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Efficient referee assignment in Argentinean professional basketball leagues using operations research methods

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Abstract

The top two divisions of the Argentinean professional basketball system have since 2014–2015 used a season schedule format similar to that used by the NBA in which games are played all through the week, replacing the previous setup where all games were scheduled on weekends. This change has confronted the Argentinean league organizers with new scheduling challenges, one of which is the assignment of referees to games. The present article addresses this assignment problem using a tool based on an integer linear programming model. The objective is to minimize the total cost of trips made by the referees while also satisfying a series of other conditions. The problem is broken down into a series of relatively small subproblems representing successive periods of the season, and the solution is obtained using a rolling horizon heuristic. The approach was tested by applying it to the First Division's 2015–2016 season, the last one before introducing the approach presented in this article, when referees were still assigned using manual methods. The travel costs simulated by the model were 26% lower than the total travel costs actually incurred under the manual assignments, and all of the restrictions that had been requested by league officials were satisfied. The model was used by the First Division for the 2016–2017 and 2017–2018 seasons and in 2017–2018 also by the Second Division.

Keywords Referee assignment · Basketball · Integer linear programming · Travelling umpire problem · Sports scheduling

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1 Introduction

Problems in sports league scheduling have been the subject of a rapidly growing literature in recent years (see the surveys in Kendall et al. (2010); Rasmussen and Trick (2008) and Durán (2020), this last one with special reference to Latin America). Most of the attention has focused on applications aimed at scheduling regular season games. For example, match calendar implementations have been reported for a number of South American football (Alarcón et al. 2017; Durán et al. 2007, 2012, 2017; Recalde et al. 2013; Ribeiro and Urrutia 2012), basketball (Durán et al. 2019) and volleyball (Bonomo et al. 2012) leagues. Relatively little work, however, has been done on scheduling referee assignments in real cases. With the exception of a few applications for cricket in England (Wright 1991), baseball in the United States (Trick et al. 2012) and soccer in Chile (Alarcón et al. 2013), published studies on referee scheduling have been confined to analyses of methodological aspects and computational experiments. In Trick et al. (2012), for instance, the authors formulate what they call the *Travelling Umpire Problem* (TUP) and in Duarte et al. (2006), the *Referee Assignment Problem* (RAP) is treated in general terms. The problems these papers address aim to capture the most essential aspects of what would be an efficient referee assignment. But real-world applications must incorporate additional ad-hoc conditions necessitated by the peculiarities of the leagues they schedule. Moreover, the particular characteristics of the referee assignment problem are such that league officials find it prudent, if not essential, to recalculate the assignments multiple times over the course of a season. This contrasts starkly with game scheduling problems, which are typically solved only once in a static setting before the season starts.

The purpose of the present article is to report on the development of a solution approach based on an integer linear programming model that determines referee assignments for the regular season games of the Argentinean professional basketball leagues. The referee assignment problem we modeled has certain similarities to the TUP, mainly in that its objective is to minimize referee travel. This is an important cost item for the leagues since the teams' home venue locations are scattered widely across the country and road trips of more than 1,000 kilometers are not unusual.

To deal with the dynamic features of the referee assignment decisions, we implemented a rolling horizon heuristic approach that solves the model sequentially by periods, incorporating the solutions so obtained as the tournament progresses. Similar rolling horizon heuristic approaches have proved to be effective in other problems of scheduling (Agra and Oliveira 2018; Barrera et al. 2020; Ramos et al. 2020) and routing (Fontecha et al. 2020).

The results generated by our work were adopted for the First Division's 2016–2017 and 2017–2018 seasons. This case is the main focus here but the adaptability of the approach to other leagues is partially corroborated by its successful application to the Second Division (known officially as the *Argentina League*), which adopted it for the 2017–2018 season.

The remainder of this article is divided into four sections. Section 2 introduces the problem to be solved, with details on the season formats of the two divisions. Section 3 formulates a linear integer programming model for the problem and describes our solution approach for practical applications. Section 4 analyses the results of the model and compares them to the last season in which First Division referee assignments were still defined manually. Section 5 presents our conclusions and indicates some possibilities for further development of the model.

City	Teams
Bahía Blanca	Weber Bahía
City of Buenos Aires	Boca Juniors
	Ferrocarril Oeste
	Obras
	San Lorenzo
Comodoro Rivadavia	Gimnasia(CR)
Concordia	Estudiantes
Córdoba	Atenas
	Instituto
Corrientes	Regatas
	San Martín
Formosa	La Unión
Junín	Argentino
Mar del Plata	Peñarol
	Quilmes
Mercedes	Comunicaciones
Río Gallegos	Hispano Americano
Salta	Salta Basket
Santiago del Estero	Ciclista Olímpico
	Quimsa

Fig. 1 Map of Argentina showing home locations of the teams in the First Division (LNB) for the 2017–2018 season

2 Background and problem description

Basketball has become one of the most popular sports in Argentina. Interest in the game received a major boost with the creation of the National Basketball League (the LNB by its initials in Spanish) in 1985, which is currently the First Division of professional basketball in the South American nation. The standard of play in the league is very high, and LNB teams are now located all around the country. In 2017–2018, for example, the 20 league teams had their home floors in 13 different cities located in 10 different provinces. Given the size of the national territory, away games often mean the teams have to travel long distances, a factor that must be taken into account when scheduling the LNB's regular season. The 20 team locations in 2017–2018 are indicated on the map in Fig. 1.

Beginning with 2014-2015, the First Division seasons have been scheduled using Operations Research techniques (Durán et al. 2019). The process begins each year with the election by each team of *desirable road trips*, that is, series of away games that attempt to take maximum advantage of the mileage covered to play as many times as possible. For example, La Union, which plays at home in the city of Formosa 1100 kilometers north of Buenos Aires, might consider a desirable road trip to be one in which it plays all four Buenos Aires area teams in succession.

Since 2015–2016, the First Division has had 20 teams divided into two conferences, North and South, each one with 10 teams. The 2015–2016 season consisted of two phases, regional and national, in both of which each team played every other one ("all-play-all") once at home and once away. In the regional phase, however, the teams played only within their own conference while in the national phase the format applied over the whole division. Altogether there were 560 regular season games, 180 in the regional phase and 380 in the national phase. At the conclusion of the regular season, the top six teams in each conference standings met in the playoffs to determine the division champion while the bottom team in each conference played a series to determine which of the two would be relegated to the Second Division. The 2016–2017 season used the same format and was the first one in which the OR tool presented here was used to assist league officials in determining the referee assignments.

Each regular season game was officiated by two referees whereas at playoff games there were three. The referees are licensed by category according to experience and performance over their entire careers. The First Division's corps of officials consisted of 20 referees, 10 with category A (main) licences and 10 with category A1 (secondary) licences. Each referee's home town was known, and is an important data item used in our model to calculate travel costs.

A slight change in the First Division's schedule format for the 2017–2018 season meant that there were about 50 fewer games. Since our approach was also adopted that season by the Second Division, 26 more referees with A2 and A3 licences working in that league had to be added to the referee corps. Initially, the Argentine Association of Basketball Clubs (Spanish initials: AdC) insisted that referees with A and A1 licences would not be permitted to officiate Second Division games, which reduces the feasible search space and thus improves model running times. Later in the season, however, this restriction was dropped so the individual assignment problems for the two divisions could no longer be solved independently.

Beginning in 2017–2018, three officials were required for each First Division game while two continued to be used in the Second Division. Apart from a small number of exceptions requested by the AdC technical committee, the trio of referee slots for First Division matches was to be filled by one official with an A licence, one with an A1 licence, and a third who could be in the A2 or A3 category.

In general terms, the objective of our model is to assign each game a referee duo or trio (as the case requires) so as to minimize the officials' travel and lodging costs while satisfying all of the conditions set by the AdC technical committee. As an example, sporting fairness criteria require that a team cannot be officiated by the same referee more than once in four consecutive games (i.e., upon officiating a given team in a game, a referee cannot do so again until that team has played at least three more times).

In practice, the problem is subject to change over the course of a season. Referees may not be assignable for a given period if, for example, they are injured or chosen to officiate at an international tournament. Or there may be changes in their licence categories. Also, new referees must be readily incorporated. This last situation arose when women referees were assigned for the first time in First Division history, a different female official joining the referee trio assigned to each of three different games. The games themselves may on occasion be rescheduled to different days.

Although complete game schedules are announced well before the season starts, referee assignments for individual games are not decided by league officials until relatively shortly before the game date in order to take account of eventualities arising as the season proceeds that are not knowable in advance. To solve the assignment problem, therefore, we formulate a model that can be solved sequentially for limited time windows, typically two weeks. With this design the model can be adjusted during each such period as different eventualities arise. As we will see, each solution generated by running the model for a given period is included in the input data for the next run.

3 Model and solution approach

In this section we present the approach developed to solve the referee assignment problem. It involves breaking down the season into smaller periods that are then solved in succession by an integer programming model. In solving each individual period, the solutions found for all the previous periods are taken into account.

As was noted earlier, the game dates and their home/away status are defined previously and constitute a key part of the model's input. Other data that must be known are the trip distances between the teams' venues (in effect, their home towns) and the pairs of venues at which a referee can and cannot officiate on consecutive days. By contrast, referees are deemed able to officiate at any pair of venues if they have at least one day off between the two assignments. Note that in such cases they may incur lodging costs in addition to the cost of travel. Recall in this regard that during the regular season there are games scheduled every day, not just on certain set days such as weekends as in some other league sport formats.

The central idea behind the model is to follow the location of each referee over time. If, on a given day, a pair of referees are to be assigned to a game between team 1 (at home) and 2 (away), the model must ensure that two referees are located in team 1's home town on that date, taking into account the travel costs involved from their immediately previous locations. To represent the location of a referee currently unassigned (i.e., at home), we add a fictitious team. Such a situation is modeled by including a home game for this fictitious team on every calendar day of the season and applying no upper bound to the number of referees that can officiate those games. What the model then decides is which games (real and fictitious) each referee officiates over the relevant period. Also, since referees must start and finish the season at home, two extra days are added at either end of the season calendar.

The model as described was coded in the OPL modeling language and solved using the Cplex 12.6.3 solver on a computer with four processors running at 3.4GHz with 16GB of RAM.

3.1 Formulation of the referee assignment model

In what follows we present the formulation of our integer linear programming model for determining which referees officiate at each game. The model generates assignments for games from the beginning of the season to a day denoted t_{window}, although a number of assignments previous to that day may be fixed. The objective function is the global total of all referees' travel and lodging costs up to t_{window}.

The data inputted to the model are the following:

- Set $\mathcal{E}$ of teams. Each of the real teams is indexed by a positive number while the fictitious team, representing the referee's home town (hereafter simply "home"), is indexed as 0. Thus, the set is expressed as $\mathcal{E} = \{0\} \cup \mathcal{E}_1 \cup \mathcal{E}_2$, where $\mathcal{E}_1$ denotes the set of teams in the First Division and $\mathcal{E}_2$ the set of teams in the Second Division. We also define $\mathcal{E}^\star = \mathcal{E} \setminus \{0\}$ to improve readability.
- Set $\mathcal{P}$ of games. Each element of $\mathcal{P}$ is a single game and is expressed as a tuple having the form $(t, k, l) =$ (day, home team, away team) that denotes the day of the game (counting from the beginning of the season), the home team index number and the away team index number. Thus, $t \in \mathbb{Z}_{\geq 0}$ and $k, l \in \mathcal{E}$.
- Set $\mathcal{A} = \mathcal{A}_I \cup \mathcal{A}_{II} \cup \mathcal{A}_{III} \cup \mathcal{A}_{IV}$ of referees, the four subsets corresponding to the different referee licence categories used by the AdC (A, A1, A2, A3) as noted in the

previous section. There is also a subset $\widehat{\mathcal{A}} \subseteq \mathcal{A}_{III} \cup \mathcal{A}_{IV}$ containing the referees who can be assigned as third referees to First Division games.

- Set $\mathcal{Q}_{\text{home}} = \{(i, k) : i \in \mathcal{A}$ of referees who cannot officiate a game involving team $k \in \mathcal{E}^\star$ as home team$\}$. Also, set $\mathcal{Q}_{\text{away}}$, defined analogously. In most cases, the elements of these sets refer to a referee and a team at home in the same city, but they may also represent cases where some sort of conflict justifies such an assignment prohibition.
- Set $\mathcal{V}$ of possible trips. Each element of $\mathcal{V}$ represents a possible trip and is expressed as a tuple having the form (s, t, k, m, n), which denotes a trip that starts from the home of team $k \in \mathcal{E}$ at time $t \in \mathbb{Z}_{\geq 0}$ and ends at the home of team $m \in \mathcal{E}$ on day $t + s$ when m plays against rival team $n \in \mathcal{E}$.
  This set is generated as follows. We take each game $(t, k, l) \in \mathcal{P}$, and for each $s \in \{1, 2\}$ we take all of the games given by $(t + s, m, n) \in \mathcal{P}$ and add trip (s, t, k, m, n) to $\mathcal{V}$. Similarly, the set $\widehat{\mathcal{V}}$ is generated by adding (s, t, k, m) to $\widehat{\mathcal{V}}$ (this definition will make sense once the variables are defined below). Note that only those trips that are truly feasible will need to be considered given that by generating $\mathcal{V}$ in this manner, for $s = 1$ many trips are generated that will not in fact be possible on consecutive days.
- Set $\mathcal{C}_E = \{(k, m)$ of team pairs such that the referee cannot officiate at the home of team $k \in \mathcal{E}^\star$ and at the home of $m \in \mathcal{E}^\star$ on consecutive days$\}$. Similarly, set $\mathcal{C}_A = \{(i, m)$ of referee/team pairs such that referee $i \in \mathcal{A}$ cannot officiate at his or her home and at the home of team $m \in \mathcal{E}^\star$ on consecutive days$\}$.
- Parameters d_{km} denote the cost of a trip from the home of team $k \in \mathcal{E}^\star$ to that of team $m \in \mathcal{E}^\star$. Also, parameters r_{ik}, defined analogically as the cost of a trip by referee $i \in \mathcal{A}$ from the corresponding home town to the home of team $k \in \mathcal{E}^\star$.
- Parameters z_i^A denote the zone where the home of referee $i \in \mathcal{A}$ is located. Also, parameters z_k^E, which denote the home zone of team $k \in \mathcal{E}^\star$. Note that more than one team may be at home in the same zone, as in the case of Buenos Aires.
- Parameter h denotes the daily lodging cost.

The model contains two binary variables. The first one is x_{ip} for each referee $i \in \mathcal{A}$ and each game $p \in \mathcal{P}$. Thus, $x_{ip} = 1$ if and only if referee $i \in \mathcal{A}$ is assigned to officiate $p \in \mathcal{P}$. The second one is z_{iv} for each referee $i \in \mathcal{A}$ and each possible trip $v \in \widehat{\mathcal{V}}$, where $z_{iv} = 1$ if and only if referee $i \in \mathcal{A}$ initiates a trip of length $s \in \{1, 2\}$ on day $t \in \mathbb{Z}_{\geq 0}$ from the home of team $k \in \mathcal{E}$ to the home of $m \in \mathcal{E}$ (recall that $v = (s, t, k, m)$).

With the above-described elements we can now specify our ILP model for resolving the referee assignment problem as follows:

The objective function minimizes the total cost of trips by the corps of referees. We assume that the cost of a trip is the same in both directions. Also, a trip between two team venues to officiate a game played outside the referee's home zone incurs a cost for lodging.

Recall in what follows that a zero index for set $\mathcal{E}$ indicates that the referee is at home, so $k > 0$ refers to a real (not fictitious) team.

$$\min : \overbrace{\sum_{i \in \mathcal{A}} \sum_{\substack{v \in \widehat{\mathcal{V}} \\ k>0 \\ m>0}} d_{km} z_{iv}}^{\text{trips between team homes}} + \overbrace{\sum_{i \in \mathcal{A}} \sum_{\substack{v \in \widehat{\mathcal{V}} \\ k>0 \\ m=0}} r_{ik} z_{iv}}^{\text{trips arriving at referee's home}}$$

$$+ \underbrace{\sum_{i\in\mathcal{A}} \sum_{\substack{v\in\widehat{\mathcal{V}}\\ k=0\\ m>0}} r_{im} z_{iv}}_{\text{trips departing referee's home}} + \underbrace{\sum_{i\in\mathcal{A}} \sum_{\substack{v\in\widehat{\mathcal{V}}\\ k>0\\ m>0\\ z_i^A \neq z_k^E}} h\ s\ z_{iv}}_{\text{lodging cost}} \tag{1}$$

In a given time window there may be upper and lower bounds set by the technical committee on the number of games a given referee can officiate. For example, if a referee cannot officiate at all in a given window (due to an injury or a trip to officiate at an international tournament), the upper bound of this restriction is the number of games the referee has officiated so far (this is so because, as noted earlier, the games a referee officiated in previous windows are indicated by setting the values of the variables involved to 1). Recall also that $p \in \mathcal{P}$ takes the form $p = (t, k, l)$.

$$\alpha_i \leq \sum_{\substack{p\in\mathcal{P}:\\ t\leq t_{\text{window}}\\ k>0}} x_{ip} \leq \beta_i \quad \forall i \in \mathcal{A}. \tag{2}$$

A referee cannot officiate a given team's game until it has played at least γ games since he or she last did so. Games $q, q+1, \ldots, q+\gamma$ inclusive of team $k \in \mathcal{E}^\star$ in chronological order are denoted $\mathcal{P}_{k,q}$. The number of games played by $k \in \mathcal{E}^\star$ over the course of the season is denoted T_k.

$$\sum_{p\in\mathcal{P}_{k,q}} x_{ip} \leq 1 \quad \forall i \in \mathcal{A}, k \in \mathcal{E}^\star, q \in \{1, \ldots, T_k - \gamma\}. \tag{3}$$

Similarly, a referee cannot officiate a given team's home game until it has played at least γ_{home} home games since he or she last did so. Analogously, denoting as T_k^{home} the number of games played at home by $k \in \mathcal{E}^\star$, and as $\mathcal{P}_{k,q}^{\text{home}}$ the home games $q, q+1, \ldots, q+\gamma_{\text{home}}$ inclusive of team $k \in \mathcal{E}^\star$ in chronological order, we have the following:

$$\sum_{p\in\mathcal{P}_{k,q}^{\text{home}}} x_{ip} \leq 1 \quad \forall i \in \mathcal{A}, k \in \mathcal{E}^\star, q \in \left\{1, \ldots, T_k^{\text{home}} - \gamma_{\text{home}}\right\}. \tag{4}$$

To satisfy rest requirements, a referee i cannot be away from home more than δ_i consecutive days. Usually, $\delta_i = 14$ for all i but this may vary depending on the referee. The fictitious games added to each calendar day to represent days when a referee is at home are denoted $p_t^0 = (t, 0, 0) \in \mathcal{P}$.

$$\sum_{q\leq t\leq q+\delta_i-1} x_{ip_t^0} \geq 1 \quad \forall i \in \mathcal{A}, q \in \{1, \ldots, t_{\text{window}} - \delta_i + 1\}. \tag{5}$$

Also part of the rest requirements is that a referee cannot officiate more than three games in a window of five days.

$$\sum_{\substack{p\in\mathcal{P}:\\ q\leq t\leq q+4\\ k>0}} x_{ip} \leq 3 \quad \forall i \in \mathcal{A}, q \in \{1, \ldots, t_{\text{window}} - 4\}. \tag{6}$$

As was noted in the definition of set $\mathcal{Q}_{\text{home}}$, a referee may be prohibited from officiating certain teams' home games.

$$\sum_{\substack{p\in\mathcal{P}:\\ k=\hat{k}}} x_{ip} = 0 \quad \forall (i, \hat{k}) \in \mathcal{Q}_{\text{home}}. \tag{7}$$

The analogous constraint for the teams' away games is modeled as follows.

$$\sum_{\substack{p\in\mathcal{P}:\\ l=\hat{l}}} x_{ip} = 0 \quad \forall (i,\hat{l}) \in \mathcal{Q}_{\text{away}}. \tag{8}$$

First Division games are currently officiated by three referees. The trio is typically composed of one from $\mathcal{A}_I$, one from $\mathcal{A}_{II}$ and a third referee from $\widehat{\mathcal{A}}$.

$$\sum_{i\in\mathcal{A}'} x_{ip} = 1 \quad \forall p \in \{(t,k,l) \in \mathcal{P} : k \in \mathcal{E}_1, t \le t_{\text{window}}\}. \tag{9}$$

There are three constraints of this type depending on whether $\mathcal{A}' = \mathcal{A}_I, \mathcal{A}_{II}$, or $\widehat{\mathcal{A}}$.

Second Division games are currently officiated by two referees. The pair is typically composed of one from $\mathcal{A}_I \cup \mathcal{A}_{II} \cup \mathcal{A}_{III}$ and one from $\mathcal{A}_{III} \cup \mathcal{A}_{IV}$.

$$\sum_{i\in\mathcal{A}'} x_{ip} = 1 \quad \forall p \in \{(t,k,l) \in \mathcal{P} : k \in \mathcal{E}_2, t \le t_{\text{window}}\}. \tag{10}$$

There are two constraints of this type depending on whether $\mathcal{A}' = \mathcal{A}_I \cup \mathcal{A}_{II} \cup \mathcal{A}_{III}$ or $\mathcal{A}_{III} \cup \mathcal{A}_{IV}$.

No referee may officiate two games in a single day.

$$\sum_{\substack{p\in\mathcal{P}:\\ t=q}} x_{ip} \le 1 \quad \forall i \in \mathcal{A}, q \in \{1,\ldots,t_{\text{window}}\}. \tag{11}$$

Similarly, no referee may initiate more than one trip on a single day.

$$\sum_{\substack{v\in\widehat{\mathcal{V}}:\\ t=q}} z_{iv} \le 1 \quad \forall i \in \mathcal{A}, q \in \{1,\ldots,t_{\text{window}}\}. \tag{12}$$

If a referee makes a trip of two days' duration across three consecutive calendar days, he or she cannot initiate a trip on that intervening day.

$$\sum_{\substack{v\in\widehat{\mathcal{V}}:\\ t=q\\ s=2}} z_{iv} + \sum_{\substack{v\in\widehat{\mathcal{V}}:\\ t=q+1}} z_{iv} \le 1 \quad \forall i \in \mathcal{A}, q \in \{1,\ldots,t_{\text{window}}\}. \tag{13}$$

If a referee officiates games $p_1 = (t,k,l)$ and $p_2 = (t+1,m,n)$ (that is, $x_{ip_1} = x_{ip_2} = 1$), we want variable $z_{i\hat{v}}$ where $\hat{v} = (1,t,k,m)$ to be 1, indicating that the trip was taken. Analogously, if $z_{i\hat{v}} = 1$, we want $x_{ip_1} = x_{ip_2} = 1$.

$$\begin{aligned}
&\begin{cases} x_{ip_1} + x_{ip_2} \le & 1 + z_{i\hat{v}} \\ \quad 2\cdot z_{i\hat{v}} & \le x_{ip_1} + x_{ip_2} \end{cases}\\
&\forall i \in \mathcal{A}, q \le t_{\text{window}} - 1, p_1 \in \{(t,k,l) \in \mathcal{P} : t = q\},\\
&v \in \left\{(s,t,\hat{k},m,n) \in \mathcal{V} : s = 1, t = q, \hat{k} = k\right\}
\end{aligned} \tag{14}$$

where $\hat{v} = (1,q,k,m)$ and $p_2 = (q+1,m,n)$.

If a referee officiates games $p_1 = (t,k,l)$ and $p_2 = (t+2,m,n)$ (that is, $x_{ip_1} = x_{ip_2} = 1$), we want variable $z_{i\hat{v}}$ where $\hat{v} = (2,t,k,m)$ to be 1, indicating that this trip was taken, unless the two equalities are due to the fact that the referee officiated three games in a row (in which

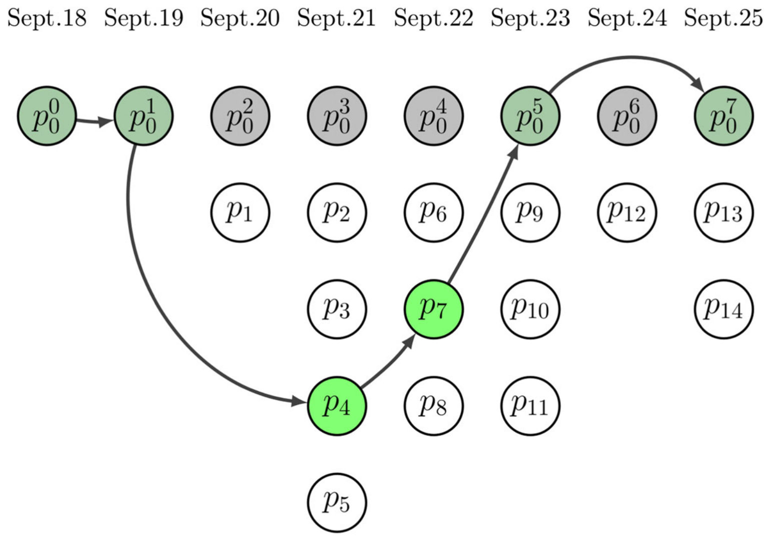


Fig. 2 Visual representation of a path followed by a referee over time

case the referee must make a trip of length $s = 1$ starting on day t to a game intermediate between p_1 and p_2), and thus made two one-day trips.

$$\begin{cases} x_{ip_1} + x_{ip_2} \leq 1 + z_{i\hat{v}} + \sum\limits_{\substack{\bar{v}\in\widehat{\mathcal{V}}: \\ t=q \\ s=1}} z_{i\bar{v}} \\ \quad 2 \cdot z_{i\hat{v}} \quad \leq \quad x_{ip_1} + x_{ip_2} \end{cases}$$
$$\forall i \in \mathcal{A}, q \leq t_{\text{window}} - 2, p_1 \in \{(t, k, l) \in \mathcal{P} : t = q\},$$
$$v \in \left\{(s, t, \hat{k}, m, n) \in \mathcal{V} : s = 2, t = q, \hat{k} = k\right\} \tag{15}$$

where $\hat{v} = (2, q, k, m)$ and $p_2 = (q + 2, m, n)$.

In order to follow the locations of the referees, they must be assigned to at least one game, real or fictitious, every two days.

$$\sum_{\substack{p\in\mathcal{P}: \\ t=q}} x_{ip} + \sum_{\substack{p\in\mathcal{P}: \\ t=q+1}} x_{ip} \geq 1 \quad \forall i \in \mathcal{A}, q \in \{1, \ldots, t_{\text{window}} - 1\}. \tag{16}$$

If a trip is undertaken without an intervening day, the trip must be feasible.

$$z_{i\hat{v}} = 0 \quad \forall i \in \mathcal{A}, (k, m) \in \mathcal{C}_E, \hat{v} \in \left\{(s, t, k, m) \in \widehat{\mathcal{V}} : s = 1\right\}, \tag{17}$$
$$z_{i\hat{v}} = 0 \quad \forall i \in \mathcal{A}, (i, m) \in \mathcal{C}_A, \hat{v} \in \left\{(s, t, k, m) \in \widehat{\mathcal{V}} : s = 1, k = 0\right\}, \tag{18}$$
$$z_{i\hat{v}} = 0 \quad \forall i \in \mathcal{A}, (i, k) \in \mathcal{C}_A, \hat{v} \in \left\{(s, t, k, m) \in \widehat{\mathcal{V}} : s = 1, m = 0\right\}. \tag{19}$$

An example of the path followed by a referee over time is shown in Fig. 2. Each node represents a game $p \in \mathcal{P}$ and each column groups all of the games (real or fictitious) played on the date indicated in the column head. The first row contains all the fictitious games. The games shown in green constitute a possible assignment for a referee i. For each game p in green, $x_{ip} = 1$, and for each arc, $z_{i\hat{v}} = 1$ where $\hat{v} \in \widehat{\mathcal{V}}$ represents a trip taken.

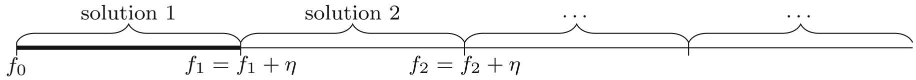


Fig. 3 Solution overlap with $\eta = 0$

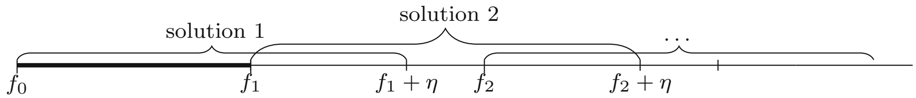


Fig. 4 Solution overlap with $\eta > 0$

3.2 Sequential solution of the referee assignment problem by periods

As was noted previously, our proposed method breaks the seasons down into limited time windows and solve each one as a subproblem. The overall solution is implemented sequentially and incrementally over time. The assignments of previous periods are essential for solving the current one. This rolling horizon approach is explained in what follows.

At the beginning of each period the AdC technical committee provides the modeler with a list of indications for the period plus any changes that may have been made in the actual implementation of the solution generated for the previous period. The indications may be inconsistent with certain of the established constraints, in which case the latter will have to be relaxed. They may also require incorporation into the model of new conditions. For example, if in the current period the referees in $\mathcal{A}_I$ cannot officiate more than one Second Division game, constraints must be added of the following type:

$$\sum_{\substack{p \in \mathcal{P}: \\ t_{\text{previous}} < t \leq t_{\text{window}} \\ k \in \mathcal{E}_2}} x_{ip} \leq 1 \quad \forall i \in \mathcal{A}_I \tag{20}$$

where t_{previous} refers to the last assigned day of the previous period.

Any ad-hoc constraints in the AdC indications must also be incorporated. Also indicated is the last day of the period to be solved, which we call f. An assignment is then generated by the model for period $t_{\text{window}} = f + \eta$, that is, up to day $f + \eta$.

An appropriate choice for the value of this *overlapping parameter* η is thus a key factor. As we will see in Sect. 4, tests were conducted for the regional phase of the First Division's 2015–2016 season using a number of different values of $\eta \in \{0, 1, 2, 3, 4, 5\}$. With $\eta \in \{0, 1\}$, it was found that intermediate periods could not be solved due to the decisions taken in earlier periods, unless perhaps the choice of f_i was changed *a posteriori* (in the i-th period, $t_{\text{window}} = f_i + \eta$). Thus, neither of these two values was a robust choice in terms of feasibility of future assignments. With $\eta = 2$, making assignments for the entire regional phase was feasible but the objective function value turned out to be considerably higher than was the case when $\eta \in \{3, 4, 5\}$. With either of these latter three there was little difference in the objective value after five periods so any one of them would be a similarly good choice for η under the particular conditions of our problem.

A visual representation of this solution approach is given in Figs. 3 and 4 for $\eta = 0$ and $\eta > 0$, respectively.

Solution of large instances

In practice, the periods are typically around 15 days so the instances solved are about 20 days (given that the overlapping parameter is set as $\eta = 5$). Another consideration is the number of games in a period, as this has a significant effect on solution times. If for a given period an optimal solution is not found within 1 h, it is divided into subperiods until each subperiod is solved optimally. The subperiods also overlap, in this case using an overlapping parameter of $\eta_2 = 3$. A feasible solution is then found for the entire period with the method described above.

This feasible solution is then subjected to a refinement procedure. The procedure begins by fixing the generated assignments that involve referees from $\mathcal{A} \setminus \mathcal{A}_I$ as input and solving the problem again for the whole period (but not for all of the referees). This can be seen as a step in a local search heuristic in which the neighborhood of an assignment is composed of all the feasible assignments that differ from the original one in its $\mathcal{A}_I$ assignments.

Taking the optimal assignments thus generated with the $\mathcal{A} \setminus \mathcal{A}_I$ referees fixed as just described, the assignments involving the $\mathcal{A} \setminus \mathcal{A}_{II}$ referees are now fixed and the procedure continues analogously. Once the procedure ends with $\mathcal{A}_{II}$, $\mathcal{A}_{III} \cup \widehat{\mathcal{A}}$ and $\mathcal{A}_{IV} \cup \widehat{\mathcal{A}}$, a single iteration has been completed. Further iterations are executed until the variation in the solution between two consecutive iterations is negligible in practical terms, at which point the solution of the last iteration is retained for the period. In practice, this entire procedure is not iterated more than three times, which is reasonable given that there is little relationship between the problems for the $\mathcal{A}_I$, $\mathcal{A}_{II}$, $\mathcal{A}_{III} \cup \widehat{\mathcal{A}}$ and $\mathcal{A}_{IV} \cup \widehat{\mathcal{A}}$ referee categories.

The complete approach is summarized in Algorithm 3, which uses Algorithms 1 and 2 as subroutines. In Algorithm 3, we introduce the notation $\mathcal{S}(\mathcal{A}')$ to denote the set of assignments in the last solution found by the model that consist of referees in the set $\mathcal{A}'$.

Algorithm 1 Rolling horizon approach up to $f_i + \eta$ for large instances.

```
Find largest t_window ≤ f_i + η such that the model runs within 1 h using binary search.
while t_window < f_i + η do
    Fix assignments obtained up to t_window − η2
    Find largest t_window ≤ f_i + η such that the model runs within 1 h.
end while
return Optimal solution found by the model.
```

Algorithm 2 Single refinement procedure for set $\mathcal{L}$.

```
Fix assignments in the model not included in L.
Apply Algorithm 1.
```

Solution times for a joint instance of the First and Second Divisions are set out in Table 1 while the same for an instance with only First Division games is displayed in Table 2.

Consider now as an example the case reported in Table 1 of a period covering 14 days. With $\eta = 5$, the solution procedure must be run for a 19-day period. However, since at the start we have no previously generated feasible assignment for the period and the instance is too big to be solved completely, we reduce the overlapping parameter to $\eta_2 = 3$. The longest time window that executes within 1 h turns out to be only 5 days, of which the solution will apply to the first $5 - \eta_2 = 2$. The next time window will then start on day 3 and extend as long

Algorithm 3 Complete solution approach.

```
previousValue ← +∞.
incumbentValue ← Solution's value obtained from Algorithm 1
while previousValue-incumbentValue > Threshold do
   previousValue ← incumbentValue.
   for all 𝓛 ∈ {𝓢(𝓐_I), 𝓢(𝓐_II), 𝓢(𝓐_III ∪ Â), 𝓢(𝓐_IV ∪ Â)} do
      Apply Algorithm 2 for set 𝓛 and update incumbentValue.
   end for
end while
return Last optimal solution found by the model.
```

Table 1 Solution times for First and Second Division instances starting 2018/2/1 and ending 2018/2/14

Days	Games	$\mathcal{A}_I$	$\mathcal{A}_{II}$	$\mathcal{A}_{III} \cup \widehat{\mathcal{A}}$	$\mathcal{A}_{IV} \cup \widehat{\mathcal{A}}$	$\mathcal{A}$
5	29	14 s	8 s	16 s	11 s	18 min
6	35	18 s	53 s	30 s	29 s	> 1 h
7	41	47 s	105 s	50 s	237 s	⋮
8	46	48 s	8 min	104 s	22 min	⋮
9	53	109 s	36 min	13 min	46 min	⋮
10	56	11 min	> 1 h	30 min	> 1 h	⋮
11	65	19 min	⋮	> 1 h	⋮	⋮
12	70	> 1 h	⋮	⋮	⋮	⋮
13	79	⋮	⋮	⋮	⋮	⋮
14	84	> 1 h	> 1 h	> 1 h	> 1 h	> 1 h

Entries in each column indicate the time to optimality for $\mathcal{A}'$ with different fixed assignment values for $\mathcal{A} \setminus \mathcal{A}'$. A feasible solution was found in every case in a matter of seconds

as possible while still being solvable within an hour, the solution obtained again applying to the period comprising the entire window less the last η_2 days. By iterating this process we arrive at the feasible solution for the aforementioned 19 days, at which point the refinement procedure can begin.

4 Case study and results

To test our model and solution approach we carried out a case study based on the regional phase of the First Division for the 2015–2016 season, which ran from September 22 to November 28, 2015. A total of 181 games (one more than the 180 mentioned in Sect. 2, for technical reasons not relevant here) were played over the approximately two-month interval. This season was chosen for testing in part because it was the last one for which referee assignments were scheduled manually before our model was adopted, but also because its game schedule was designed so as to maximize the teams' road trip preferences, the same criterion underlying the solutions generated by the Division's game scheduling model used

Table 2 Solution times for instances starting 2018/4/11 and ending 2018/4/24 of First Division games only

Days	Games	$\mathcal{A}_I$	$\mathcal{A}_{II}$	$\widehat{\mathcal{A}}$	$\mathcal{A}$
5	15	10 s	11 s	14 s	25 s
6	19	12 s	15 s	21 s	54 s
7	20	13 s	16 s	23 s	85 s
8	24	16 s	26 s	35 s	159 s
9	26	17 s	28 s	42 s	11 min
10	32	29 s	53 s	89 s	> 1 h
11	35	48 s	88 s	103 s	⋮
12	39	11 min	5 minutes	20 min	⋮
13	43	18 min	26 min	54 min	⋮
14	46	> 1 h	> 1 h	> 1 h	> 1 h

Entries in each column indicate the time to optimality for $\mathcal{A}'$ when setting the values for $\mathcal{A} \setminus \mathcal{A}'$. A feasible solution was found in every case in a matter of seconds. Since the Second Division games are not included, only $\widehat{\mathcal{A}}$ need be considered

ever since. Thus, to measure the model's performance we contrasted the solutions it would have generated for the indicated period with those actually implemented using the manual methods, which were graciously made available to use by the AdC technical committee.

In order to ensure the model results were truly comparable with the manual assignments, in which the number of games officiated by each referee was bounded between 14 and 22, the model was subjected to the same restriction. Had this not been done the model could, for example, choose not to assign a certain referee and the comparisons would not then have been reliable.

Before applying our model to the case study period, however, we first had to experiment with the 2015–2016 season data to determine what would be the most appropriate value of the overlapping parameter η. For this purpose we derived the optimal solution for a time horizon of 11 days, the longest solvable within the 1-hour criterion established in the previous section, then generated an initial solution for the entire season, and finally, applied the refinement procedure also presented in the previous section. These steps are detailed below, followed by the comparisons of the model and the manual methods.

4.1 Overlapping parameter experiment

We began our experimentation with the overlapping parameter by deriving the optimal solution for an 11-day period. The total estimated cost for this solution was U$S 4,404. We then solved the problem for the same 11 days using our method of proceeding sequentially over a series of shorter periods. For each period i we set $f_i = i$ and $\eta = 5$ where $1 \leq i \leq 6$. Thus, a sequence of solutions was generated for 6-day time windows, each one consisting of a 1-day period followed by a 5-day overlap. The first window started on day 1 of the 11 days and the sixth and last window started on day 6. As each of the six solutions was found, its assignments were inputted to the first day of the next one.

The result of this procedure was a feasible solution with a total cost of U$S 4,453. Considering the small gap between this figure and the U$S 4,404 for the optimal solution, this

Table 3 Estimated total cost and total kilometers travelled for the initial solution obtained by the model with values of $\eta \in \{0, \ldots, 5\}$

η	Estimated cost	Kilometers travelled
0	**U$S 44,938**	**318,456 km.**
1	**U$S 43,632**	**312,728 km**
2	U$S 44,023	312,342 km
3	U$S 42,266	299,859 km
4	U$S 42,117	292,363 km
5	U$S 42,842	298,142 km

The results for $\eta = 0$ and $\eta = 1$ are shown in bold given that during the process of generating the solution, one of the periods could not be solved until the choice of f_i was changed *a posteriori*

outcome was encouraging so we immediately applied our method to the whole season to obtain a feasible initial solution.

Obtaining an initial solution

The time windows used for the generation of the initial solution were the longest possible periods that still satisfied the 1-hour criterion. In practice, however, the AdC technical committee rather than the modeler determines the f_i values, that is, the assignment periods, so the choice of a value for η is key to ensuring a level of solution robustness that will prevent any unfeasible scenarios from arising in future assignment periods.

As we saw in Sect. 3.2, an overlapping parameter of $\eta \in \{0, 1\}$ does not give robust results in that with those values, feasibility is highly dependent on the value chosen for f_i. Whenever the solution for a period turned out to be unfeasible, we returned to the previous period and solved it using a time window one day shorter, repeating this procedure until a feasible solution was found.

The results obtained with sequential executions of the model in terms of cost and travel distance for the various periods with $\eta \in \{0, 1, 2, 3, 4, 5\}$ are set out in Table 3. They are consistent with the idea that the overlapping parameter must be chosen carefully to avoid the feasibility problems that arise when $\eta \in \{0, 1\}$ and the relatively high objective function value resulting when $\eta \in \{2\}$.

Refining the solution

Having obtained an initial feasible assignment of referees for the complete set of games $\mathcal{P}$, we proceeded to refine the assignment using the same model by searching in a neighborhood of that solution. To define this neighborhood we took, at each step of the refinement procedure, a subset of referee slots denoted $\mathcal{L}$ and considered as assignments neighboring the initial solution all those feasible assignments that were different from the ones in the initial solution but within $\mathcal{L}$ (thereby fixing the set of assignments not in $\mathcal{L}$). For example, in the first step in Sect. 3.2 we took the set of slots involving referees in $\mathcal{A}_I$ for the entire period as $\mathcal{L}_1$, and once a refined solution was obtained, the slots involving referees in $\mathcal{A}_{II}$ were included in $\mathcal{L}_2$, continuing analogously for the referees in $\mathcal{A}_{III} \cup \widehat{\mathcal{A}}$ and $\mathcal{A}_{IV} \cup \widehat{\mathcal{A}}$ at each step. In short, at each step we generated an optimal reassignment for referee slots in $\mathcal{L}$ whose total cost could not be worse than the one obtained previously given that the incumbent solution is a feasible reassignment.

Table 4 Estimated total cost and kilometers travelled after refinement of the initial solution for different values of $\eta \in \{0, \ldots, 5\}$

η	Estimated cost	Kilometers travelled
0	U$S 42,165	296,658 km
1	U$S 42,474	297,566 km
2	U$S 41,323	293,030 km
3	U$S 41,386	288,698 km
4	U$S 40,728	283,532 km
5	U$S 40,816	286,309 km

Applying this procedure to the 2015–2016 regional phase, we performed the following refinement steps on all of the initial solutions obtained for values of $\eta \in \{0, \ldots, 5\}$ and for each referee licence category (recall that each First Division game that year was officiated by two referees and all were in categories $\mathcal{A}_I$ or $\mathcal{A}_{II}$). In the first step, $\mathcal{L}_1$ was taken as all referee slots involving the category in question for games within a time window centered on f_1 and as long as possible. In the second step, $\mathcal{L}_2$ was taken as in the first step but starting with the first games of the phase (as was the case at the start of the initial solution but now with just one referee category and the future assignments fixed). From the third step onwards, $\mathcal{L}_i$ was taken as all slots for games in a time window overlapping 3 days in $\mathcal{L}_{i-1}$. Thus, we advanced incrementally until the entire regional phase was refined for slots involving referees in the category. Lastly, the analogous procedure was repeated with the other referee category.

Upon completing this procedure we obtained, for each η value, a new solution that improved on the initial one. The cost and travel distance results obtained by this process are displayed in Table 4. They suggest that overlapping parameter values of $\eta \in \{3, 4, 5\}$ would be the reasonable alternatives. This conclusion is based not only on the assignment costs but also on the robustness these values lend to the whole solution generation process. In practice, we opted for $\eta = 5$, and in the cases where the size of the period did not allow the assignments to be completely solved, the period was divided into overlapping subperiods with an overlapping parameter of $\eta_2 = 3$.

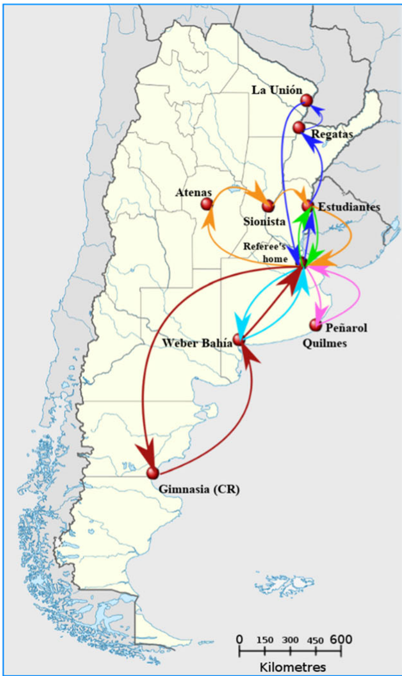
4.2 Comparison with the manual assignment

We now turn to the comparison of the assignments identified by our model for the 2015–2016 season regional phase with the manual assignments actually implemented for that set of games. This involved simulating the proposed solution procedure for the whole regional phase using periods of 14 or 15 days. More specifically, the final days of the periods were $f_1 = 14$, $f_2 = 28$, $f_3 = 42$, $f_4 = 57$ and $f_5 = 71$. Since our overlapping parameter was $\eta = 5$, the first time window solution was the regional phase's first 19 days. A solution was first generated for the window up to $f_1 + \eta$ and the assignments so obtained were applied to the period up to f_1. The procedure was then iterated for the following periods until the assignments for the entire phase were generated.

The results produced by our method satisfied all of the problem constraints specified here in earlier sections. The estimated global trip and lodging costs incurred by the referee corps, set forth in Table 5, were U$S 41,488 on a total of 291,177 kilometers travelled compared to the actual figures for the manual assignments of U$S 56,151 and 405,540 kilometers. Our model's simulated solutions thus show an improvement over the actual assignments of 26%

Table 5 Comparison of costs and distance travelled for the manual and model-simulated referee assignments, regional phase 2015–2016

	Estimated cost	Kilometers travelled
Model	U$S 41,488	291,177 km
Manual	U$S 56,151	405,540 km



Date	Home Team	Away Team
23/09	Estudiantes	Regatas
27/09	Weber Bahía	Quilmes
02/10	Obras	Lanús
06/10	Boca Juniors	San Lorenzo
08/10	Quilmes	Weber Bahía
14/10	Estudiantes	Atenas
16/10	Regatas	Instituto
18/10	La Unión	Quimsa
24/10	Peñarol	San Lorenzo
28/10	Ferrocarril Oeste	Quilmes
29/10	Obras	Argentino
31/10	Atenas	Quimsa
02/11	Sionista	San Martín
03/11	Estudiantes	Instituto
15/11	Gimnasia (CR)	Boca Juniors
17/11	Weber Bahía	Peñarol
25/11	Quilmes	Obras
27/11	San Lorenzo	Lanús
28/11	Estudiantes	Sionista

Fig. 5 Travel patterns generated by the model for a referee living in Buenos Aires, home of First Division teams Obras, Boca Juniors, Ferrocarril Oeste and San Lorenzo in 2015–2016. Road trips of one, two or three games are shown in non-white colors in consecutive rows of the table and arrows on the map of Argentina; games in Buenos Aires are shown in the table in white

in costs and 28% in travel distance. Note also that the manual solutions did not satisfy all of the constraints.

Finally, note that the slight discrepancies between the model results in Tables 4 and 5 for the case where $\eta = 5$ are due to a pair of technical changes that had to be incorporated to ensure the model simulations were subject to the same conditions as the manual scheduling process for the 2015–2016 regional phase. The first change was that the final day of each period was set using criteria similar to those applied by the AdC at that time. The second change was that in solving for each period, the refinement process was applied only for that period's games given that the games for previous periods had already been played so their assignments obviously could no longer be modified. An example of the travel schedule generated by the model for a referee is illustrated in Fig. 5.

5 Conclusions and future work

This article has reported on a new real-world sports scheduling application that assigns referees to games, a problem much less studied than the scheduling of the games themselves. An integer linear programming model was developed using a rolling-horizon heuristic approach that significantly improved referee assignments in the Argentinean professional basketball leagues. In a simulation test on a real case, the model reduced referee travel costs and total distance travelled under the assignments arrived at with manual methods by some 26% and 28% respectively. The solid performance of the model led to its implementation by the First Division of Argentinean professional basketball in 2016–2017 and 2017–2018 and by the Second Division in 2017–2018.

Since the season schedules for the two divisions are designed to maximize the number of preferred road trips requested by the teams, the rule prohibiting referees from officiating the same team more than once every so many games means referee assignments can never follow the same travel patterns as the teams. An interesting problem for future research would therefore be to integrate the referee assignment problem with the game scheduling problem. For example, suppose that a team is scheduled for a three-game road trip, playing away against the two Mar del Plata teams on a Monday and the following Wednesday, respectively, then travelling to play the third game on Friday in Buenos Aires, one day's journey (some 400 kms) to the north, before returning home. Under the above-mentioned rule the referee officiating at the Monday game would not be able to officiate at the same team's Wednesday (or Friday) game. If, however, other teams are assigned road trips with games on the Tuesday and the Thursday that the same referee is permitted to officiate in cities such as Buenos Aires or Bahía Blanca, the latter also a one-day trip from Mar del Plata (450 kms to the west), the referee would then be able to officiate on consecutive days following the Monday game. In other words, the teams' trips could be distributed over the week in such a way as to be complementary to the referee assignments by incorporating the criteria just described, without substantially modifying the game scheduling procedure.

It is also possible, of course, that integrating the two assignment problems would pose formidable difficulties given that solving them separately is already challenging. Developing a combined approach would therefore be a major task for future research, but a workable formulation would have general application beyond the particular case of Argentinean basketball. At present, only a few studies reported in the literature have attempted to develop such an approach (Atan and Hüseyinoğlu 2017; Bender and Westphal 2016; Linfati et al. 2019).

Devising integer programming formulations that solve the referee assignment problem for longer time periods should also be of considerable interest. One such formulation might involve the inclusion of binary variables associated with all series of feasible games a referee can officiate that start and end at his or her home town. This would be similar to a branch-and-price formulation except that the number of feasible trips is likely to be small enough that all of them could be listed, thus obviating the need to generate them dynamically. Thorough experimentation with this approach would be required to determine whether it could yield significant improvements.

Yet another possibility for future research would be to address the dynamic aspect of referee assignment decisions, a practical issue we encountered in our work with the Argentinean league that departs from the setup in the benchmark problems in the literature (Trick et al. 2012; Duarte et al. 2006). This was dealt with in our work reported here using a rolling horizon heuristic approach that sequentially assigns referees as the season progresses, but alternative approaches could, for example, incorporate uncertainty regarding future situations at the start of the season or propose an entire season assignment as a base solution that will be modified as the season progresses round by round.

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