Critical behaviour of the surface tension in the 3D Ising model

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Summary

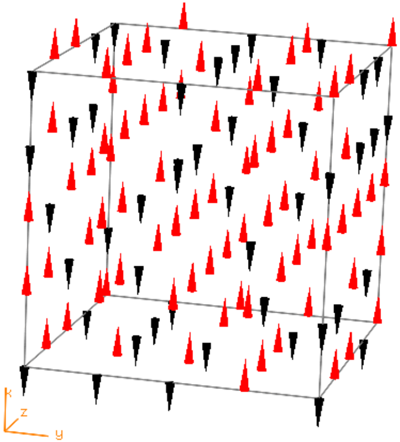
- 3D Ising models
- Definition of the surface tension
- Cluster algorithm and boundary flip
- (Notes on the implementation?)
- Estimation of the errors and autocorrelation
- Fit of the free energy
- Fit of the critical behaviour
- Conclusion

3D Ising model

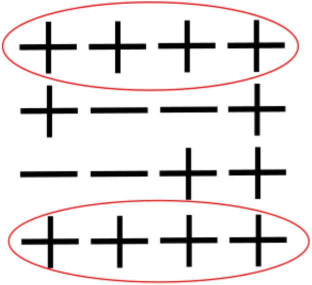
$$\mathcal{H} = -\sum_{\langle x,y \rangle} J_{\langle x,y \rangle} s_x s_y$$

$J_{\langle x,y \rangle} = 1$ ferromagnetic

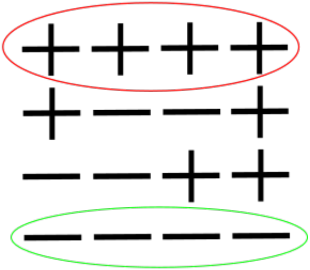
$J_{\langle x,y \rangle} = -1$ antiferromagnetic



Definition of the surface tension

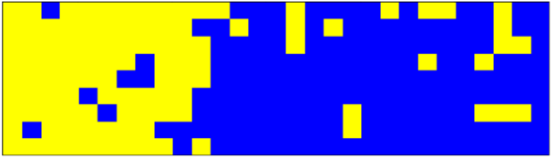


Z_{++}

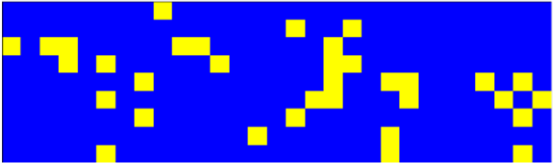


Z_{+-}

$$\sigma = -\lim_{L\to\infty} \frac{1}{L^2} \log \frac{Z_{+-}}{Z_{++}} \qquad L \times L \times T,\ T = cL$$



Z_{+-}



Z_{++}

$$\sigma = -\lim_{L\to\infty} \frac{1}{L^2} \log \frac{Z_{+-}}{Z_{++}} = \lim_{L\to\infty} \frac{1}{L^2} \left(F_{+-} - F_{++} \right) = \lim_{L\to\infty} \frac{F_s}{L^2}$$

$\sigma =$ interface free energy per unit area

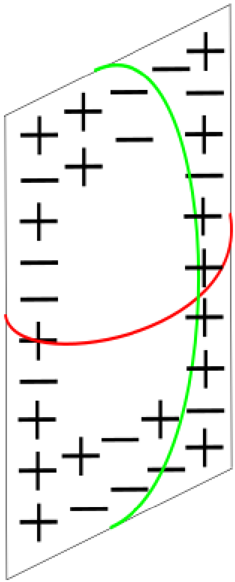
Redefinition of Z_{++} and Z_{+-}.

$Z_{++} \to$ ferromagnetic link between **top and bottom**.
$Z_{+-} \to$ **anti**ferromagnetic link between **top and bottom**.

Always ferromagnetic link in x and y.

Same definition for σ.

Periodic boundary conditions reduce the finite size effect.



Montecarlo simulations can't measure Z!

Solution: $J_{\langle x,y \rangle}$ between top and bottom becomes a **dinamical variable** that is summed over in Z.

$J_{\langle x,y \rangle} = 1$ (periodic b.c.) $J_{\langle x,y \rangle} = -1$ (antiperiodic b.c.)

Other $J_{\langle x,y \rangle}$ remains ferromagnetic.

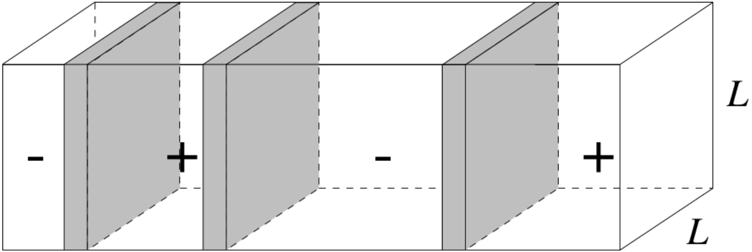
$$Z = \sum_{\{s\},J} \exp\left(\beta \sum_{\langle x,y \rangle} J_{\langle x,y \rangle} s_x s_y\right)$$

$$\frac{Z_{+-}}{Z_{++}} = \frac{\frac{Z_{+-}}{Z}}{\frac{Z_{++}}{Z}} = \frac{\left\langle \delta_{J=-1} \right\rangle}{\left\langle \delta_{J=+1} \right\rangle}$$

Ratio of measurable expectation values.

We redefine the free energy of the interface in order to improve the convergence proprieties of $\frac{F_s}{L^2}$ to σ when $L \to \infty$.
Thermodynamic limit $\longrightarrow$ only **one** interface

For finite L multiple interface can be present. An even number for Z_{++} and odd for Z_{+-}.



F_s is the free energy of a single surface. There are $\sim T$ different position for the interface.

$$Z_1 = T \exp\left(-F_s\right) = \exp\left(-F_S + \ln T\right)$$

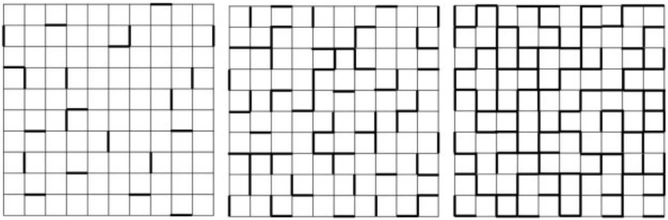
$$\frac{Z_{+-}}{Z_{++}} = \frac{Z_1 + \frac{Z_1^3}{3!} + \frac{Z_1^3}{5!} + \ldots}{1 + \frac{Z_1^2}{2!} + \frac{Z_1^4}{4!} + \ldots} = \tanh\left(\exp\left(-F_S + \ln T\right)\right)$$

$$F_S = \ln\left(T\right) - \ln\left(\frac{1}{2}\ln\left(\frac{1+\frac{Z_{+-}}{Z_{++}}}{1-\frac{Z_{+-}}{Z_{++}}}\right)\right) \quad \sigma = \lim_{L\to\infty}\frac{F_s}{L^2}$$

Cluster algorithm and boundary flip

Cluster algorithms allow for simultaneous updates of large parts of the lattice. Thus reducing the autocorrelation time and the critical slowling down. Swendsen and Wang (1987).

Introduce link variables $\sigma_{\langle x,y \rangle} = \{0, 1\}$ on the lattice:



$$Z = \sum_{\{s\},J} \exp\left(\beta \sum_{\langle x,y\rangle} J_{\langle x,y\rangle} s_x s_y\right) = \sum_{\{s\}} \prod_{\langle x,y\rangle} \exp\left(\beta s_x s_y\right)$$