

# Adaptive Observer-Based Fault Diagnosis with Application to Satellite Attitude Control Systems

Ke Zhang, Bin Jiang  
College of Automation Engineering, Nanjing  
University of Aeronautics and Astronautics  
Nanjing 210016, P.R. China  
binjiang@nuaa.edu.cn

Peng Shi  
Faculty of Advanced Technology, University of  
Glamorgan Pontypridd, CF37 1DL, UK  
pshi@glam.ac.uk

## Abstract

*In this paper, a novel adaptive approach for fault diagnosis is proposed. In order to enhance performance of fault estimation, the proposed fault estimator is composed of proportional term and integral one, which enables to improve rapidity and accuracy for fault estimation. Finally, a satellite attitude control example is presented to illustrate the efficiency of the presented techniques for fault diagnosis.*

## 1. Introduction

In practical industrial process, control engineers are faced with increasingly complex systems where dependability considerations are sometimes more important than performance. Faults may occur in any location, such as actuators, sensors, etc. In order to improve efficiency, the reliability can be achieved by fault-tolerant control, which relies on early detection of faults, using fault detection and isolation (FDI) procedures. So FDI has become an attractive topic over the past ten years. Fruitful results can be found in [1-3] and the references therein. In general, fault tolerance can be achieved in two ways: 1) passively, using feedback control laws that are robust with respect to possible systems fault, or 2) actively, using a fault detection and isolation (FDI) and accommodation technique.

Once a fault occurs, fault accommodation can be activated to ensure system performance. FDI is the first step in fault accommodation to monitor the system and determine the location of the fault. Then, fault estimation is utilized to on-line generate magnitude of the fault, but, in general, this is not an easy task. Finally, using the obtained fault information, an additive controller can be designed to compensate the fault. Since the controller designed is based on fault information, it is more reliable for active fault-tolerant control. From the above procedures, it can be seen that fault

estimation is the key technique in fault accommodation [4].

During the last decade, various methods for fault estimation have been developed such as adaptive techniques [5, 6], sliding mode observer approaches using the equivalent output injection signal [7, 8], and learning methods based on neural network [9, 10], etc. The motivation of the study is to enhance the performance of fault estimation. The novelty of the proposed adaptive fault estimator consists of two terms, namely, a proportional one and an integral one. These can eliminate steady estimation error and improve systemic rapidity, simultaneously. Therefore, both satisfactory dynamical and steady state performances can be achieved.

The paper is organized as follows. Section 2 gives the system description for linear systems with actuator faults. A novel adaptive algorithm for fault estimation is presented in Section 3. In Section 4, a satellite attitude control system is given to demonstrate the effectiveness of the new algorithm. Finally, conclusions are given in Section 5.

## 2. System description

Consider the following linear system with actuator fault

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where  $x(t) \in R^n$  is the state vector,  $u(t) \in R^m$  is the input vector,  $y(t) \in R^p$  is the output vector and  $f(t) \in R^r$  represents the actuator fault.  $A, B, E$  and  $C$  are known constant real matrices of appropriate dimensions, the pair  $(A, C)$  is assumed to be observable and  $rank(E) = r$ . The failure  $f(t) = \beta(t - t_s)f_0$  can be treated as an additive signal, where the function  $\beta(t - t_s)$  is given by

$$\beta(t - t_s) = \begin{cases} 0 & t \leq t_s \\ 1 & t > t_s \end{cases} \quad (3)$$

That is,  $f(t)$  is zero prior to the failure time ( $t \leq t_s$ ) and is a constant vector after the failure occurs ( $t > t_s$ ).

*Remark 1:* Note that only actuator fault is considered in this paper. As for sensor fault case, if its model is available, it can be reformulated as actuator fault.

### 3. Adaptive fault diagnosis observer design

In this section, a modified adaptive fault diagnosis observer is investigated, which is constructed be described as follows

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + E\hat{f}(t) - L(\hat{y}(t) - y(t)) \quad (4)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (5)$$

where  $\hat{x}(t) \in R^n$  is the observer state vector,  $\hat{y}(t) \in R^p$  is the observer output vector and  $\hat{f}(t)$  is an estimate of  $f(t)$ .  $L \in R^{n \times p}$  is observer gain.

Denote the estimation error  $e_x(t) = \hat{x}(t) - x(t)$ , and then the estimation error dynamics is described by

$$\dot{e}_x(t) = (A - LC)e_x(t) + Ee_f(t) \quad (6)$$

where fault estimation error  $e_f(t) = \hat{f}(t) - f_0$ .

*Remark 2:* Since actuator fault  $f_0$  is constant, the derivative of  $e_f(t)$  with respect to time can be written as

$$\dot{e}_f(t) = \dot{\hat{f}}(t) \quad (7)$$

Denote

$$s_x(t) = \dot{e}_x(t) + \Sigma e_x(t) \quad (8)$$

$$s_y(t) = Cs_x(t) \quad (9)$$

where  $\Sigma = \text{diag}(\sigma, \dots, \sigma)$ ,  $\sigma$  is a positive constant. From (5) and (8), one obtains

$$\dot{s}_x(t) = (A - LC)s_x(t) + E(\dot{e}_f(t) + \sigma e_f(t)) \quad (10)$$

Based on (10), a novel algorithm will be proposed to improve the performance of fault estimation.

The purpose of the adaptive observer is to not only detect and isolate faults, but also obtain the information of fault, which is useful for fault accommodation. Now, we are ready to present our main result in this paper, which is an adaptive diagnostic algorithm for estimating the fault.

*Theorem 1:* For a given matrix  $Q_{(n \times n)} > 0$ , if there exist two matrices  $P_{n \times n}$  and  $F_{r \times p}$ , such that

$$P(A - LC) + (A - LC)^T P - 2\sigma P E \Gamma E^T P \leq -Q \quad (11)$$

$$P E = C^T F^T \quad (12)$$

then the observer described by (4) and the following adaptive fault estimation algorithm

$$\dot{\hat{f}}(t) = -\sigma \Gamma F s_y(t) \quad (13)$$

can realize  $\lim_{t \rightarrow \infty} e_x(t) = 0$  and  $\lim_{t \rightarrow \infty} e_f(t) = 0$ , where  $\Gamma > 0$  is a weighting matrix.

*Proof:* Consider the following Lyapunov function

$$V(t) = s_x^T(t) P s_x(t) + e_f^T(t) \Gamma^{-1} e_f(t) \quad (14)$$

From (7) and (9), its derivate with respect to time is

$$\begin{aligned} \dot{V}(t) &= s_x^T(t) [P(A - LC) + (A - LC)^T P] s_x(t) \\ &\quad + 2s_x^T(t) P E [\dot{e}_f(t) + \sigma e_f(t)] + 2e_f^T(t) \Gamma^{-1} \dot{e}_f(t) \\ &= s_x^T(t) [P(A - LC) + (A - LC)^T P] s_x(t) \\ &\quad - 2\sigma s_x^T(t) P E \Gamma E^T P s_x(t) + 2\sigma s_x^T(t) P E e_f(t) \\ &\quad - 2\sigma e_f^T(t) F s_y(t) \end{aligned} \quad (15)$$

According to (11) and (12), it is easy to show that

$$\dot{V}(t) \leq -s_x^T(t) Q s_x(t) \leq 0 \quad (16)$$

which guarantees  $\lim_{t \rightarrow \infty} s_x(t) = 0$  and  $\lim_{t \rightarrow \infty} e_f(t) = 0$ . From (7), we obtain  $\dot{e}_x(t) + \Sigma e_x(t) = 0$ . From  $\Sigma > 0$ , one has  $\lim_{t \rightarrow \infty} e_x(t) = 0$ . Furthermore, the fact of  $\lim_{t \rightarrow \infty} e_f(t) = 0$  makes  $\hat{f}(t)$  converge to  $f_0$  without any estimate bias.

This completes the proof.  $\square$

*Remark 3:* The conventional adaptive approach is pure integral term, which can guarantee that the fault estimation is unbiased at the cost of rapidity. Otherwise, fault estimation using sliding mode observers is pure proportional term in essence, and steady state error is inevitable.

$$\hat{f}(t) = -\Gamma F \int_{t_s}^t e_y(\tau) d\tau \quad (17)$$

$$\hat{f}(t) = -\rho \frac{P_0 e_y(t)}{\|P_0 e_y(t)\| + \delta} \quad (18)$$

where  $P_0$  is a positive definite matrix,  $\rho$  is an upper bound on the fault,  $\delta$  is a small positive scale.

The proposed adaptive identification strategy can be expressed as follows

$$\dot{\hat{f}}(t) = -\sigma \Gamma F \left[ e_y(t) + \sigma \int_{t_s}^t e_y(\tau) d\tau \right] \quad (19)$$

Compared with two fault estimation methods (17) and (18), it can be seen that the proposed adaptive estimation algorithm (19) combines proportional term with integral one. The proportional one can improve systemic rapidity, while the integral one can eliminate estimation error. Therefore, the algorithm (19) is superior to the above two approaches, which can improve rapidity and accuracy for fault estimation.

*Remark 4:* A nice feature of the adaptive method proposed in this paper is that it not only enables fault detection, but also provides the shape of the fault, which will be used for fault accommodation.

#### 4. Simulation results

In this section, we consider a satellite attitude control system [11, 12] to show the effectiveness of the proposed method. For small attitude deviation from local vertical local horizontal orientation, the angle velocity can be expressed by yaw angle  $\psi$ , pitch angle  $\theta$ , roll angle  $\phi$  and orbital rate  $\omega_0$ . Thus, through transformation, the nominal model of a satellite attitude dynamics is written as

$$\begin{aligned} I_1 \ddot{\phi} - \omega_0(I_1 - I_2 + I_3)\dot{\psi} + 4\omega_0^2(I_2 - I_3)\phi &= T_1 \\ I_2 \ddot{\theta} + 3\omega_0^2(I_1 - I_3)\theta &= T_2 \\ I_3 \ddot{\psi} + \omega_0(I_1 - I_2 + I_3)\dot{\phi} + \omega_0^2(I_2 - I_1)\psi &= T_3 \end{aligned}$$

The motion of inertia about principal axes of the satellite are  $I_1 = 17.581 \text{ kgm}^2$ ,  $I_2 = 16.533 \text{ kgm}^2$ ,  $I_3 = 3.529 \text{ kgm}^2$ . The orbital rate  $\omega_0$  is  $7.2722 \times 10^{-5} \text{ rad/s}$ .  $u = [T_1 \ T_2 \ T_3]^T$  is the control torque vector.

The state vector is chosen as  $x = [\phi \ \theta \ \psi \ \dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T$ , therefore  $A$ ,  $B$  can be obtained. For satellite attitude control system, the six variables are available, so  $C$  is identity matrix. The fault distribution matrix is selected as

$$E = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T$$

We choose  $\Sigma = 5 \times I_6$  in (8) and the weighting matrix  $\Gamma = 10 \times I_2$ . By solving equations (11) and (12), one can obtain the observer gain  $L = 5 \times I_6$  and

$$P = \begin{bmatrix} 1.91 & 0 & 0 & 0.20 & 0 & 0 \\ 0 & 1.91 & 0 & 0 & 0.20 & 0 \\ 0 & 0 & 1.91 & 0 & 0 & 0.20 \\ 0.20 & 0 & 0 & 1.95 & 0 & 0 \\ 0 & 0.20 & 0 & 0 & 1.95 & 0 \\ 0 & 0 & 0.20 & 0 & 0 & 1.95 \end{bmatrix}$$

In the simulation, it is assumed that faults occur at state  $\psi$  and  $\dot{\phi}$  simultaneously, which will be considered as following

$$f_1(t) = \begin{cases} 0 & 0 \leq t \leq 4 \\ 0.3 & 4 < t \leq 8 \\ -0.1 & 8 < t \leq 10 \end{cases}$$

$$f_2(t) = \begin{cases} 0 & 0 \leq t \leq 5 \\ -0.5 & 5 < t \leq 10 \end{cases}$$

Figure 1 and 2 show the estimation of fault  $f_1(t)$  and  $f_2(t)$  using the proposed method in (19). It can be seen that using the adaptive fault diagnosis observer, actuator fault can be detected and isolated accurately. Furthermore, the designed algorithm can improve the performance of fault estimation and guarantee the estimated  $\hat{f}_1(t)$  and  $\hat{f}_2(t)$  are unbiased with high rapidity.

The conventional adaptive identification algorithm (17) also can guarantee the identified faults are unbiased. However, when a larger weighting matrix is chosen, the performance of system is rapid response, but big overshoot is unavoidable. On the other hand, when a small weighting matrix is selected, the overshoot can be overcome at the cost of the rapid response. These are demonstrated in Figure 3 and 4. In Figure 5, Applying sliding mode observer method (18), the response is fast, but estimated error is inevitable.

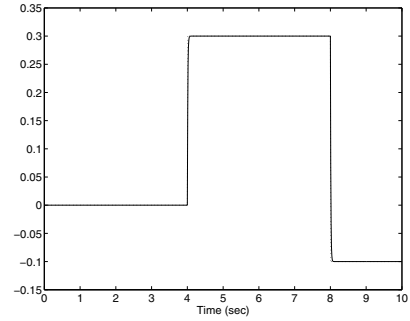


Figure 1. Fault  $f_1(t)$  (dotted) and its estimation  $\hat{f}_1(t)$  (solid).

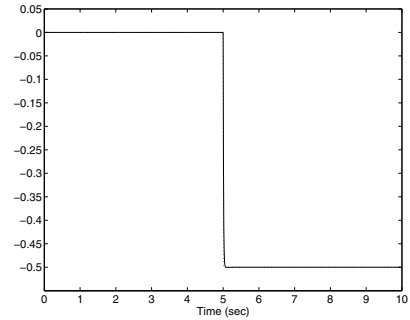
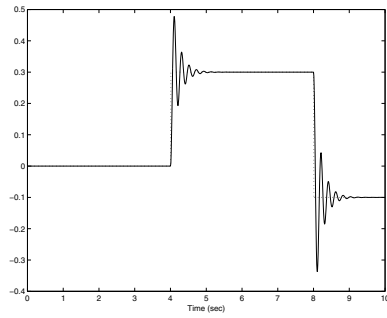


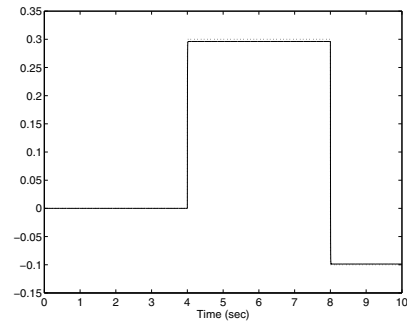
Figure 2. Fault  $f_2(t)$  (dotted) and its estimation  $\hat{f}_2(t)$  (solid).

#### 5. Conclusion

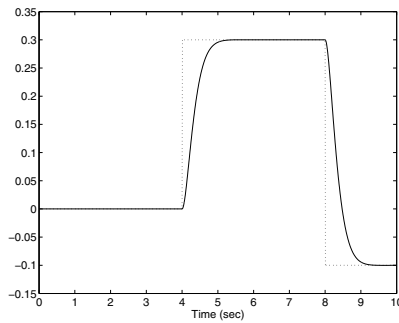
In this paper, an adaptive fault diagnosis observer is designed. The proposed method improves rapidity and accuracy on fault detection and estimation. The application of this design scheme to a satellite attitude control system shows that the systemic fault can be detected and estimated with satisfactory performance.



**Figure 3. Using fault estimation algorithm (17) with large weighting matrix, fault  $f_1(t)$  (dotted) and its estimation  $\hat{f}_1(t)$  (solid).**



**Figure 5. Using fault estimation algorithm (18), fault  $f_1(t)$  (dotted) and its estimation  $\hat{f}_1(t)$  (solid).**



**Figure 4. Using fault estimation algorithm (17) with small weighting matrix, fault  $f_1(t)$  (dotted) and its estimation  $\hat{f}_1(t)$  (solid).**

## 6. Acknowledgment

This work is partially supported by National Natural Science Foundation of China (60574083) and Innovation Scientific Research Fund of NUAA (Y0508-031).

## References

- [1] D.M. Frank, "Fault detection in dynamic systems using analytical and knowledge-based redundancy-a survey and some new results", *Automatica*, March 1990, vol. 26, pp. 450-472.
- [2] J.J. Gertler, *Fault Detection and Diagnosis in Engineering Systems*, Marcel Dekker, New York, 1998.
- [3] J. Chen, and R.J. Patton, *Robust Model-based Fault Diagnosis for Dynamic Systems*, MA: Kluwer, Boston, 1999.
- [4] B. Jiang, M. Staroswiecki, and V. Cocquempot, "Fault accommodation for a class of nonlinear systems", *IEEE Trans. Automatic Control*, Sept. 2006, vol. 51, pp. 1578-1583.
- [5] B. Jiang, M. Staroswiecki, and V. Cocquempot, "Fault identification for a class of time-delay systems", In *Proc. American Control Conference*, Jan. 2002, vol.151, pp. 2239-2244.
- [6] B. Jiang, J.L. Wang, and Y.C. Soh, "An adaptive technique for robust diagnosis of faults with independent effects on system outputs", *Int. J. Control*, Nov. 2002, vol.75, pp. 792-802.
- [7] C. Edwards, S.K. Spurgeon, and R.J. Patton, "Sliding mode observers for fault detection and isolation", *Automatica*, Apr. 2000, vol. 36, pp. 541-553.
- [8] C.P. Tan, and C. Edwards, "Sliding mode observers for detection and reconstruction of sensor faults", *Automatica*, Oct. 2002, vol. 38, pp. 1815-1821.
- [9] M.M. Polycarpou, "Fault accommodation of a class of multivariable nonlinear dynamical systems using a learning approach", *IEEE Trans. Automatic Control*, May 2001, vol. 46, pp. 736-741.
- [10] M.A. Demetriou, and M.M. Polycarpou, "Incipient fault diagnosis of dynamical systems using online approximators", *IEEE Trans. Automatic Control*, Nov. 1998, vol. 43, pp. 1612-1617.
- [11] C.D. Yang, and Y.P. Sun, "Mixed  $H_2/H_\infty$  state-feedback design for microsatellite attitude control", *Control Engineering Practice*, 2002, vol.10, pp.951-970.
- [12] Q. Wu, and M. Saif, "Robust fault diagnosis for satellite attitude systems using neural state space models", In *IEEE International Conf. Systems, Man and Cybernetics*, Sept. 2005, pp. 1955-1960.