Linear Optical Quantum Computing

Rishabh Sahu*

*Roll number: 13817575, email: srishabh@iitk.ac.in

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Optical quantum computers provide a variety of advantages over other types of quantum computers including seamless integration of computational and communication part of the quantum technology. Single-qubit unitary operations on photons have been shown to be rather trivial, however, two-qubit gates become a challenge due to non-interacting nature of photons. Although several theoretical two-qubit gates were proposed using non-linear optical elements, any physical realisation of these gates was not seen due to unavailability of required highly non-linear materials for the scheme. In this report, we discuss implementation of optical quantum computer, especially two-qubit gates, using only linear optical elements.

1. INTRODUCTION

Various physical entities can be used as a qubit including ion traps, superconducting charges, spin of atomic nucleus etc. However, each of them have some or the other disadvantage that stems realisation of a fully scalable quantum computer using that entity. In this race of quantum computers, photons also provide several advantages compared to other entities as qubits.

The biggest advantage that an optical quantum computer can offer over other computing systems would be that they provide a seamless integration of quantum communication, that is imagined to be done using photons, and quantum computation. Apart from that photons don't decohere easily as compared to many other systems. Also, unlike other contenders, they need not be operated at extremely low temperatures to keep the thermal noise at minimum.

However, these advantages come with one big disadvantage. Photons don't interact among themselves. This is a desired property to have when building single-bit gates but it creates a challenge when designing two-qubit gates.

In this report, we will describe the approaches used to make two-qubit gates. We start with introducing photons as qubits and some properties of their properties. We, then, move on to describe common optical elements that can be used to make single-bit unitary gates. Thereafter, we address the problem of two-qubit gates. We start with some earlier approaches that tried to implement two-qubit gates but were either not realisable or scalable. We, then, describe a protocol originally given by Knill, Laflamme and Milburn in 2001[1]. The protocol describes implementation of scalable two-qubit gates.

2. PHOTONS AS QUBITS

There are various properties of photons that allow them to be used as qubits. Major ones include they travel in straight line, they travel with constant speed in a given media and their polarization, that is, the field of vibration of electric field. Using these properties we can construct two main types of photonic qubits,

Dual-rail qubits Photons travel in straight line. Hence, if we choose two paths of travel and put photon in either one of them, depending in which path do we detect the photon, we can call that either a $|0\rangle$ or $|1\rangle$. Figure 1 shows a dual-rail bit.

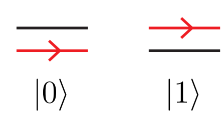

Fig. 1. A dual-rail photonic qubit. Depending on the channel in which the photon is put, the qubit is in either in state $|0\rangle$ or $|1\rangle$.

A photon state that has a specific number of photons is called a fock-state. A single photon fock-state is represented by $|1\rangle$, a two-photon fock state by $|2\rangle$ and so on. A fock-state with no photons is called a vacuum state is represented by $|0\rangle$. Raising, $\hat{a}^\dagger$, and lowering operator, $\hat{a}$, can be used receptively to switch from one fock-state to other. They function as,

$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle$$
$$\hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

Hence, we can write $|1\rangle$ as $\hat{a}^\dagger\,|0\rangle$ and so on. These operators will be used extensively for analytical treatment of further optical elements that we discuss in next section.

Polarization qubit It turns out that the plane of vibration of electric field can be at any angle independent of direction of propagation of light (as long it is perpendicular to direction of propagation in 3 dimensions). The electric field can hence take any direction in 2-dimensional space. This two dimensional space can be represented by a basis consisting of horizontal polarization, $\langle H|$, and vertical polarization, $\langle V|$. Any other direction can then be written as a superposition of these two.

3. SINGLE-QUBIT UNITARY OPERATIONS

Single-bit unitary operations become rather trivial on the photonic qubits. They can be implemented using a few simple optical elements. Here, we discuss the optical elements along with their operation on qubits.

Phase Shifter These are simplest optical elements that introduce a phase shift to any photons passing through it. The amount of phase introduced, ϕ, can be controlled by changing the refractive index or length of glass used. Analytically, we write this in terms of operators introduced in previous section,

$$\hat{a}^\dagger_{out} = e^{i\phi}\hat{a}^\dagger_{in}$$

Note that in a dual-rail qubit, if we put the phase shifter only in the second path, we can perform a rotation of qubit about Z axis in Bloch sphere.

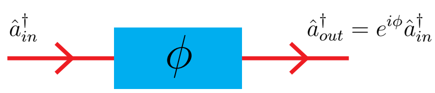

Fig. 2. A phase-shifter is just an optical element with different refractive index. It introduces a pre-determined phase to all incoming modes.

Beam Splitter The second important element in linear optics is beam splitter. It mixes two incoming modes $\hat{a}_{in}$ and $\hat{b}_{in}$. Working of a generalised beam splitter can be given as follows,

$$\hat{a}^\dagger_{out} = cos\theta\hat{a}^\dagger_{in} + ie^{-i\phi}sin\theta\hat{b}^\dagger_{in} \tag{1}$$
$$\hat{b}^\dagger_{out} = ie^{i\phi}sin\theta\hat{a}^\dagger_{in} + cos\theta\hat{b}^\dagger_{in} \tag{2}$$

where θ and ϕ are parameters of a general beam-splitter. Reflection and transmission coefficients are $sin^2\theta$ and $cos^2\theta$ respectively which add upto 1. It is worth noting here that if we substitute $\phi = 0$ here, a beam splitter with θ parameter will perform a rotation of dual-rail qubit about Y axis by an amount of -2θ.

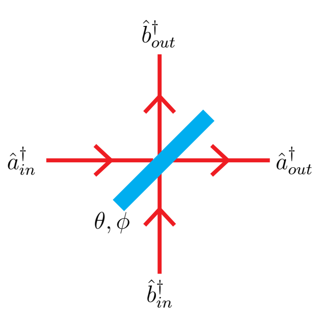

Fig. 3. A generalised beam splitter with θ and ϕ as characteristic parameters. The action is given by equations 1 and 2.

We also note that a rotation about X axis can be written in terms of rotation about Y and Z axis as,

$$R_x(\theta) = R_z(\frac{\pi}{2})R_y(-\theta)R_z(-\frac{\pi}{2})$$

We can, hence, use phase shifters and beam splitters to perform any rotation of the qubit in Bloch sphere.

Finally, it can be observed that any single-bit unitary gate can be expanded as,

$$U = e^{i\alpha}R_z(\alpha)R_y(\beta)R_x(\gamma) \tag{3}$$

This shows that we can implement any single-bit unitary operation on a dual-rail qubit using a combination of beam splitters and phase shifters. However, to include polarization qubit also, we use another optical element,

Polarising Beam Splitter This is another kind of beam splitter with a slight difference. Instead of splitting the beam in some specific ratio, it splits the path of incoming photon according to its polarization. Horizontally polarised components pass without reflection and vertically polarised components suffer a reflection. It is trivial to see how polarising beam splitter can be used to switch between a dual-rail qubit and a polarization qubit.

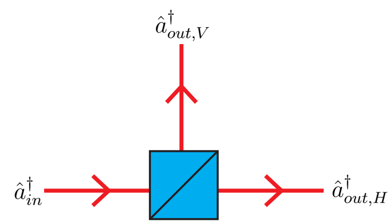

Fig. 4. A polarising beam splitter. Horizontally polarized part passes without reflection while the vertically polarised part gets reflected.

There are other optical elements such a polarization rotators and wave-plates that can be used to modify a polarization qubit. However, in this report we mostly deal with dual-rail qubits and would hence skip any details about these elements.

4. EARLIER ATTEMPTS AT TWO-QUBIT GATES

The above section shows how to do single qubit unitary operation with photonic qubits. Use of simple linear optical elements make the task rather trivial. However, realisation of optical quantum computer is stemmed by two-qubit gates whose implementation with photonic qubits becomes extremely difficult because of non-interacting nature of photons.

Many attempts have been made to implement two-qubit gates but all of them failed to provide a scalable and physically realisable solution. We discuss some of these attempts here.

A. Cerf, Adami and Kwait protocol (1998)

In 1998, Cerf *et al.*[2] came up with a technique to implement CNOT gate using linear optical elements such as beam splitters and phase shifters only. In this protocol, n-qubits were represented by a single photon in 2^n different paths. For example, a two qubit system would be represented by a single photon in 4 different paths representing $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$.

As all the representation were comfortably separated, they can be easily manipulated separately. A CNOT implementation,

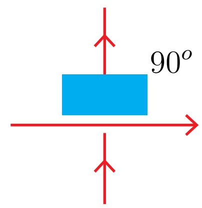

Fig. 5. CNOT as implemented by Cerf *et al.* The control bit is represented by spatial degree of freedom while the target qubit is represented by polarisation degree of freedom.

hence, became trivial in this representation. It is shown in figure 5. In this figure, photon's spatial degree of freedom represents control qubit and its polarization represents the target qubit. Changing between spatial and polarization degree of freedom is not a big issue as they can switched between easily using polarising beam splitter.

However, this implementation was not scalable as it required exponentially large number of resources for increasing number of qubits. In fact, it was a simulation of a quantum computer rather than a quantum computer itself. In 2001, Grover's algorithm was experimentally realised using this protocol[3].

B. Non-linear quantum computers

One other way to make two-qubit gates would be to use non-linear elements. These are the elements that have a non-linear response or in other words the response of the material (such as phase shift) can depend on some parameter. A Kerr-nonlinear medium is one whose properties depend on the intensity of light passing through it or the number of photons it is interacting with. These materials can be used to make two-qubit gates such as Controlled-NOT or Controlled-Z gate.

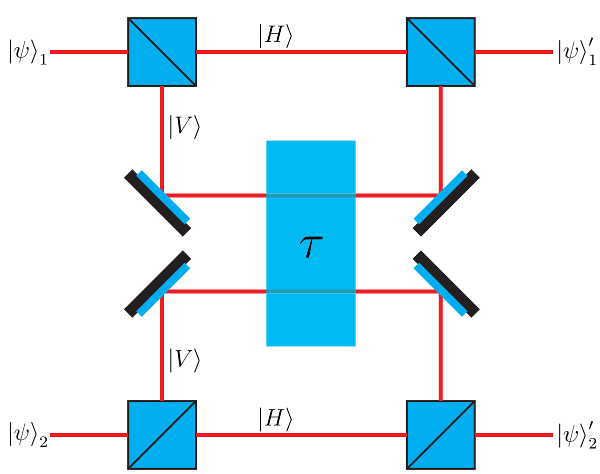

Fig. 6. CZ gate implemented using Kerr nonlinear material for polarization qubit. The material only flips the phase when 2 photons are passing through it. In all other cases it leaves the qubit unchanged. Hence, only the $|VV\rangle$ gets a phase flip.

Figure 6 shows an implementation of CZ gate with Kerr nonlinear medium. The implementation utilises two qubits in their polarization basis that need to pass through a polarising beam splitter. Then the vertical polarization or the $|1\rangle$ part of both the qubits passes through a nonlinear medium. Now, this medium, being nonlinear, is designed such that it will only introduce a phase shift of $\tau = \pi$ if two photons pass through it. In any other case, it won't introduce any phase shift in the photons. After this interaction with nonlinear medium, the vertical and horizontal part of both the qubits is then combined back. The only change that occurs, as a result, is $|1,1\rangle$ part gets a phase shift of π while the rest remains the same as required by the CZ gate.

However, this approach suffers from one big problem. Even the most nonlinear materials available are extremely weak and can introduce a phase shift of only about $\tau = 10^{-18}$ while for this scheme to work, required phase shift is $\tau = \pi$. As a result, a nonlinear optical quantum computer cannot be realised until highly nonlinear materials are found. Further research is needed in this topic.

5. KLM PROTOCOL

In 2001, Knill, Laflamme and Milburn[1] came up with a protocol that can implement two-qubit gates and thus optical quantum computer using only linear optical elements. Till now, the problem that stemmed development of two-qubits gates was non-interacting nature of photons. One way, as mentioned in earlier section, to make photons interact is to use nonlinear medium. However, as it turns out there was one effect discovered by Hong *et al.* in 1987[4] in which photons interacted owing to their bosonic nature. We elucidate the effect in detail.

Hong-Ou-Mandel Effect This effect is probably the only known way where photons influence each other directly. It is observed when two photons simultaneously reach two ports of a 50-50 beam splitter ($\theta = 45^o, \phi = 0^0$). More analytically,

$$|1,1\rangle_{a,b} = \hat{a}^\dagger \hat{b}^\dagger |0\rangle_{a,b} \to \frac{1}{2}(\hat{c}^\dagger + \hat{d}^\dagger)(\hat{c}^\dagger - \hat{d}^\dagger)\,|0\rangle_{c,d}$$

$\left[\text{where, } \hat{c}^\dagger = \hat{a}^\dagger_{out} \text{ and } \hat{d}^\dagger = \hat{b}^\dagger_{out} \text{ (transformed using eqn. 1, 2)}\right]$

$$= \frac{1}{2}(\hat{c}^{\dagger 2} - \hat{d}^{\dagger 2})\,|0\rangle_{c,d}$$

$$= \frac{1}{\sqrt{2}}(|2,0\rangle_{c,d} - |0,2\rangle_{c,d})$$

As can be observed from the last equation, either both the photons are observed in first output channel of beam splitter or second output channel. The amplitudes of two cases where single photon emerges from either output channel of beam splitter cancel each other out. The effect can, hence, be seen as bunching of photons, a kind of direct interaction.

Knill *et al.* used this effect to implement a CZ-gate. However, their implementation required a nonlinear sign gate or NS-gate. This gate was required to function as follows,

$$\alpha\,|0\rangle + \beta\,|1\rangle + \gamma\,|2\rangle \to \alpha\,|0\rangle + \beta\,|1\rangle - \gamma\,|2\rangle$$

i.e, only introduce a phase of π to two-photon fock state. This is extremely similar to the use of nonlinear Kerr medium we saw in last section. To implement such a gate which is fundamentally nonlinear using only linear optical elements, a probabilistic approach is opted.

Nonlinear sign gate The first implementation of probabilistic NS-gate was given by Knill *et al.* in 2001 with success probability of 1/4. It was followed by many others who came up with another version of the probabilistic gate with better success probabilities[5, 6] and in other basis[7]. We discuss the original implementation here. Figure 7 shows the original circuit of NS-gate given in KLM protocol.

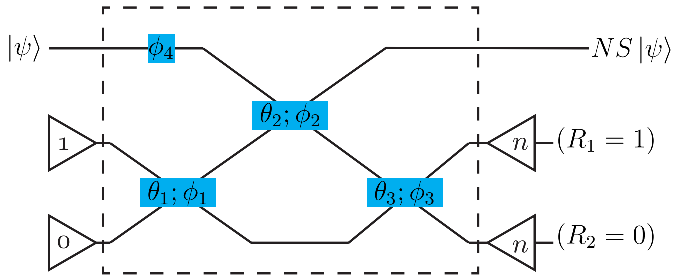

Fig. 7. For $\theta_1 = 22.5^o$, $\theta_2 = 65.53^o$, $\theta_3 = -22.5^o$ and $\phi_1 = \phi_2 = \phi_3 = 0^o$; $\phi_4 = \pi$, we get a nonlinear sign gate probabilistically. The successful case is when exactly one photon is detected in first detector and no photons are detected in second detector. The probability of a successful case is 1/4.

The circuit uses two ancilla qubits and certain rather unusual parameters for various beam splitters. However, solving the circuit analytically, it is observed that in the case when the first of the two detectors gets a single photon and the other one doesn't detect a photon at all, we get $NS\,|\psi\rangle$ in the output port. The probability of finding this exact number of photons in the two detectors is 1/4 in this case which gives the success probability of the gate.

Using this gate, a rather straightforward implementation of CZ-gate was put forward was Knill *et al.*

CZ-gate This CZ-gate is implemented using the dual-rail representation of qubit in which $|1\rangle$ is represented by a photon in second channel. The circuit for the gate is given in figure 8.

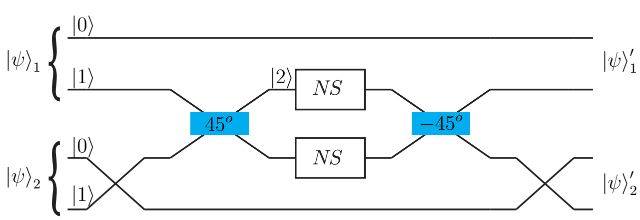

Fig. 8. Implementation of CZ-gate as given by Knill *et al.* in dual-rail representation. The NS-gate only flips the phase $|2\rangle$ state and leaves the other two states unchanged. Hence, only the $|11\rangle$ gets a phase flip.

The second channel of both the qubits is firstly fed to two input ports of 50-50 beam splitter. Only when both the qubits are in $|1\rangle$ state, bunching will happen at the beam splitter. The state produced as a result of this would be $|2\rangle$ which after passing through a NS-gate would get a phase flip. In any other case, NS-gate would not change the outcome and another 50-50 beam splitter will cancel out the effects introduced by the first one. Hence, as a result, only the $|1,1\rangle$ state will get a phase flip as required by the CZ-gate.

However, CZ-gate uses two NS-gates which are probabilistic with success probability of 1/4. CZ-gate will hence only succeed when both the NS-gates succeed together. The probability of that happening is 1/16. Hence, the CZ-gate proposed by Knill *et al.* succeeds with a probability of 1/16.

This undesirably low probability makes this CZ-gate almost unusable. If any quantum algorithm that n CZ-gates, it would end up having a minuscule success probability of only $\left(\frac{1}{16}\right)^n$. With such a low probability, any advantage the quantum algorithm has over the classical one would be diminished. However, in KLM protocol a solution to this problem was presented. Using quantum teleportation, it was shown that the success probability of CZ-gate can be pushed arbitrarily close to 1. We discuss the solution given in the protocol in the next subsection.

A. Quantum Teleportation Fix

To deal with any probabilistic gate, Gottesman and Chuang proposed a trick in 2001[8]. Using their proposed method, one can turn any probabilistic gate into deterministic gate upto a reasonable limit. The trick was based on the effect of quantum teleportation. We discuss it here.

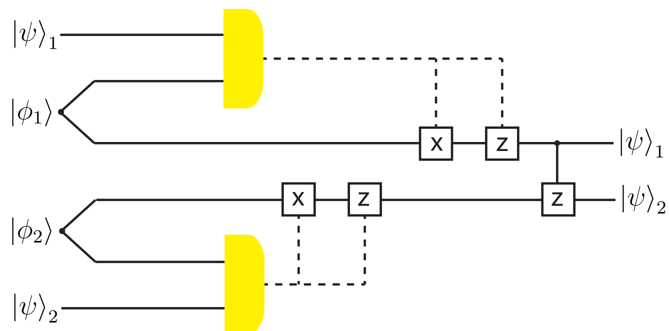

Fig. 9. CZ-gate implemented after teleporting two states $|\psi_1\rangle$ and $|\psi_2\rangle$. The yellow rounded rectangle on two corners represents bell measurements with two classical outputs. These outputs determine whether Pauli X and Z gates need to be applied or not.

As shown in figure 9, two states $|\psi_1\rangle$ and $|\psi_2\rangle$ are teleported using two entangled resources $|\phi_1\rangle$ and $|\phi_2\rangle$. The rectangle rounded at two corners represents bell measurements with two classical outputs used to control the application of Pauli X and Z gate, as is required by quantum teleportation protocol. The probabilistic gate, in this case CZ-gate, is then applied after the states have been teleported.

The idea is to bring the probabilistic gate before any teleportation happens or in other words before any Pauli X and Z gate is required. To do this, we use several identities like,

$$(\sigma_x \bigotimes \mathbb{1})CZ = CZ(\sigma_x \bigotimes \sigma_z) \tag{4}$$

The proof is rather trivial and includes only multiplication of various matrices to manually verify the equality. However, the proof entails something shown in figure 10, *i.e,* the order of Pauli X and CZ can be switched if another Pauli Z gate is included. Similar, identities relate to other types switches required and are given as follows,

$$(\mathbb{1} \bigotimes \sigma_x)CZ = CZ(\sigma_z \bigotimes \sigma_x) \tag{5}$$
$$(\sigma_z \bigotimes \mathbb{1})CZ = CZ(\sigma_z \bigotimes \mathbb{1}) \tag{6}$$
$$(\mathbb{1} \bigotimes \sigma_z)CZ = CZ(\mathbb{1} \bigotimes \sigma_z) \tag{7}$$

Using equation 4-7, a situation represented in figure 9 can be transformed into a new situation represented in figure 11. Now, the probabilistic gate CZ is not directly connected to states $|\psi_1\rangle$ and $|\psi_2\rangle$ making it independent of them and can be applied using only entangled states. In this new set-up, one can now allow the gate to fail as many times as one wants with the entangled

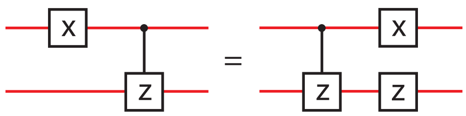

Fig. 10. The ordering of CZ-gate can be switched with Pauli-X gate using the identity given by equation 4.

states as they can be generated independent of $|\psi_1\rangle$ and $|\psi_2\rangle$. We, then, store any instances of entangled state for which the probabilistic gate succeeded. Whenever, the probabilistic gate needs to applied to $|\psi_1\rangle$ and $|\psi_2\rangle$, we use the entangled resource that succeeded with probabilistic gate and go ahead with further gates that are assumed to be deterministic. Hence, using this trick we can apply a non-deterministic gate deterministically.

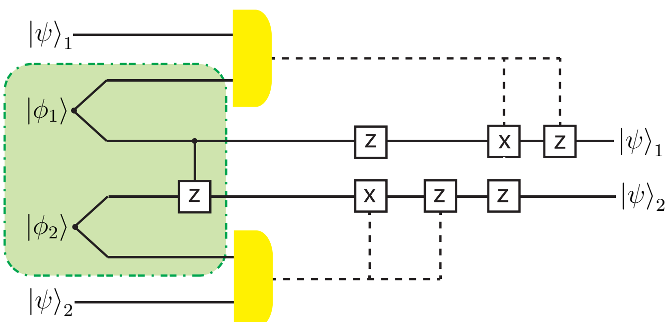

Fig. 11. Using equations 4-7, CZ-gate can be brought before application of any Pauli gate. This allows for a offline system (highlighted in green) that is independent of $|\psi_1\rangle$ and $|\psi_2\rangle$. The offline system can, hence, be run independently and only the successful outputs will be used when actual application is required.

However, this trick assumes several things. First, that Pauli gates can be applied deterministically which is true with optical quantum computers. Second, that Bell measurements can be made deterministically. Unfortunately, this is not possible with linear optics. It should not come as a surprise because basic protocol of quantum teleportation itself includes a CNOT gate for Bell measurement. As, CNOT cannot be applied deterministically using linear optics, Bell measurement also cannot be done deterministically. It was formally proved in 1999[9] that the four Bell states cannot be distinguished using just linear optical elements.

To solve this problem, we now focus on fixing the success probability of quantum teleportation using linear optics alone.

B. Quantum Teleportation using Linear Optics

Before trying to increase the success probability of quantum teleportation, let us look at the traditional way of attempting teleportation using linear optics. Figure 12 shows the set-up used for teleporting a arbitrary dual-rail qubit $|\psi\rangle = \alpha\,|01\rangle + \beta\,|10\rangle$. An entangled resource is produced first using a single photon, $|1\rangle$ and a vacuum mode, $|0\rangle$ imparted on two input modes of a 50-50 beam splitter. This produces an entangled resource,

$$\frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

One channel of this entangled resource is then imparted onto another 50-50 beam splitter along with one channel of dual-rail

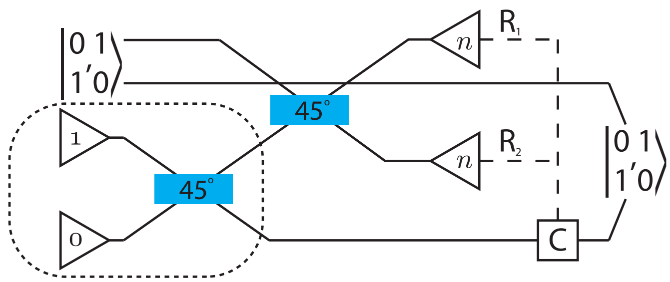

Fig. 12. Quantum teleportation using linear optical elements. It is probabilistic and succeeds only when the two detectors R_1 and R_2 detect a total of 1 photon. The probability of success is 1/2.

bit. Further analytical calculation show the following outcomes,

$$R_1 = 0, R_2 = 1: \ \frac{1}{2}(\alpha\,|01\rangle + \beta\,|10\rangle)$$
$$R_1 = 1, R_2 = 0: \ \frac{1}{2}(-\alpha\,|01\rangle + \beta\,|10\rangle)$$

This means that if we face either of these two situation with detectors R_1 and R_2, we can apply some operation (represented by C) to get back the original $|\psi\rangle$ back. The combined probability of above two situations is 1/2. In all other cases the information is lost and the state collapses into either of the two basis states.

Hence, we see that the success probability of quantum teleportation using only linear optics is 1/2. However, in KLM protocol, it was shown that this probability can be boosted arbitrarily close to one using more sophisticated type of teleportation.The original teleportation protocol uses entangled state resource of 2 dimensions. It was shown that if we use an n-dimensional entangled resource along with n-dimensional QFT as opposed to 2-dimensional QFT (or Hadammard applied by 50-50 beam splitter), we can boost the success probability of quantum teleportation to $1 - \frac{1}{n}$.

Instead of discussing a generalised quantum teleportation protocol using n dimensions, treatment of which can get fairly complicated, we discuss 3-dimensional teleportation protocol and show that its success probability is 2/3 as compared to a worse probability of 1/2.

Figure 13 shows the schematic of a teleportation protocol using a 3-dimensional entangled resource. A 3-dimensional entangled resource is given by,

$$|\phi_3\rangle = \frac{1}{\sqrt{3}}(|0011\rangle + |1001\rangle + |1100\rangle)$$

and F_3 is the 3-dimensional QFT gate. One other difference this circuit has opposed to the previous one is that instead of teleporting the state to just one channel, in this case, teleportation now occurs to one of the two output modes. Skipping the calculations, we report the successful cases in the following table,

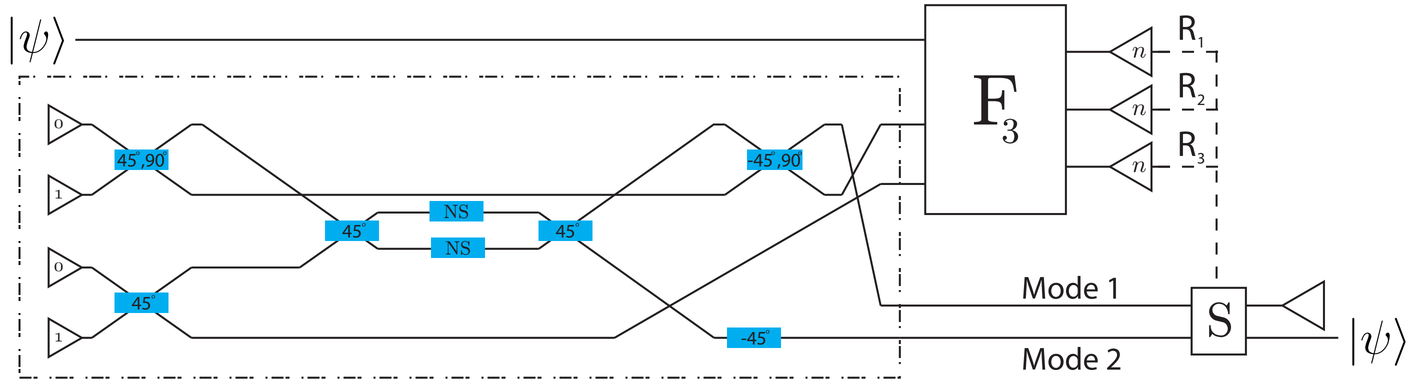

Fig. 13. Quantum teleportation using 3-dimensional entangled state and respective QFT-gate. The circuit in dashed rectangle prepares a 3-dimensional entangled state. F_3 is 3-dimensional OFT gate and $R_1,\ R_2,\ R_3$ represent three detectors. Using this more sophisticated scheme increases the total success probability to 2/3.

Measurement	Output	Mode	Probability
$\|100\rangle$	$\frac{1}{3}(\alpha\|0\rangle+\beta\|1\rangle)$	1	1/9
$\|010\rangle$	$\frac{1}{3}(e^{\frac{2\pi i}{3}}\alpha\|0\rangle+\beta\|1\rangle)$	1	1/9
$\|001\rangle$	$\frac{1}{3}(e^{\frac{-2\pi i}{3}}\alpha\|0\rangle+\beta\|1\rangle)$	1	1/9
$\|200\rangle$	$\frac{\sqrt{2}}{3\sqrt{3}}(\alpha\|0\rangle+\beta\|1\rangle)$	2	2/27
$\|020\rangle$	$\frac{\sqrt{2}}{3\sqrt{3}}(\alpha\|0\rangle+e^{\frac{2\pi i}{3}}\beta\|1\rangle)$	2	2/27
$\|002\rangle$	$\frac{\sqrt{2}}{3\sqrt{3}}(\alpha\|0\rangle+e^{\frac{-2\pi i}{3}}\beta\|1\rangle)$	2	2/27
$\|110\rangle$	$\frac{1}{3\sqrt{3}}(-\alpha\|0\rangle+e^{\frac{2\pi i}{3}}\beta\|1\rangle)$	2	1/27
$\|101\rangle$	$\frac{1}{3\sqrt{3}}(-\alpha\|0\rangle+e^{\frac{-2\pi i}{3}}\beta\|1\rangle)$	2	1/27
$\|011\rangle$	$\frac{1}{3\sqrt{3}}(-\alpha\|0\rangle-\beta\|1\rangle)$	2	1/27

where, the first column measurement denotes the number of photons detected in the 3 detectors respectively. The third column tells the output mode (out of two) in which the mode as denoted by the second column will be obtained. The final column tells the probability of that situation occurring. Summing up all the probabilities, we get 2/3 overall success probability of quantum teleportation.

Thus, using this teleportation protocol, we can increase the success probability of teleportation arbitrarily close to 1. Combining this with the technique discussed in previous subsection, we can make a two-qubit gate with very high success probability using only linear optical elements.

6. CONCLUSION

Optical quantum computers offer a range of advantages over other forms of quantum computers. Although, unitary operations on single photonic qubit is rather trivial, the problem of non-interacting nature of photons had been one of the major problems that prevented development of two-qubit gates. We discussed some attempts at making a two-qubit photonic gates but they failed to be scalable or even physically realisable. We, then, discussed the first successful attempt by Knill *et al.* in 2001 that improved the success probability of quantum teleportation using only linear optical elements and using that to boost the success probability of non-deterministic two-qubit gates.

Since, it's publication in 2001, KLM protocol has been experimentally realised to implement Shor's algorithm in 2012[10] and in many more applications mostly revolving around two-qubit photonic gates.

REFERENCES

1. E. Knill, R. Laflamme, and G. J. Milburn, "A scheme for efficient quantum computation with linear optics," nature **409**, 46–52 (2001).
2. N. J. Cerf, C. Adami, and P. G. Kwiat, "Optical simulation of quantum logic," Physical Review A **57**, R1477 (1998).
3. P. Kwiat, J. Mitchell, P. Schwindt, and A. White, "Grover's search algorithm: an optical approach," Journal of Modern Optics **47**, 257–266 (2000).
4. C. Hong, Z.-Y. Ou, and L. Mandel, "Measurement of subpicosecond time intervals between two photons by interference," Physical review letters **59**, 2044 (1987).
5. S. Scheel and N. Lütkenhaus, "Upper bounds on success probabilities in linear optics," New Journal of Physics **6**, 51 (2004).
6. X. Zou, K. Pahlke, and W. Mathis, "Teleportation implementation of nondeterministic quantum logic operations by using linear optical elements," Physical Review A **65**, 064305 (2002).
7. T. Ralph, A. White, W. Munro, and G. Milburn, "Simple scheme for efficient linear optics quantum gates," Physical Review A **65**, 012314 (2001).
8. D. Gottesman and I. L. Chuang, "Quantum teleportation is a universal computational primitive," arXiv preprint quant-ph/9908010 (1999).
9. N. Lütkenhaus, J. Calsamiglia, and K.-A. Suominen, "Bell measurements for teleportation," Physical Review A **59**, 3295 (1999).
10. E. Martin-Lopez, A. Laing, T. Lawson, R. Alvarez, X.-Q. Zhou, and J. L. O'brien, "Experimental realization of shor's quantum factoring algorithm using qubit recycling," Nature Photonics **6**, 773–776 (2012).
11. C. Myers and R. Laflamme, "Linear optics quantum computation: an overview," arXiv preprint quant-ph/0512104 (2005).
12. P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, "Linear optical quantum computing with photonic qubits," Reviews of Modern Physics **79**, 135 (2007).