• ASSIOMI PROBABILITÀ

1) $P(E) \geq \emptyset$

2) La somma delle probabilità di uno spazio campionario $= 1$.

3) $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

• Dimostra che $E[aX+b] = aE[X]+b$

$$E[aX+b] = \sum_x (aX+b) \cdot p(x)$$

$$= a\sum_x X \cdot P(X) + \sum_x b \cdot p(x)$$

$$= a\underbrace{\sum_x X \cdot p(X)}_{E[X]} + b\underbrace{\sum_x p(x)}_{1}$$

$$= aE[X] + b$$

• Dimostra che $Var(aX+b) = a^2 Var(x)$

$$Var(aX+b) = E[(ax+b)^2] - E[aX+b]^2$$

$$= E[ax^2 + 2abX + b^2] - (aE[X]+b)^2$$

$$= a^2E[X^2] + 2abE[X] + b^2 - (aE[X]+b)^2$$

$$= a^2\left(E[x^2] - E[x]^2\right)$$

$$= a^2 Var(x)$$

• Teorema di Bayes per eventi E, E^c e F

$$P(E|F) = \frac{P(F|E)\cdot P(E)}{P(F)} = \frac{P(F|E)\cdot P(E)}{P(F|E)\cdot P(E) + P(F|E^c)\cdot P(E^c)}$$

• Se $Var(X) = a^2$ calcola $Var(aX)$.

$$Var(aX) = E((aX)^2)\cdot(E[aX])^2$$

$$= a^2 E[X^2] \cdot a^2 \cdot E[X]^2$$

$$= a^2 \cdot Var(X)$$

Se $a^2 = Var(x)$ allora

$$= Var(x)\cdot Var(x) = (Var(x))^2$$