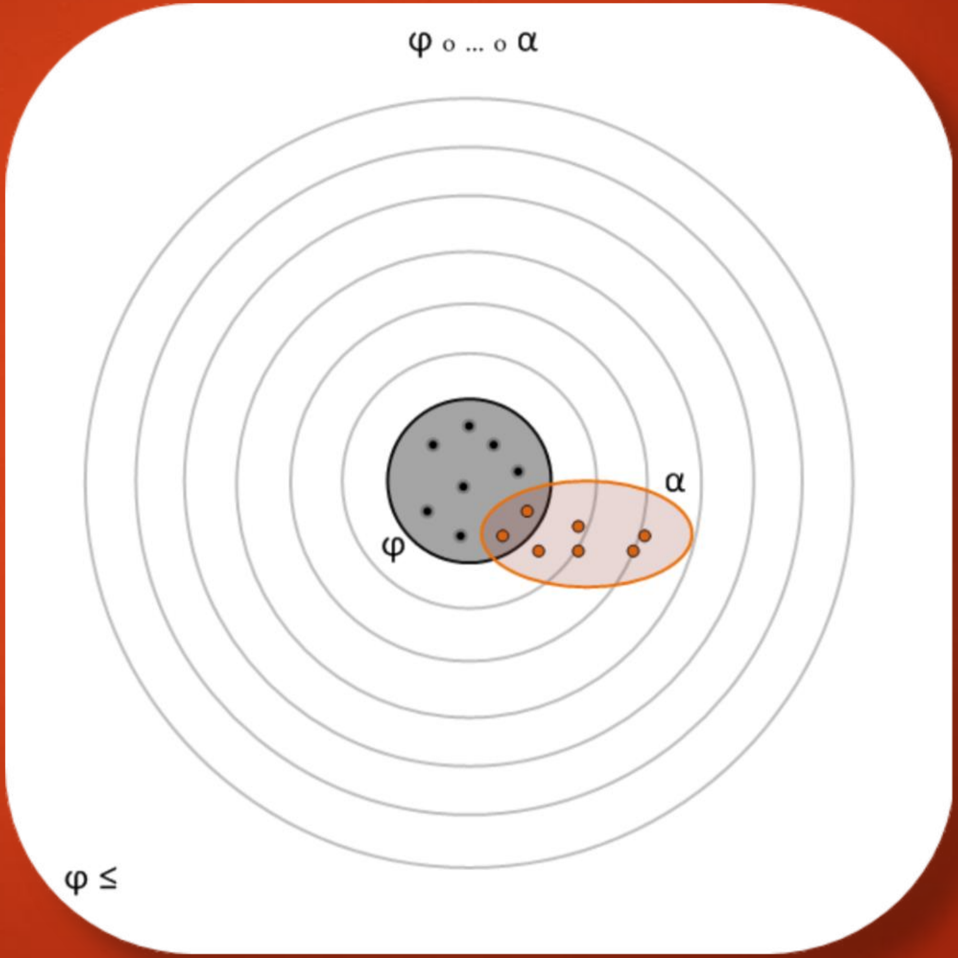
Improvement

Improvement



Improvement

Improvement



CLIO (Credibility-Limited Improvement Operators

Perspectives

Hot topics

- Iteration
- Ontologies
- Horn Contraction
- Argumentation
- Paraconsistent Models
- Implementations
- … (incomplete list)

Ontologies

- An ontology in computer science is an explicit, formal specification of the terms of a domain of application, along with the relations among these terms.
- An ontology provides a (structured) vocabulary which forms the basis for the representation of general knowledge.
- Ontologies have found extensive application in Artificial Intelligence and the Semantic Web, as well as in areas such as software engineering, bioinformatics, and database systems

Ontologies: Description Logics

Research in ontologies in Artificial Intelligence has focussed on description logics (DL), a (decidable) fragment of first order logic.

Two components,

TBox , for expressing concepts.
Characterises a domain of application.

ABox , that contains assertions about specific individuals and instances.
Contains information on a specific instance of a domain

Ontologies and belief revision

Crucially, an ontology will be expected to evolve,

- either as domain information is corrected and refined, or
- in response to a change in the underlying domain.

In a description logic, such change may come in two different forms:

- the background knowledge, traditionally stored in the TBox, may require modification, or
- the ground facts or data, traditionally stored in the ABox, may be modified.

Horn Belief Change

Address belief change in the expressively weaker language of *Horn clauses*, where a Horn clause can be written as a rule in the form $a_1 \wedge a_2 \wedge \cdots \wedge a_n \rightarrow a$ for $n \geq 0$, and where a, a_i $(1 \leq i \leq n)$ are atoms.

An agent's beliefs are represented by a Horn clause knowledge base, and the input is a conjunction of Horn clauses.

Horn Belief Change

This topic is interesting for several reasons. It sheds light on the theoretical underpinnings of belief change, in that it weakens the assumption that the underlying logic contains propositional logic. As well, Horn clauses have found extensive use in artificial intelligence and database theory, in areas such as logic programming, truth maintenance systems, and deductive databases.

Argumentation

Argumentation deals with strategies agents employ for their own reasoning or to change the beliefs of other agents

Argument: A set of statements, composed of three parts: a set of premises, a conclusion, and an inference.

Defeasibility: When a conclusion is defeated by new information, that reasoning is defeasible

The attack/refutation question: Arguments can attack or support other arguments

Argumentative Systems: A way to formalize common-sense reasoning. Formalisms that intend to support Argumentation and standardize it

Argumentation

Dung's Abstract Framework

- Abstract notion of argument and big possibility of extension
- Argumentation framework: **AF ⟨Arg, Attacks⟩**
- Attack relation between arguments: An argument **A** will be defeated if it is possible to find at least one defeater for it that is not defeated

Argumentation

Different ways of applying BR in Argumentation

- Changing by adding or deleting an argument or a set of them .
- Changing the attack (defeat) relation among arguments
- Changing the status of beliefs
- Changing the type of an argument (strict to defeasible, or vice versa)

Implementations

- Actually there exists very few “real” implementations.
- Is the future of Belief Revision in Computer Science (and its big challenge !).
- It will require to combine different models:
  - Multi-agents systems
  - Horn Belief Change
  - Argumentations (by means of programs like DeLP Defeasible Logic Programming)
  - Belief Bases
  - ...

Research in AGM

One usual way ...

- We start with an idea/intuition/problem in one of the five AGM presentation.
- After that we complete our intuition in the presentation selected
- Then we try to formalize it in one or some of the others presentations

One usual way ...

- From instance

- From postulates we try later to obtain a constructive method or a semantic

- From a constructive method sometimes we try to obtain the axiomatic (or at list a list of axioms that capture the behaviour of the model).

But …

- Sometimes a simple idea in one presentation require a hard work in the others …

- Example: Revision by comparison

"Revision by Comparison". Eduardo Fermé, Hans Rott. Artificial Intelligence. Elsevier Publisher. 157 pp 5 -- 47. 2004

Revision by Comparison

Key idea:

- **We use reference beliefs in order to determine the degree of entrenchment of the newly accepted piece of information.**

Or more easy …

- **accept β with a degree of plausibility that at least equals that of a.**

Revision by Comparison

The model in Epistemic Entrenchment:

Case	Entrenchment	Kind of change
The intended case: AGM-style revision	$\beta < \alpha$ and $\neg\beta < \alpha$	$K \circ_\alpha \beta = K * \beta$
The vacuous case: No change	$\alpha \leq \beta$	$K \circ_\alpha \beta = K$
The unsuccessful case: Withdrawal	$\alpha \leq \neg\beta$ and $\alpha < \top$	$K \circ_\alpha \beta = K \div \alpha$
The epistemic collapse:	$\top \leq \alpha$ and $\top \leq \neg\beta$	$K \circ_\alpha \beta = K_\perp$

The four basic cases.

Revision by Comparison

Finding an axiomatic

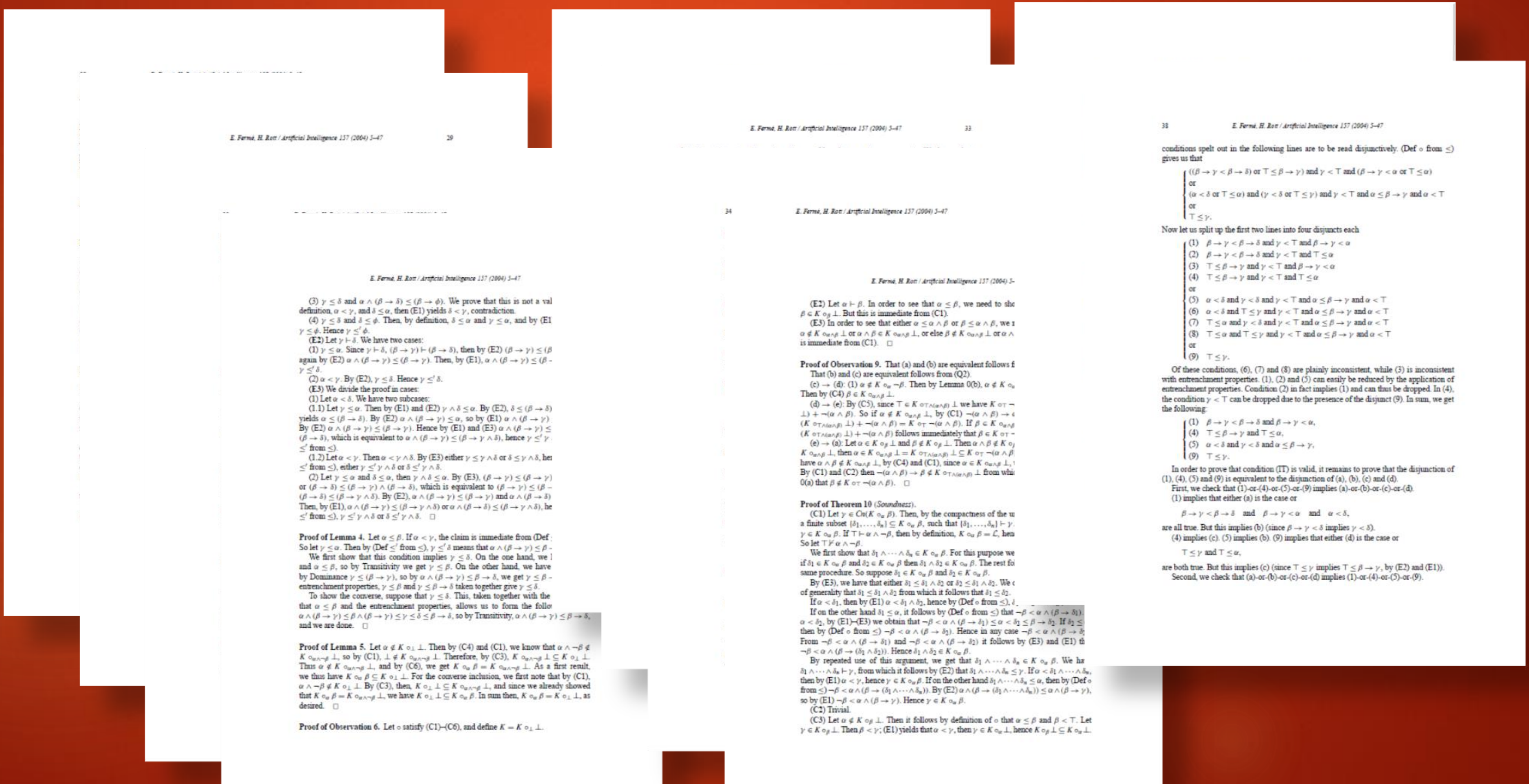
(C1) $K \circ_\alpha \beta = Cn(K \circ_\alpha \beta)$. (Closure)
(C2) If $Cn(\alpha) = Cn(\gamma)$ and $Cn(\beta) = Cn(\delta)$, then $K \circ_\alpha \beta = K \circ_\gamma \delta$. (Extensionality)
(C3) If $\alpha \notin K \circ_\beta \bot$, then $K \circ_\beta \bot \subseteq K \circ_\alpha \bot$. (Strong Inclusion)
(C4) If $\alpha \in K \circ_\alpha \bot$, then $\alpha \in K \circ_\beta \gamma$. (Irrevocability)
(C5) If $\alpha \in K \circ_{\alpha \wedge \neg\beta} \bot$, then $K \circ_\alpha \beta = (K \circ_{\alpha \wedge \neg\beta} \bot) + \beta$. (Reduction 1)
(C6) If $\alpha \notin K \circ_{\alpha \wedge \neg\beta} \bot$, then $K \circ_\alpha \beta = K \circ_{\alpha \wedge \neg\beta} \bot$. (Reduction 2)

Theorem 10 (Soundness). *Let $\leq$ be an entrenchment ordering that satisfies* (E1)–(E3). *Furthermore let $\circ$ be the entrenchment-based revision-by-comparison function defined by condition* (Def $\circ$ from $\leq$). *Then $\circ$ satisfies* (C1)–(C6). *Moreover, $K_\circ = K_\leq$, and $\leq$ can be retrieved from $\circ$ with the help of* (Def $\leq$ from $\circ$).

Theorem 11 (Completeness). *Let $\circ$ be a revision-by-comparison function satisfying* (C1)–(C6). *Then there is an entrenchment relation $\leq$ satisfying* (E1)–(E3) *such that $\circ$ can be represented as being generated from $\leq$ with the help of* (Def $\circ$ from $\leq$), *and $K_\leq = K_\circ$.*

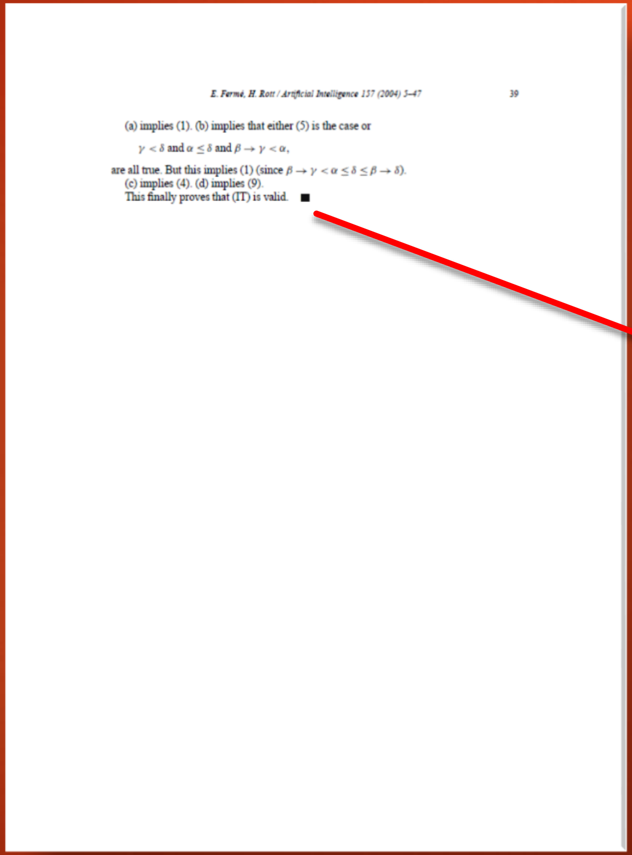
Revision by Comparison

Proving the axiomatic



Revision by Comparison

and after 3 years of hard work…

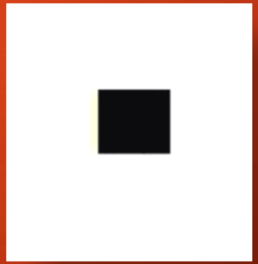


(a) implies (1). (b) implies that either (5) is the case or

$$\gamma < \delta \text{ and } \alpha \leq \delta \text{ and } \beta \to \gamma < \alpha,$$

are all true. But this implies (1) (since $\beta \to \gamma < \alpha \leq \delta \leq \beta \to \delta$).
(c) implies (4). (d) implies (9).
This finally proves that (IT) is valid. ■

The most exciting symbol in our área …



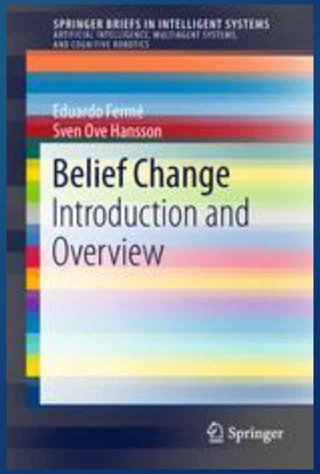
Conclusions

Conclusions

This tutorial was based on

Eduardo Fermé and Sven Ove Hansson.

1. "AGM 25 Years: Twenty-Five Years of Research in Belief Change" Journal of Philosophical Logic 40, (2) : 113-114. 2011
2. "*Belief Change: Introduction and Overview*". Springer Briefs in Computer Science Series. Springer. 2018.



Conclusions

- This tutorial was an attempt to summarize (in 4 hours) major developments in 35 years of AGM-inspired research.
- In preparing our works we benefited from the help of more than fifty colleagues who answered our queries and provided us with information.
- I also received many suggestions and material from friends and colleagues that help me to improve the presentation.
- In addition to its academic excellence, the belief revision community is a remarkably generous one.
- This bodes well for the future of belief revision research.

Thank you!

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