Critical behaviour of the surface tension in the 3D Ising model

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Summary

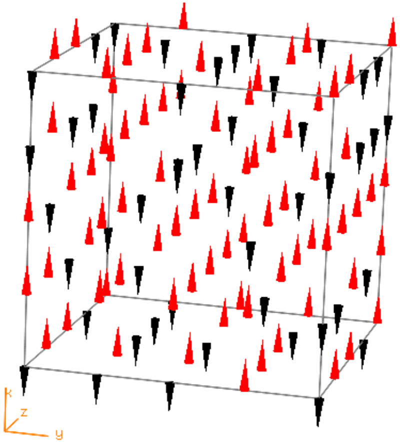
- Definition of the surface tension
- Algorithm for generating the Markov chain
- (Notes on the implementation?)
- Estimation of the errors and autocorrelation
- Fit of the free energy
- Fit of the critical behaviour
- Conclusion

3D Ising model

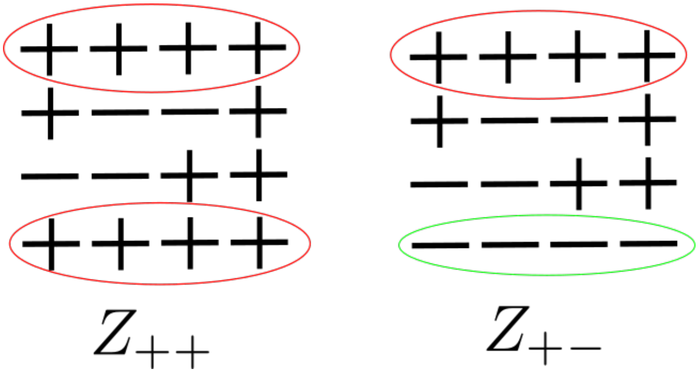
$$\mathcal{H} = -\sum_{\langle x,y \rangle} J_{\langle x,y \rangle} s_x s_y$$

$J_{\langle x,y \rangle} = 1$ ferromagnetic

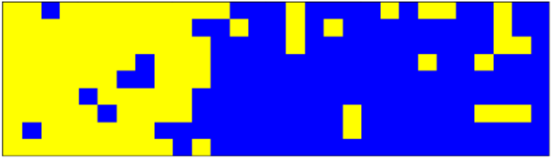
$J_{\langle x,y \rangle} = -1$ antiferromagnetic



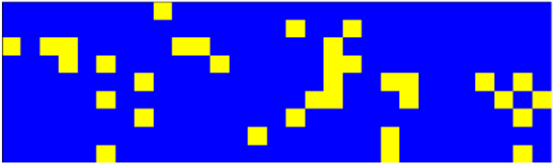
Definition of the surface tension



$$\sigma = -\lim_{L \to +\infty} \frac{1}{L^2} \log \frac{Z_{+-}}{Z_{++}} \quad L \times L \times cL \quad c = \text{const}$$



Z_{+-}



Z_{++}

$$\sigma = -\lim_{L\to+\infty} \frac{1}{L^2} \log \frac{Z_{+-}}{Z_{++}} = \lim_{L\to+\infty} \frac{1}{L^2} \left(F_{+-} - F_{++}\right)$$

$\sigma =$ difference of the free energies per unit area

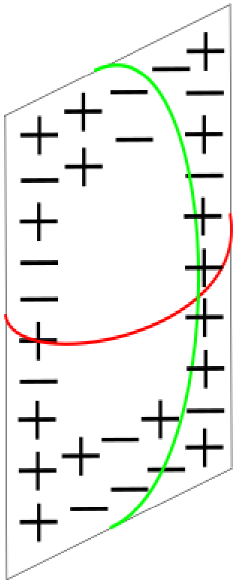
Redefinition of Z_{++} and Z_{+-}.

$Z_{++} \to$ ferromagnetic link between **top and bottom**.
$Z_{+-} \to$ **anti**ferromagnetic link between **top and bottom**.

Always ferromagnetic link in x and y.

Same definition for σ.

Periodic boundary conditions reduce the finite size effect.



Montecarlo simulations can't measure Z!

Solution: $J_{\langle x,y \rangle}$ between top and bottom becomes a **dinamical variable** that is summed over in Z.
$J_{\langle x,y \rangle} = 1$ (periodic b.c.) $J_{\langle x,y \rangle} = -1$ (antiperiodic b.c.)
Other $J_{\langle x,y \rangle}$ remains ferromagnetic.

$$Z = \sum_{\{s\},J} \exp\left(\beta \sum_{\langle x,y \rangle} J_{\langle x,y \rangle} s_x s_y\right)$$

$$\frac{Z_{+-}}{Z_{++}} = \frac{\frac{Z_{+-}}{Z}}{\frac{Z_{++}}{Z}} = \frac{\left\langle \delta_{J=-1} \right\rangle}{\left\langle \delta_{J=+1} \right\rangle}$$

We want to define the free energy of the interface in order to improve the convergence proprieties of $\frac{F_s}{L^2}$ in the thermodynamic limit.
In $F_{+-} - F_{++}$ there is an **entropic** contribution which we sutract:

$$F_s = F_{+-} - F_{++} + \ln(cL)$$