APPROSSIMAZIONE: minimi quadrati

ESEMPIO:

$$g(x) = \alpha x^2 + 2\beta x + \gamma$$

Calcolare i coifficenti per approssimare ai minimi quadrati i seguenti

X	-2	-1	0	1	0
Y	8	5	-7	-1	1

✓ Punti base ↗

✓ Spazio: polinomio di secondo grado

• Base per V_n

$$\{1, 2X, X^2\}$$

$$A = \begin{bmatrix} 1 & -4 & 4 \\ 1 & -2 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 4 & 4 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -2 & 0 & 2 & 4 \\ 4 & 1 & 0 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -4 & 4 \\ 1 & -2 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 10 \\ 0 & 40 & 0 \\ 10 & 0 & 34 \end{bmatrix}$$

$$A^T \cdot y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -2 & 0 & 2 & 4 \\ 4 & 1 & 0 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 5 \\ -7 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -40 \\ 40 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 10 \\ 0 & 40 & 0 \\ 10 & 0 & 34 \end{bmatrix} \cdot \begin{bmatrix} \gamma \\ \beta \\ \alpha \end{bmatrix} = \begin{bmatrix} 6 \\ -40 \\ 40 \end{bmatrix}$$

$R_3 \to R_3 - 2 \cdot R_1$ $\begin{bmatrix} 5 & 0 & 10 & | & 6 \\ 0 & 40 & 0 & | & -40 \\ 0 & 0 & 14 & | & 28 \end{bmatrix}$ $\begin{cases} \gamma = -\dfrac{14}{5} \\ \beta = -1 \\ \alpha = 2 \end{cases}$ $\begin{cases} \gamma = -2,8 \\ \beta = -1 \\ \alpha = 2 \end{cases}$

$$g(x) = 2X^2 + -2X + 2,8$$

ESEMPIO:

$$g(x) = \alpha x^2 + \beta x + y$$

x	-1	0	0	1
y	0	0	-2	0

$$\{1, X, X^2\} \qquad \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$A^T \cdot Y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} y \\ \beta \\ \alpha \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 4y + 2\alpha = -2 \\ 2\beta = 0 \\ 2y + 2\alpha = 0 \end{cases} \qquad \begin{cases} \alpha = 1 \\ \beta = 0 \\ y = -1 \end{cases} \qquad g(x) = 1X^2 - 1$$