Observer-Based Fault Diagnosis of Satellite Systems Subject to Time-Varying Thruster Faults

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This paper presents a novel fault diagnosis approach in satellite systems for identifying time-varying thruster faults. To overcome the difficulty in identifying time-varying thruster faults by adaptive observers, an iterative learning observer (ILO) is designed to achieve estimation of time-varying faults. The proposed ILO-based fault-identification strategy uses a learning mechanism to perform fault estimation instead of using integrators that are commonly used in classical adaptive observers. The stability of estimation-error dynamics is established and proved. An illustrative example clearly shows that time-varying thruster faults can be accurately identified. [DOI: 10.1115/1.2719773]

Keywords: iterative learning observer, time-varying thruster faults, fault identification, satellite systems

1 Introduction

Satellites have been used for various purposes, such as remote sensing, television, telephony, data communication, etc. These important services cannot be affected or degraded by faults that may occur in components, actuators, and sensors. Hence, a spacecraft control system should include fault detection and identification subsystems in order for the spacecraft to achieve prompt and accurate fault detection and identification (FDI). On the basis of this, fault accommodation that can reduce human intervention for maintenance and decrease hardware complexity can be realized [1,2].

Current approaches to FDI and fault accommodation in satellites are based on a set of "if-then" rules and require long-time testing with a great amount of investment [2]. Control reconfiguration capability of the current strategies in the event of actuator or sensor faults is also fairly weak. Motivated by these considerations, a model-based thruster leakage monitoring approach is proposed in [3] for Cassini spacecraft. The leakage will typically lead to a freezing type of faults in thrusters. Instead of monitoring some measurable quantities, the entire Euler's equations are monitored under the assumption that estimates of most Euler's equation terms are available onboard. Reference [2] deals with faulttolerant control of spacecraft with actuator-stuck faults, i.e., constant actuator faults. The constant faults are estimated by an adaptive observer for the purpose of fault-tolerant control. In addition, fault detection in gyro rates and earth sensor error is considered in [1]. A Luenberger observer is constructed to achieve this task. Incipient faults are often time varying and the aforementioned

techniques do not deal with such type of faults. Apart from this particular application, the ability to detect time-varying actuator faults that do occur in practice is important [4].

The purpose of this paper is to explore the issue of FDI for satellite systems subject to time-varying thruster faults. In the literature, adaptive observers have been used to detect and estimate faults [5–7]. However, their ability is limited to estimate constant faults because the fault estimation is based on integrators that require persistent excitation [8]. This is the main deficiency of the adaptive-observer-based fault identification. The iterative learning observer (ILO) is a robust observer that can simultaneously estimate both system states and disturbances or uncertainties [9,10]. To overcome the difficulty in identifying time-varying faults, an ILO is designed to perform detection and estimation of time-varying thruster faults. The ability of estimating time-varying faults by the ILO is facilitated through the use of a learning mechanism; that is, previous information is employed to update the fault estimates law.

The remainder of this paper is organized as follows: In Sec. 2, the problem of interest is stated. Main results are presented in Sec. 3, where an ILO is designed, and its stability and convergence are also proved. An illustrative example is used to show the effectiveness of the ILO-based approach in Sec. 4. Finally, conclusions are drawn in Sec. 5.

Problem Statement

A satellite is assumed to be a rigid body with thrusters providing torques. Its dynamics is presented as follows [2,11]:

$$J\dot{w} = -w^{x}Jw + Bu \tag{1}$$

where $u \in R^3$, the torque generated from thrusters, $w \in R^3$ denotes the inertial angular velocity, $J = J^T$ indicates a positive definite inertia matrix, where T denotes transpose, w^x represents the skew symmetric matrix. More precisely, let $w = [w_1 \ w_2 \ w_3]^T$, then

$$w^{x} = \begin{bmatrix} 0 & -w_{3} & w_{2} \\ w_{3} & 0 & -w_{1} \\ -w_{2} & w_{1} & 0 \end{bmatrix}$$

It is worth noting that Euler's equations are omitted in system (1) because they are not associated with thrusters. Please refer to [2,11] for a complete dynamic model of satellite.

Fault estimation is important because once a fault is detected and identified, fault accommodation can be easily achieved. Given that, the goal of this paper is to design an ILO for the purpose of estimating time-varying thruster faults. In previous studies, adaptive-observer-based fault estimation approach has been broadly adopted [2,5,6]. But, the limitation has been that only constant or slowly varying faults can be dealt with because the adaptive law requires persistent excitation for fault estimation [6]. To overcome this obstacle, an ILO that was first proposed in [12] is used in this paper to tackle this problem. Instead of using integrators, the principle of fault estimation by the ILO is through the use of a learning mechanism, i.e., the ILO will learn from the time-varying faults. The learning makes it possible for the ILO to estimate time-varying faults.

3 Main Results

Once a thruster fault occurs, system (1) has the following form:

$$J\dot{w} = -w^{X}Jw + Bu(t) + B_{f}u_{f}(t)$$
 (2)

where $u_f(t) \in R^3$, representing an additive time-varying thruster fault [4,13], B_f is the 3×3 fault distribution matrix. Signal $u_f(t)$ may represent a constant thruster fault, a periodic or an aperiodic time-varying fault.

3.1 Design of the Iterative Learning Observer. The ILO was first used in [12] to estimate constant faults. Herein, we shall

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exhibit its ability of estimating time-varying thruster faults while estimating system states. In line with this, an ILO is designed as follows:

$$J\dot{\hat{w}} = -\hat{w}^{x}J\hat{w} + \Lambda(w - \hat{w}) + Bu + B_{f}v(t)$$

$$v(t) = K_1 v(t - \tau) + K_2 [w(t) - \hat{w}(t)]$$

where X_1 and K_2 are gain matrices. Parameter τ is the updating interval. It may be taken as the sampling-time interval in a sampled-data control system, or as an integer multiple of the sampling-time interval. The parameter Λ is a positive definite matrix. Signal v(t) is called ILO input that is used to estimate the time-varying fault. The signal v(t) is updated by both its past information and state estimation error. The angular velocity v is assumed measurable.

The estimation error can be obtained by subtracting (3) from (2)

$$J\widetilde{\widetilde{w}} = (-w^{x}Jw + \widehat{w}^{x}J\widehat{w}) + B_{f}[u_{f}(t) - v(t)] - \Lambda\widetilde{w}$$
(4)

where $\widetilde{w} = w - \hat{w}$.

Remark 1. Compared to the adaptive-observer-based scheme [14], the ILO-based fault detection and estimation approach possesses the following features:

- Can detect and estimate actuator faults that are constant, periodic, or aperiodically time-varying by learning from the faults.
- Requires less on-line computing power than the adaptive observer because the updating law of fault estimation is an algebraic equation. This makes it possible for the ILO to be easily realized in practice.
- Is robust to measurement noise and other high-frequency disturbances because it makes infrequent adjustments of the control parameters at discrete instants even as continuous signals are being processed in real time. Compared to this, continuous adaptive algorithms suffer from a lack of robustness since the control parameters are continuously adjusted, using an adaptive law involving pure integration [15].

This ILO will be tested, in Sec. 4, to estimate a constant fault, a periodic fault, and an aperiodic time-varying fault, respectively. At last, a mixed actuator fault, combining a constant, a periodic and an aperiodic fault, will be used to inspect the ILO's estimation ability.

3.2 Stability Analysis of the ILO. To derive the stability and convergence of the ILO's estimation error, the following assumptions are made:

ASSUMPTION 1. The time-varying signal $u_f(t)$ is bounded, and $||u_f(t) - K_1 u_f(t-\tau)||_{\infty} \leq k_d$.

ASSUMPTION 2. Nonlinear term w^xJw satisfies $||w^xJw - \hat{w}^xJ\hat{w}|| \le m||\widetilde{w}||$.

Remark 2. Assumption 2 is reasonable because system state w(t) is first-order differentiable and its derivative is bounded.

Lemma 1 is used for the proof of Theorem 1.

LEMMA 1. If ILO input v(t) is defined in (3), then the following inequality holds:

$$e^{\mathsf{T}}(t)e(t) \leq \alpha_1 e^{\mathsf{T}}(t-\tau)K_1^{\mathsf{T}}K_1 e(t-\tau) + \alpha_2 \widetilde{w}^{\mathsf{T}}(t)K_2^{\mathsf{T}}K_2 \widetilde{w}(t) + \alpha_3 d^{\mathsf{T}}(t)d(t)$$

$$(5)$$

where $e(\cdot) = u_f(\cdot) - v(\cdot)$, α_i is a positive constant, i = 1, 2, 3, and $d(t) = u_f(t) - K_1 u_f(t - \tau)$

Proof. Starting with

$$e(t) = u_f(t) - K_1 v(t - \tau) - K_2(w - \hat{w})$$

= $K_1 e(t - \tau) - K_2(w - \hat{w}) + d(t)$ (6)

it follows that

$$e^{T}(t)e(t) = [K_{1}e(t-\tau) - K_{2}(w-\hat{w}) + d(t)]^{T}[K_{1}e(t-\tau) - K_{2}(w-\hat{w}) + d(t)] = e^{T}(t-\tau)K_{1}^{T}K_{1}e(t-\tau) + \widetilde{w}^{T}(t)K_{2}^{T}K_{2}\widetilde{w}(t) + d^{T}(t)d(t) - 2e^{T}(t-\tau)K_{1}^{T}K_{2}\widetilde{w}(t) + 2e^{T}(t-\tau)K_{1}^{T}d(t) - 2\widetilde{w}^{T}(t)K_{2}^{T}d(t)$$

$$(7)$$

Furthermore, according to [16] (p. 295),

$$\begin{aligned} 2|e^{\mathrm{T}}(t-\tau)K_1^{\mathrm{T}}K_2\widetilde{w}(t)| &\leq \gamma_1 e^{\mathrm{T}}(t-\tau)K_1^{\mathrm{T}}K_1 e(t-\tau) \\ &+ \frac{1}{\gamma_1}\widetilde{w}^{\mathrm{T}}(t)K_2^{\mathrm{T}}K_2\widetilde{w}(t), \quad \gamma_1 > 0 \end{aligned} \tag{8}$$

$$2e^{T}(t-\tau)K_{1}^{T}d(t) \leq \gamma_{2}e^{T}(t-\tau)K_{1}^{T}K_{1}e(t-\tau) + \frac{1}{\gamma_{2}}d^{T}(t)d(t),$$
(9)

$$|2\widetilde{w}^{\mathrm{T}}(t)K_{2}^{\mathrm{T}}d(t)| \leq \gamma_{3}\widetilde{w}^{\mathrm{T}}(t)K_{2}^{\mathrm{T}}K_{2}\widetilde{w}(t) + \frac{1}{\gamma_{3}}d^{\mathrm{T}}(t)d(t), \quad \gamma_{3} > 0$$

$$\tag{10}$$

Combining (8)-(10), into (7) leads to

$$e^{\mathsf{T}}(t)e(t) \leq \alpha_1 e^{\mathsf{T}}(t-\tau)K_1^{\mathsf{T}}K_1 e(t-\tau) + \alpha_2 \widetilde{w}^{\mathsf{T}}(t)K_2^{\mathsf{T}}K_2 \widetilde{w}(t) + \alpha_3 d^{\mathsf{T}}(t)d(t)$$
(11)

where $\alpha_1 = \gamma_1 + \gamma_2 + 1$, $\alpha_2 = 1 + \gamma_3 + 1/\gamma_1$, and $\alpha_3 = 1 + 1/\gamma_2 + 1/\gamma_3$. This completes the proof.

Remark 3. If u_f is a constant, then d(t) can be zero by selecting a $K_1 = I$, an identity matrix. As a result, inequality (11) can be reduced to

$$e^{\mathsf{T}}(t)e(t) \leq \alpha_4 e^{\mathsf{T}}(t-\tau)K_1^{\mathsf{T}}K_1 e(t-\tau) + \alpha_5 \widetilde{w}^{\mathsf{T}}(t)K_2^{\mathsf{T}}K_2 \widetilde{w}(t). \tag{12}$$

THEOREM 1 (Boundedness). Consider estimation error equation (4) satisfying Lemma 1. If $\epsilon_1 = 2\lambda_{\min}(\Lambda) - 2\eta - \gamma_4 - \alpha_2[1 + \rho_1 + (1/\gamma_4)||B_f^T B_f||_{\infty}]||K_2^T K_2|| > 0$ and $\epsilon_2 K_1^T K_1 \leq I$, then the estimation errors \widetilde{w} and e(t) are bounded.

Proof. Consider the following Lyapunov function candidate:

$$V = \widetilde{w}^{\mathrm{T}} J \widetilde{w} + \int_{-\infty}^{t} e^{\mathrm{T}}(s) e(s) ds$$
 (13)

Substituting (4) into the derivative of the Lyapunov candidate leads to

$$\begin{split} \dot{V} &= 2\widetilde{w}^{\mathrm{T}}J\dot{\widetilde{w}} + e^{\mathrm{T}}(t)e(t) - e^{\mathrm{T}}(t-\tau)e(t-\tau) = 2\widetilde{w}^{\mathrm{T}}[-w^{\mathrm{X}}Jw + \hat{w}^{\mathrm{X}}J\hat{w} \\ &+ B_{f}e(t) - \Lambda\widetilde{w}] + e^{\mathrm{T}}(t)e(t) - e^{\mathrm{T}}(t-\tau)e(t-\tau) \leq 2\eta \|\widetilde{w}\|^{2} \\ &+ 2\widetilde{w}^{\mathrm{T}}B_{f}e(t) - 2\lambda_{\min}(\Lambda)\|\widetilde{w}\|^{2} + e^{\mathrm{T}}(t)e(t) - e^{\mathrm{T}}(t-\tau)e(t-\tau) \end{split}$$

$$(14)$$

Considering the following inequality

$$2\widetilde{w}^{\mathsf{T}}B_{f}e(t) \leq \gamma_{4}\widetilde{w}^{\mathsf{T}}\widetilde{w} + \frac{1}{\gamma_{4}}e^{\mathsf{T}}(t)B_{f}^{\mathsf{T}}B_{f}e(t), \quad \gamma_{4} > 0$$
 (15)

inequality (14) can be further reorganized as follows

where $\epsilon_1 = 2\lambda_{\min}(\Lambda) - 2\eta - \gamma_4 - \alpha_2[1 + \rho_1 + (1/\gamma_4)||B_f^T B_f||_{\infty}]||K_2^T K_2||$ > 0, $\epsilon_2 = \alpha_1[1 + \rho_1 + (1/\gamma_4)||B_f^T B_f||_{\infty}], \quad \epsilon_3 = \alpha_3[1 + \rho_1]$

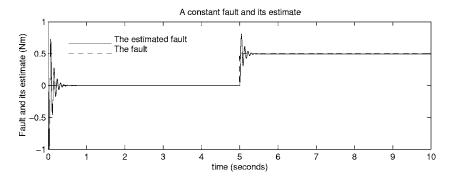


Fig. 1 Fault estimation by the ILO: A constant fault

+ $(1/\gamma_4)\|B_f^{\mathrm{T}}B_f\|_{\infty}$], and $\rho_1 > 0$. Selecting K_1 such that $\epsilon_2 K_1^{\mathrm{T}}K_1 \leq I$, then

$$\dot{V} \leq -\epsilon_1 \|\widetilde{w}\|^2 - \rho_1 e^{\mathsf{T}}(t) e(t) + \epsilon_3 k_d^2 \tag{17}$$

According to [17],

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T \left[\epsilon_1 \|\widetilde{w}(t)\|^2 + \rho_1 \|e(t)\|^2 \right] dt \le \epsilon_3 k_d^2$$
 (18)

this constitutes the boundedness of $\widetilde{w}(t)$ and e(t).

Remark 4. If a thruster is stuck, then $u_f(t)$ is a constant. The estimation errors $\widetilde{w}(t)$ and e(t) will approach to zero as $t \to \infty$. This is due to the fact that d(t) = 0, as stated in Remark 3.

Remark 5. The ILO can be designed to start running as soon as the satellite systems begin to work. Once a thruster fault occurs, the ILO can detect and identify it.

Remark 6. If $\epsilon_3 k_d^2$ is a small perturbation, then the system will only be perturbed slightly away from the equilibrium. In addition, the estimation errors as shown in (18) will be also very small.

4 Illustrative Example

The following example illustrates the effectiveness of the ILO-based fault detection and estimation. Parameter J and B are taken as

$$J = \begin{bmatrix} 36 & 1.5 & 0 \\ 1.5 & 17 & 0 \\ 0 & 0 & 26 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 (19)

The thruster faults are considered herein, therefore, the fault unit is Newton meters (Nm). A constant (stuck or freezing) thruster fault is first considered to test the ILO-based fault identification. It is assumed that the ILO starts running at t=0 and that thruster 1 gets stuck at time instant 5 s with a value equal to 0.5. The ILO input v(t) exhibits a zero fault estimate of thruster 1 before 5 s. After the occurrence of the stuck thruster 1, the ILO can accurately estimate the stuck value and the fault estimation error approaches to zero as shown in Fig. 1. From practical significance, the ILO input v(t) can be designated as a residual because the ILO

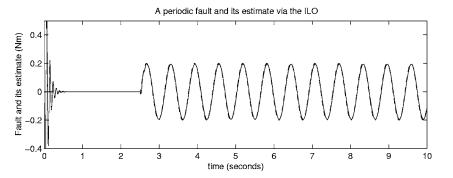


Fig. 2 Fault estimation by the ILO: A periodic fault.

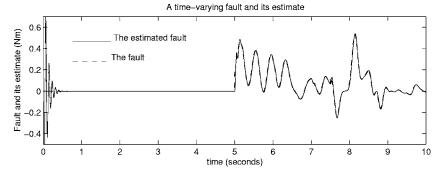


Fig. 3 Fault estimation by the ILO: An arbitrary time-varying fault.

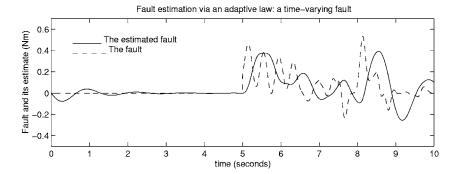


Fig. 4 Fault estimation by an adaptive observer: A time-varying fault

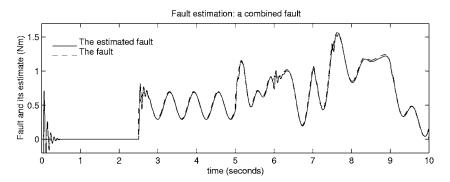


Fig. 5 Fault estimation: A combined time-varying fault

can start running from time t=0, and as such can be used for monitoring and diagnostic purposes. Another interesting observation is that the updating rate can be low once the stuck value has been accurately estimated; in other words, the fault estimation law does not need to be adjusted frequently.

A sinusoidal fault $\sin(10t)$ of thruster 1, which occurs at time instant 2.5 s, is then used to test the ILO-based fault estimation. Figure 2 demonstrates that the sinusoidal thruster fault can be accurately reconstructed via the proposed ILO.

An aperiodic time-varying fault that occurs at time instant 5 s and that varies aperiodically and randomly is further employed to show the effectiveness of the ILO-based fault identification. Accurate fault estimate can also be accomplished as exhibited in Fig. 3. Note that the ILO can learn the time-varying thruster fault on-line. This is why the ILO can identify any time-varying fault. If the time-varying fault varies fast, to ensure accuracy of estimation, the updating interval τ needs to be selected small; that is, the fault estimate v(t) should be adjusted more frequently. The estimated time-varying fault as shown in Fig. 3 is accurate enough to be used in fault-tolerant control to ensure safety and reliability in the safety-critical satellite systems.

As comparison, an adaptive-observer-based fault estimation method proposed in [14], using a differential equation, is tested to identify the same time-varying fault. Figure 4 indicates the impossibility of estimating an arbitrary time-varying fault via an adaptive observer using an integrator; the fault estimate cannot converge to the real fault at all except the zero-fault case from time instant 0–5 s. This reveals that the adaptive observer can accurately identify constant faults, but cannot identify time-varying faults.

In the above experiments, the estimation ability of the ILO has been explored using a constant, periodic, and an aperiodic fault, respectively. A more complex fault by combining the constant fault, the periodic fault, and the time-varying fault together is used to further investigate the ILO's fault-estimation ability. The fault scenario is described as: The system is fault-free from time 0 to time instant 2.5 s; at time instant 2.5 s, a constant and a periodic

fault occur simultaneously; at time instant 5 s, an aperiodic timevarying fault is added to them so that the system is subject to a combined fault. Figure 5 shows that the combined fault can also be accurately estimated from time instant 2.5 s to time instant 10 s. This has clearly shown the ILO's extremely strong ability of fault estimation. In summary, the strong estimation ability of the ILO makes it an applicable approach in practice.

5 Conclusions

This paper has proposed an ILO-based fault diagnosis approach in satellite systems subject to time-varying thruster faults. An ILO is designed to circumvent the deficiency of dealing with constant faults by the adaptive observers. The use of a learning mechanism enables the ILO to estimate time-varying faults. This is the main advantage of the ILO-based fault identification. In addition, the ILO requires less on-line computing power than the adaptive observer does because fault estimation is realized by an algebraic equation. By virtue of the ILO, time-varying thruster faults, including a constant fault, a periodic fault and an aperiodic fault, can all be accurately estimated. An illustrative example clearly demonstrated the proposed approach.

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