PROBLEM #1

MATRICES GIVEN: a, b & f

$$T_1^0 = A_1 = A_a = \begin{array}{c} \begin{array}{cccc} & & z_1 & o_1 \end{array} \\ \begin{pmatrix} C\theta_a & -S\theta_a & 0 & 0 \\ S\theta_a & C\theta_a & 0 & 0 \\ 0 & 0 & 1 & d_a \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

$a_1 C\theta_1 = 0$
$a_1 S\theta_1 = 0$] $a_1 = 0$

$d_1 = d_a$

$S\alpha_1 = 0$
$C\alpha_1 = 1$] $\alpha_1 = 0$

$\theta_1 = \theta_a$.

$q_1 = \theta_a$

→ THIS LINK IS REVOLUTE

$$T_2^0 = A_1 A_2 = A_a A_b = \begin{pmatrix} C\theta_a & -S\theta_a & 0 & 0 \\ S\theta_a & C\theta_a & 0 & 0 \\ 0 & 0 & 1 & d_a \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_b \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_2^0 = \begin{array}{c} \begin{array}{cccc} & & z_2 & o_2 \end{array} \\ \begin{pmatrix} C\theta_a & 0 & -S\theta_a & 0 \\ S\theta_a & 0 & C\theta_a & 0 \\ 0 & -1 & 0 & d_a + d_b \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

$a_2 C\theta_2 = 0$
$a_2 S\theta_2 = 0$] $a_2 = 0$

$d_2 = d_a + d_b$

$S\alpha_2 = -1$
$C\alpha_2 = 0$] $\alpha_2 = 3\pi/2$

$\theta_2 = \theta_a$

$q_2 = \alpha_2 = 3\pi/2$

→ THIS LINK IS REVOLUTE

$$T_3^0 = A_1A_2A_3 = A_aA_bA_f = T_2^0 A_f = \begin{pmatrix} C\theta_a & 0 & -S\theta_a & 0 \\ S\theta_a & 0 & C\theta_a & 0 \\ 0 & -1 & 0 & d_a+d_b \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C\theta_f & -S\theta_f & 0 & a_f C\theta_f \\ S\theta_f & C\theta_f & 0 & a_f S\theta_f \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_3^0 = \begin{pmatrix} C\theta_a C\theta_f & -C\theta_a S\theta_f & -S\theta_a & a_f C\theta_a C\theta_f \\ S\theta_a C\theta_f & -S\theta_a S\theta_f & C\theta_a & a_f S\theta_a C\theta_f \\ -S\theta_f & -C\theta_f & 0 & -a_f S\theta_f + d_a + d_b \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\left.\begin{matrix} S\alpha_3 = -C\theta_f \\ C\alpha_3 = 0 \end{matrix}\right] -$ SINCE TO COMPLY WITH THE DH CONVENTION WE NEED $-S\theta_f = 0$

WE OBSERVE:

$C\alpha_3 = 0$. THEN

$\alpha_3 = +/- \pi/2$

THEREFORE $\boxed{S\alpha_3 = +/- 1 = -C\theta_f}$

$\theta_f = 0$ OR $\theta_f = \pi$

$\sin(0) = 0$ AND $\sin(\pi) = 0$

THEREFORE $\boxed{S\theta_f = 0}$

→ SINCE FROM T_2^0 WE HAVE:

$\left.\begin{matrix} S\alpha_2 = -1 \\ C\alpha_2 = 0 \end{matrix}\right] - \alpha_2 = \frac{3\pi}{2}$

WE WILL ASSUME THIS IS UNCHANGED

IE $\boxed{\alpha_2 = \alpha_3 = \frac{3\pi}{2}}$

AS A CONSEQUENCE: $\boxed{C\theta_f = 1}$ & $\boxed{S\theta_f = 0}$

T_3^0 CAN BE SIMPLIFIED AS FOLLOWS:

$$T_3^0 = \begin{pmatrix} C\theta_a & 0 & -S\theta_a & a_f C\theta_a \\ S\theta_a & 0 & C\theta_a & a_f S\theta_a \\ 0 & -1 & 0 & d_a + d_b \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

o_3

z_3

$\theta_3 = \theta_a$

$d_3 = d_a + d_b$

$$\left.\begin{matrix} S\alpha_3 = -1 \\ C\alpha_3 = 0 \end{matrix}\right] \alpha_3 = \frac{3\pi}{2}$$

$$\left.\begin{matrix} a_F C\theta_a \\ a_F S\theta_a \end{matrix}\right] a_3 = a_F$$

$q_3 = a_F$ → THIS LINK IS PRISMATIC

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \theta_a \\ \alpha = 3\pi/2 \\ a_F \end{pmatrix}$$

PROBLEM # 2

FINDING THE JACOBIANS:

$$J = \begin{pmatrix} J_v \\ J_w \end{pmatrix} \quad \text{WHERE} \quad J_{v_i} = \begin{cases} z_{i-1} \times (O_n - O_{i-1}) & \text{REVOLUTE} \\ z_{i-1} & \text{PRISMATIC} \end{cases}$$

$$J_{w_i} = \begin{cases} z_{i-1} & \text{REVOLUTE} \\ 0 & \text{PRISMATIC} \end{cases}$$

THUS:

$$J = \begin{cases} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{cases}$$

FROM PROBLEM #1:

$$O_3 = \begin{pmatrix} a_f c\theta_a \\ a_f s\theta_a \\ d_a + d_b \end{pmatrix} \quad O_2 = \begin{pmatrix} 0 \\ 0 \\ d_a + d_b \end{pmatrix} \quad O_1 = \begin{pmatrix} 0 \\ 0 \\ d_a \end{pmatrix} \quad O_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$z_3 = \begin{pmatrix} -s\theta_a \\ c\theta_a \\ 0 \end{pmatrix} \quad z_2 = \begin{pmatrix} -s\theta_a \\ c\theta_a \\ 0 \end{pmatrix} \quad z_1 = z_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

CALCULATING ENTRIES OF J:

$$z_0 \times (O_3 - O_0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \left\{ \begin{pmatrix} a_f c\theta a \\ a_f s\theta a \\ d_a + d_b \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\} = \begin{pmatrix} a_f s\theta a \\ -a_f c\theta a \\ 0 \end{pmatrix}$$

$$z_1 \times (O_3 - O_1) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \left\{ \begin{pmatrix} a_f c\theta a \\ a_f s\theta a \\ d_a + d_b \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ d_a \end{pmatrix} \right\} = \begin{pmatrix} a_f s\theta a \\ -a_f c\theta a \\ 0 \end{pmatrix}$$

FILLING THE JACOBIAN:

$$J = \left(\begin{array}{c|c|c} J_{v_1} & J_{v_2} & J_{v_3} \\ \hline J_{\omega_1} & J_{\omega_2} & J_{\omega_3} \end{array} \right)$$

$$J = \left(\begin{array}{c|c|c} a_f s\theta a & a_f s\theta a & -s\theta a \\ -a_f c\theta a & -a_f c\theta a & c\theta a \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{array} \right)$$

FINDING THE ROTATION MATRICES:

FROM PROBLEM #1:

$$T_1^0 = A_1 = A_a = \begin{pmatrix} C\theta_a & -S\theta_a & 0 & 0 \\ S\theta_a & C\theta_a & 0 & 0 \\ 0 & 0 & 1 & d_a \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{aligned} a_1 &= 0 \\ d_1 &= d_a \\ \alpha_1 &= 0 \\ \theta_1 &= \theta_a \end{aligned}$$

THEREFORE $R_1^0 = R_{z,\theta_a} = \begin{pmatrix} C\theta_a & -S\theta_a & 0 \\ S\theta_a & C\theta_a & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$T_2^0 = A_1 A_2 = A_a A_b = \begin{pmatrix} C\theta_a & 0 & -S\theta_a & 0 \\ S\theta_a & 0 & C\theta_a & 0 \\ 0 & -1 & 0 & d_a+d_b \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{aligned} a_2 &= 0 \\ d_2 &= d_a + d_b \\ \alpha_2 &= 3\pi/2 \\ \theta_2 &= \theta_a \end{aligned}$$

THEREFORE $R_2^0 = R_1^0 R_2^1 = R_{z,\theta_a} R_{x,3\pi/2}$

$$R_2^0 = \begin{pmatrix} C\theta_a & -S\theta_a & 0 \\ S\theta_a & C\theta_a & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} C\theta_a & 0 & -S\theta_a \\ S\theta_a & 0 & C\theta_a \\ 0 & -1 & 0 \end{pmatrix}$$

$$T_3^0 = \begin{pmatrix} C\theta_a & 0 & -S\theta_a & a_f C\theta_a \\ S\theta_a & 0 & C\theta_a & a_f S\theta_2 \\ 0 & -1 & 0 & d_a+d_b \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{aligned} a_3 &= a_f \\ d_3 &= d_a+d_b \\ \alpha_3 &= 3\pi/2 \\ \theta_3 &= \theta_a \end{aligned}$$

SINCE $\alpha_2 = \alpha_3$ & $\theta_3 = \theta_2$, THERE IS NO ROTATION BETWEEN T_2^0 & T_3^0, THEN $R_2^0 = R_3^0$

FINDING THE INERTIA MATRICES:

LINK 1/A IS ASSUMED TO BE:

- A SQUARE CROSS SECTION OF SIDE d_a
- HAVE HEIGHT $2d_a$

$$I_{xx} = \int_{-d_a}^{d_a} \int_{-d_a/2}^{d_a/2} \int_{-d_a/2}^{d_a/2} y^2\rho + z^2\rho \; dx\,dy\,dz$$

$$I_{xx} = \int_{-d_a}^{d_a} \int_{-d_a/2}^{d_a/2} y^2\rho \cdot d_a + z^2\rho_i d_a \; dy\,dz$$

$$I_{xx} = \int_{-d_a}^{d_a} z^2\rho \, d_a^2 + \frac{1}{12}\rho_i d_a^4 \; dz$$

$$\boxed{I_{xx} = \frac{5}{6}\rho_i d_a^5}$$

SIMILARLY $I_{yy} = \int_{-d_a}^{d_a} \int_{-d_a/2}^{d_a/2} \int_{-d_a/2}^{d_a/2} \rho_i (x^2 + z^2) \, dx\,dy\,dz$

$$\boxed{I_{yy} = \frac{5}{6}\rho \, d_a^5}$$

$$I_{zz} = \int_{-d_a}^{d_a} \int_{-d_a/2}^{d_a/2} \int_{-d_a/2}^{d_a/2} \rho (x^2 + y^2) \, dx\,dy\,dz$$

$$I_{zz} = \int_{-d_a}^{d_a} \int_{-d_a/2}^{d_a/2} \frac{1}{12}\rho d_a^3 + \rho d_a y^2 \; dy\,dz$$

$$I_{zz} = \int_{-da}^{da} \int_{-da/2}^{da/2} \frac{1}{12} \rho da^3 + \rho da y^2 \, dy \, dz$$

$$I_{zz} = \int_{-da}^{da} \frac{1}{12} \rho da^4 + \frac{1}{12} \rho da^4 \, dz$$

$$\boxed{I_{zz} = \frac{1}{3} d^5}$$

$$I_{xy} = I_{yx} = -\int_{-da}^{da} \int_{-da/2}^{da/2} \int_{-da/2}^{da/2} xy\rho \, dx \, dy \, dz = 0$$

$$\boxed{I_{xy} = I_{yx} = 0}$$

$$I_{xz} = I_{zx} = -\int_{-da}^{da} \int_{-da/2}^{da/2} \int_{-da/2}^{da/2} xz\rho \, dx \, dy \, dz = 0$$

$$\boxed{I_{xz} = I_{zx} = 0}$$

$$I_{yz} = I_{zy} = -\int_{-da}^{da} \int_{-da/2}^{da/2} \int_{-da/2}^{da/2} yz\rho \, dx \, dy \, dz = 0$$

$$\boxed{I_{yz} = I_{zy} = 0}$$

THEREFORE

$$I_1 = \begin{pmatrix} \frac{5}{12} m_1 da^2 & 0 & 0 \\ 0 & \frac{5}{12} m_1 da^2 & 0 \\ 0 & 0 & \frac{1}{6} m_1 da^2 \end{pmatrix}$$

NOTE:

$m_1 = \rho (da)(da)(2da)$

$m_1 = 2\rho da^3$

SIMILARLY, LINK 2/B IS ASSUMED TO BE:

- A SQUARE CROSS SECTION OF SIDE d_b
- HAVE HEIGHT $2\,d_b$.

THEREFORE

$$I_2 = \begin{pmatrix} \frac{5}{12} m_2 d_b^2 & 0 & 0 \\ 0 & \frac{5}{12} m_2 d_b^2 & 0 \\ 0 & 0 & \frac{1}{6} m_2 d_b^2 \end{pmatrix}$$

NOTE:
$m_2 = \rho (d_b)(d_b)(2d_b)$
$m_2 = 2\rho\, d_b^3$

LINK 3/F IS ASSUMED TO BE:

- A SQUARE CROSS SECTION OF SIDE a_f
- HAVE HEIGHT $2\,a_f$

$$I_3 = \begin{pmatrix} \frac{1}{6} m_3 a_f^2 & 0 & 0 \\ 0 & \frac{5}{12} m_3 a_f^2 & 0 \\ 0 & 0 & \frac{5}{12} m_3 a_f^2 \end{pmatrix}$$

NOTE:
$m_3 = \rho (a_f)(a_f)(2a_f)$
$m_3 = 2\rho\, a_f^3$

FINDING THE KINETIC ENERGY:

$$K = \frac{1}{2}\dot{q}^T \left[\sum_{i=1}^{n} \left(m_i J_{v_i}(q)^T J_{v_i}(q) + J_{\omega_i}(q)^T R_i(q) I_i R_i(q)^T J_{\omega_i}(q)\right)\right]\dot{q}$$

$$K = \frac{1}{2}\dot{q}^T D(q)\,\dot{q}$$

USING MATLAB:

$$D = \begin{pmatrix} I_1+I_2+I_3 + a_f^2(m_1+m_2+m_3) & I_2+I_3 + a_f^2(m_2+m_3) & -a_f m_3 \\ I_2+I_3 + a_f^2(m_2+m_3) & I_2+I_3 + a_f^2(m_2+m_3) & -a_f m_3 \\ -a_f^2 m_3 & -a_f m_3 & m_3 \end{pmatrix}$$

SUBSTITUTING I_1, I_2, I_3

$$D = \begin{pmatrix} (1/6)d_a^2 m_1 + (5/12)d_b^2 m_2 + (5/12)a_f^2 m_3 + a_f^2(m_1+m_2+m_3) & (5/12)d_b^2 m_2 + (5/12)a_f^2 m_3 + a_f^2(m_2+m_3) & -a_f m_3 \\ (5/12)d_b^2 m_2 + (5/12)a_f^2 m_3 + a_f^2(m_2+m_3) & (5/12)d_b^2 m_2 + (5/12)a_f^2 m_3 + a_f^2(m_2+m_3) & -a_f m_3 \\ -a_f^2 m_3 & -a_f^2 m_3 & m_3 \end{pmatrix}$$

SINCE $q = \begin{pmatrix} \theta_a \\ 3\pi/2 \\ a_f \end{pmatrix}$ $\dot{q} = \begin{pmatrix} \dot{\theta}_a \\ 0 \\ \dot{a}_f \end{pmatrix}$

USING MATLAB:

$$K = \frac{1}{2}\dot{q}^T D(q)\dot{q} = \frac{\dot{\theta}_a^2}{2}\left[I_1+I_2+I_3 + a_f^2(m_1+m_2+m_3)\right] + (\dot{\theta}_a)\left(\frac{-\dot{a}_f}{2}\right)\left[(a_f m_3) + \dot{\theta}_a a_f m_3 - \dot{a}_f m_3\right]$$

FINDING THE POTENTIAL ENERGY:

$$P = \sum m_i g^T r_{ci}$$

THE ROTATION ABOUT $\alpha_2 = 3\pi/2$ PUT THE LINK "UPSIDE DOWN" NO EFFECT ON GRAVITY.

NOTE:

- LINK 1/A DIMENSIONS $(da) \times (da) \times (2da)$
- LINK 2/B DIMENSIONS $(db) \times (db) \times (2db)$
- LINK 3/F DIMENSIONS $(af) \times (af) \times (2af)$

$$P = m_1 g \left(\frac{2da}{2}\right) + m_2 g \left(2da + \frac{2db}{2}\right) + m_3 g \underbrace{\left(2da + \frac{2db}{2}\right)}_{\text{NO CHANGE IN } z \text{ IN THE 3RD LINK.}}$$

$$\boxed{P = m_1 g\, da + (m_2 + m_3) g (2da + db).}$$

PROBLEM #3

FINDING THE MATRIX FORM OF $C(q)$:

FROM PROBLEM 2:

$$D(q) = \begin{pmatrix} (1/6)d_a^2 m_1 + (5/12)d_b^2 m_2 + (5/12)a_f^2 m_3 + a_f^2(m_1+m_2+m_3) & (5/12)d_b^2 m_2 + (5/12)a_f^2 m_3 + a_f^2(m_2+m_3) & -a_f m_3 \\ (5/12)d_b^2 m_2 + (5/12)a_f^2 m_3 + a_f^2(m_2+m_3) & (5/12)d_b^2 m_2 + (5/12)a_f^2 m_3 + a_f^2(m_2+m_3) & -a_f m_3 \\ -a_f^2 m_3 & -a_f^2 m_3 & m_3 \end{pmatrix}$$

PER p. 183:

$$C_{kj} = \sum_{i=1}^{n} c_{ijk}(q)\dot{q}_i = \sum_{i=1}^{n} \left(\frac{1}{2}\right)\left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\} \dot{q}_i$$

$$q = \begin{pmatrix} \theta_a \\ 3\pi/2 \\ a_f \end{pmatrix} \quad \dot{q} = \begin{pmatrix} \dot{\theta}_a \\ 0 \\ \dot{a}_f \end{pmatrix}$$

$$C_{11} = c_{111}\dot{q}_1 + c_{211}\dot{q}_2 + c_{311}\dot{q}_3 \qquad j=1 \quad k=1$$

$$C_{11} = \frac{1}{2}\left[\cancel{\frac{\partial d_{11}}{\partial q_1}}^{\,0} + \cancel{\frac{\partial d_{11}}{\partial q_1}}^{\,0} - \cancel{\frac{\partial d_{11}}{\partial q_1}}^{\,0} \right]\dot{q}_1 \qquad i=1,\ j=1,\ k=1$$

$$+ \frac{1}{2}\left[\frac{\partial_{11}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_1} - \frac{\partial d_{21}}{\partial q_1} \right]\cancel{\dot{q}_2}^{\,0} \qquad i=2,\ j=1,\ k=1$$

$$+ \frac{1}{2}\left[\frac{\partial_{11}}{\partial q_3} + \cancel{\frac{\partial d_{13}}{\partial q_1}}^{\,0} - \cancel{\frac{\partial d_{31}}{\partial q_1}}^{\,0} \right]\dot{q}_3 \qquad i=3,\ j=1,\ k=1$$

$$\boxed{c_{11} = \frac{1}{2}(2af)\left(m_1+m_2+\frac{17}{12}m_3\right)\dot{a}_f}$$

$$c_{12} = c_{121}\dot{q}_1 + c_{221}\dot{q}_2 + c_{321}\dot{q}_3 \qquad j=2,\ k=1$$

$$c_{12} = \frac{1}{2}\left[\frac{\partial d_{12}}{\partial q_1} + \cancel{\frac{\partial d_{11}}{\partial q_2}} - \cancel{\frac{\partial d_{12}}{\partial q_1}}\right]\cancel{\dot{q}_1} \qquad i=1 \quad j=2 \quad k=1$$

$$+ \frac{1}{2}\left[\frac{\partial_{12}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_2} - \frac{\partial_{22}}{\partial q_1}\right]\cancel{\dot{q}_2} \qquad i=2 \quad j=2 \quad k=1$$

$$+ \frac{1}{2}\left[\frac{\partial_{12}}{\partial q_3} + \cancel{\frac{\partial d_{13}}{\partial q_2}} - \cancel{\frac{\partial_{32}}{\partial q_1}}\right]\dot{q}_3 \qquad i=3 \quad j=2 \quad k=1$$

$$\boxed{c_{12} = \frac{1}{2}(2af)\,\dot{a}_f\left(m_2 + \frac{17}{12}m_3\right)}$$

$$c_{13} = c_{131}\dot{q}_1 + c_{231}\dot{q}_2 + c_{331}\dot{q}_3 \qquad j=3 \quad k=1$$

$$c_{13} = \frac{1}{2}\left[\cancel{\frac{\partial_{13}}{\partial q_1}} + \frac{\partial_{11}}{\partial q_3} - \cancel{\frac{\partial_{13}}{\partial q_1}}\right]\dot{q}_1 \qquad i=1 \quad j=3 \quad k=1$$

$$+ \frac{1}{2}\left[\cancel{\frac{\partial_{13}}{\partial q_2}} + \frac{\partial d_{12}}{\partial q_3} - \cancel{\frac{\partial d_{23}}{\partial q_1}}\right]\cancel{\dot{q}_2} \qquad i=2 \quad j=3 \quad k=1$$

$$+ \frac{1}{2}\left[\frac{\partial_{13}}{\partial q_3} + \frac{\partial d_{13}}{\partial q_3} - \cancel{\frac{\partial d_{33}}{\partial q_1}}\right]\dot{q}_3 \qquad i=3 \quad j=3 \quad k=1$$

$$c_{13} = \frac{1}{2}(2af)\left(m_1+m_2+\frac{17}{12}m_3\right)\dot{\theta}_a + \left(\frac{1}{2}(-2af)m_3\right)2\dot{a}_f$$

$$\boxed{c_{13} = af\left(m_1+m_2+\frac{17}{12}m_3\right)\dot{\theta}_a - 2af\,m_3\,\dot{a}_f}$$

$$C_{21} = c_{112}\dot{q}_1 + c_{212}\dot{q}_2 + c_{312}\dot{q}_3 \qquad j=1,\ k=2$$

$$C_{21} = \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{21}}{\partial q_1}} + \cancelto{0}{\frac{\partial d_{11}}{\partial q_1}} - \cancelto{0}{\frac{\partial d_{11}}{\partial q_2}}\right]\dot{q}_1 \qquad i=1,\ j=1,\ k=2$$

$$+ \frac{1}{2}\left[\frac{\partial d_{21}}{\partial q_2} + \frac{\partial d_{22}}{\partial q_1} - \frac{\partial d_{21}}{\partial q_2}\right]\cancelto{0}{\dot{q}_2} \qquad i=2,\ j=1,\ k=2$$

$$+ \frac{1}{2}\left[\frac{\partial d_{21}}{\partial q_3} + \cancelto{0}{\frac{\partial d_{13}}{\partial q_1}} - \cancelto{0}{\frac{\partial d_{31}}{\partial q_2}}\right]\dot{q}_3 \qquad i=3,\ j=1,\ k=2$$

$$\boxed{C_{21} = \frac{1}{2}\left[2af\left(m_2 + \frac{17}{12}m_3\right)\right]\dot{q}_f}$$

$$C_{22} = c_{122}\dot{q}_1 + c_{222}\dot{q}_2 + c_{322}\dot{q}_3 \qquad j=2,\ k=2$$

$$= \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{22}}{\partial q_1}} + \cancelto{0}{\frac{\partial d_{21}}{\partial q_2}} - \cancelto{0}{\frac{\partial d_{12}}{\partial q_2}}\right]\dot{q}_1 \qquad i=1,\ j=2,\ k=2$$

$$+ \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{22}}{\partial q_2}} + \cancelto{0}{\frac{\partial d_{22}}{\partial q_2}} - \cancelto{0}{\frac{\partial d_{22}}{\partial q_2}}\right]\cancelto{0}{\dot{q}_2} \qquad i=2,\ j=2,\ k=2$$

$$+ \frac{1}{2}\left[\frac{\partial d_{22}}{\partial q_3} + \cancelto{0}{\frac{\partial d_{23}}{\partial q_2}} - \cancelto{0}{\frac{\partial d_{32}}{\partial q_2}}\right]\dot{q}_3 \qquad i=3,\ j=2,\ k=2$$

$$\boxed{C_{22} = \frac{1}{2}(2af)\left[m_2 + \frac{17}{12}m_3\right]\dot{q}_f}$$

$$C_{23} = c_{132}\dot{q}_1 + c_{232}\dot{q}_2 + c_{332}\dot{q}_3 \qquad j=3 \quad k=2$$

$$= \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{23}}{\partial q_1}} + \frac{\partial d_{21}}{\partial q_3} - \cancelto{0}{\frac{\partial d_{13}}{\partial q_2}}\right]\dot{q}_1 \qquad i=1 \quad j=3 \quad k=2$$

$$+ \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{23}}{\partial q_1}} + \cancelto{0}{\frac{\partial d_{22}}{\partial q_3}} - \cancelto{0}{\frac{\partial d_{23}}{\partial q_2}}\right]\cancelto{0}{\dot{q}_2} \qquad i=2 \quad j=3 \quad k=2$$

$$+ \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{23}}{\partial q_1}} + \frac{\partial d_{23}}{\partial q_3} - \cancelto{0}{\frac{\partial d_{33}}{\partial q_2}}\right]\dot{q}_3 \qquad i=3 \quad j=3 \quad k=2$$

$$\boxed{C_{23} = \frac{1}{2}(2af)\left[m_2 + \frac{17}{12}m_3\right]\dot{\theta}_a + \frac{1}{2}(2af)\,m_3\,\dot{q}_f}$$

$$C_{31} = c_{113}\dot{q}_1 + c_{213}\dot{q}_2 + c_{313}\dot{q}_3 \qquad j=1 \quad k=3$$

$$C_{31} = \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{31}}{\partial q_1}} + \cancelto{0}{\frac{\partial d_{31}}{\partial q_1}} - \frac{\partial d_{11}}{\partial q_3}\right]\dot{q}_1 \qquad i=1 \quad j=1 \quad k=3$$

$$+ \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{31}}{\partial q_2}} + \cancelto{0}{\frac{\partial d_{32}}{\partial q_1}} - \cancelto{0}{\frac{\partial d_{21}}{\partial q_3}}\right]\cancelto{0}{\dot{q}_2} \qquad i=2 \quad j=1 \quad k=3$$

$$+ \frac{1}{2}\left[\cancelto{\neq 0}{\frac{\partial d_{31}}{\partial q_3}} + \cancelto{0}{\frac{\partial d_{33}}{\partial q_1}} - \cancelto{0}{\frac{\partial d_{31}}{\partial q_3}}\right]\dot{q}_3 \qquad i=3 \quad j=1 \quad k=3$$

$$\boxed{C_{31} = -\frac{1}{2}\left[(2af)\left(m_1 + m_2 + \frac{17}{12}m_3\right)\right]\dot{\theta}_a}$$

$$C_{32} = c_{123}\dot{q}_1 + c_{223}\dot{q}_2 + c_{323}\dot{q}_3 \qquad j=2 \quad k=3$$

$$C_{32} = \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{32}}{\partial q_1}} + \cancelto{0}{\frac{\partial d_{31}}{\partial q_2}} - \frac{\partial d_{12}}{\partial q_3}\right]\dot{q}_1 \qquad i=1 \quad j=2 \quad k=3$$

$$+ \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{32}}{\partial q_2}} + \cancelto{0}{\frac{\partial d_{32}}{\partial q_2}} - \frac{\partial d_{22}}{\partial q_3}\right]\cancelto{0}{\dot{q}_2} \qquad i=2 \quad j=2 \quad k=3$$

$$+ \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{32}}{\partial q_3}} + \cancelto{0}{\frac{\partial d_{33}}{\partial q_2}} - \cancelto{0}{\frac{\partial d_{32}}{\partial q_3}}\right]\dot{q}_3 \qquad i=3 \quad j=2 \quad k=3$$

$$\boxed{C_{32} = -\frac{1}{2}(2af)\left[m_2 + \frac{17}{12}m_3\right]\dot{\theta}_a}$$

$$C_{33} = c_{133}\dot{q}_1 + c_{233}\dot{q}_2 + c_{333}\dot{q}_3 \qquad j=3 \quad k=3$$

$$C_{33} = \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{33}}{\partial q_1}} + \frac{\partial d_{31}}{\partial q_3} - \frac{\partial d_{13}}{\partial q_3}\right]\dot{q}_1 \qquad i=1 \quad j=3 \quad k=3$$

$$+ \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{33}}{\partial q_2}} + \frac{\partial d_{32}}{\partial q_3} - \frac{\partial d_{23}}{\partial q_3}\right]\cancelto{0}{\dot{q}_2} \qquad i=2 \quad j=3 \quad k=3$$

$$+ \frac{1}{2}\left[\cancelto{0}{\frac{\partial d_{33}}{\partial q_3}} + \cancelto{0}{\frac{\partial d_{33}}{\partial q_3}} - \cancelto{0}{\frac{\partial d_{33}}{\partial q_3}}\right]\dot{q}_3 \qquad i=3 \quad j=3 \quad k=3$$

$$\boxed{C_{33} = \frac{1}{2}(-m_3)\dot{\theta}_a - \left(\frac{1}{2}\right)(-2af)(m_3)\dot{\theta}_a}$$

$$C = \begin{pmatrix} (\dot{a}_f)(a_f)(m_1+m_2+\frac{17}{12}m_3) & (\dot{a}_f)(a_f)(m_2+\frac{17}{12}m_3) & -2\dot{a}_f a_f m_3 \\ & & \dot{\Theta}_a a_f(m_1+m_2+\frac{17}{12}m_3) \\ (\dot{a}_f)(a_f)(m_2+\frac{17}{12}m_3) & (\dot{a}_f)(a_f)(m_2+\frac{17}{12}m_3) & -(\dot{a}_f)(a_f)m_3 \\ & & (\dot{\Theta}_a)(a_f)(m_2+\frac{17}{12}m_3) \\ (\dot{\Theta}_a)(a_f)(m_1+m_2+\frac{17}{12}m_3) & -(\dot{\Theta}_a)(a_f)(m_2+\frac{17}{12}m_3) & -\frac{1}{2}m_3\dot{\Theta}_a + \\ & & (\dot{\Theta}_a)(a_f)m_3 \end{pmatrix}$$

FINDING $g(q)$:

FROM PROBLEM # 2:

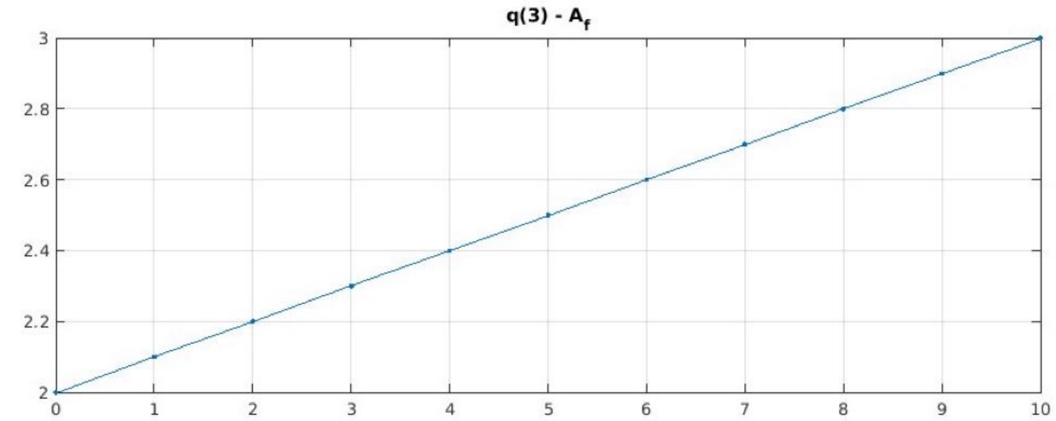
$$P = m_1 g d_a + (m_2+m_3)g(2d_a+d_b)$$

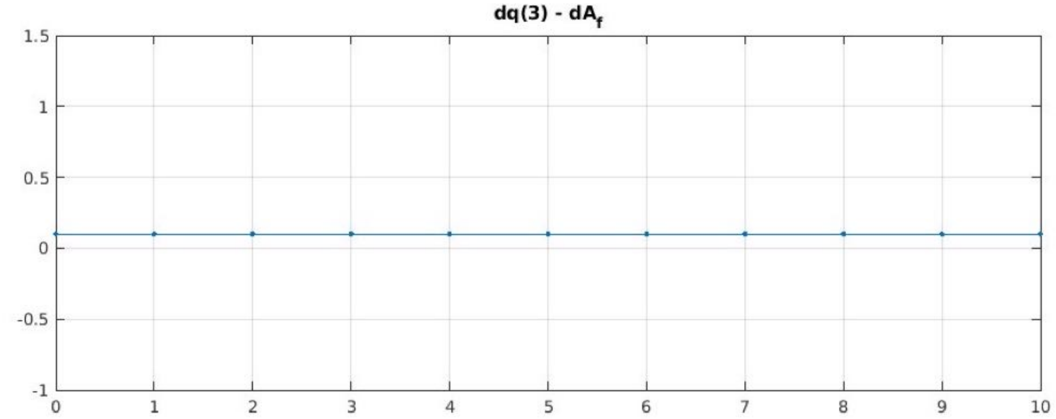
$$g(q) = \begin{pmatrix} g_1(q) & g_2(q) & g_3(q) \end{pmatrix} = \begin{pmatrix} \frac{\partial P}{\partial q_1} & \frac{\partial P}{\partial q_2} & \frac{\partial P}{\partial q_3} \end{pmatrix}$$

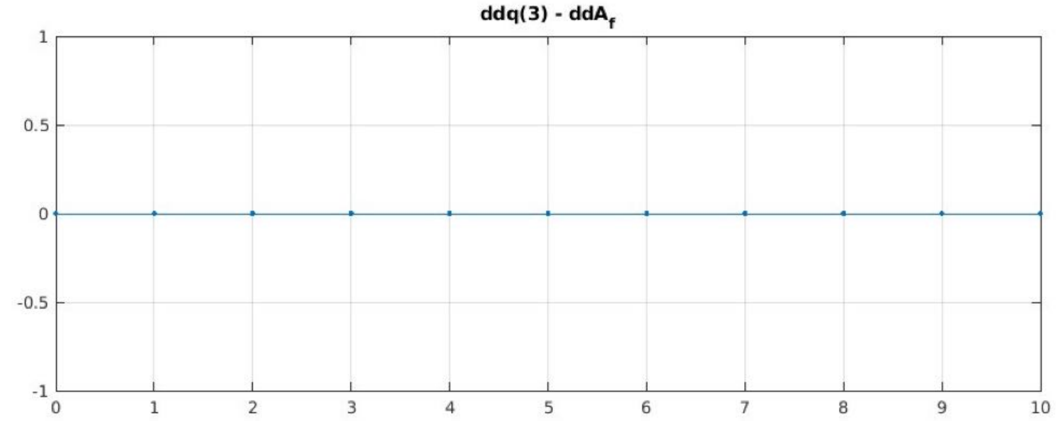
$$\boxed{g(q) = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}}$$

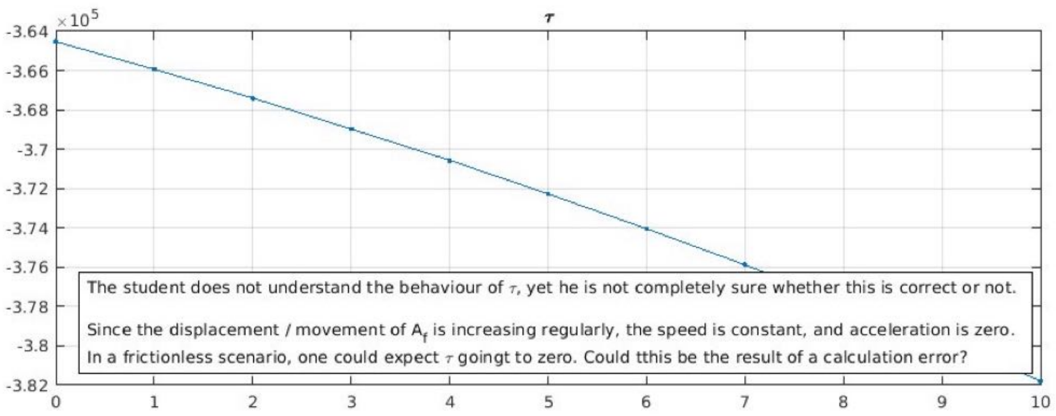
PROBLEM #4

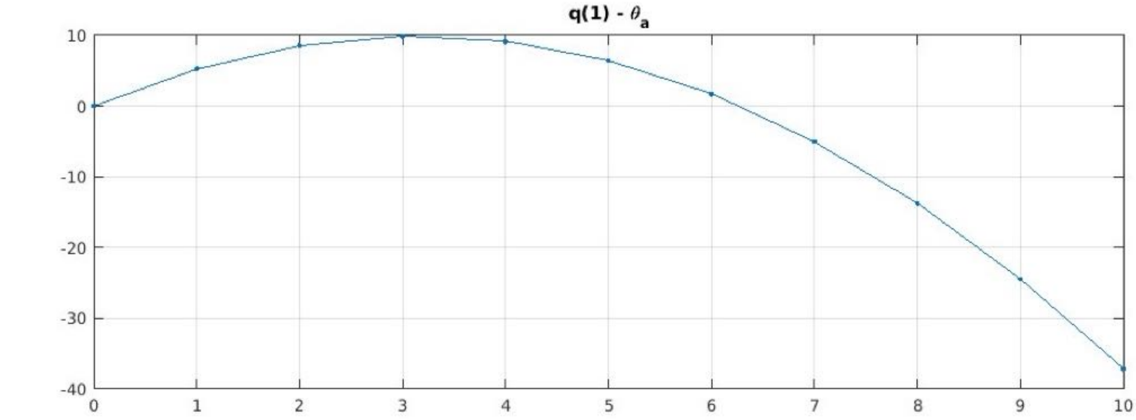
SEE MATLAB SCRIPT.

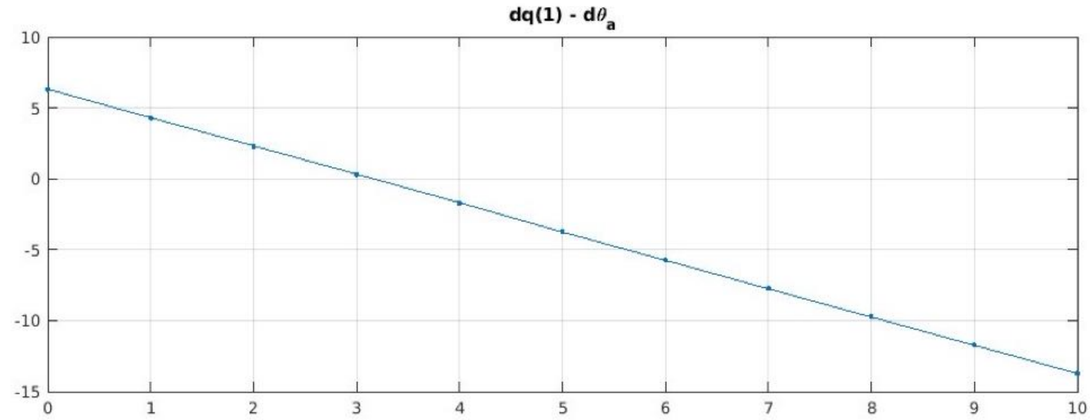

q(3) - A_f
3
2.8
2.6
2.4
2.2
2
0 1 2 3 4 5 6 7 8 9 10

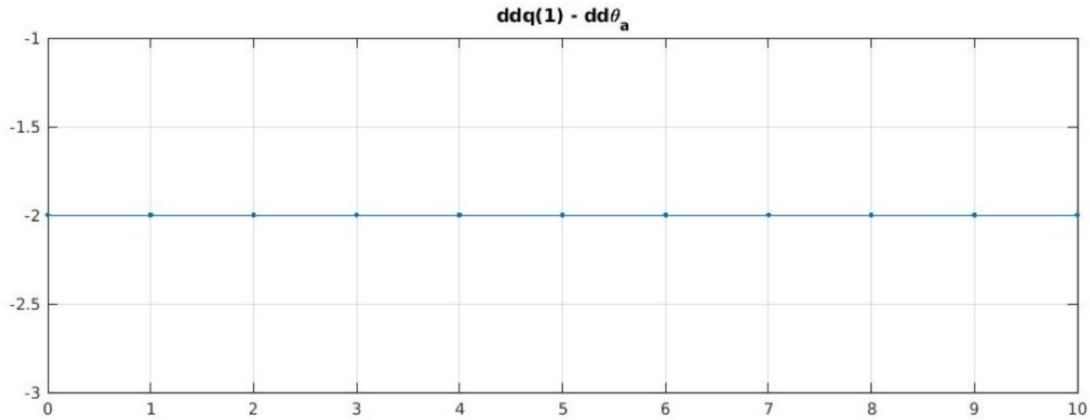

dq(3) - dA_f
1.5
1
0.5
0
-0.5
-1
0 1 2 3 4 5 6 7 8 9 10

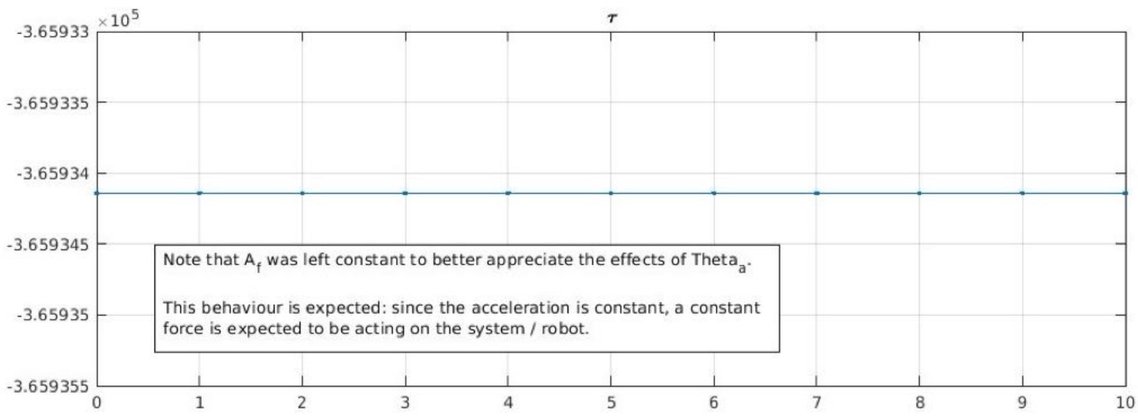

ddq(3) - ddA_f
1
0.5
0
-0.5
-1
0 1 2 3 4 5 6 7 8 9 10


τ
$\times 10^5$
-3.64
-3.66
-3.68
-3.7
-3.72
-3.74
-3.76
-3.78
-3.8
-3.82
0 1 2 3 4 5 6 7 8 9 10
The student does not understand the behaviour of τ, yet he is not completely sure whether this is correct or not.
Since the displacement / movement of A_f is increasing regularly, the speed is constant, and acceleration is zero.
In a frictionless scenario, one could expect τ goingt to zero. Could tthis be the result of a calculation error?


q(1) - θ_a
10
0
-10
-20
-30
-40
0 1 2 3 4 5 6 7 8 9 10


dq(1) - dθ_a
10
5
0
-5
-10
-15
0 1 2 3 4 5 6 7 8 9 10


ddq(1) - ddθ_a
-1
-1.5
-2
-2.5
-3
0 1 2 3 4 5 6 7 8 9 10


τ
$\times 10^5$
-3.65933
-3.659335
-3.65934
-3.659345
-3.65935
-3.659355
0 1 2 3 4 5 6 7 8 9 10
Note that A_f was left constant to better appreciate the effects of Theta_a.
This behaviour is expected: since the acceleration is constant, a constant force is expected to be acting on the system / robot.