

An accurate empirical correlation for predicting natural gas compressibility factors

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[Manuscript received July 15, 2011; revised November 10, 2011]

Abstract

The compressibility factor of natural gas is an important parameter in many gas and petroleum engineering calculations. This study presents a new empirical model for quick calculation of natural gas compressibility factors. The model was derived from 5844 experimental data of compressibility factors for a range of pseudo reduced pressures from 0.01 to 15 and pseudo reduced temperatures from 1 to 3. The accuracy of the new empirical correlation has been compared with commonly used existing methods. The comparison indicates the superiority of the new empirical model over the other methods used to calculate compressibility factor of natural gas with average absolute relative deviation percent (AARD%) of 0.6535.

Key words

natural gas; compressibility factor; correlation; empirical model; equation of state

1. Introduction

Natural gas is a subcategory of petroleum that is a naturally occurring, complex mixture of hydrocarbons, with a small amount of inorganic compounds [1]. Knowledge of the pressure-volume-temperature (*PVT*) behavior of natural gases is necessary to solve many petroleum engineering problems, design and analyze natural gas production and processing systems.

In most petroleum and natural gas engineering calculations, the compressibility factor and density of fluids are necessary to gas metering, gas compression, design of pipelines and surface facilities [2]. Petroleum engineers need to predict pressure-volume temperature (*PVT*) properties of petroleum fluids under given conditions of temperature and pressure. This can be determined through *PVT* analysis of fluid sample tests or can be calculated by Equations of State (EOS) based on computer codes if the fluid composition is known. This information is often unavailable particularly at the early stage of field development or needs to be verified, supported and supplemented during the course of field development. Then it is the task of empirical correlations to estimate the petroleum fluid properties as a function of the reservoir's readily available characteristics [3]. In this study, a simple, quick and accurate empirical model has been derived from experimental

data and the accuracy of the model has been compared with commonly used empirical correlations as well as classical and statistical equations of state.

2. Natural gas compressibility factor

In the real gas equation, *Z*-factor can be obtained as follow:

$$PV = ZnRT \quad (1)$$

where, *P* is pressure in kPa, *V* is the volume in L, *n* is the number of gas mol, *R* is universal gas constant, *T* is temperature in K, and *Z* is the compressibility factor of gas.

The theory of corresponding states dictates that the *Z*-factor can be uniquely defined as a function of reduced pressure and reduced temperature. The pseudo reduced pressure and temperature are defined by Equations 2 and 3, respectively.

$$T_{pr} = T/T_{pc} \quad (2)$$

$$P_{pr} = P/P_{pc} \quad (3)$$

where, *P_{pr}* and *T_{pr}* are pseudo reduced pressure and temperature, *T_{pc}* is pseudo critical temperature and *P_{pc}* is pseudo critical pressure that calculated as follow:

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This work was supported by the University of Kashan (Grant No. 65460).

$$P_{pc} = \sum_{i=1}^n P_{ci} y_i \quad (4)$$

$$T_{pc} = \sum_{i=1}^n T_{ci} y_i \quad (5)$$

where, P_{ci} is critical pressure, T_{ci} is critical temperature, and y_i is the mole fraction of component i .

By knowing the composition of natural gas, pseudo pressure and temperature of gas can be obtained and used in em-

pirical models to predict compressibility factor. Some of commonly used empirical correlations and equations of state for calculating of Z-factor are presented as follow:

2.1. Dranchuk and Abu-Kassem correlation and Heidaryan-Salarabadi-Moghadasi (HMS) correlation

In 1975, Dranchuk and Abu-Kassem (DA) proposed Equation 6 with eleven-constant for calculating the gas compressibility factors [4].

$$Z = \left[A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}^3} + \frac{A_4}{T_{pr}^4} + \frac{A_5}{T_{pr}^5} \right] \rho_r + \left[A_6 + \frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2} \right] \rho_r^2 - A_9 \left[\frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2} \right] \rho_r^5 + A_{10} (1 + A_{11} \rho_r^2) \frac{\rho_r^2}{T_{pr}^3} \exp[-A_{11} \rho_r^2] + 1 \quad (6)$$

where, ρ_r is reduced density of gas.

In 2010, Heidaryan et al. presented a correlation based

on Standing and Katz chart for rapid estimation of natural gas Z-factor as Equation 7 [5]:

$$Z = \frac{A_1 + A_2 \ln(P_{pr}) + A_3 (\ln P_{pr})^2 + A_4 (\ln P_{pr})^3 + \frac{A_5}{T_{pr}} + \frac{A_6}{T_{pr}^2}}{1 + A_7 \ln(P_{pr}) + A_8 (\ln P_{pr})^2 + \frac{A_9}{T_{pr}} + \frac{A_{10}}{T_{pr}^2}} \quad (7)$$

2.2. Azizi-Behbahani-Isazadeh (ABI) correlation

Azizi, Behbahani and Isazadeh in 2010 presented a new correlation for calculating compressibility factor of natural gases based on 3038 points from Standing and Katz Z-factor chart:

$$Z = A + \frac{B + C}{D + E} \quad (8)$$

where,

$$A = aT_r^{2.16} + bP_r^{1.028} + cP_r^{1.58}T_r^{-2.1} + d \ln(T_r)^{-0.5} \quad (9)$$

$$B = e + fT_r^{2.4} + gP_r^{1.56} + hP_r^{0.124}T_r^{3.033} \quad (10)$$

$$C = i \ln(T_r)^{-1.28} + j \ln(T_r)^{1.37} + k \ln(P_r) + l \ln(P_r)^2 + m \ln(P_r) \ln(T_r) \quad (11)$$

$$D = 1 + nT_r^{5.55} + oP_r^{0.68}T_r^{0.33} \quad (12)$$

$$E = p \ln(T_r)^{1.18} + q \ln(T_r)^{2.1} + r \ln(P_r) + s \ln(P_r)^2 + t \ln(P_r) \ln(T_r) \quad (13)$$

The tuned coefficients in Equations 8 to 13 are constant independent parameters which can be calculated by curve fitting method [6].

2.3. Peng-Robinson equation of state

The Peng-Robinson (PR) equation of state (EOS) is very effective for predicting K-values for hydrocarbon systems at

medium to high pressures. Good results have been obtained by this method for demethanizers, de-ethanizers, depropanizers, debutanizers and wellhead processes. The compressibility factors for both vapor and liquid phases can be calculated by the PR equation of state. The PR EOS has been presented by Equation 14 [7]:

$$P = \frac{RT}{V - b} - \frac{a}{V(V + b) - b(V - b)} \quad (14)$$

2.4. Statistical association fluid theory (SAFT) equation of state

The SAFT equation of state is similar to perturbed hard chain equations, but provides competitive accuracy with UNIQUAC, NRTL, or Wilson's equation for hydrogen bonding mixtures. Unlike the interpolation methods, the SAFT equation is based on treating the hydrogen bonding interactions as chemical reactions. The compressibility factor can be obtained from SAFT EOS as Equation 15 [8]:

$$Z = \frac{PV}{RT} = 1 + Z^{\text{seg}} + Z^{\text{chain}} + Z^{\text{assoc}} \quad (15)$$

3. New proposed compressibility factor correlation

This work tried to find a rapid method to calculate compressibility factor based on experimental data with high accuracy compared with other empirical methods by reduced pressure and reduced temperature. By considering 5844 experimental data points for Z-factors of natural gas mixtures and using all of them in multiple regression analysis, an empirical correlation based on Virial equation of state is suggested to accurately predict natural gas compressibility factor in the range

of $1.01 \leq T_{pr} \leq 3.0$ and $0.01 \leq P_{pr} \leq 15.0$. This new correlation has two dependent variables (P_{pr} and T_{pr}) and eight independent variables ($A_1 - A_8$). The Z-factor from this correlation is expressed as follow:

$$Z = 1 + A_1 P_{pr} + A_2 P_{pr}^2 + \frac{A_3 P_{pr}^{A_4}}{T_{pr}^{A_5}} + \frac{A_6 P_{pr}^{(A_4+1)}}{T_{pr}^{A_7}} + \frac{A_8 P_{pr}^{(A_4+2)}}{T_{pr}^{(A_7+1)}} \quad (16)$$

For further accuracy of this model, the independent variables in reduced pressures from 0.01 to 3.0 are different from 3.0 to 15, which are presented in Table 1.

Table 1. Tuned coefficients used in Equation 16

Coefficients	$0.01 < P_{pr} < 3.0$	$3.0 < P_{pr} < 15$
A_1	0.007698	0.015642
A_2	0.003839	0.000701
A_3	-0.467212	2.341511
A_4	1.018801	-0.657903
A_5	3.805723	8.902112
A_6	-0.087361	-1.136000
A_7	7.138305	3.543614
A_8	0.083440	0.134041

The presented correlation in this study is obtained by considering of a large number of compressibility factor data for different natural gas mixtures. These data contain 5844 points of different natural gas mixtures that measured by many researchers from 1939 to 2009 [9–33]. The gas mixtures have different molecular weight from 11.3 to 30 or gas gravity from 0.389 to 1.04. The properties of gas mixture used in this work have been reported in Table 2.

Table 2. Properties of different natural gas used in this study

Composition (mol%)	Minimum	Maximum	Average
Methane	30.64	100	88.29
Carbon dioxide	0	50	3.4970
Ethane	0	64.56	2.2368
Propane	0	21	0.8194
<i>i</i> -butane	0	0.755	0.0236
<i>n</i> -butane	0	0.755	0.0221
<i>i</i> -pentane	0	0.299	0.0092
<i>n</i> -pentane	0	0.304	0.0067
Hexane	0	0.058	0.0003
Heptane plus	0	0.043	0.0004
Nitrogen	0	50	4.1563
Hydrogen	0	33.75	0.9382
Pressure (bar)	0.1	973	173.51
Temperature (K)	240	598	372.73
Z-factor	0.5221	1.431	0.9439

4. Results and discussion

To compare the accuracy of proposed correlation, calculated Z-factor versus 5844 experimental compressibility factor data has been presented in Figure 1. Figure 1 also shows the difference between Z-factor of experimental data and output of the correlation in the range of $0.01 \leq P_{pr} \leq 15.0$ and $1.01 \leq T_{pr} \leq 3.0$.

Table 3 reports the average absolute relative deviation percent (AARD%) of the new empirical correlation compared with commonly used empirical models and equations of state for different natural gas mixtures. The compositions of natural gas namely “N.G.1” to “N.G.10” are listed in Table 4.

Table 3. Average absolute relative deviation of this study compared with other empirical models and EOSs

Methane (mol%)	Other components	Data points	P range (MPa)	T range (K)	DA	HMS	ABI	PR-EOS	SAFT-EOS	This study	Ref.
0.5–0.9	CO ₂	277	0.01–80	307–673	2.8762	5.1101	1.3853	2.7114	0.9405	1.4399	[9,18,27]
0.5–0.9	N ₂	455	0.6–80	240–673	1.6106	2.3193	2.3344	2.1464	0.7913	0.6804	[9,18,24,31]
0.6–0.8	CO ₂ -N ₂	56	20–80	373–573	1.5438	1.4754	4.3232	4.0402	0.6472	1.2978	[19]
0.54–0.94	H ₂ O	70	7.5–18.8	430–698	4.3349	5.377	2.4659	0.3891	0.3565	2.2396	[20]
0.35–0.85	C ₂	558	0.1–24.1	294–394	1.9081	5.0589	2.1445	2.2584	0.9406	0.9107	[10,23]
0.79–0.93	C ₃	56	0.9–10	278–313	0.6971	3.9476	0.9912	2.311	0.8687	1.4817	[21]
0.91–0.94	C ₃ -C ₆	30	0.8–10.1	293–313	0.5511	5.5574	1.0182	2.558	0.1939	0.7617	[22]
0.3–0.34	C ₂ -CO ₂	60	0.7–7.5	283–333	3.0628	2.9833	0.8676	2.4633	1.4118	0.9113	[25]
0.35–0.53	C ₄	71	8.8–48.1	316–479	1.5802	3.9886	3.5132	1.487	0.6939	0.8601	[26]
0.55–0.81	H ₂	109	0.4–51	273–348	1.4738	3.3327	2.485	1.8357505	0.6931	0.7994	[11,15]
1	–	3336	0.1–100	250–573	2.1609	2.5584	2.2452	1.6887	0.5913	0.4841	[9–17]
0.8484	N. G.1	40	0.24–14	241–455	0.8099	11.3732	3.4509	0.9832	0.1301	0.2029	[32]
0.9016	N. G.2	45	4.46–25.1	263–303	4.0135	11.6532	1.3212	1.1834	0.8112	0.5242	[33]
0.8001	N. G.3	34	13.3–25.3	262–304	13.2343	17.8473	1.1322	4.4563	1.6011	2.0733	[33]
0.922	N. G.4	45	4.9–25.2	262–304	4.0129	7.0848	4.7866	3.0021	0.7353	0.6965	[33]
0.8957	N. G.5	224	0.1–20.1	260–320	3.5317	8.2178	2.87	3.6513	0.9919	0.6289	[16]
0.8433	N. G.6	224	0.1–20.2	269–320	2.6531	8.7632	2.6523	1.9734	1.1032	0.3569	[16]
0.9835	N. G.7	28	1.0–15	253–323	0.9843	1.8322	0.8546	0.8812	0.4512	0.3212	[28]
0.9036	N. G.8	28	1.0–15	253–323	0.7916	2.0003	1.6598	0.8113	0.5087	0.4457	[28]
0.9244	N. G.9	28	1.0–15	253–323	1.1569	3.1456	1.1232	1.7632	0.4257	0.8322	[28]
0.9099	N. G.10	70	2.2–34.5	263–350	3.4362	3.8321	3.0013	2.3421	0.9588	0.7144	[30,31]
Average		5844			2.2684	3.7086	2.4063	1.9566	0.7097	0.6535	

Table 4. Compositions of 10 different natural gas mixtures mentioned in Table 3

	N.G.1	N.G.2	N.G.3	N.G.4	N.G.5	N.G.6	N.G.7	N.G.8	N.G.9	N.G.10
Methane	84.84	90.16	80.0079	92.2045	89.57	84.33	98.352	90.362	92.436	90.991
Ethane	8.4	6.3077	9.3063	4.3373	6.1464	3.4085	0.511	5.708	1.285	2.949
Propane	0.5	0.801	4.963	0.5396	1.2532	0.6023	0.153	1.124	0.348	1.513
<i>n</i> -Butane	—	0.0643	1.2791	0.0771	0.1924	0.1282	0.031	0.169	0.046	0.755
<i>i</i> -Butane	—	0.0446	0.7188	0.2562	0.2857	0.1033	0.021	0.301	0.041	0.755
<i>n</i> -Pentane	—	0.0044	0.2499	0.0198	0.0324	0.035	0.011	0.029	0.014	0.304
<i>i</i> -Pentane	—	0.0054	0.2556	0.0468	0.0565	0.0375	0.008	0.059	0.015	0.299
Hexane	—	0.0014	0.1793	0.0606	0.0572	0.0388	0.005	0.058	0.012	—
Heptane	—	0.0005	0.101	0.0364	0.034	0.0174	0.001	0.035	—	—
Heptane ⁺	—	0.0001	0.0621	0.0091	0.0095	0.0072	0.0003	0.008	—	—
Hydrogen	—	—	0.001	0.0005	0.0007	0.0005	—	—	—	—
Helium	—	0.0068	0.0084	0.0168	0.0137	0.052	—	0.015	—	—
Water	—	0.0006	—	—	—	—	—	—	—	—
Argon	—	0.0514	—	—	0.0419	0.01	—	0.011	—	—
Nitrogen	5.6	0.7587	0.6601	1.3916	1.5324	9.752	0.841	1.474	5.751	2.031
Carbon dioxide	0.66	1.7947	2.1902	1.003	0.774	1.4523	0.066	0.647	0.052	0.403
Benzene	—	0.0002	0.0173	0.0007	—	0.025	—	—	—	—

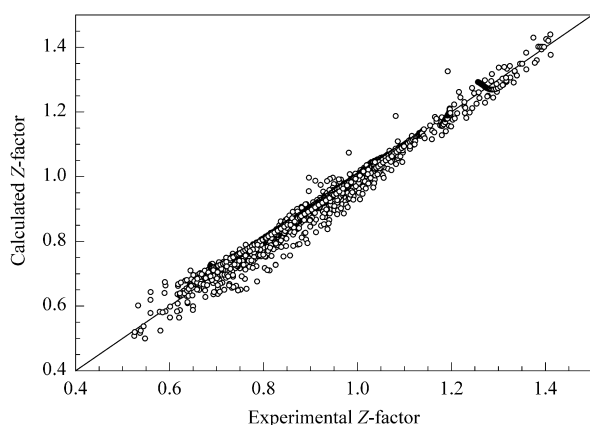
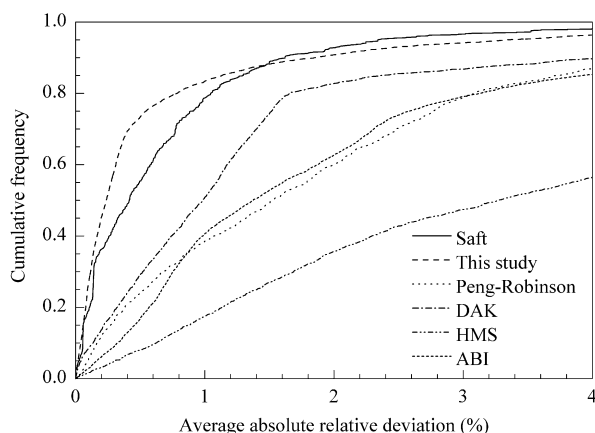
**Figure 1.** Accuracy of this study versus 5844 experimental data points [9–33]

Figure 2 shows the cumulative frequency of this study correlation versus AARD% of method compared with empirical models of DA, HMS and ABI as well as equations state of PR and SAFT.

**Figure 2.** AARD% of various methods in calculating natural gas Z-factor as function of cumulative frequency

According to Figure 2, the new proposed empirical correlation is more accurate than DA, HMS, ABI, PR and SAFT models. This figure shows the accuracy of the new model in prediction of compressibility factor of 5844 measurements in comparison with other models used in this work. The new method has successfully predicted 73% of all the measurements with AARD% less than 0.5% and 84% of the data with AARD% less than 1%. Only 3.5% of the 5844 Z-factor measurements were predicted with AARD% higher than 4%. SAFT EOS, that is the second accurate method, predicted 56% of the compressibility factor measurements with AARD% less than 0.5% and 79% of the measurements with AARD% less than 1%. Hence this new correlation shows its priority over all the methods considered in this study.

To estimate the applicability of this new correlation for calculating compressibility factor of various natural gases, the experimental data and results of Equation (16) and other mentioned models for Z-factor versus pseudo reduced pressure at constant temperature for different natural gas mixtures data obtained from Khangiran Refinery [34] are presented in Figures 3. It should be noted that the experimental gas mixture compressibility factor data of Khangiran Refinery were not employed in regression analysis of new proposed empirical correlation. The composition of Kangiran gas mixture is reported in Table 5.

Table 5. Composition of Khangiran Refinery natural gas

Components	Composition (mol%)
Methane	98.64
Ethane	0.593
Propane	0.065
<i>n</i> -Butane	0.034
<i>i</i> -Butane	0.015
<i>n</i> -Pentane	0.019
<i>i</i> -Pentane	0.026
<i>n</i> -Hexane	0.125
Nitrogen	0.428
Carbon dioxide	0.055

As shown in Figure 3, the new empirical correlation presented in this study is much more accurate than other empirical methods and equation of states for prediction of compressibility factors with average absolute relative deviation percent of 0.3789.

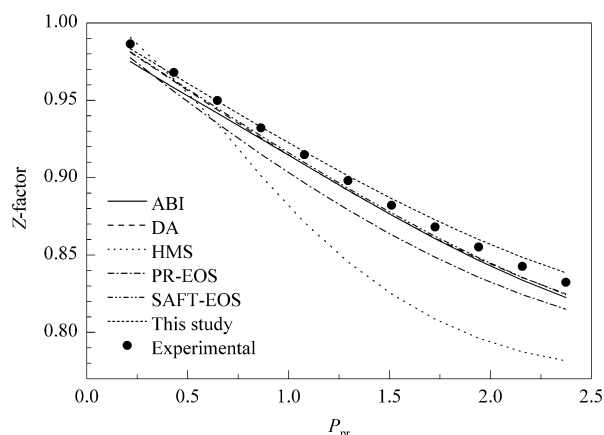


Figure 3. Calculated and measured Z-factor for Khangiran Refinery data ($T = 298\text{ K}$)

5. Conclusions

In this study, a simple empirical correlation for rapid prediction of compressibility factor of natural gas is presented. This correlation is provided based on 5844 natural gas compressibility factor experimental data as a function of pseudo reduced pressure and pseudo reduced temperature. The accuracy of the new empirical correlation has been compared with the commonly used empirical models, namely DA, HMS and ABI as well as equations of state of PR and SAFT. The comparison indicates the superiority of the new method over the other methods used to calculate compressibility factor of natural gas with average absolute relative deviation percent (AARD%) of 0.6535.

Acknowledgements

The authors are grateful to University of Kashan for supporting this work by Grant No.65460.

References

- [1] Nikkholgh M R, Moghadassi A R, Parvizian F. The 6th International Chemical Engineering Congress & Exhibition, Kish Island, 2009
- [2] Elsharkawy A M. *Fluid Phase Equilib*, 2004, 218: 1
- [3] Ahmed T H. *Hydrocarbon Phase Behavior*. Gulf Publishing Company: Houston, TX, 1989. 79
- [4] Dranchuk P M, Abou-Kassem J H. *J Can Petrol Technol*, 1975, 14: 34
- [5] Heidaryan E, Salarabadi A, Moghadassi, J. *J Nat Gas Chem*, 2010, 19: 189
- [6] Azizi N, Behbahani R, Isazadeh M A. *J Nat Gas Chem*, 2010, 19: 642
- [7] Peng D Y, Robinson D B. *Ind Eng Chem Fundamen*, 1976, 15: 59
- [8] Huang S H, Radosz M. *Ind Eng Chem Res*, 1990, 29: 2284
- [9] Seitz J C, Blencoe J G, Bodnar R J. *J Chem Thermodyn*, 1996, 28: 521
- [10] Hou H W, Holste J C, Hall K R, Marsh K N, Gammon B E. *J Chem Eng Data*, 1996, 41: 344
- [11] Mihara S, Sagara H, Arai Y, Saito S. *J Chem Eng Japan*, 1977, 10: 395
- [12] Douslin D R, Harrison R H, Moore R T, McCullough J P. *J Chem Eng Data*, 1964, 9: 358
- [13] Achtermann H J, Hong J G, Wanger W, Pruss A. *J Chem Eng Data*, 1992, 37: 414
- [14] Goodwin A R H. *J Chem Eng Data*, 2009, 54: 536
- [15] Chuang S Y, Chapple P S, Kobayashi R. *J Chem Eng Data*, 1976, 21: 403
- [16] Schley P, Jaeschke M, Kuchenmeister C, Vogel E. *Int J Thermophys*, 2004, 22: 1623
- [17] Younglove B A, Ely J F. *J Phys Chem Ref Data*, 1987, 16: 577
- [18] Seitz J C, Blencoe J G. *J Chem Thermodyn*, 1996, 28: 1207
- [19] Seitz J C, Blencoe J G, Bodnar R J. *J Chem Thermodyn*, 1996, 28: 539
- [20] Fenghour A, Wakeham W A, Watson J T R. *J Chem Thermodyn*, 1996, 28: 447
- [21] May E F, Miller R C, Goodwin A R H. *J Chem Eng Data*, 2002, 47: 102
- [22] May E F, Miller R C, Shan Z J. *J Chem Eng Data*, 2001, 46: 1160
- [23] Sage B H, Lacey W N. *Ind Eng Chem*, 1939, 31: 1497
- [24] Ababio B D, McElroy P J, Williamson C J. *J Chem Thermodyn*, 2001, 33: 413
- [25] McElroy P J, Fang J, Williamson C J. *J Chem Thermodyn*, 2001, 33: 155
- [26] Fenghour A, Trusler J P M, Wakeham W A. *Fluid Phase Equilib*, 1999, 163: 139
- [27] Reamer H H, Olds R H, Sage B H, Lacey W N. *Ind Eng Chem*, 1944, 36: 88
- [28] Capla L, Buryan P, Jedelsky J, Rottner M, Linek J. *J Chem Thermodyn*, 2002, 34: 657
- [29] Patil P, Ejaz S, Atilhan M, Cristancho D, Holste J C, Hall K R. *J Chem Thermodyn*, 2007, 39: 1157
- [30] Zhou J J, Patil P, Ejaz S, Atilhan M, Holste J C, Hall K R. *J Chem Thermodyn*, 2006, 38: 1489
- [31] Chamorro C R, Segovia J J, Martín M C, Villamañán M A, Estela-Urbe J F, Trusler J P M. *J Chem Thermodyn*, 2006, 38: 916
- [32] Assael M J, Dalaouti N K, Vesovic V. *Int J Thermophys*, 2001, 22: 61
- [33] Langelandsvik L I, Solvang S, Rousselet M, Mextaxa I N, Assael M J. *Int J Thermophys*, 2007, 28: 1120
- [34] Farzanehgard M, Hashemi S, Khamforoosh S A, PoorKhadem H. 2nd ed, National Iranian Petroleum Engineering. February, 2008