Dobal, F.

Mate. 3. - Práctica 7

1) $X_i \sim N(\mu, \sigma^2)$; $\sigma^2 = 25$, $i = 1, 2, \dots, 20.$

a) $\alpha = \frac{5}{100} \rightarrow 1 - \alpha = 0{,}95.$

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \rightarrow P(-z \leq Z \leq z) = 0{,}95$$

$$= \text{por simetría} = \phi(z) - \phi(-z)$$

$$= 2\phi(z) - 1 = 0{,}95$$

$$2\phi(z) = 1{,}95$$

$$\underbrace{\phi(z) = 0{,}975}_{\text{por tabla}} \Rightarrow z = 1{,}96$$

$$\therefore P(-1{,}96 \leq Z \leq 1{,}96) = P\left(-1{,}96 \leq \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq 1{,}96\right) = 0{,}95$$

$$\rightarrow \bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

intervalo a usar: $\left[\bar{X} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right]$

$\underbrace{z_{\frac{\alpha}{2}}}_{\text{por tabla}} = 1{,}96 \Rightarrow$ Reemplazamos: $\left[1040 - 1{,}96 \cdot \frac{\sqrt{25}}{\sqrt{20}}, 1040 + 1{,}96 \cdot \frac{\sqrt{25}}{\sqrt{20}}\right]$

$$\underbrace{[1037{,}8 , 1042{,}19]}_{\text{intervalo de confianza del 95\%}} =$$

b) $n = \left(\dfrac{z_{\frac{\alpha}{2}} \sigma}{l}\right)^2 \quad , \ l = 5$

$= \left(\dfrac{1{,}96 \cdot 25}{5}\right)^2 \approx 97$

c) $n = \left(\dfrac{2 z_{\frac{\alpha}{2}} \sigma}{l}\right)^2 \quad , \ l = 6$

$= \left(\dfrac{2 \cdot 1{,}96 \cdot 25}{6}\right)^2 = 16{,}\hat{3}^2 = 266{,}\hat{7} \approx 267$

2) a) $\bar{X} = (0{,}63 + 2{,}64 + 1{,}85 + 1{,}68 + 1{,}09 + 1{,}67 + 0{,}73 + 1{,}04 + 0{,}68) \dfrac{1}{9}$

$= 12{,}01 \cdot \dfrac{1}{9} = 1{,}33\hat{4}$

$S^2 = \dfrac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{X})^2$

$= \dfrac{1}{8} \sum_{i=1}^{8} (x_i - 1{,}33\hat{4})^2 =$

$= \dfrac{1}{8}(0{,}496 + 1{,}704 + 0{,}265 + 0{,}119 + 0{,}059 + 0{,}112 +$
$+ 0{,}365 + 0{,}086 + 0{,}428) = \dfrac{3{,}6385}{8} \approx 0{,}4548$

$\hookrightarrow S = \sqrt{0{,}4548} \approx 0{,}6743$

b) $\left[1{,}33\hat{4} - 1{,}96 \cdot \dfrac{0{,}6743}{\sqrt{9}} \ , \ 1{,}33\hat{4} + 1{,}96 \cdot \dfrac{0{,}6743}{\sqrt{9}}\right]$

$= [0{,}893 \ , \ 1{,}774]$

3) $(4{,}01 \;,\; 6{,}02)$, $(4{,}2 \;,\; 5{,}83)$, $(3{,}57 \;,\; 6{,}46)$.

↳ $6{,}02 - 4{,}01 = 2{,}01$

↳ $5{,}83 - 4{,}2 = 1{,}63$

↳ $6{,}46 - 3{,}57 = 2{,}89$

$99\% > 95\% > 90\% \longrightarrow 2{,}89 > 2{,}01 > 1{,}63$

$\therefore$ * $(3{,}57 \;,\; 6{,}46)$ es el intervalo de 99%.

* $(4{,}01 \;,\; 6{,}02)$ es el intervalo de 95%.

* $(4{,}2 \;,\; 5{,}83)$ es el intervalo de 90%.

4) X_i = 'tiempo de vida de la i-ésima batería producida'.

$i = 1..100$.

$\bar{X} = 150$.

$S = 25$.

a) $Z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \approx N(0,1)$

↓ por TCL → porque $n = 100 > 30$.

intervalo: $\left[\bar{X} - z_{\frac{\alpha}{2}} \dfrac{S}{\sqrt{n}} \;,\; \bar{X} + z_{\frac{\alpha}{2}} \dfrac{S}{\sqrt{n}}\right]$

$1-\alpha = 0,95$

$\alpha = 0,05 \Rightarrow \frac{\alpha}{2} = 0,025$

$$\left[150 - 0,025 \cdot \frac{25}{\sqrt{100}} \;,\; 150 + 0,025 \cdot \frac{25}{\sqrt{100}}\right] =$$

$$= [145,1001 \;,\; 154,8999]$$

b) $(147, 153)$?

$$\frac{\text{longitud del intervalo}}{2} = \frac{L}{2} = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \Rightarrow \frac{153-147}{2} = z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \Rightarrow$$

$$\Rightarrow 3 = z_{\frac{\alpha}{2}} \frac{25}{10}$$

$$\frac{3}{2{,}5} = z_{\frac{\alpha}{2}}$$

$$z_{\frac{\alpha}{2}} = 1,2 \Rightarrow \phi(1,2) = 0,8849$$

5) x_i = 'tiempo de secado (en horas) de cierta marca de pintura de látex en la i-ésima medición'. $i=1..n$; $n=15$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$; $x_i \sim N(\mu, \sigma^2)$

(Int. de confianza para la media de una dist. normal, varianza desconocida). $IC_\mu^{99\%} =$

$$T = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

$$S = \sqrt{\frac{1}{14}\sum_{i=1}^{15}(x_i - \bar{X})^2} = 0,942\hat{6}$$

$1-\alpha = 0,99$

$-\alpha = -0,01$

$\alpha = 0,01$

$$\left[\bar{X} - t_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}} \;,\; \bar{X} + t_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}}\right]$$

$$\left[\frac{56,8}{15} - 2,977 \cdot \frac{0,942\hat{6}}{\sqrt{15}} \;,\; \frac{56,8}{15} + 2,977 \cdot \frac{0,942\hat{6}}{\sqrt{15}}\right]$$

$$[3,0621 \;,\; 4,5113]$$

6) $\sigma_1^2 = 1{,}5$; $\sigma_2^2 = 1{,}2$; $m_1 = 15$; $m_2 = 20$; $\bar{X}_1 = 89{,}6$; $\bar{X}_2 = 92{,}5$.

a) σ_1^2, σ_2^2 conocidos, dist. normales.

$$\bar{X}_1 \sim N(\mu_1, \sigma_1^2) \; ; \; \bar{X}_2 \sim N(\mu_2, \sigma_2^2).$$

$$1-\alpha = 95\% = 0{,}95$$
$$-\alpha = 0{,}95-1$$
$$\alpha = 0{,}05$$

$$\left[(\bar{X}_1-\bar{X}_2) - z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \; , \; (\bar{X}_1-\bar{X}_2) + z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right] \Rightarrow$$

$$IC^{95\%}_{\mu_1-\mu_2} = \left[(89{,}6-92{,}5) - z_{0{,}025}\sqrt{\frac{1{,}5}{15}+\frac{1{,}2}{20}} \; , \; (89{,}6-92{,}5) + z_{0{,}025}\sqrt{\frac{1{,}5}{15}+\frac{1{,}2}{20}}\right]$$

$$= \left[-2{,}9 - 1{,}96\sqrt{0{,}16} \; , \; -2{,}9 + 1{,}96 \sqrt{0{,}16}\right] = \left[-3{,}684 \; , \; -2{,}116\right]$$

b) $L = 2z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \longrightarrow L = 2\cdot 1{,}96\sqrt{\frac{1{,}5}{15}+\frac{1{,}2}{20}}$

$$= 2\cdot 1{,}96 \cdot 0{,}4 = 1{,}568$$

$$l = \frac{L}{2} = \frac{1{,}568}{2} = 0{,}784 \Rightarrow 2z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{n}} \leq l \rightarrow$$

$$\rightarrow 2\cdot 1{,}96\sqrt{\frac{\sigma_1^2+\sigma_2^2}{n}} \leq l \rightarrow 3{,}92\frac{\sqrt{\sigma_1^2+\sigma_2^2}}{\sqrt{n}} \leq l \rightarrow$$

$$\rightarrow \frac{3{,}92\sqrt{1{,}5+1{,}2}}{l} \leq \sqrt{n} \rightarrow \left(\frac{6{,}441}{0{,}784}\right)^2 \leq n \rightarrow 67{,}5 \leq n$$

7) Marca → A (50 PC) — Resistencia a la t. promedio: 78,3 kg.
— Desviación estándar: 5,6 kg. (σ_1^2)

→ B (50 PC) — Resistencia a la t. promedio: 87,2 kg.
— Desviación estándar: 6,3 kg. (σ_2^2)

→ I.C. para la dif. de los medias poblacionales → 95% requerido.

$1-\alpha = 0,95$

$-\alpha = -0,05$

$\alpha = 0,05$

I.C. para la dif. $\mu_1-\mu_2$ de nivel 95%:

$$\left[(\bar{X}_1-\bar{X}_2) - z_{0,025}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}, (\bar{X}_1-\bar{X}_2+z_{0,025}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right]$$

Además, $\bar{X}_{1,2} \sim N(\mu_{1,2}, \sigma_{1,2}^2)$

$\downarrow$

Por T.C.L. ya que $n = 50 > 30$.

$$\Longrightarrow \left[(78,3-87,2) - 1,96\sqrt{\frac{5,6^2}{50}+\frac{6,3^2}{50}}, \right.$$

$$\left.(78,3-87,2)+1,96\sqrt{\frac{5,6^2}{50}+\frac{6,3^2}{50}}\right] =$$

$$= [-8.9 - 2,3364 , -8.9 + 2,3364]$$

$$= [-11,2364 , -6,5636]$$

8) $\bar{x}_A$ = 'media muestral del proveedor A' $= 5459$

$x_{A,i}$ = 'nro de hojas impresas del proveedor A en la i-ésima observación'

$\bar{x}_B$ = 'media muestral del proveedor B' = $= 5162$

$x_{B,i}$ = 'nro de hojas impresas del proveedor B en la i-ésima observación'.

$x_{A,i} \sim N(\mu_A, \sigma_A^2 = \sigma^2)$, $i = 1..12$.

$x_{B,i} \sim N(\mu_A, \sigma_B^2 = \sigma^2)$, $i = 1..12$.

$S_A^2 = 33703$

$S_B^2 = 199928$

$$\bar{X}_A \sim N\left(\mu_A, \frac{\sigma_A^2}{n_A} = \frac{\sigma^2}{n_A}\right) ; \bar{X}_B \sim N\left(\mu_B, \frac{\sigma_B^2}{n_B} = \frac{\sigma^2}{n_B}\right)$$

$$\bar{X}_A - \bar{X}_B \sim N\left(\mu_A - \mu_B, \frac{\sigma_A^2+\sigma_B^2}{n_A+n_B} = \frac{\sigma^2+\sigma^2}{n_A+n_B}\right)...$$

dónde... $S_p^2 = \dfrac{(n_1-1)S_A^2+(n_2-1)S_B^2}{n_1+n_2-2}$ y... $\begin{cases} S_A^2 = \dfrac{1}{11}\sum_{i=1}^{12}(x_{A,i}-\bar{x}_A)^2 \\ S_B^2 = \dfrac{1}{11}\sum_{i=1}^{12}(x_{B,i}-\bar{x}_B)^2 \end{cases}$

Tenemos: I.C. para la diferencia de dos medias, varianzas desconocidas.

$S_A^2 = 33703$ y $S_B^2 = 199928.$

$$\Rightarrow S_p^2 = \frac{11 \cdot 33703 + 11 \cdot 199928}{22}$$

$1-\alpha = 0,95$

$\alpha = 0,05$

$$S_p^2 = 116815,5$$

$$S_p = \sqrt{116815,5} = 341,78$$

$$I.C.^{95\%}_{\mu_A-\mu_B} = \left[(5459-5162) - t_{0,025,22} \cdot S_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \; , \; (5459-5162 + t_{0,025,22} \cdot S_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right]$$

$$= \left[297 - 2,074 \cdot 341,78 \sqrt{0,1\hat{6}} \; , \; 297 + 2,074 \cdot 341,78 \sqrt{0,16}\right]$$

$$= \left[297 - 289,38 \; , \; 297 + 289,38\right]$$

$$= \left[7,62 \; , \; 586,38\right]$$