$$\underbrace{\lim_{n\to\infty}\left(1+\frac{1}{a_n}\right)^{a_n}}_{\parallel} = e \text{ , si } a_n \xrightarrow[n\to\infty]{} +\infty$$

$$\lim_{n\to\infty}\left(1+\frac{1}{\text{algo}}\right)^{\text{algo}} = e \text{ , si algo} \xrightarrow[n\to\infty]{} +\infty .$$

Ej 2) $\displaystyle\lim_{n\to\infty}\underbrace{\left(1+\underbrace{\frac{1}{3n}}_{\to 0}\right)}_{\to 1}^{\overset{\longrightarrow\infty}{n}} \longrightarrow 1^{\infty}$ ¡indet!

$$\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^{n} = \lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^{\overbrace{3n}\cdot\overbrace{\tfrac{1}{3}}} = \lim_{n\to\infty}\Bigg[\overbrace{\underbrace{\left(1+\frac{1}{3n}\right)^{3n}}_{\to 1}}^{\to e}\Bigg]^{\frac{1}{3}} = e^{\frac{1}{3}} = \sqrt[3]{e} \;/\!/$$

$$(a^b)^c = a^{b.c} \qquad n = 3n.\frac{1}{3} \qquad n = n\cdot 1 = \frac{n\cdot 3}{3} \qquad 3n \xrightarrow[n\to\infty]{} +\infty$$

Ej 3) $\displaystyle\lim_{n\to\infty}\underbrace{\left(1+\underbrace{\frac{5}{n^2}}_{\to 0}\right)}_{\to 1}^{\overset{\longrightarrow\infty}{n+1}} \longrightarrow 1^{\infty}$ ¡indet!

$$\boxed{n+1 = (n+1)\frac{n^2}{5}\cdot\frac{5}{n^2} = \frac{n^2}{5}\cdot\frac{5(n+1)}{n^2}}$$

$$\lim_{n\to\infty}\left(1+\frac{5}{n^2}\right)^{n+1} = \lim_{n\to\infty}\left(1+\frac{1}{\frac{n^2}{5}}\right)^{n+1} = \lim_{n\to\infty}\underbrace{\left(1+\frac{1}{\frac{n^2}{5}}\right)}^{\frac{n^2}{5}\cdot\frac{5}{n^2}(n+1)} =$$

$$\frac{5}{n^2} = \frac{1}{\frac{1}{5}\cdot n^2}$$

$$\frac{1}{\frac{1}{5}n^2} = \frac{1}{\frac{n^2}{5}} = 1 : \frac{n^2}{5} = \frac{5}{n^2}$$

$$\frac{1}{\frac{a}{b}} = \frac{b}{a}$$

$$= \lim_{n\to\infty}\Bigg[\underbrace{\left(1+\frac{1}{\frac{n^2}{5}}\right)^{\frac{n^2}{5}}}_{\xrightarrow[n\to\infty]{} e}\Bigg]^{\underbrace{\frac{5n+5}{n^2}}_{\xrightarrow[n\to ?]{} ?}} = e^{0} = 1 \;/\!/$$

Caux $\displaystyle\lim_{n\to\infty}\frac{5n+5}{n^2} \xrightarrow{} \frac{\infty}{\infty}$ indet

$$\lim_{n\to\infty}\frac{\cancel{n}\left(5+\frac{5}{n}\right)}{n^{\cancel{2}}} = \lim_{n\to\infty}\frac{5+\overset{\nearrow 0}{\frac{5}{n}}}{n} = 0$$

EXTRA: $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{3n} = \lim_{n\to\infty}\Bigg[\underbrace{\left(1+\frac{1}{n}\right)^{n}}_{\longrightarrow e}\Bigg]^{3} = e^{3} \;/\!/.$$

miro

Ej4) $\lim\limits_{n\to\infty}\left(\underbrace{\frac{2n^3+1}{2n^3-3}}_{\to \frac{2}{2}=1}\right)^{n^4} \quad \begin{matrix}\longrightarrow \infty \\ \longrightarrow 1\end{matrix}$ indet!

$1+\frac{1}{*}$

$\underbrace{1+\frac{1}{\text{algo}}}$

$$\frac{2n^3+1}{2n^3-3} = \frac{2n^3-3+3+1}{2n^3-3} = \frac{2n^3-3\ +4}{2n^3-3} = \frac{2n^3-3}{2n^3-3}+\frac{4}{2n^3-3} = 1+\frac{4}{2n^3-3} = 1+\frac{1}{\frac{2n^3-3}{4}}$$

$$\frac{2n^3+1}{2n^3-3} = 1+\frac{2n^3+1}{2n^3-3}-1 = 1+\frac{2n^3+1-(2n^3-3)}{2n^3-3} = 1+\frac{4}{2n^3-3}$$

$$1+\frac{2n^3+1}{2n^3-3}-\frac{2n^3-3}{2n^3-3}$$

$$\lim_{n\to\infty}\left(\frac{2n^3+1}{2n^3-3}\right)^{n^4} = \lim_{n\to\infty}\left(1+\frac{1}{\frac{2n^3-3}{4}}\right)^{n^4} =$$

$$= \lim_{n\to\infty}\left[\underbrace{\left(1+\frac{1}{\frac{2n^3-3}{4}\to+\infty}\right)^{\frac{2n^3-3}{4}}}_{\longrightarrow e}\right]^{\underbrace{\frac{4}{2n^3-3}\cdot n^4}_{\longrightarrow+\infty}} = +\infty$$

∞

CAUX

$$\frac{4n^4}{2n^3-3} = \frac{4n^4}{n^3\left(2-\frac{3}{n^3}\right)} = \frac{4n}{2-\frac{3}{n^3}} \xrightarrow[n\to\infty]{} +\infty$$

miro

$\underline{r > 0}$

$\longrightarrow 2$ $\longrightarrow \mp\infty$

$$r^n = 1^n = 1 \xrightarrow[n \to \infty]{} 1$$

$$\left(r^n \xrightarrow[n \to \infty]{} +\infty \quad , \quad \text{si } r > 1 \right)$$

$2^n : \quad 2 \quad 4 \quad 8 \quad 16 \quad 32$

$$\left[r^n \xrightarrow[n]{} 0 \quad , \text{ si } \quad 0 < r < 1 \right] \qquad \left(\frac{1}{e}\right)^{\overbrace{\text{algo}}^{\to +\infty}} \longrightarrow 0$$

$\left(\frac{1}{2}\right)^n : \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \text{---}$

$\%2 \quad 2^{-2} = \frac{1}{4}$

$\%2 \quad 2^{-1} = \frac{1}{2}$

$\%2 \quad 2^0 = 1$

$\%2 \quad 2^1 = 2 \quad \times 2$

$\%2 \quad 2^2 = 4 \quad \times 2$

$\%2 \quad 2^3 = 8 \quad \times 2$

$2^4 = 16$

$! \quad \underbrace{1 + \frac{1}{\text{algo}}}$

Ej 5) $\displaystyle \lim_{n \to \infty} \left(\underbrace{\frac{2n+5}{2n-1}}_{\to 1} \right)^{\overbrace{3n+1}^{\to \infty}} \longrightarrow 1 \quad$ indet!

Vol $\quad \dfrac{2n+5}{2n-1} = 1 + \dfrac{2n+5}{2n-1} - 1 = 1 + \dfrac{2n+5-(2n-1)}{2n-1} = 1 + \dfrac{\overbrace{6}^{\to \infty}}{2n-1}$

$1 + \text{algo}$

$= 1 + \dfrac{1}{\frac{2n-1}{6}}$

$$\lim \left(1 + \frac{1}{\frac{2n-1}{6}} \right)^{\frac{2n-1}{6} \cdot \frac{6}{2n-1}(3n+1)} = e^9$$

$\underbrace{\frac{6}{2n-1}(3n+1)}_{\to ? = 9}$

$$\lim_{n\to\infty}\left(\frac{2n+5}{2n-1}\right)^{3n+1} = \lim_{n\to\infty}\left(1+\frac{1}{\frac{2n-1}{6}}\right)^{3n+1} = \lim_{n\to\infty}\Bigg[\underbrace{\left(1+\frac{1}{\frac{2n-1}{6}}\right)^{\frac{2n-1}{6}}}_{\longrightarrow e}\Bigg]^{\underbrace{\frac{6}{2n-1}(3n+1)}_{\to ?=9}} = e^9$$

$$\underbrace{\text{xq' } \frac{2n-1}{6} \xrightarrow[n\to\infty]{} +\infty}$$

<u>CAux</u> $\frac{6(3n+1)}{2n-1} = \frac{18n+6}{2n-1} = \frac{\cancel{n}(18+6/n)}{\cancel{n}(2-1/n)} =$

$$= \frac{18+\overbrace{6/n}^{\to 0}}{2-\underbrace{1/n}_{\to 0}} \xrightarrow[n\to\infty]{} \frac{18}{2} = 9$$

$$\underbrace{\lim_{n\to\infty}\left(1+\frac{1}{a_n}\right)^{a_n} = e,} \quad \text{si } \underbrace{a_n \xrightarrow[n\to\infty]{} +\infty.}$$

$$\underbrace{0 \neq b_n = \frac{1}{a_n} \xrightarrow[n\to\infty]{} 0}$$

$a_n b_n = 1$

$\underbrace{a_n = \frac{1}{b_n}}$

$$\left[\begin{array}{l} \lim_{n\to\infty}(1+b_n)^{\frac{1}{b_n}} = e, \\ \text{si } \underbrace{b_n \xrightarrow[n\to\infty]{} 0} \end{array}\right]$$

<u>FACTORIAL</u> $3! = 1.2.3 = 6.$

$4! = \underbrace{1.2.3}_{3!}.4 = 24$

$\underbrace{4! = 3!\,4}$

$5! = 5.4.3.2.1 = 120$

<u>$n\in\mathbb{N}$</u> $n! = \underline{n.(n-1).(n-2)\ldots 3.2.1.}$

$0! = 1$ (Def)

<u>PROP</u> $(n+1)! = (n+1)\underbrace{n(n-1)\ldots 3.2.1}_{n!}$

$= (n+1)\,n!$

$$\boxed{(n+1)! = (n+1)\,n!}$$

miro

CRITERIO DE D'ALEMBERT — DIDEROT
LAPLACE

→ $\lim_{n\to\infty} a_n$

1°) $\lim_{n\to\infty} \underbrace{\left|\frac{a_{n+1}}{a_n}\right|}_{\geq 0} = L$ → $+\infty$ / real ≥ 0.

Entonces:

i) Si $L \in [0,1)$, $\lim_{n\to\infty} a_n = 0$

ii) Si $L > 1$ ó L es $+\infty$, $\lim_{n\to\infty} |a_n| = +\infty$

iii) Si $L = 1$, EL CRIT. NO DECIDE.

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} : \frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c}$$

Ej 1) $\lim_{n\to\infty} \frac{3^{n+2}}{n!}$ $\frac{\to\infty}{\to\infty}$ indet!

$$\frac{3^{n+3}}{3^{n+2}} = 3^{n+3-(n+2)} = 3^{n+3-n-2} = 3.$$

$$= \frac{3^{n+2+1}}{3^{n+2}} = \frac{3\cdot\cancel{3^{n+2}}}{\cancel{3^{n+2}}} = 3.$$

USO D'AL $a_n = \frac{3^{n+2}}{n!} \geq 0$ $(|a_n| = a_n)$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{3^{n+1+2}}{(n+1)!}}{\frac{3^{n+2}}{n!}} = \lim_{n\to\infty} \frac{3^{n+3}}{\underbrace{(n+1)!}_{(n+1)\,n!}}\cdot\frac{n!}{3^{n+2}} = \lim_{n\to\infty}\frac{3\,\cancel{n!}}{(n+1)\cancel{n!}} =$$

$a_n = \frac{3^{n+2}}{n!}$

$a_{n+1} = \frac{3^{(n+1)+2}}{(n+1)!}$

$$= \lim_{n\to\infty}\frac{3}{n+1} = 0 = L$$

$$\lim_{n\to\infty} a_n = 0$$

(xq' $L \in [0,1)$).

$$\lim_{n\to\infty}\frac{3^{n+2}}{n!} = 0.$$

CRITERIO DE CAUCHY (COYÍ)

¿$\lim_{n\to\infty} a_n$?

1°) $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$ → $+\infty$ / real ≥ 0

1) $L \in [0,1) \Longrightarrow \lim_n a_n = 0$

→ 2) $L > 1$ ó L es $+\infty \Longrightarrow \lim_n |a_n| = +\infty$

3) $L = 1$, NO DECIDE.

Ej 2) $\lim\limits_{n\to\infty} \dfrac{5^{n+2}}{n} \to \dfrac{\infty}{\infty}$ indet.

$5^{n+2} = 5^n \cdot 5^2 = 5^n \cdot 25$

CAUCHY $\quad a_n = \dfrac{5^{n+2}}{n} > 0 \quad (|a_n| = a_n)$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{5^{n+2}}{n}} =$$

$$= \lim_{n\to\infty} \sqrt[n]{\frac{5^n\, 25}{n}} = \lim_{n\to\infty} \frac{\sqrt[\cancel{n}]{5^{\cancel{n}}}\,\sqrt[n]{25}}{\sqrt[n]{n}} =$$

$$= \lim_{n\to\infty} \frac{5\overbrace{\sqrt[n]{25}}^{\to 1}}{\underbrace{\sqrt[n]{n}}_{\to 1}} = \underline{5 = L}$$

$|3| = 3$

$$|x| = \begin{cases} x, & \text{si } x \geq 0 \\ -x, & \text{si } x < 0 \end{cases}$$

$\sqrt[n]{25} = 25^{\overbrace{\frac{1}{n}}^{\to 0}} \xrightarrow[n\to\infty]{} 1$

$|\overset{<0}{a_n}| = -a_n \geq 0$

$|\overset{x}{-5}| = -(-5) = 5$

POR CAUCHY, como $L > 1$,

$\lim\limits_{n\to\infty} |a_n| = +\infty$.

$\Longrightarrow \lim\limits_{n\to\infty} a_n = +\infty$ //.

Ej3) $\lim\limits_{n\to\infty} \dfrac{(n+2)!}{3n} \to \dfrac{\infty}{\infty}$ indet.

D'Alem. $\quad a_n = \dfrac{(n+2)!}{3n} > 0 \quad$ // $\quad a_{n+1} = \dfrac{(n+1+2)!}{3(n+1)}$

$(n+1+2)! = (n+1+2)\cdot(n+2)!$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1+2)!}{3(n+1)}}{\frac{(n+2)!}{3n}} = \lim_{n\to\infty} \frac{(n+3)!}{3n+3} \cdot \frac{3n}{(n+2)!} =$$

$$= \lim_{n\to\infty} \frac{3n}{3n+3} \cdot \frac{(n+3)\cancel{(n+2)!}}{\cancel{(n+2)!}} =$$

$$= \lim_{n\to\infty} \frac{3n^2+9n}{3n+3} = +\infty. \quad \underline{L}$$

¿POR QUÉ?

POR D'Alembert, como L es $+\infty$,

$\lim\limits_{n\to\infty} |a_n| = +\infty$.

$\Longrightarrow \lim\limits_{n\to\infty} a_n = +\infty$

$\lim\limits_{n\to\infty} \dfrac{(n+2)!}{3n} = +\infty.$

miro