Nambu–Goto string model and interfaces

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Outline

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2. Interfaces

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3. The Nambu Goto bosonic string

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5. Conclusions and overview

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Based on: M. Billó, M. Caselle and L. Ferro, "The partition function of interfaces from the Nambu−Goto effective string theory", JHEP 0602 (2006) 070 [arXiv:hep−th/0601191]

Overview

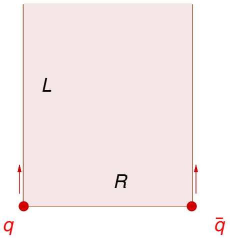
First evidence: confinement of quarks

$\swarrow$ $\searrow$

gauge theory point of view | effective string representation

$\boxed{\text{string theory} \leftrightarrow \text{confining gauge theories}}$

Overview



- Wilson loop: $q\bar{q}$ potential

$$< W(L,R) > \sim e^{-LV(R)}$$

- for large R: $V(R) \sim \sigma R + \ldots$

$\boxed{\text{area law}}$

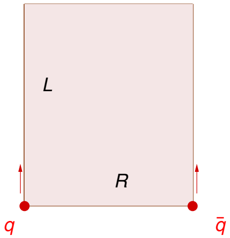
- σ: string tension

- Fluctuations of the colour flux tube$\rightsquigarrow$string d.o.f.
- Nambu$-$Goto with fixed endpoints: asymptotic expansion gives a confining term and a leading correction from quantum fluctuations [Lüscher, Symanzik and Weisz, 1980]

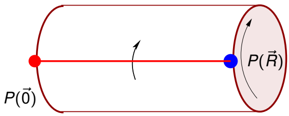
$$V(R) = \sigma R - \frac{\pi(D-2)}{24R} + \ldots$$

Overview

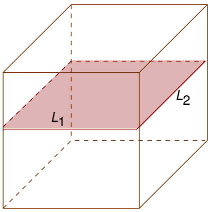
- Wilson loop
  ↓
  disk



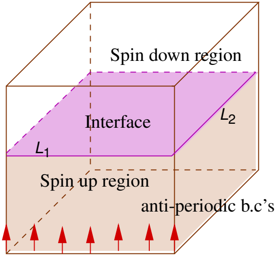
- Polyakov loop correlator
  ↓
  cylinder



- interface in a compact target space
  ↓
  torus



Interfaces



- surface between two regions of different magnetization
- T^d compact target space
- anti-periodic boundary conditions in one direction

fluctuations given by an effective string theory in a toroidal geometry

The Nambu-Goto action

- Generalization of the point particle action: proportional to the area of the surface traced by the string as it travels through spacetime
- The most natural model to describe surfaces

$$S_{NG} = \sigma \int d\xi^0 d\xi^1 \sqrt{-det(\partial_\alpha X_i \partial_\beta X^i)}$$

where $\xi^1 \in [0, \pi]$ and $\xi^0 \in [0, +\infty)$ are the world-sheet coordinates

Perturbative evaluation

- Invariance under reparametrizations → fix a physical gauge: the proper coordinates are identified with two target space coordinates
- The world-sheet is directly the minimal interface→torus
- After gauge fixing, functional integration over the $d-2$ transverse bosonic fields

$$
\begin{aligned}
Z_\Sigma &= \int_{(\partial\Sigma)} DX^i \, e^{-\sigma \int_\Sigma dX^0 dX^1 \left(1+(\partial_0\vec{X})^2+(\partial_1\vec{X})^2+(\partial_0\vec{X}\wedge\partial_1\vec{X})^2\right)^{\frac{1}{2}}} \\
&= \int_{(\partial\Sigma)} DX^i \, e^{-\sigma \int_\Sigma dX^0 dX^1 \left[1+\frac{1}{2}(\partial_0\vec{X})^2+\frac{1}{2}(\partial_1\vec{X})^2+\text{interactions}\right]}
\end{aligned}
$$

- $\frac{1}{\sigma\mathcal{A}}$: loop expansion parameter ($\mathcal{A}$ area of the minimal surface Σ) [Dietz and Filk, 1982]

First order formulation

Alternative formulation: Polyakov action

$$S_P = \sigma \int d\xi^0 \int_0^{2\pi} d\xi^1 \, h^{\alpha\beta} \partial_\alpha X^i \partial_\beta X_i$$

- $h_{\alpha\beta}$: independent world-sheet metric
- $\xi^1 \in [0, 2\pi]$ parametrizes the spatial extension of the string
- ξ^0 parametrizes the proper time evolution of the string
- gauge fixing: $h_{\alpha\beta} \to \eta_{\alpha\beta}$
- covariant quantization
  - appearance of ghosts
  - anomaly in the conformal algebra for $d \neq 26$ (Liouville mode)

Exact partition function

Interface described by closed string theory$\rightsquigarrow$the toroidal world-sheet is mapped into the target space T^d in different topological ways.

Partition function in the first order formulation:

$$\mathcal{Z}^{(d)} = \int \frac{d^2\tau}{\tau_2} \, Z^{(d)}(q, \bar{q}) \, Z^{\mathrm{gh}}(q, \bar{q})$$

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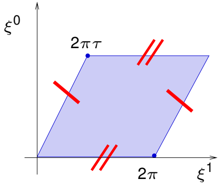
- $\tau = \tau_1 + \mathrm{i}\tau_2$ is the modular parameter of the world-sheet
- $q = e^{2\pi i\tau}, \bar{q} = e^{-2\pi i\bar{\tau}}$



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- $Z^{(d)}(q,\bar{q})$: partition function of the d compact bosons X^i:

$$Z^{(d)}(q,\bar{q}) = \mathrm{Tr}\; q^{L_0-\frac{d}{24}}\,\bar{q}^{\tilde{L}_0-\frac{d}{24}}$$

Exact partition function

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- $Z^{\mathrm{gh}}(q,\bar{q})$: partition function for the ghost system

Exact partition function

- Partition function of a single boson defined on a circle $X(\xi^0,\xi^1) \sim X(\xi^0,\xi^1) + L$

$$Z(q,\bar{q}) = \sum_{n,w\in\mathbb{Z}} q^{\frac{1}{8\pi\sigma}\left(\frac{2\pi n}{L}+\sigma w L\right)^2} \bar{q}^{\frac{1}{8\pi\sigma}\left(\frac{2\pi n}{L}-\sigma w L\right)^2} \frac{1}{\eta(q)}\,\frac{1}{\eta(\bar{q})}$$

  - n: discrete momentum $p = \frac{2\pi n}{L}$
  - w: winding around the compact target space

- Poisson resum over the integer n: from momenta to topological index

$$Z(q,\bar{q}) = \sqrt{\frac{\sigma}{2\pi}}L \sum_{m,w\in\mathbb{Z}} \mathrm{e}^{-\frac{\sigma L^2}{2\tau_2}|m-\tau w|^2} \frac{1}{\sqrt{\tau_2}\eta(q)\eta(\bar{q})}$$

Exact partition function

- Partition function for the ghost system

$$Z^{\mathrm{gh}}(q,\bar{q}) = (\eta(q)\eta(\bar{q}))^2$$

- Generalization to d bosons: the product over the d partition functions contains the sum over n^i and m^i, $i = 1...d$

Modular invariance

$$\downarrow$$

$$\boxed{\text{perform the integral on the fundamental modular domain of } \tau}$$

Exact partition function

- Partition function for the ghost system

$$Z^{\mathrm{gh}}(q,\bar{q}) = (\eta(q)\eta(\bar{q}))^2$$

- Generalization to d bosons: the product over the d partition functions contains the sum over n^i and m^i, $i = 1 \dots d$

Modular invariance

$$\downarrow$$

$$\boxed{\text{choose the winding numbers } w^i \text{ and } m^i}$$

$$\boxed{\text{integrate } \tau \text{ over all the upper half plane}}$$

Exact partition function

Interface aligned along a T^2 in the x^1, x^2 directions with minimal area

Many choices of w_1 , w_2 , m_1 , m_2

- We fix

$$w_1 = 1 \ , \quad w_2 = w_3 = \ldots = w_d = 0$$

$$\downarrow$$

string winding once in the x^1 direction

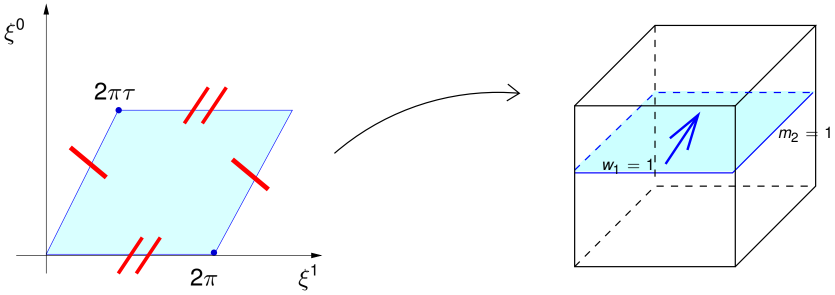
- Furthermore

$$m_2 = 1 \ , \quad m_3 = m_4 = \ldots = m_d = 0$$

$$\downarrow$$

string winding once in the x^2 direction

Exact partition function



$$w_1 = 1\ , \quad w_2 = w_3 = \ldots = w_d = 0$$

$$m_2 = 1\ , \quad m_3 = m_4 = \ldots = m_d = 0$$

Exact partition function

With this choice the partition function becomes:

$$\mathcal{I}^{(d)} = \prod_{i=2}^{d} \left(\sqrt{\frac{\sigma}{2\pi}} L_i \right) \sum_{k,k'=0}^{\infty} \sum_{m_1 \in \mathbb{Z}} c_k c_{k'} \int_{-\infty}^{\infty} d\tau_1 \mathrm{e}^{2\pi \mathrm{i}(k-k'+m_1)} \int_0^{\infty} \frac{d\tau_2}{(\tau_2)^{\frac{d+1}{2}}}$$

$$\times \exp\left\{ -\tau_2 \left[\frac{\sigma L_1^2}{2} + \frac{2\pi^2 m_1^2}{\sigma L_1^2} + 2\pi(k + k' - \frac{d-2}{12}) \right] - \frac{1}{\tau_2} \left[\frac{\sigma L_2^2}{2} \right] \right\}$$

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$$\times \exp\left\{ -\tau_2 \left[\frac{\sigma L_1^2}{2} + \frac{2\pi^2 m_1^2}{\sigma L_1^2} + 2\pi (k + k' - \frac{d-2}{12}) \right] - \frac{1}{\tau_2} \left[\frac{\sigma L_2^2}{2} \right] \right\}$$

Integration over τ_1 produces $\delta(k - k' + m_1)$

Level$-$matching condition

Exact partition function

With this choice the partition function becomes:

$$\mathcal{I}^{(d)} = \prod_{i=2}^{d}\left(\sqrt{\frac{\sigma}{2\pi}}L_i\right)\sum_{k,k'=0}^{\infty}\sum_{m_1\in\mathbb{Z}} c_k c_{k'}\int_{-\infty}^{\infty} d\tau_1 \mathrm{e}^{2\pi\mathrm{i}(k-k'+m_1)}\int_0^{\infty}\frac{d\tau_2}{(\tau_2)^{\frac{d+1}{2}}}$$

$$\times \exp\left\{-\tau_2\left[\frac{\sigma L_1^2}{2}+\frac{2\pi^2 m_1^2}{\sigma L_1^2}+2\pi(k+k'-\frac{d-2}{12})\right]-\frac{1}{\tau_2}\left[\frac{\sigma L_2^2}{2}\right]\right\}$$

Integration over τ_2 produces Bessel functions of type K_ν

Exact partition function

The final result is

$$\mathcal{I}^{(d)} = 2\left(\frac{\sigma}{2\pi}\right)^{\frac{d-2}{2}} V_T \sqrt{\sigma \mathcal{A} u} \sum_{k,k'=0}^{\infty} c_k c_{k'} \left(\frac{\mathcal{E}}{u}\right)^{\frac{d-1}{2}} K_{\frac{d-1}{2}}(\sigma \mathcal{A} \mathcal{E})$$

with the "spectrum" given by

$$\mathcal{E} = \sqrt{1 + \frac{4\pi u}{\sigma \mathcal{A}}(k + k' - \frac{d-2}{12}) + \frac{4\pi^2 u^2 (k-k')^2}{(\sigma \mathcal{A})^2}}$$

$$\mathcal{A} = L_1 L_2 \ , \quad u = \frac{L_2}{L_1}$$

It should resum the loop expansion of the functional integral

Functional Integral

Check with the perturbative result [Dietz and Filk, 1982]

- Physical gauge fixing: $d-2$ bosonic d.o.f. corresponding to the transverse fluctuations of the interface. Perturbative evaluation of the path integral:

$$\mathcal{I}^{(d)} \propto \sigma^{\frac{d-2}{2}} \frac{\mathrm{e}^{-\sigma\mathcal{A}}}{\left[\sqrt{u}\eta^2(\mathrm{i}u)\right]^{d-2}} \left\{1 + \frac{f_1(u)}{\sigma\mathcal{A}} + \dots\right\}$$

  where the two loop term is given by

$$f_1 = \left\{\frac{(d-2)^2}{2}\left[\left(\frac{\pi}{6}\right)^2 u^2 E_2^2(\mathrm{i}u) - \frac{\pi}{6} u E_2(\mathrm{i}u)\right] + \frac{d(d-2)}{8}\right\}$$

- Our exact expression reproduces the perturbative expansion when asymptotically expanded for large $\sigma\mathcal{A}$.

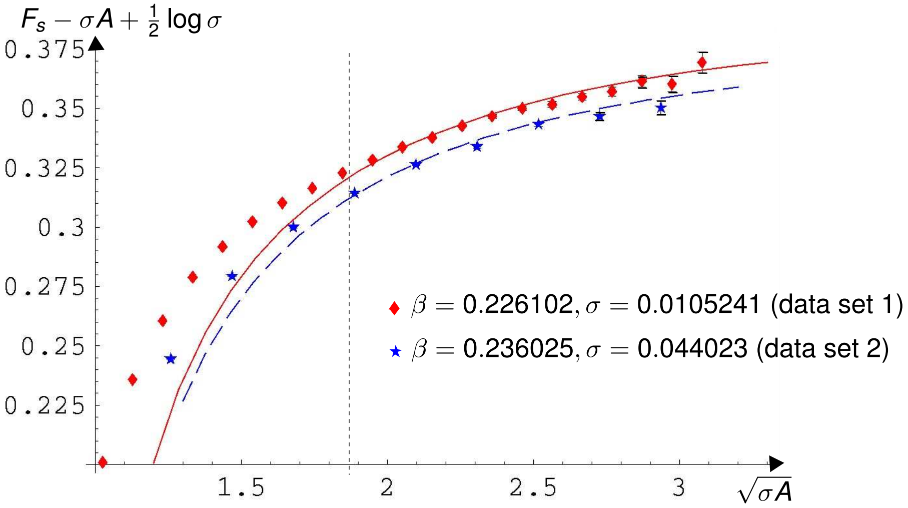
Comparison with Monte Carlo data

- Two sets of Monte Carlo data for the interface free energy in 3d and square lattice ($u = 1$) [Caselle et al., 2006] are compared to our theoretical predictions (solid and dashed lines)
- Free energy corresponding to our partition function

$$F = -\log\left(\frac{\mathcal{I}^{(3)}}{V_T}\right) + \mathcal{N}$$

- $\mathcal{N}$: free parameter $\rightarrow$ an overall normalization of the NG partition function
- The MC data are very well accounted for when we consider lattices of sufficiently large sizes, typically with $L \geq \frac{2}{\sqrt{\sigma}}$. For smaller ones, deviations are observed

Comparison with Monte Carlo data

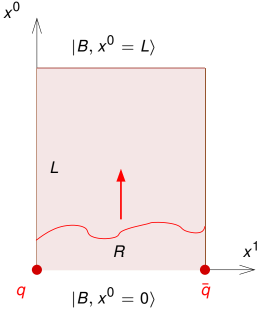


Next step

- Take in exam the Wilson loop
- As in the case of interfaces, the goal is to find an exact expression which resums the functional integration result
- Use the operatorial approach of open boundary states

Next step

Define boundary states for open strings, in analogy with the ones for closed [Imamura et al., 2005]



$$\langle B, x^0 = 0|\Delta|B, x^0 = L\rangle$$

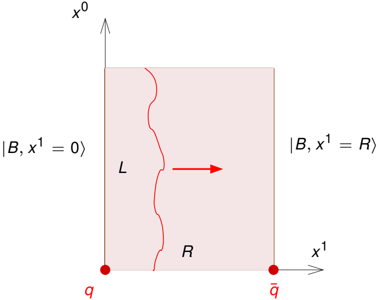
- $\langle B, x^0|$: open boundary state
- Δ: open string propagator

$$\int dt e^{-2\pi t L_0}$$

We expect invariance under $L \leftrightarrow R$

Next step

Define boundary states for open strings, in analogy with the ones for closed [Imamura et al., 2005]



$$\langle B, x^1 = 0|\Delta|B, x^1 = R\rangle$$

Not invariant: we should change measure in Δ. Writing $\Delta' = \int \frac{dt}{t^w} e^{-2\pi t L_0}$, we can choose the power w in order to make the result invariant.

Our expression fits the functional one only up at 1 loop...

Conclusions so far...

- We studied interfaces in a T^d space with Nambu$-$Goto string in the first order formulation
- We found an exact result which is the resummation of the perturbative one
- In the 3d square lattice case Nambu$-$Goto is a reliable effective model (for lattices of sufficiently large sizes). Also in the case of asymmetric lattices ($u \neq 1$) it fits well the MC data
- Liouville mode is to be taken into account for smaller sizes to avoid the breaking of conformal invariance
- The explanation of the Wilson loop in terms of open boundary states is not yet clear