

# Sensor fault detection and recovery in satellite attitude control

Seiied Saeed Nasrolahi<sup>b</sup>, Farzaneh Abdollahi<sup>a,b,\*</sup>

<sup>a</sup> The Center of Excellence on Control and Robotics, Department of Electrical Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran 15914, Iran

<sup>b</sup> Department of Electrical and Computer Engineering, Concordia University, Montreal, Quebec, H3G 1M8, Canada

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## ABSTRACT

This paper proposes an integrated sensor fault detection and recovery for the satellite attitude control system. By introducing a nonlinear observer, the healthy sensor measurements are provided. Considering attitude dynamics and kinematic, a novel observer is developed to detect the fault in angular rate as well as attitude sensors individually or simultaneously. There is no limit on type and configuration of attitude sensors. By designing a state feedback based control signal and Lyapunov stability criterion, the uniformly ultimately boundedness of tracking errors in the presence of sensor faults is guaranteed. Finally, simulation results are presented to illustrate the performance of the integrated scheme.

## 1. Introduction

The healthy level of attitude control system (ACS) plays a key role on satellite mission success. Attitude sensor faults are critical in the faults logged in space fields which potentially can lead to satellite mission failure [1]. Therefore, sensor fault detection is essential to increase the ACS reliability. Employing sensor measurement to provide appropriate control signal, it is essential to develop a sensor fault recovery control algorithm to compensate the fault effects. In this article, a sensor fault detection and recovery scheme is developed on ACS.

Generally, fault manifests an abnormal situation between actual and expected behavior. To detect sensor fault, hardware and analytical redundancy approaches are utilized [2]. In hardware redundancy, additional attitude sensors are required. By developing an on-board software, their measurements are compared with each other to detect the fault. Due to imposing extra cost, mass and dimension, these approaches are inappropriate to implement on space missions, especially for small satellites [3].

Analytical redundancy strategies without any extra hardware requirement, utilize analytical relations and actual outputs to generate residual. These methods are mostly model-based approaches [4]. Model-based sensor fault detection methods could be categorized based on the sensor type; 1) Methods that focus on angular rate sensors fault such as those on eigen structure assignment technique [5], parity equation [6], and independent component analysis [7]. 2) Methods that concentrate on attitude sensors fault such as magnetometer, sun sensor,

star tracker and infrared earth sensor [8,9].

Some researches have focused on angular rate as well as attitude sensor fault by presenting two dedicated approaches independently. For instance, three Kalman filters in Ref. [10], six parallel extended Kalman filters (EKF) in Ref. [11] and two dedicated nonlinear observers in Ref. [12] have been utilized. Obviously, applying multiple observers leads to more complexity and computational load.

Alongside model-based methods, some model-free approaches have been presented for ACS sensor fault detection. Most of them are data-driven, such as possibility theory and fuzzy sets [13], fault diagnostic tree [14] and neural networks [15]. As a major deficiency, these methods have suitable performance in the range of available data. To overcome it [16], utilizes multiple rotation matrices to generate several Euler sets independently from available sensors outputs and compares them. However, it restricts to only sun sensor and magnetometer as attitude sensors.

We intend to develop a nonlinear observer to detect fault in both attitude and rate sensors. After detecting sensor faults, a recovery strategy is provided to prevent any unstable behavior in ACS performance.

In hardware redundancy approach, as a fault recovery technique, attitude determination algorithm switches to healthy back-up sensor. However, it is a costly solution and the switching operation could have inappropriate effects on performance continuity. Analytical approaches could be proposed to estimate the sensor fault and recover fault side effects. In Ref. [17], an angular rate sensor and actuator faults method have been presented by utilizing four observers. Furthermore, Kalman

\* Corresponding author. The Center of Excellence on Control and Robotics, Department of Electrical Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran 15914, Iran.

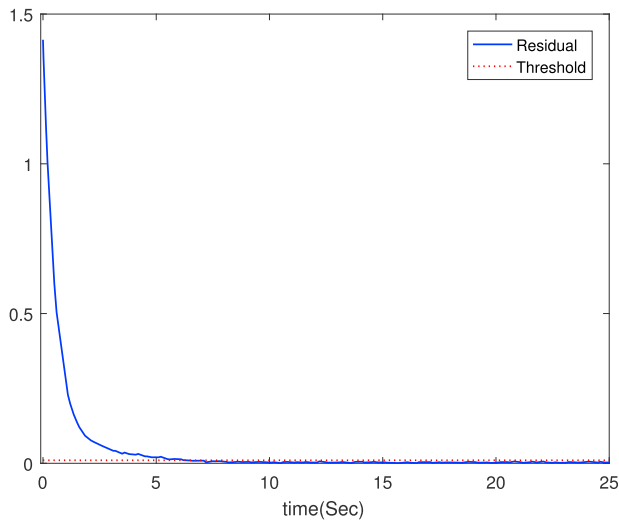
E-mail address: [f.abdollahi@eee.org](mailto:f.abdollahi@eee.org) (F. Abdollahi).

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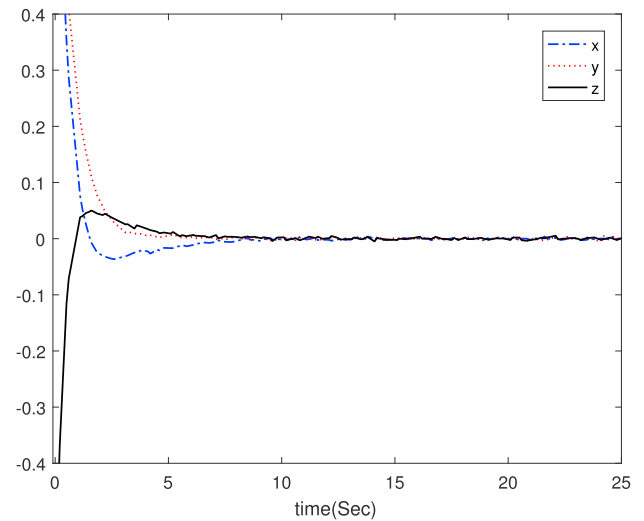
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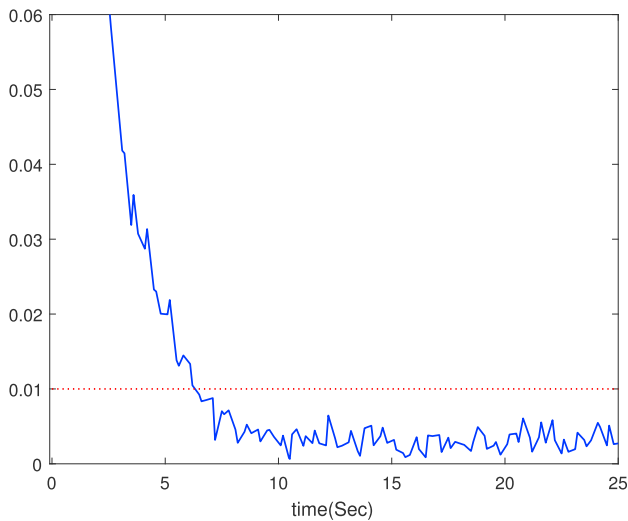
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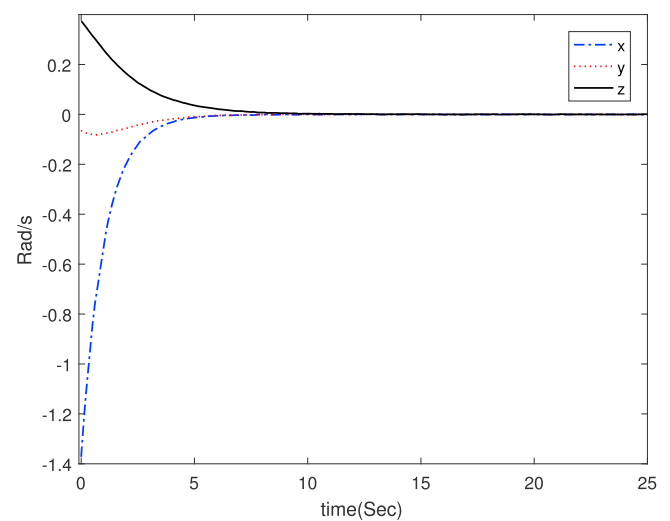
(a) Residual value



(a) Attitude in MRP representation



(b) Threshold selection



(b) Angular velocity

**Fig. 1.** Residual value in healthy mode with considering enviromental disturbances and sensors measurement accuracy.

filter-based approaches have been presented for sensor fault estimation [18]. In Refs. [19,20] unscented Kalman filters (UKF) are utilized to tolerate sensor fault. Federated UKFs have been proposed for sensor fault detection and isolation in Refs. [21,22].

In addition, descriptor system approach has been utilized to diagnose the sensor fault for nonlinear systems including ACS [23]. However, limiting constraints on dynamics relations should be met to use the descriptor system. Generally, the observer-based fault detection methods for nonlinear systems are not matured [12]. In order to avoid losing information in linearization approaches, nonlinear observers have been investigated [24]. Nonlinear adaptive observers [25,26], unknown input observers [27,28], sliding mode observers [29–31], nonlinear extended state observer [32] and high-gain nonlinear observers [33] are some solutions to name. The introduced complicated observers in the literature are not general to utilize in every nonlinear dynamics such as ACS.

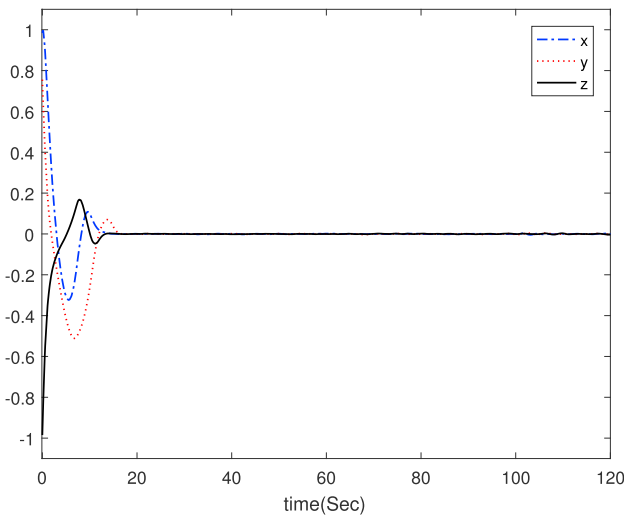
All of the mentioned estimation algorithms suffer from high computational efforts. Obviously, the computations load is a critical issue for satellite on-board computer subsystem to run the attitude control algorithms. In addition, some of them suffer from lack stability analysis in a

**Fig. 2.** Observation error in healthy mode.

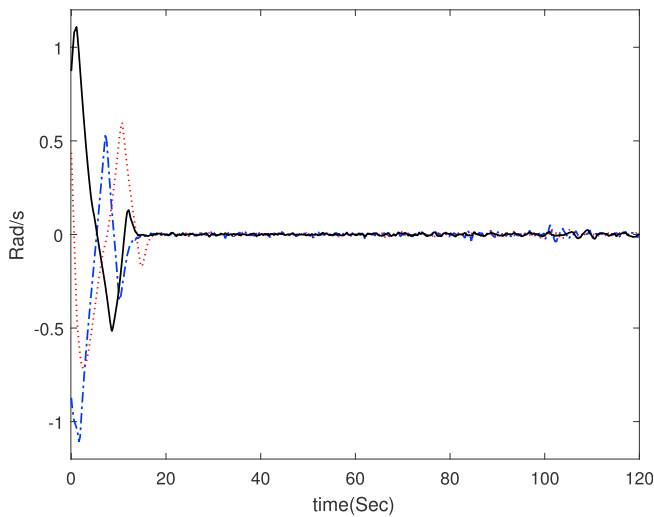
closed loop system where control signal is generated from estimated fault-eliminated sensor outputs.

In this paper, a control approach is proposed for sensor faulty ACS system which guarantees uniformly ultimately bounded (UUB) of tracking errors in a proportional-derivative (PD) control outline. PD controller is a popular approach to implement on ACS and has acceptable performance and less complexity in comparison with nonlinear attitude control methods in practice [34–36]. In PD control method as a state feedback scheme, states are divided to two categories; firstly, attitude parameters which are measured by orientation sensors and represented by Modified Rodriguez Parameters (MRP) in this paper and secondly angular velocity vector measured by rate sensors. The contributions of this work are summarized as follows:

- 1) By proposing a nonlinear observer, any rate and attitude sensor fault is detectable.
- 2) Utilizing modified Rodriguez parameters, the attitude sensor type such as sun sensor, magnetometer, star tracker and with any configuration, has no effect on observer performance.



(a) Attitude in MRP representation



(b) Angular velocity

Fig. 3. Tracking error in healthy mode.

- 3) The presented scheme focuses on fault recovery after fault detection with no isolation process. This feature facilitates less computational load and complexity.
- 4) The proposed controller guarantees the asymptotically stability and UUB for closed loop system in healthy and faulty modes, respectively.

This paper is organized as follows; In Section 2, the preliminaries are presented. We state the problem and propose sensor fault detection and recovery schemes in Section 3. The numerical simulation results are given in Section 4, followed by conclusions in Section 5.

Throughout this paper, the following algebraic matrix notations are considered,  $I_n$  is an  $n \times n$  identity matrix,  $0_n$  is an  $n \times n$  zero matrix,  $\odot$  denotes the cross of two vectors in the MRP representation,  $\lambda_{\min}(\cdot)$  and  $\lambda_{\max}(\cdot)$  are the minimum and maximum eigenvalues, respectively.

## 2. Preliminaries

### 2.1. Attitude representation by MRP

To manifest the satellite orientation in orbit, two different frames

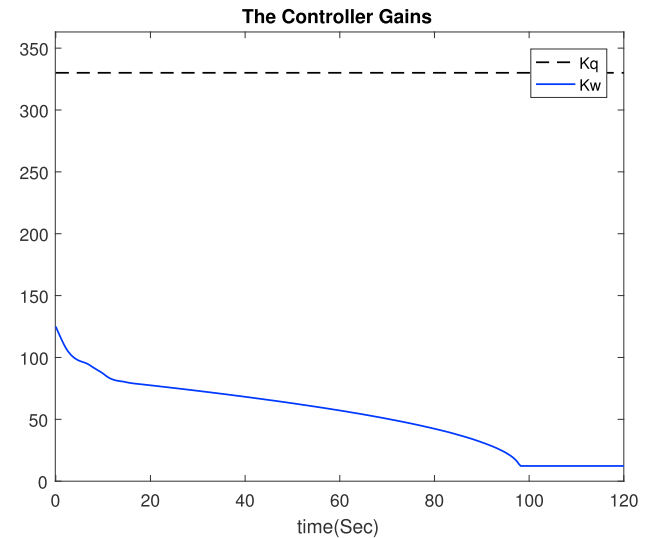


Fig. 4. Controller gains in healthy mode.

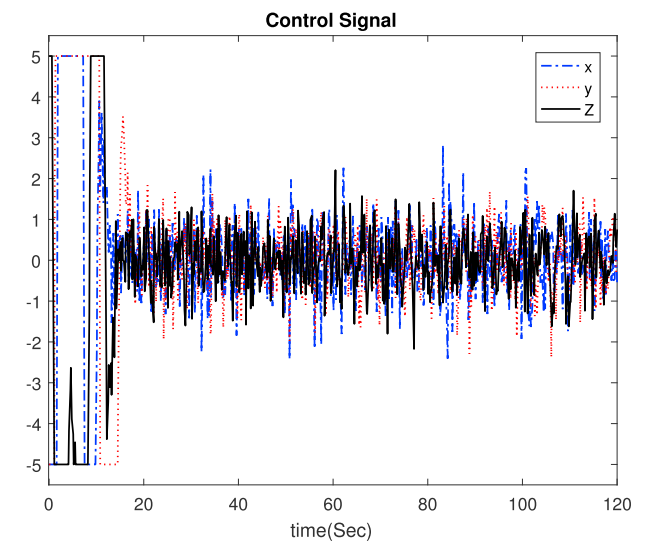


Fig. 5. Control signal in healthy mode.

should be utilized. A body-fixed frame with its origin at the center of mass and axes pointing along the principal axes of the satellite inertia, and a reference frame such as orbital frame which has its origin at the center of the earth and axes pointing to the center of the earth, along the orbital velocity and the direction opposite to the orbit plane. The attitude expresses the orientation of satellite body-fixed frame toward reference frame which can be demonstrated by several representations such as Euler angles, direction cosine matrix, quaternion vector and Modified Rodriguez Parameters (MRP). Euler angles has trigonometric function and singularity problem. In direction cosine matrix too many dependent state variables are involved. Moreover, using 4-element quaternion vector will result in ambiguity and unwinding phenomena [37]. Therefore, we use MRP vector to represent the satellite attitude deriving from the Eulers principal rotation theorem as follows

$$q = \tan(\theta/4)n$$

where  $q$ ,  $n$  and  $\theta$  represent 3-element MRP vector, the unit vector as Euler's rotation axes and rotation angle, respectively [38]. Note that MRPs have a singularity when  $\theta$  belongs to  $[0, 2\pi]$ . The singularity can be avoided by using the MRP shadow set ( $q^s$ ) which has a relation with MRP

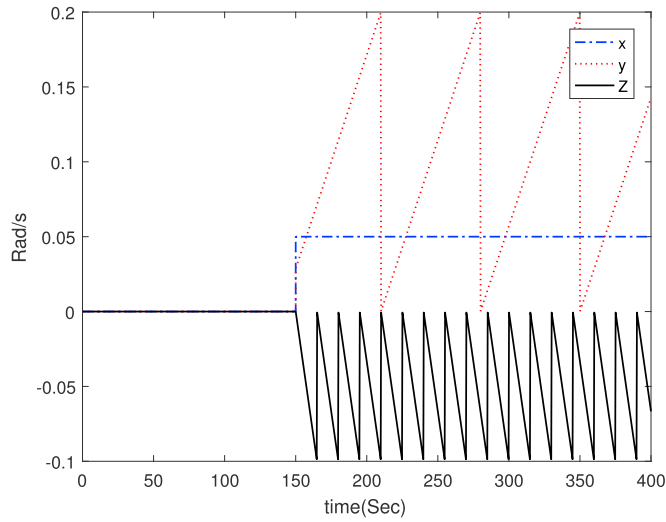
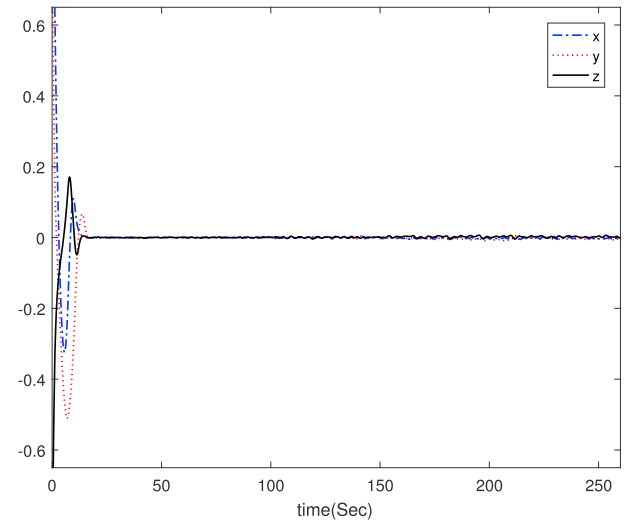


Fig. 6. The gyro fault in Scenario 1.



(a) Attitude in MRP representation

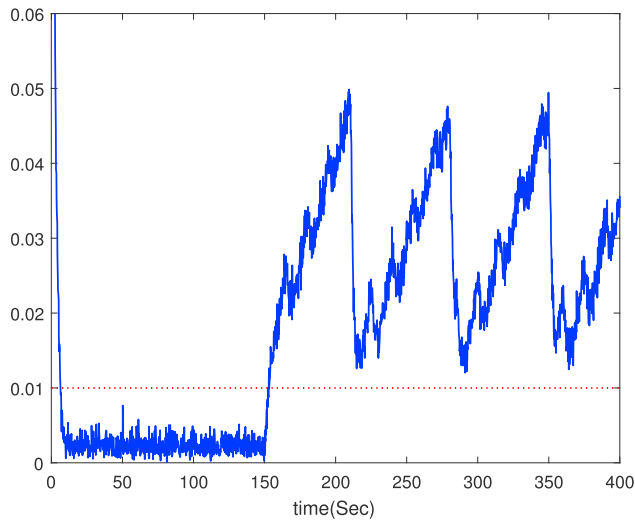
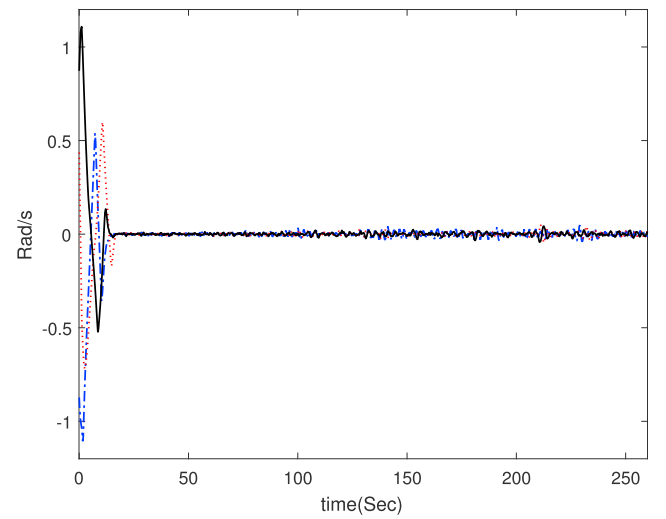


Fig. 7. The residual for Scenario 1.



(b) Angular velocity

Fig. 8. Tracking error in Scenario 1.

as follows [39]:

$$\dot{q}^s = -q/q^\top q$$

In particular, the MRPs are used when  $0 \leq \theta < \pi$ , and when  $\pi \leq \theta < 2\pi$ , MRP shadow set are used and the approaches can be extended for MRP shadow set.

## 2.2. Satellite kinematic and dynamics

The satellite relative motion around its center of mass is described by kinematic. By using MRP set, the kinematic equation can be expressed as [40]:

$$\dot{q} = G(q)\omega \quad (1)$$

where  $\omega$  is the angular velocity of the body frame with respect to the reference frame expressed in the body frame and

$$G(q) = \frac{1}{4}((1 - q^\top q)I_3 + 2S(q) + 2qq^\top)$$

which has the properties of;

$$q^\top G(q) = \left(\frac{1 + q^\top q}{4}\right)q^\top \quad (2)$$

and

$$\|G(q)\| \leq \frac{1}{4}(3 + 2\|q\| + 3\|q\|^2) \quad (3)$$

and the matrix  $S(\cdot)$  is a skew-symmetric matrix

$$S(q) = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}$$

Attitude dynamics represents the relation between satellite attitude motion and applied torques on it. Dynamics equation can be expressed in body frame using Euler's equation as follows

$$J\dot{\omega} = S(\omega)J\omega + u \quad (4)$$

where  $J$  and  $u$  represent satellite inertia symmetric matrix and applied torque, respectively [41].

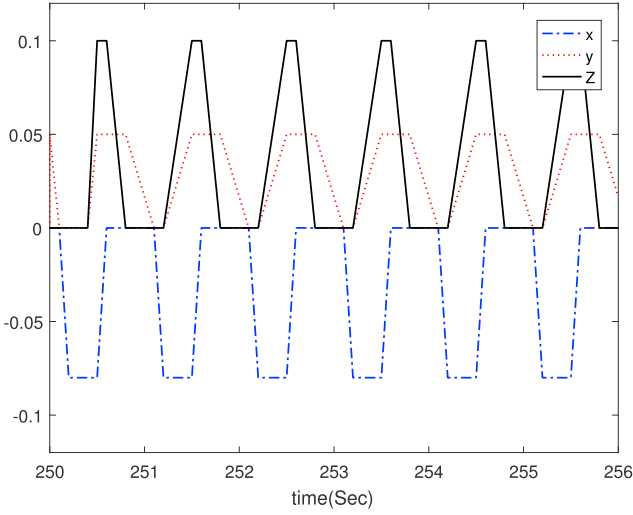


Fig. 9. The considered attitude sensor fault in Scenario 2 in a sample interval.

### 2.3. Tracking error dynamics

Consider  $q_d$  and  $\omega_d$  as desired MRP and angular velocity vectors. Also,  $E = [e_q^T \ e_\omega^T]^T$  represents the tracking error consisting orientation error ( $e_q$ ) and rate error ( $e_\omega$ ) which can be given as [42].

$$\begin{aligned} e_q &= q \odot q_d \\ e_\omega &= \omega - \omega_d \end{aligned} \quad (5)$$

where  $\odot$  denotes the multiplication of MRP and it is defined as

$$q \odot q_d = \frac{q_d(q^T q - 1) - q(q_d^T q_d - 1) - 2(q \times q_d)}{1 + \|q_d\|^2 \|q\|^2 + 2q \cdot q_d}$$

**Assumption 1;** The satellite mission is observing a certain point on the earth or space with slow rate. It means that  $\omega_d, \dot{q}_d \simeq 0$ . This assumption is required in missions with subject of capturing an image or monitoring a predefined surface.

Therefore, the nonlinear tracking error dynamics with regards to satellite attitude kinematic (1) and dynamics (4) can be obtained as

$$\begin{cases} \dot{e}_q = G(e_q)e_\omega \\ J\dot{e}_\omega = S(e_\omega)J e_\omega + u \end{cases} \quad (6)$$

### 2.4. Schur complements

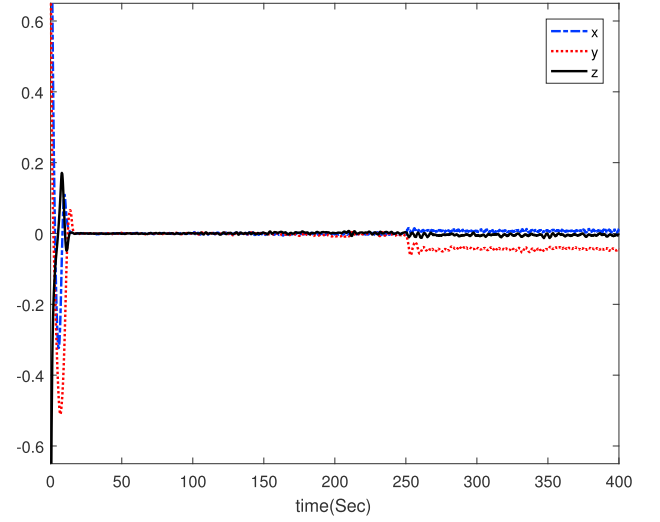
For any  $2n \times 2n$  symmetric matrix of the form  $M = \begin{pmatrix} A & B \\ B^T & C \end{pmatrix}$ , the following properties hold [43];

$$\begin{aligned} &\text{If } A \text{ is invertible Then} \\ &M > 0 \text{ if and only if } A > 0 \text{ and } C - B^T A^{-1} B > 0 \\ &\text{If } C \text{ is invertible Then} \\ &M > 0 \text{ if and only if } C > 0 \text{ and } A - B C^{-1} B^T > 0 \end{aligned} \quad (7)$$

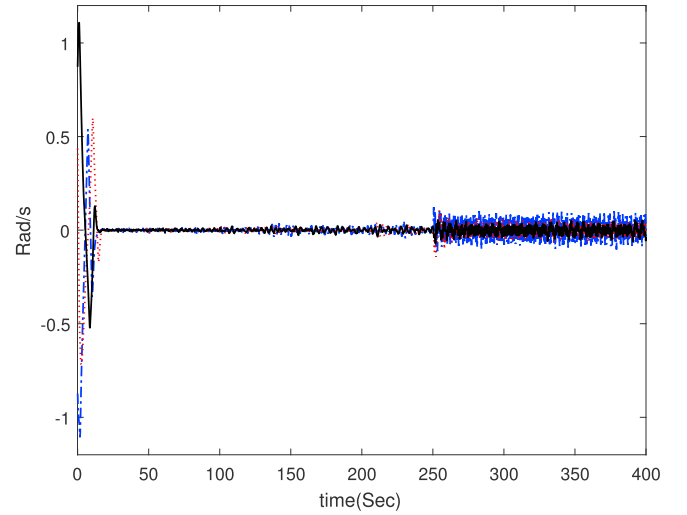
The problem formulation and proposed approach with stability analysis are presented in the next section.

## 3. Problem statement and the proposed fault detection and recovery method

By considering  $x = [q^T \ \omega^T]^T$  as state variable, the state-space model of ACS according to attitude kinematic (1) and dynamics (4) can be expressed as



(a) Attitude in MRP representation



(b) Angular velocity

Fig. 10. Tracking error in Scenario 2.

$$\begin{cases} \dot{x}(t) = \phi(x)x(t) + Bu(t) \\ y(t) = Cx(t) + f_s(t) \end{cases}$$

where

$$\phi(x) = \begin{bmatrix} 0_3 & G(q) \\ 0_3 & J^{-1}S(\omega)J \end{bmatrix}, \quad B = \begin{bmatrix} 0_3 \\ J^{-1} \end{bmatrix}, \quad C = I_6$$

and  $y(t)$  is measured states by attitude and rate sensors and  $f_s = [f_q^T \ f_\omega^T]^T$  denotes unknown bounded sensor fault ( $\|f_s(t)\| < m$ ).

### 3.1. Fault detection observer

To detect any rate and attitude sensor fault without any restrictions in attitude sensor configuration, a nonlinear observer is presented to estimate modified Rodriguez parameters and angular velocity vector as follows

$$\begin{cases} \dot{\hat{x}}(t) = \psi(\hat{x}, y) + \theta(y, \alpha, \beta)(y(t) - \hat{y}(t)) + Bu(t) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \quad (8)$$

where  $\hat{x}$  and  $\hat{y}$  are estimated states and output, respectively and

$$\psi(\hat{x}, y) = \begin{bmatrix} G(q)\hat{\omega} \\ J^{-1}S(\hat{\omega})J\omega \end{bmatrix}$$

and  $\theta(y, \alpha, \beta)$  is the observer gain matrix including  $\alpha, \beta > 0$  defined as follows

$$\theta(y, \alpha, \beta) = \begin{bmatrix} \alpha I_3 & 0_3 \\ J^{-1}G^T(q) & \beta J^{-1} \end{bmatrix}$$

Therefore, the extended form of observer dynamics in (8) can be given as

$$\begin{cases} \dot{\hat{q}} = G(q)\hat{\omega} + \alpha \xi_q \\ J\dot{\hat{\omega}} = S(\hat{\omega})J\omega + \beta \xi_\omega + G^T(q)\xi_q + u \end{cases} \quad (9)$$

and  $\hat{y} = [\hat{q}^T \quad \hat{\omega}^T]^T$ . It is expected that in fault-free situation, estimated output  $\hat{y}(t)$  converges asymptotically to sensor output  $y(t)$ . The output estimation error  $\xi = \begin{bmatrix} \xi_q^T & \xi_\omega^T \end{bmatrix}^T$  can be described as

$$\xi(t) = y(t) - \hat{y}(t)$$

which its norm ( $\|\xi(t)\|$ ) is chosen as the residual evaluation function. Obviously, when the residual exceeds a predetermined threshold, the fault occurrence would be declared. The following theorem demonstrates the asymptotic stability of the observer for healthy situation.

**Theorem 1.** By choosing the observer gains  $\alpha > 0$  and  $\beta > 0$  in nonlinear observer (9), the output estimation error  $\xi(t)$  converges to zero asymptotically in healthy mode  $f_s(t) = 0$ .

*Proof*

Let us choose the Lyapunov candidate as

$$V(\xi) = \frac{1}{2} \xi^T A \xi \quad (10)$$

where

$$A = \begin{bmatrix} I_3 & 0_3 \\ 0_3 & J \end{bmatrix}$$

The time derivative of (10) is obtained as

$$\begin{aligned} \dot{V}(\xi) &= \xi_q^T \dot{\xi}_q + \xi_\omega^T J \dot{\xi}_\omega \\ &= \xi_q^T \left( \dot{q} - \dot{\hat{q}} \right) + \xi_\omega^T J \left( \dot{\omega} - \dot{\hat{\omega}} \right) \end{aligned} \quad (11)$$

Considering  $x^T S(x) = 0$  and designed observer (9), attitude kinematic (1) and dynamics (4), the derivative of Lyapunov candidate (11) can be expressed as

$$\begin{aligned} \dot{V}(\xi) &= \xi_q^T (G(q)\omega - G(q)\hat{\omega} - \alpha \xi_q) \\ &\quad + \xi_\omega^T (S(\omega)J\omega + u - S(\hat{\omega})J\omega - \beta \xi_\omega - G^T(q)\xi_q - u) \\ &= -\alpha \|\xi_q\|^2 - \|\xi_\omega\|^2 < 0 \end{aligned}$$

Hence if  $\alpha, \beta > 0$ , the sensor output estimation error converges zero, if there is no sensor fault. This completes the proof.

### 3.2. Fault recovery control

A state feedback control scheme, according to tracking error dynamics in (6) can be given as

$$u = -KE \quad (12)$$

where  $K = [k_q I_3 \quad k_\omega I_3]$  represents the controller gains.

The asymptotic stability proof of closed loop system (6) and (12) in sensor fault free mode has been performed in Refs. [35,44] by utilizing

the Lyapunov's direct method. However, in the presence of sensor fault, the control signal transforms from the standard scheme (12) to unfavorable form as follows

$$u = -k_q e_q - k_\omega e_\omega + d \quad (13)$$

where by considering the bound of fault ( $\|f_s(t)\| < m$ ), the bound of  $d$  can be obtained as

$$\|d\| \leq m(k_q + k_\omega) \quad (14)$$

Obviously, it concludes that  $d$  affects on closed loop system as a disturbance signal. However, its magnitude depends on controller gains and the boundedness of fault directly. In other words, choosing the controller gains is a critical issue in the presence of sensor fault. Therefore, there should be a re-configurable algorithm to tune controller gains and preserve the stability of closed loop system in the presence of sensor fault. The following theorem guarantees the UUB of tracking error in the presence of faulty sensor by deriving the proper controller gains.

**Theorem 2.** Considering control signal given in (13), there are  $k_q(t) > 0$  and  $k_\omega(t) > 0$  to make the tracking error with dynamics given in (6) uniformly ultimately boundedness (UUB) in the presence of rate and attitude sensor faults.

*Proof*

The proof is accomplished by introducing the following Lyapunov candidate:

$$V(E) = \frac{1}{2} E^T Q E + 2(k_q + \varepsilon k_\omega) \ln(1 + e_q^T e_q) \quad (15)$$

where

$$Q = \begin{bmatrix} J & \varepsilon J \\ \varepsilon J & 4I_3 \end{bmatrix}$$

Due to the fact that  $Q$  should be positive definite, based on Schur complement (7),  $\varepsilon$  is chosen as

$$\varepsilon < \frac{2}{\sqrt{\lambda_{\max}(J)}}$$

where  $\lambda_{\max}(J)$  is the largest eigenvalue of  $J$ . Taking the derivative of Lyapunov function in (15) leads to

$$\begin{aligned} \dot{V}(E) &= e_\omega^T J \dot{e}_\omega + 4e_q^T \dot{e}_q + \varepsilon e_\omega^T J \dot{e}_q + \varepsilon e_q^T J \dot{e}_\omega \\ &\quad + 4(k_q + \varepsilon k_\omega) \frac{e_q^T \dot{e}_q}{1 + e_q^T e_q} + 2(\dot{k}_q + \varepsilon \dot{k}_\omega) \ln(1 + e_q^T e_q) \end{aligned}$$

where along the trajectories of (6) and (13) and property of (2), one can obtain:

$$\begin{aligned} \dot{V}(E) &= -k_\omega e_\omega^T e_\omega + 4e_q^T G(e_q) e_\omega + \varepsilon e_\omega^T J G(e_q) e_\omega + \varepsilon e_q^T S(e_\omega) J e_\omega \\ &\quad - \varepsilon k_q e_q^T e_q + e_\omega^T d + \varepsilon e_q^T d + 2(\dot{k}_q + \varepsilon \dot{k}_\omega) \ln(1 + e_q^T e_q) \end{aligned} \quad (16)$$

Considering

$$\forall x > 0; \ln(1 + x) < x$$

and (2) and by defining  $D = [d, \dot{d}]^T$ , (16) can be expressed by

$$\dot{V}(E) \leq -E^T T_1 E + E^T T_2 D \quad (17)$$

where  $T_1$  and  $T_2$  are  $2 \times 2$  block matrices as follows

$$T_1(1, 1) = k_\omega I_3 - \varepsilon J G(e_q)$$

$$T_1(1, 2) = T_1^T(2, 1) = 0.5(\varepsilon J S(e_\omega) - (1 + \|e_q\|^2) I_3)$$

$$T_1(2, 2) = (\varepsilon k_q - 2\dot{k}_q - 2\varepsilon \dot{k}_\omega)I_3$$

and

$$T_2 = \begin{bmatrix} I_3 & 0_3 \\ 0_3 & \varepsilon I_3 \end{bmatrix}$$

Using the fact that for any vector  $c_1$  and  $c_2$ , we have

$$(c_1 - c_2)^\top (c_1 - c_2) \geq 0 \Rightarrow c_1^\top c_2 \leq \frac{1}{2}c_1^\top c_1 + \frac{1}{2}c_2^\top c_2$$

and by considering symmetric property of  $T_2$  and  $c_1^\top = E^\top T_2$  and  $c_2 = D$ , the following inequality can be derived from (17)

$$\dot{V}(E) \leq -E^\top T_1 E + \frac{1}{2}E^\top T_2^2 E + \frac{1}{2}D^\top D$$

$$\leq -E^\top \left( T_1 - \frac{1}{2}T_2^2 \right) E + \frac{1}{2}D^\top D$$

Therefore, by defining  $T = T_1 - \frac{1}{2}T_2^2$  the following inequality can be derived

$$\dot{V}(E) \leq -\lambda_{\min}(T)\|E\|^2 + \frac{1}{2}\|D\|^2 \quad (18)$$

The controller gains  $k_q$  and  $k_\omega$  should be tuned to make  $T > I > 0$  which leads to  $\lambda_{\min}(T) > 1 > 0$ . Defining  $T_I = T - I$  leads to  $T_I > 0$  which its components can be obtained as

$$T_I(1, 1) = k_\omega I_3 - \varepsilon J G(e_q) - 1.5I_3$$

$$T_I(1, 2) = T_I^\top(2, 1) = 0.5(\varepsilon J S(e_\omega) - (1 + \|e_q\|^2)I_3)$$

$$T_I(2, 2) = (\varepsilon k_q - 2\dot{k}_q - 2\varepsilon \dot{k}_\omega - 0.5\varepsilon^2 - 1)I_3$$

Employing Schur complements (7) and considering (3), it is sufficient to have

$$k_\omega > h(e_q) \quad (19)$$

where

$$h(e_q) = \frac{\varepsilon \lambda_{\max}(J)}{4} (3 + 2\|e_q\| + 3\|e_q\|^2) + \frac{3}{2}$$

and with respect to  $\|S(e_\omega)\| = \|e_\omega\|$ , the controller gains update law should meet the condition

$$\varepsilon k_q - 2\dot{k}_q - 2\varepsilon \dot{k}_\omega - \frac{1}{2}\varepsilon^2 - 1 - \frac{(\varepsilon \lambda_{\max}(J)\|e_\omega\| + 1 + \|e_q\|^2)^2}{k_\omega - h(e_q)} > 0$$

Now, the adaptation laws can be obtained individually as follows

$$\begin{aligned} i) \quad & 2\dot{k}_q < \varepsilon k_q - \frac{1}{2}\varepsilon^2 - 1 \\ ii) \quad & 2\varepsilon \dot{k}_\omega \leq -\frac{(\varepsilon \lambda_{\max}(J)\|e_\omega\| + 1 + \|e_q\|^2)^2}{k_\omega - h(e_q)} \end{aligned} \quad (20)$$

Without loss of generality, we can assume that  $\dot{k}_q = 0$ . Therefore,  $k_q$  can be obtained constantly as

$$k_q > \frac{\varepsilon}{2} + \frac{1}{\varepsilon} \quad (21)$$

Obviously, it can be concluded from (19) and (20-ii) that  $\dot{k}_\omega \leq 0$ . Therefore, the adaptation law for  $k_\omega$  can be obtained as

$$\dot{k}_\omega = \begin{cases} -\frac{1}{2\varepsilon(k_\omega - h(e_q))}(\varepsilon \lambda_{\max}(J)\|e_\omega\| + 1 + \|e_q\|^2)^2, & k_\omega > h(e_q) \\ 0, & \text{Otherwise} \end{cases} \quad (22)$$

It should be noted that as the switching condition is satisfied, while  $\dot{k}_\omega \rightarrow -\infty$ , since  $k_\omega > 0$ , it still remains bounded. Therefore, based on (18)

$$\|E\| > \frac{\|D\|}{\sqrt{2\lambda_{\min}(T)}}$$

guarantees  $\dot{V}(E) < 0$ . With respect to  $\|D\| = \sqrt{2}\|d\|$ , the boundedness of  $\|d\|$  in (14) and  $\lambda_{\min}(T) > 1$  leading from conditions (21) and (22),  $\|E\| > m(k_q + k_\omega)$  guarantees  $\dot{V}(E) < 0$ . Obviously, the constant  $k_q$  and decreasing  $k_\omega$  lead to UUB of tracking errors. This completes the proof.

**Remark 1** The proposed controller guarantees the asymptotic stability of the closed loop system in healthy mode ( $m = 0$ ).

**Remark 2** Due to faulty sensor output, it should be considered that true values of states  $e_q$  and  $e_\omega$  are not accessible. Therefore, we can replace the  $\|e_q\|$  with  $\|e_q\| + m$  and  $\|e_\omega\|$  with  $\|e_\omega\| + m$  in (22) to obtain an accessible boundary in updating  $k_\omega$ .

**Remark 3** By considering sensor fault with known bound, the proposed controller guarantees the UUB of attitude tracking errors to point the satellite to a fixed-target. As be concluded in the proof of Theorem 2, the bound of stability is obtained as  $\|E\| > m(k_q + k_\omega)$ . Obviously, when there is no fault ( $m = 0$ ), the controller guarantees the asymptotically stability of the attitude tracking error. Consequently, the proposed controller guarantees the stability of satellite attitude in sensor healthy and faulty situations with a known bound.

#### 4. Simulation

In this section, the proposed sensor fault detection and recovery control schemes are applied to the ACS of a three-axis stabilized micro-satellite. It has a 3-axis gyro sensor to measure angular velocity. To obtain more realistic simulation results, maximum control torque of actuator, the environmental disturbance torques and sensor measurement accuracy are considered. It is considered that the satellite flies on a circular low earth orbit (LEO) with altitude of 300 km. In LEO orbit the main sources to make disturbance torques are aerodynamics, solar radiation, Earth's magnetic field and gravity. Generally, the maximum magnitude of the mentioned disturbances for a micro-satellite with 300 km altitude is in order of  $[1 \ 1 \ 1]^\top \times 10^{-4} \text{ N.m}$  [40]. It should be noted that the obtained results from the proposed theorems have no restriction on the type of orbit and can be generalized on every orbit such as geosynchronous orbit (GEO). Moreover, we assume that maximum applicable torque for actuator is 5N.m and the sensor measurement accuracy for gyro as 0.1°/s and for attitude sensor as 1° which is approximately equal to 0.005 in MRP representation. This transformation can be obtained by a simple static procedure. Other specified parameters are:

The matrix of moment inertia;

$$J = \begin{bmatrix} 15 & 2 & -1 \\ 2 & 27 & 3 \\ -1 & 3 & 30 \end{bmatrix}$$

Initial attitude;  $q_0 = [0.4 \ 0.153 \ -0.634]^\top$

Desired attitude;  $q_d = [0 \ 0 \ 0]^\top$

Initial angular velocity;  $\omega_0 = [-0.87 \ 0.44 \ 0.87]^\top \text{ rad/s}$

The satellite points on a fixed target to perform an observation mission, that is, desired angular velocity;  $\omega_d = [0 \ 0 \ 0]^\top$

We select the observer gains in (9) as  $\alpha = 1.1$  and  $\beta = 15$ . To determine the threshold value for residual, healthy mode as presented in Fig. 1



is referenced. Regardless of initial interval for observer convergence, we can choose threshold value  $\xi_v = 0.01$ . Obviously, due to consider environmental disturbances and measurement accuracy, the threshold is nonzero. The controller gain  $k_q$  is selected as 330 according to (21). In addition,  $k_\omega$  is initialized as 125 and updated according to (22). The state estimation and tracking errors are presented in Fig. 2 and Fig. 3, respectively which represent the effectiveness of the proposed observer and controller. Moreover, Fig. 4 presents the controller gains along the time. The control signal considering actuator saturation is shown in Fig. 5.

There are several reasons to occur a sensor fault, such as power fluctuation, component aging or environmental disturbances. To model a sensor fault, we utilize a difference between the measured sensor output and fault-free output. More detail for modeling of sensor faults are addressed in Ref. [45]. Two scenarios will be presented to illustrate the effectiveness of the proposed schemes.

**Scenario 1** Periodical ramp-bias-type fault is introduced in the gyro at the time  $t_{f1} = 150s$  with different periods in its components and maximum magnitude of  $0.2 \text{ rad/s}$  which is shown in Fig. 6. The residual behavior is illustrated in Fig. 7 exceeding the threshold after fault occurrence. Fig. 8 shows the attitude and angular velocity tracking error converge to a bound around zero.

**Scenario 2** During Scenario 1, we assume the fault occurs in one or more attitude sensors at the time  $t_{f2} = 250s$ . Fig. 9 illustrates a model of abnormal situation for attitude sensors faults in MRP representation in a sample interval. This vector with different period and magnitude values in three components is introduced in  $q$ . Due to simultaneous rate and attitude sensors faults in Scenario 2, the bound of tracking error becomes wider than the interval of  $[t_{f1}, t_{f2}]$ . Fig. 10 shows the effectiveness of the proposed controller in simultaneous sensor fault mode.

## 5. Conclusions

This paper addressed the sensor fault detection and recovery control in the satellite attitude control system. To detect the fault in attitude and rate sensors, a nonlinear observer has been presented. Moreover, a compensator is developed to recover destructive effect of faulty measured states. The observer convergence and UUB of tracking errors in closed loop control system are guaranteed based on the Lyapunov stability criterion. The external disturbances, sensor measurement accuracy and actuator saturation have been applied in the simulation to evaluate the given controller for more practical cases.

## Appendix A. Supplementary data

Supplementary data related to this article can be found at <https://doi.org/10.1016/j.actaastro.2018.01.002>.

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