# Visualizing Fortune's Sweepline Algorithm for Planar Voronoi Diagrams

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## Abstract

A visualization of Fortune's sweepline algorithm for planar Voronoi diagrams is presented. The algorithm sweeps upward through the set of input sites, maintaining a parabolic wavefront comprised of points equidistant from the sweepline and the sites. Voronoi edges are traced wherever adjacent parabolic arcs intersect; Voronoi vertices are left behind wherever an arc is overtaken by its neighbors. In fact, the algorithm computes a transformation of the diagram, in which the wavefront maps to the sweepline itself, and all events (i.e., arc disappearances) occur above the sweepline. Some applications of the Voronoi diagram are demonstrated, including nearest neighbor-finding, Delaunay triangulation, and planar convex hull. Finally, an intriguing connection between $d$-dimensional Voronoi diagrams and $(d+1)$-dimensional convex hulls is depicted (for $d = 2$).

## 1 Visualization

Given a set of *sites* in $d$ dimensions, the Voronoi diagram of the sites is a partition of space into convex regions, one per site, such that each region is the locus of points closer to the region's associated site than to any other site. An elegant sweepline algorithm for the construction of planar Voronoi diagrams was presented in [For87]. The sites are first sorted in the direction of sweepline motion. The algorithm then maintains a parabolic wavefront comprised of points equidistant from the set of input sites and the sweepline. Each site encountered by the sweepline contributes one or more sections of arc to the wavefront. The intersections of adjacent arcs trace the *edges* of the sites' Voronoi diagram. When an arc is overtaken by its neighbors, the existence of a *triple-point*, or point equidistant from three sites, is established and a Voronoi *vertex* is generated. The algorithm itself operates discretely, by inferring the presence of each arc disappearance or site, and handling each in turn. Our visualization smoothly animates the evolution of the sweepline between these discrete events.

Fortune's algorithm uses a coordinate transformation (the "*-map") that maps the parabolic wavefront to the sweepline itself. Consequently, all triple-points scheduled by the algorithm occur *above* the sweepline, where they can be correctly processed. The action of this coordinate transformation on the wavefront, and on the Voronoi diagram, is depicted.

Several applications of the Voronoi diagram are demonstrated. A site's nearest neighbors are those with which it shares a Voronoi edge. The Delaunay triangulation is the straight-line dual of the Voronoi diagram [Del34]. The convex hull of the sites consists of those triangulation edges that occur as part of exactly one triangle.

Finally, the visualization depicts a connection between $d$-dimensional Voronoi diagrams and $(d+1)$-dimensional convex hulls, described in [PS85]. Suppose a right paraboloid is erected on the $z = 0$ plane. Lift each site to the paraboloid, and consider the tangent plane there, oriented so that its positive halfspace contains the paraboloid. The intersection of these positive halfspaces is a polyhedral set whose face structure corresponds to the face structure of the Voronoi diagram.

The visualization was produced with graphics hardware and software provided by Silicon Graphics, Inc. The author is grateful to Silicon Graphics for their generous support of this visualization effort.

## References

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