1.1

Anagrammi di TERRIERI: $\frac{8!}{2!2!3!}$

1.2

- $P(Y=3|X=1) = \frac{P(Y=3 \cap X=1)}{P(X=1)} = \frac{1/5}{3/5} = \frac{1}{\cancel{5}} \cdot \frac{\cancel{5}}{3} = \frac{1}{3}$
- $P(Y=2|X=1) = \frac{P(Y=2 \cap X=1)}{P(X=1)} = \frac{2/5}{3/5} = \frac{2}{\cancel{5}} \cdot \frac{\cancel{5}}{3} = \frac{2}{3}$
- Se $P(Y=2) = P(Y=3) = 1/2$ allora X e Y sono dipendenti perché

  $P(Y=2|X=1) \neq P(Y=2) \Rightarrow \frac{2}{3} \neq \frac{1}{2}$

$\boxed{1.3}$

$$f(x) = \begin{cases} 1/4 & 0 \le x < 1 \\ \frac{\alpha}{4} & 1 \le x \le 2 \\ 0 & \text{altrimenti} \end{cases}$$

$$\int_0^1 \frac{1}{4} + \int_1^2 \frac{\alpha}{4} = 1$$

$$\frac{1}{4} - 0 + \frac{2\alpha}{4} - \frac{\alpha}{4} = 1$$

$$\frac{1}{4} + \frac{\alpha}{4} = 1$$

$$\boxed{\alpha = 3}$$

- **C.D.F**

da $\emptyset$ a 1 $\longrightarrow \int_0^x \frac{1}{4} = \boxed{\frac{x}{4}}$

da 1 a 2 $\longrightarrow \frac{1}{4} + \int_1^x \frac{3}{4} = \frac{1}{4} + \frac{3x}{4} - \frac{3}{4} = \boxed{-\frac{2}{4} + \frac{3x}{4}}$

$$\begin{cases} 0 & x < 0 \\ x/4 & 0 \le x \le 1 \\ -2/4 + 3x/4 & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

2.1

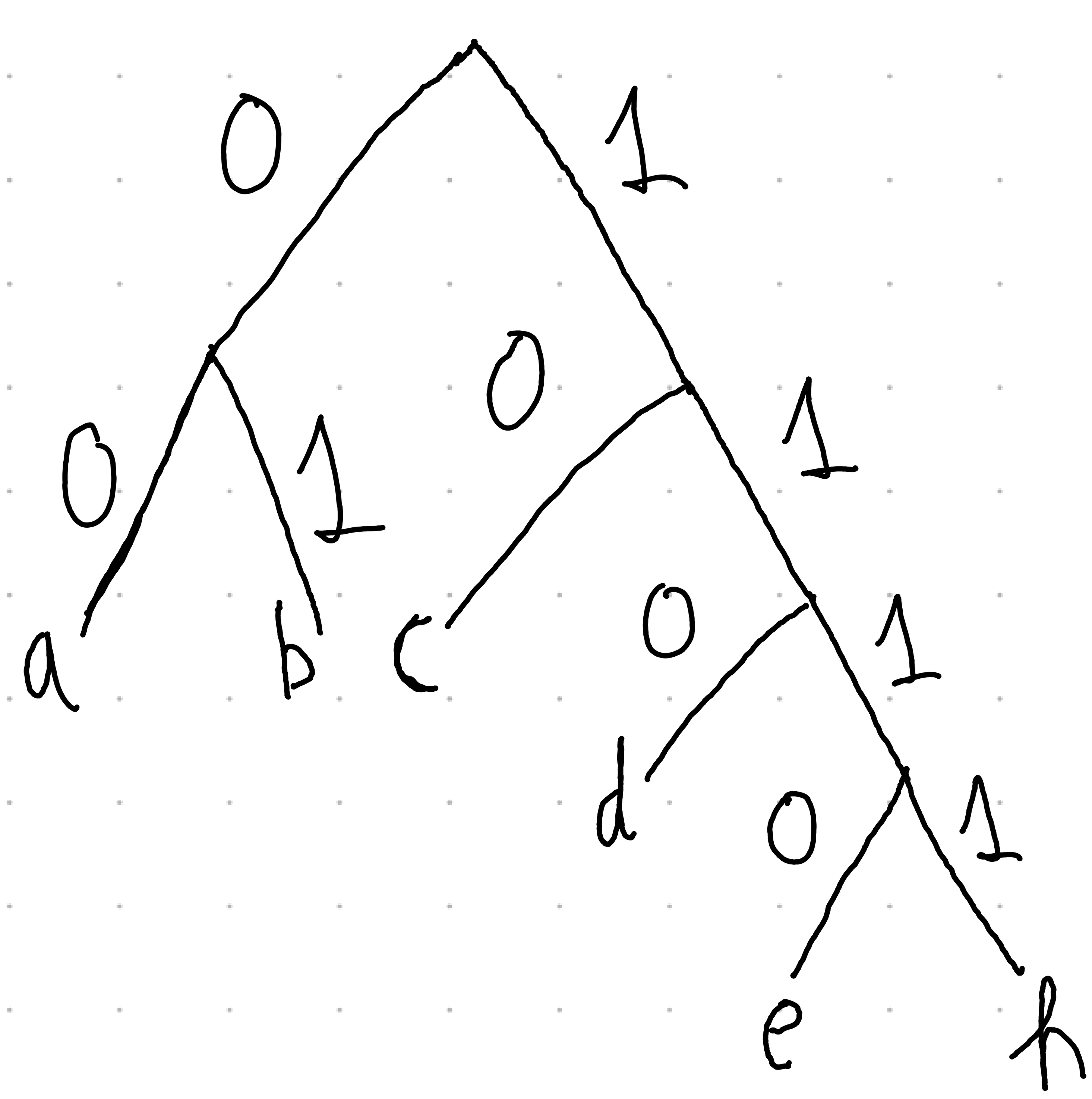
$$\log_2 256 - \log_2 128 = 1 \text{ bit}$$

abbiamo quindi ottenuto 1 bit d'informazione.

2.2

$$p(a) = \frac{7}{20} \quad p(b) = p(c) = \frac{5}{20} \quad p(d) = p(e) = p(f) = \frac{1}{20}$$

$p(a) = \frac{7}{20}$	$p(a) = \frac{7}{20}$	$p(a) = \frac{7}{20}$	$p(e)+p(f)+p(d)+p(c) = \frac{8}{20}$
$p(b) = \frac{5}{20}$	$p(b) = \frac{5}{20}$	$p(b) = \frac{5}{20}$	$p(a) = \frac{7}{20}$
$p(c) = \frac{5}{20}$	$p(c) = \frac{5}{20}$	$p(c) = \frac{5}{20}$	$p(b) = \frac{5}{20}$
$p(d) = \frac{1}{20}$	$p(e)+p(f) = 2/20$	$p(e)+p(f)+p(d) = \frac{3}{20}$	
$p(e) = 1/20$	$p(d) = \frac{1}{20}$		
$p(f) = \frac{1}{20}$			

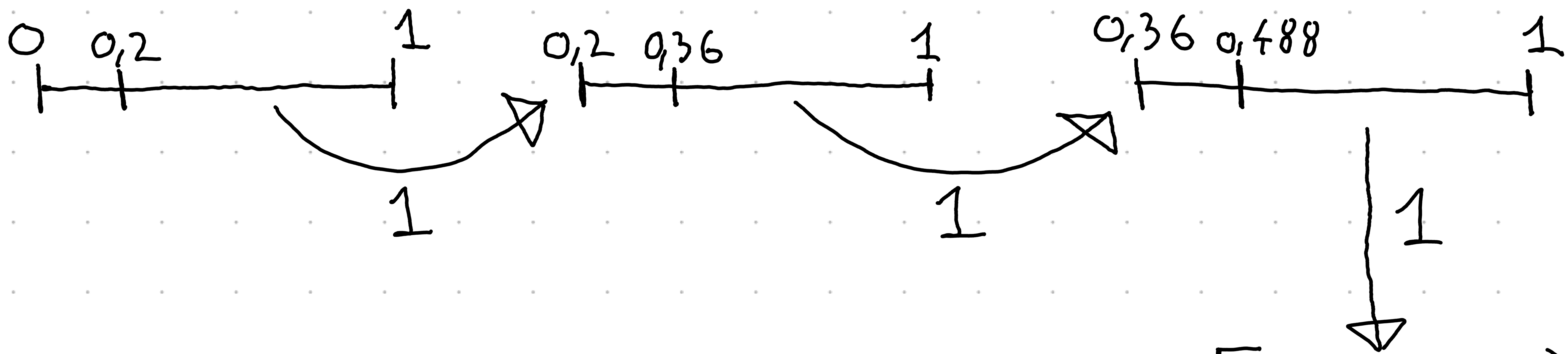


$a = 00$
$b = 01$
$c = 10$
$d = 110$
$e = 1110$
$f = 1111$

2.3

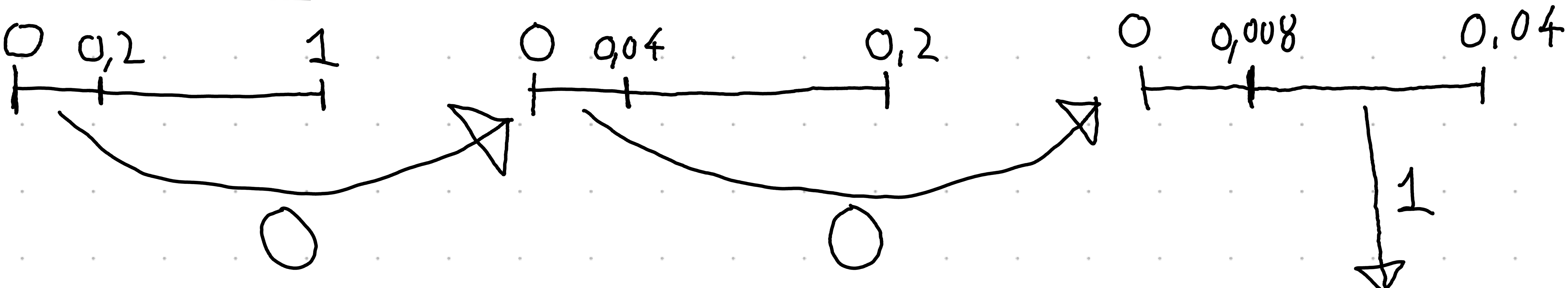
Codifica aritmetica se $p(0)=\frac{1}{5}$ e $p(1)=\frac{4}{5}$

- **Di 111**



$[0{,}488,\ 1)$

- **Di 001**



$[0{,}008,\ 0{,}04)$

2.4

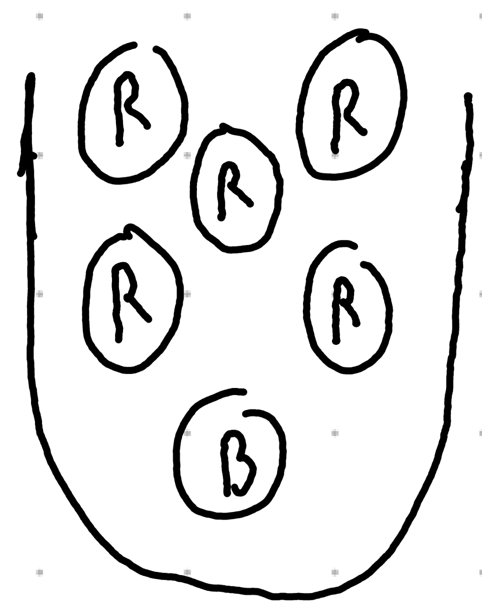
3.1

$$\mathcal{V}(x|10) = \frac{1}{10} \cdot \frac{1}{10} \cdot 0 = 0$$

$$\mathcal{V}(x|20) = \frac{1}{20} \cdot \frac{1}{20} \cdot \frac{1}{20} = \boxed{\frac{1}{8000}}$$

Da quanto emerge dalle verosimiglianze, 20 è più verosimile mentre 10 non è proprio accettabile.

3.2



$$P(T|\text{ESCE } R) = \frac{1}{4}$$

$$P(T|\text{ESCE } B) = \frac{1}{2}$$

• $P(T)$

$$P(T) = P(T|\text{ESCE } R) \cdot P(\text{ESCE } R) + P(T|\text{ESCE } B) \cdot P(\text{ESCE } B) = \frac{1}{4} \cdot \frac{5}{6} + \frac{1}{2} \cdot \frac{1}{6} = \boxed{\frac{7}{24}}$$

• $P(C_2|T_1)$

$$P(C_2|T_1) = \underbrace{P(C_2|T_1 \text{ da rossa})}_{\frac{3}{4}} \cdot \underbrace{P(R|T_1)}_{\frac{P(T_1|R) \cdot P(R)}{P(T_1)}} + P(C_2|T_1 \text{ da bianca}) \cdot P(B|T_1)$$

3.3

Calcola la probabilità di passare dallo stato 1 allo stato 2 in 2 passi.

$$P^2 = \begin{bmatrix} 0,4 & 0,6 \\ 0,8 & 0,2 \end{bmatrix} \cdot \begin{bmatrix} 0,4 & 0,6 \\ 0,8 & 0,2 \end{bmatrix} = \begin{bmatrix} 0,64 & 0,36 \\ & \end{bmatrix}$$