finding euler equation

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- Optimization problem: ($F(x_{t+i}, x_{t+1+i})$ is the objective function)

$$\max_{\{x_x\}_{s=t}^{\infty}} \sum_{i=0}^{\infty} \beta^i F(x_{t+i}, x_{t+1+i}).$$

- This is really the sequence

$$\beta^0 F(x_t, x_{t+1}) + \beta^1 F(x_{t+1}, x_{t+2}) + \beta^2 F(x_{t+2}, x_{t+3}) + \dots$$

- Suppose we want to optimize in some period s+1, then the part of the infinite sequence that is of interest are those with time s+1 variables in them
- For some random s+1, those components with x_{s+1} are simply

$$\beta^{s-t}F\left(x_s, x_{s+1}\right) + \beta^{s-t+1}F_1\left(x_{s+1}, x_{s+2}\right)$$

  and no others

So when I take the derivative with respect to x_{s+1} it is just

$$\beta^{s-t}F_2\left(x_s, x_{s+1}\right) + \beta^{s-t+1}F\left(x_{s+1}, x_{s+2}\right)$$

- We set the derivative equal to zero and get

$$0=\beta^{s-t}F_2\left(x_s,x_{s+1}\right)+\beta^{s-t+1}F\left(x_{s+1},x_{s+2}\right)$$

or, more simply,

$$0=F_2\left(x_s,x_{s+1}\right)+\beta F\left(x_{s+1},x_{s+2}\right)$$

and if the conditions for the implicit function theorem hold, we can find a function H such that the optimal x_{s+1} is

$$x_{s+1}=H\left(x_s,x_{s+2}\right)$$