

Savings in an OLG Model

One major weakness of the standard Solow model of growth is that the savings rate is given exogenously, that is to say, it is not determined as part of an optimization problem of the agents of the economy. Models with overlapping generations of individuals, each of whom lives a finite number of years, offer a relatively simple way of including the savings decision in a growth model.

Peter Diamond [38] produced a version of the overlapping generations models introduced by Samuelson [70] in which the savings rate is endogenous and can change with other parameters of the economy. The model is comprised of individuals who live two periods and who must make decisions in the first period of their lives about their consumption in both periods of life. Individuals who have substantial income in the first period of life save some of this in the form of capital or lending and are able to consume more than they otherwise might in the second period of life. People live two periods in these economies because this is the smallest number of periods that permits a savings decision and is therefore likely to be the simplest.

The basics of the model are quite simple. In each period, a new group of individuals (called a generation) is born. Members of this group live, work, and consume in the period of their birth and in the next. After this they die and disappear from the economy. The utility they receive comes only from their consumption during the two periods of their life. In the second period of their life, another generation is born and lives during that period and the next. This process continues indefinitely, and in each period there is a generation that has just been born, the young, and another in the second and last period of life, the old. While the model goes on infinitely, each person makes decisions

only about what they will do in two periods, so the savings problem, which is the problem of interest, is relatively simple.

2.1 THE BASIC OLG MODEL

First we need to define the physical environment of the economy. There is an infinite sequence of discrete periods denominated by the integers. In general we are interested in periods $t = 0, 1, 2, \dots \infty$. For the generation born in period t , know as *generation t* , what occurred in periods $t - 1$ or earlier is the past and is taken as given. There are $N(t)$ members of generation t . As mentioned above, people live for two periods and the members of generation t are young in period t and old in period $t + 1$. In period $t + 2$, members of generation t no longer exist and no longer participate in the economy.

There is only one aggregate good in the economy, and an individual's utility comes from his/her consumption of that good in the two periods of life. A member h of generation t has the utility function.

$$u_t^h(c_t^h(t), c_t^h(t + 1)),$$

where $c_t^h(s)$ is the consumption of the one aggregate good by individual h of generation t in period s .

Production takes place in competitive firms that use a homogeneous-of-degree-one production technology. Total production in period t is

$$Y(t) = F(K(t), H(t)),$$

where $H(t)$ is the total labor used in production and $K(t)$ is the total capital, and the production function, $F(K(t), H(t))$, has the same properties we assumed in Chapter 1.

Individuals are endowed with a lifetime endowment of labor given by the ordered pair

$$h_t^h = [h_t^h(t), h_t^h(t + 1)],$$

where $h_t^h(s)$ is the labor endowment of individual h of generation t in period s . Individuals are assumed to supply all of their labor endowment to firms at the competitive wage. The total labor that is used in period t is the sum of the labor supply of the young and the old who are alive in that period. Therefore,

$$H(t) = \sum_{h=1}^{N(t)} h_t^h(t) + \sum_{h=1}^{N(t-1)} h_{t-1}^h(t).$$

It will be useful to define the aggregate labor endowment of the young at time t as

$$H_t(t) = \sum_{h=1}^{N(t)} h_t^h(t),$$

and the aggregate labor endowment of the old at time t as

$$H_{t-1}(t) = \sum_{h=1}^{N(t-1)} h_{t-1}^h(t).$$

In period t , the economy has an amount of capital $K(t)$ that it inherits from period $t - 1$ and that cannot be changed in period t . This capital depreciates completely during its use in period t . This assumption removes the complication of a capital market between members of different generations.

Feasibility constraints for period t imply that

$$Y(t) = F(K(t), H(t)) \geq \sum_{h=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) + K(t+1).$$

The production of period t goes either to consumption of the young or the old or to capital for use in period $t + 1$.

The economic organization that we assume for this economy is one of perfectly competitive markets where individuals are owners of their own labor. Members of generation t earn income in period t by offering all their labor endowment to firms at the market wage and use that income for consumption in period t , for private lending to other members of generation t , and for the accumulation of private capital. Their budget constraint when young is

$$w_t h_t^h(t) = c_t^h(t) + l^h(t) + k^h(t+1), \quad (2.1)$$

where w_t is the market wage at time t and $l^h(t)$ is the amount of loans that they make to other members of generation t ($l^h(t) < 0$ if individual h is borrowing). Because of the overlapping nature of the generations, borrowing and lending can occur only among members of the same generation.

Suppose that a young person of generation t lends some goods to an old member of generation $t - 1$ in period t with the expectation of being paid back in period $t + 1$. In period $t + 1$, this rather naive young person hunts for the member of generation $t - 1$ so that he/she can be paid back. Unfortunately, members of generation $t - 1$ are now all dead and the dead cannot be forced to pay back their debts. Individuals know this and will not make loans to members of other generations. Members of the same generation are alive in the same two periods, so a young person can make a loan to a member of his/her own

generation with expectation of collecting on that loan in the next period when both lender and borrower are old. Since members of one generation lend only to other members of that same generation, we have the condition that

$$0 = \sum_{h=1}^{N(t)} l_h(t).$$

Net lending of any generation t is equal to zero.

In period $t + 1$, a member of generation t has income from the labor supplied in period $t + 1$, from interest earned on any loans that were made in period t , and from the rent on capital that they accumulated in period t . Since this is the last period of life, all income will be consumed. Therefore, the budget constraint for individual h of generation t in period $t + 1$ is

$$c_t^h(t + 1) = w_{t+1}h_t^h(t + 1) + r_t l^h(t) + \text{rental}_{t+1}k^h(t + 1), \quad (2.2)$$

where r_t is the interest paid on loans between period t and $t + 1$ and rental_{t+1} is the competitive rent paid on capital during period $t + 1$.

Individuals are assumed to have perfect foresight in the sense that they know, when young, what wages and rentals will be when they are old. In addition, no fraud is permitted so that all loans are paid back with the agreed-upon interest.

The assumption of perfect competition means that market wages in period t will be equal to the marginal product of labor in that period and that the rentals in period t will be equal to that period's marginal product of capital. So

$$w_t = F_H(K(t), H(t)),$$

and

$$\text{rental}_t = F_K(K(t), H(t)),$$

where $F_i(\cdot, \cdot)$ is the partial derivative of the production function with respect to its i th component.

One can combine the budget constraint when young (equation 2.1) and the budget constraint when old (equation 2.2), by substituting loans, to get a lifetime budget constraint,

$$c_t^h(t) + \frac{c_t^h(t + 1)}{r_t} = w_t h_t^h(t) + \frac{w_{t+1} h_t^h(t + 1)}{r_t} - k^h(t + 1) \left[1 - \frac{\text{rental}_{t+1}}{r_t} \right].$$

A no-arbitrage condition implies that, in a perfect foresight equilibrium,

$$\text{rental}_{t+1} = r_t.$$

To see why this is an arbitrage condition, suppose that it does not hold and that instead

$$\text{rental}_{t+1} > r_t.$$

Since this is perfectly known (in a perfect foresight equilibrium), every member of generation t will want to borrow an infinite amount at time t and hold it as capital into period $t + 1$. For each unit of borrowing that gets converted to capital, a profit of $\text{rental}_{t+1} - r_t$ will be made. Since infinite borrowing by all members of a generation is impossible, this inequality cannot hold in equilibrium. Suppose that the opposite condition holds,

$$\text{rental}_{t+1} < r_t.$$

Then everyone will want to lend and no one will hold capital. Since the conditions of the production function are such that the marginal product of capital goes to infinity as the quantity of capital goes to zero, the rental on capital in period $t + 1$ will be infinite and production in period $t + 1$ will be zero. Since we are prohibiting fraud (again, by our assumption of perfect foresight), this cannot be an equilibrium since the loans plus interest cannot be paid back.

Since, in equilibrium, rental_{t+1} can be neither smaller nor greater than r_t , it must be equal to it.

With this no-arbitrage condition, the lifetime budget constraint can be written as

$$c_t^h(t) + \frac{c_t^h(t+1)}{r_t} = w_t h_t^h(t) + \frac{w_{t+1} h_t^h(t+1)}{r_t},$$

or that the present value of lifetime consumption must equal the present value of lifetime wage income.

DEFINITION 2.1 *A competitive equilibrium for this economy is defined as a sequence of prices,*

$$\{w_t, \text{rental}_t, r_t\}_{t=0}^{\infty},$$

and quantities,

$$\left\{ \left\{ c_t^h(t) \right\}_{h=1}^{N(t)}, \left\{ c_{t-1}^h(t) \right\}_{h=1}^{N(t-1)}, K(t+1) \right\}_{t=0}^{\infty},$$

such that each member h of each generation $t > 0$ maximizes the utility function

$$u_t^h(c_t^h(t), c_t^h(t+1)),$$

subject to the lifetime budget constraint,

$$c_t^h(t) + \frac{c_t^h(t+1)}{r_t} = w_t h_t^h(t) + \frac{w_{t+1} h_t^h(t+1)}{r_t},$$

and the equilibrium conditions,

$$\text{rental}_{t+1} = r_t$$

$$w_t = F_H(K(t), H(t)),$$

$$\text{rental}_t = F_K(K(t), H(t)),$$

$$H(t) = \sum_{h=1}^{N(t)} h_t^h(t) + \sum_{h=1}^{N(t-1)} h_{t-1}^h(t),$$

hold in every period.

Notice that in the above definition we did not define the individual holdings of either lending or of capital. This is because they offer exactly the same return and there are an infinite number of distributions of lending and capital holdings among members of a generation that would meet the equilibrium conditions. Two example distributions for an economy where all members of a generation are identical are 1) person $h = 1$ borrows from everyone else and holds all the capital and 2) no one borrows and each person holds $K(t+1)/N(t)$ units of capital. These two distributions would result in the same total capital stock and the same equilibrium by the above definition.

Substituting the lifetime budget constraint into the utility function, the utility maximization problem for person h of generation t can be written as

$$\max_{c_t^h(t)} u(c_t^h(t), r_t w_t h_t^h(t) + w_{t+1} h_t^h(t+1) - r_t c_t^h(t)),$$

where, for individual h , the assumption of perfect foresight means that the values of all the other parameters are known. The first-order condition is

$$\begin{aligned} u_1(c_t^h(t), r_t w_t h_t^h(t) + w_{t+1} h_t^h(t+1) - r_t c_t^h(t)) \\ = r_t u_2(c_t^h(t), r_t w_t h_t^h(t) + w_{t+1} h_t^h(t+1) - r_t c_t^h(t)), \end{aligned} \quad (2.3)$$

where $u_i(\cdot)$ is the partial derivative of the utility function with respect to its i th element. Using the budget constraint when young (equation 2.1), we can

find a savings function for individual h of generation t , $s_t^h(\cdot)$, where

$$s_t^h(w_t, w_{t+1}, r_t) = l^h(t) + k^h(t+1).$$

Summing the savings of all members of generation t , we define an aggregate savings function $S_t(\cdot)$, as equal to

$$S_t(\cdot) = \sum_{h=1}^{N(t)} s_t^h(\cdot) = \sum_{h=1}^{N(t)} l^h(t) + \sum_{h=1}^{N(t)} k^h(t+1).$$

Given that, in equilibrium,

$$\sum_{h=1}^{N(t)} l^h(t) = 0$$

and

$$K(t+1) = \sum_{h=1}^{N(t)} k^h(t+1),$$

the aggregate savings equation can be written as

$$S_t(w_t, w_{t+1}, r_t) = K(t+1).$$

Substituting rental _{$t+1$} for r_t and using the equilibrium conditions for the factor markets in periods t and $t+1$, we can write the aggregate savings equation as

$$S_t(F_H(K(t), H(t)), F_H(K(t+1), H(t+1)), F_K(K(t+1), H(t+1))) = K(t+1).$$

The above expression gives $K(t+1)$ as an implicit function of the labor supplies in each period, $H_t(t)$, $H_{t-1}(t)$, $H_t(t+1)$, $H_{t+1}(t+1)$, the parameters of the utility functions and the production function, and $K(t)$. Since, as the model is constructed, all of these except $K(t)$ are constants through time, one can find the capital stock in time $t+1$ as a function of the capital stock in time t ,

$$K(t+1) = G(K(t)). \quad (2.4)$$

This is a first-order difference equation that describes the growth path of the economy.

2.1.1 An Example Economy

For an example economy where the utility function is of the form

$$u_t^h = u(c_t^h(t), c_t^h(t+1)) = c_t^h(t)c_t^h(t+1)^\beta,$$

and the production function is a simple Cobb-Douglas

$$Y_t = F(K(t), H(t)) = K(t)^\theta H(t)^{1-\theta},$$

with $0 < \beta$ and $0 < \theta < 1$, the function $G()$ in equation 2.4 can be written explicitly as¹

$$K(t+1) = G(K(t)) = \frac{\theta\beta \frac{H_t(t)}{H(t)^\theta}}{\left[\frac{H_t(t+1)}{H(t+1)}\right] + \frac{\theta(1+\beta)}{(1-\theta)}} K(t)^\theta = \kappa K(t)^\theta, \quad (2.5)$$

where all of the elements that make up κ are constants so that κ is a positive constant. This first-order difference equation has stationary states at $\bar{K} = 0$ and at $\bar{K} = \kappa^{\frac{1}{1-\theta}}$. The constant κ is a function of the exponent on consumption in the second period of life, β , and can be rewritten as

$$\kappa = \frac{\beta\psi}{(1+\beta) + \rho},$$

where $\psi = (1-\theta) \left[\frac{1}{H(t)}\right]^\theta H_t(t)$ and $\rho = \frac{1-\theta}{\theta} \left[\frac{H_t(t+1)}{H(t+1)}\right]$ are positive constants. Recall that the larger is β , the more weight is given to second-period consumption in determining utility. Taking the derivative of κ with respect to β yields

$$\frac{d\kappa}{d\beta} = \frac{\psi(1+\rho)}{(1+\beta+\rho)^2} > 0.$$

As one would expect, savings and capital (which are equal when we have complete depreciation of capital) in the positive stationary state, $\kappa^{\frac{1}{1-\theta}}$, are an increasing function of the weight put on second-period consumption in the utility function, β .

EXERCISE 2.1 Work out the example economy and find equation 2.5.

1. See McCandless and Wallace [61], Chapter 9, for a detailed development of this model.

2.2 DYNAMICS

The behavior of this model out of a stationary state is similar to that of the Solow model. If the initial capital stock is between the two stationary states, $0 < K(0) < \kappa^{\frac{1}{1-\theta}}$, the capital stock will grow, converging on the positive stationary state. This can be seen by simply looking for the range of initial capital stocks for which $K(t+1) > K(t)$, or where

$$K(t+1) - K(t) = \kappa K(t)^\theta - K(t) > 0.$$

Simple manipulation of this equation leads to the result that this condition holds for positive $K(t)$ when $K(t) < \kappa^{\frac{1}{1-\theta}}$. In addition, the rate of growth of the capital stock declines as it grows. Define the gross rate of growth of capital as $\Delta K(t) = K(t+1)/K(t)$. This can be written as

$$\Delta K(t) = \frac{\kappa K(t)^\theta}{K(t)}.$$

Taking the derivative of the growth rate with respect to the capital stock yields

$$\frac{d\Delta K(t)}{dK(t)} = (\theta - 1)\kappa K(t)^{\theta-2} < 0.$$

As in the earlier version of the Solow model, the larger the initial capital stock, the slower the growth rate of capital. In addition, since output is defined as $Y(t) = K(t)^\theta H(t)^{1-\theta}$, the gross growth rate of output, $\Delta Y(t) = Y(t+1)/Y(t)$, is equal to

$$\Delta Y(t) = \frac{K(t+1)^\theta H(t+1)^{1-\theta}}{K(t)^\theta H(t)^{1-\theta}} = \frac{K(t+1)^\theta}{K(t)^\theta} = \Delta K(t)^\theta,$$

where the second part of the expression comes about because labor supply is constant. The derivative of the gross growth rate of output with respect to the capital stock is

$$\frac{d\Delta Y(t)}{dK(t)} = \frac{d\left[\frac{\kappa K(t)^\theta}{K(t)}\right]^\theta}{dK(t)} = \theta \left[\frac{\kappa K(t)^\theta}{K(t)}\right]^{\theta-1} (\theta - 1)\kappa K(t)^{\theta-2} < 0,$$

so output growth slows as the capital stock increases.

The dynamics of the model can also be seen from a graph of equation 2.5. Using values of $\kappa = 4$ and $\theta = .36$, the equation is shown in Figure 2.1 as the function labeled $G(K(t))$. In addition, a 45 degree line (the line of stationary states) is drawn in. The graph indicates that for initial values of $K(t)$ below the stationary state (where $G(K(t))$ crosses the 45 degree line),

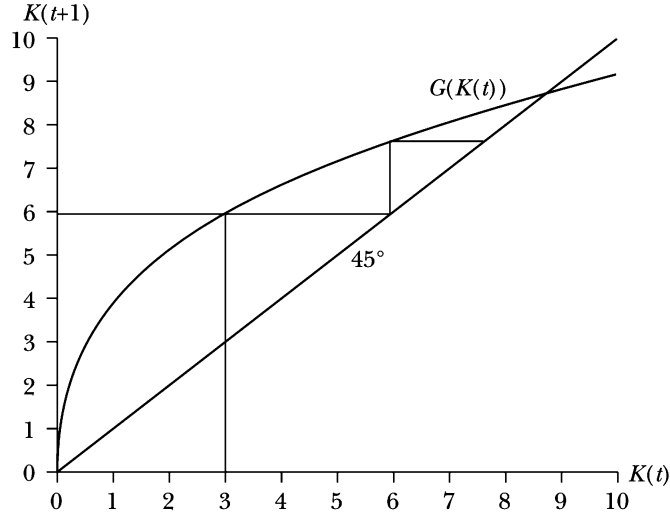


FIGURE 2.1 Graph of dynamics of OLG model

the value of $K(t+1) = G(K(t))$ is greater than $K(t)$ so the capital stock is increasing. As can be seen from the graph, the relative increase of $K(t+1)$ is smaller the greater is $K(t)$. This graphical example confirms the dynamics obtained analytically above.

EXERCISE 2.2 Show how this model would change if capital depreciation in each period were $0 < \delta < 1$ and secondary markets for capital existed.

EXERCISE 2.3 Consider a version of the model with full capital depreciation but where the production function is $Y(t) = (1 + g)K(t)^\theta H(t)^{1-\theta}$, where $g > 0$ is net technology growth. Show how you would find a balanced growth path for this economy.

2.3 A STOCHASTIC VERSION

Suppose that production follows a stochastic process given by

$$Y_t = \lambda_t K(t)^\theta H(t)^{1-\theta},$$

where

$$\lambda_t = (1 - \gamma) + \gamma \lambda_{t-1} + \varepsilon_t,$$

with $0 < \gamma < 1$ and ε_t has $E_t \varepsilon_{t+1} = 0$, is bounded below by $-(1 - \gamma)$ and is bounded above. Notice that these assumptions mean that technology will always be positive and will have a stationary state or nonstochastic value (which is when all $\varepsilon_t = 0$) of one.

It is easier to work with the same utility function as above but now expressed in terms of logs. Therefore, we assume that a member of generation t maximizes an expected utility of

$$\begin{aligned} u_t^h &= E_t u(c_t^h(t), c_t^h(t+1)) \\ &= E_t [\ln c_t^h(t) + \beta \ln c_t^h(t+1)] \\ &= \ln c_t^h(t) + \beta E_t \ln c_t^h(t+1) \end{aligned}$$

Using logs makes the utility function additive and this simplifies computation. Since the time t , technology shock is known at time t , output at time t is known and so is time t consumption. Time $t+1$ output and consumption are not known at time t , but their expected values can be calculated.

Individual budget constraints are

$$c_t^h(t) = w_t h_t^h(t) - k_t^h(t+1),$$

when young, and

$$c_t^h(t+1) = w_{t+1} h_t^h(t+1) + \text{rental}_{t+1} k_t^h(t+1).$$

The individual's optimization problem is to max

$$\ln c_t^h(t) + \beta E_t \ln c_t^h(t+1)$$

subject to the two budget constraints. We can solve this as a Lagrangian (being careful with the expectations operators in the utility function) and get the first-order condition of the individual maximization problem as

$$\frac{1}{c_t^h(t)} = \beta E_t \frac{\text{rental}_{t+1}}{c_t^h(t+1)}.$$

Competitive factor markets imply that wages and rentals equal their marginal products, so

$$w_t = (1 - \theta) \lambda_t K(t)^\theta H(t)^{-\theta},$$

and

$$\text{rental}_{t+1} = \theta \lambda_{t+1} K(t+1)^{\theta-1} H(t+1)^{1-\theta}.$$

The capital that will be carried into period $t + 1$ is observed in period t as the solution to the optimization problem. The amount of labor that the young and old provide is fixed in this model, so it is also known at date t . The only time $t + 1$ variable that is not known at date t is the value of the technology shock in period $t + 1$. Interestingly, knowing this exactly is not important for solving the optimization problem of the time t young.

Taking the first-order condition for the time t young, we substitute the budget constraints, writing everything in aggregate terms since the young at time t are identical, and get

$$\frac{1}{w_t H_t(t) - K(t+1)} = \beta E_t \frac{\text{rental}_{t+1}}{w_{t+1} H_t(t+1) + \text{rental}_{t+1} K(t+1)}.$$

Recall that $H_t(s)$ is the labor provided by generation t in period $s = t, t + 1$. We divide through by rental_{t+1} on the right-hand side of this equation and get

$$\frac{1}{w_t H_t(t) - K(t+1)} = \beta E_t \frac{1}{\frac{w_{t+1}}{\text{rental}_{t+1}} H_t(t+1) + K(t+1)}.$$

Replacing wages and rentals by their marginal products, we get

$$\begin{aligned} & \frac{1}{(1-\theta) \lambda_t K(t)^\theta H(t)^{-\theta} H_t(t) - K(t+1)} \\ &= \beta E_t \frac{1}{\frac{(1-\theta) \lambda_{t+1} K(t+1)^\theta H(t+1)^{-\theta}}{\theta \lambda_{t+1} K(t+1)^{\theta-1} H(t+1)^{1-\theta}} H_t(t+1) + K(t+1)}. \end{aligned}$$

Simplifying the right-hand side yields

$$\frac{1}{(1-\theta) \lambda_t K(t)^\theta H(t)^{-\theta} H_t(t) - K(t+1)} = \beta \frac{1}{\left[\frac{(1-\theta)}{\theta} \frac{H_t(t+1)}{H(t+1)} + 1 \right] K(t+1)}.$$

The expectation operator drops out because, in this last step, we canceled λ_{t+1} from the right-hand side. There is no longer a need to take expectations to find the stochastic growth path. With a bit of algebra, the model can be written as

$$K(t+1) = \kappa \lambda_t K(t)^\theta,$$

where

$$\kappa = \frac{\theta \beta \frac{H_t(t)}{H(t)^\theta}}{\frac{H_t(t+1)}{H(t+1)} + \frac{\theta(1+\beta)}{(1-\theta)}}$$

does not change through time. Note that κ is the same as that in the non-stochastic version of the model. Given some initial $K(0)$, the time path for a stochastic version of this economy can be found by simulating the stochastic shocks for λ_t .

For an example economy where $\beta = .99$, $\theta = .36$, $H(t) = 65$, $H_t(t) = 60.2$, and $H_t(t+1) = 4.8$, we get $\kappa = 4.00$. Assuming that $\gamma = .6$ and a uniform distribution for ε_t over $[-.02, .02]$, and starting from the three values of $K(0) = \{6.9792, 8.7241, 10.4689\}$, we get the time paths for capital and output shown in Figure 2.2. Output was found using the production function and the labor supply of $H(t) = 65$. Notice that with $\theta = .36$, the differences in the initial capital stock disappear relatively quickly. Since the technology shock enters directly into the production function, we observe more movement of the output than we do of the capital stock. In our overlapping generations model with complete depreciation, the return on capital needs to be high to make one want to hold it. The marginal product of capital is high enough only when the amount of capital (equal to around 9 units of goods) is much smaller than output (around 32 units of goods in the example economy). These results might not seem very consistent with capital-output ratios found

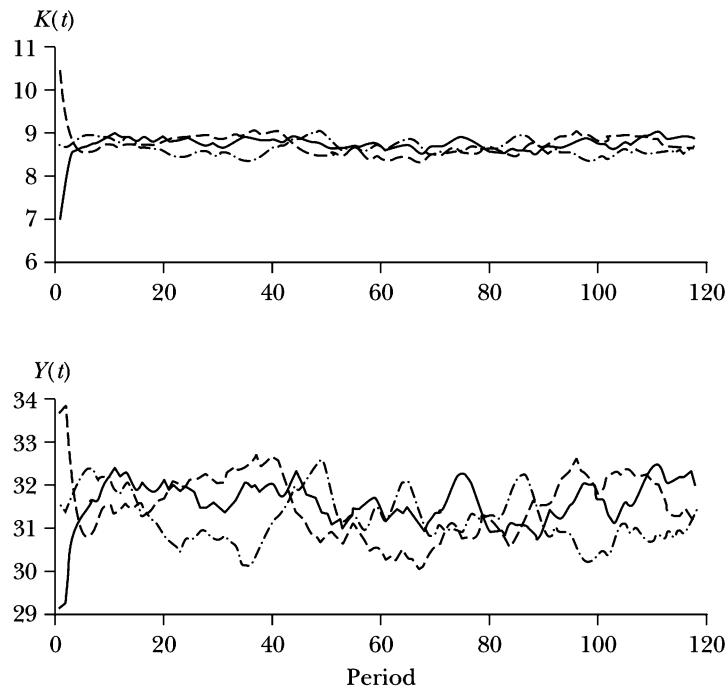


FIGURE 2.2 Simulations of three runs of the stochastic OLG model

in the literature that normally fall between 2 and 3. Here, however, there are only two periods of life, so each one is approximately 30 years. Thirty years of output measured against the stock of capital might not be very different from the approximately 1/3 that we find for this model. Capital is a stock, and it matters a lot the length of the output flow we are using for a capital-output ratio.

2.4 REPRISE

An overlapping generations model allows us to make the savings decision endogenous to the model in a relatively simple way. Since agents live only two periods, their optimization problem involves only those two periods. The overlapping nature of the economy and the fact that the old are holding all the capital that they saved from the previous period gives the model some persistence. In the version shown here, we did not make the labor supply decision endogenous, but this can be done relatively easily, since it adds only two more variables to the decision problem of each agent: the labor to supply when young and that when old.

For more information on overlapping generations models, one can refer to, in increasing level of difficulty, McCandless and Wallace [61], Sargent [72], and Azariadis [4]. A nice introduction to the use of OLG models for studying monetary economics can be found in Champ and Freeman [25].

2.5 MATLAB CODE USED TO PRODUCE FIGURE 2.2

The following program was used to generate the simulated time series for capital and output using the OLG model. This program is quite simple, but might be instructive to those learning Matlab.

```
kappa=4;
theta=.36;
roe=.9;
K0=[.8*kappa^(1/(1-theta))
    kappa^(1/(1-theta))
    1.2*kappa^(1/(1-theta))];
lambda(1:3,1)=[1 1 1]';
K(1:3,1)=K0;
Y(:,1)=lambda(:,1).*K(:,1).^theta.*65.^(1-theta);
for k=2:118
    lambda(:,k)=(1-roe)+roe.*lambda(:,k-1)+.02.*(rand(3,1)-.5);
    K(:,k)=kappa.*lambda(:,k).*K(:,k-1).^theta;
    Y(:,k)=lambda(:,k).*K(:,k-1).^theta.*65.^(1-theta);
end
subplot(2,1,1),plot(K')
subplot(2,1,2),plot(Y')
```