

APPROSSIMAZIONE: Minimi quadrati

ESEMPIO:

$$g(x) = \alpha x^2 + 2\beta x + \gamma$$

Calcolare i coefficienti per approssimare ai minimi quadrati i seguenti

X	-2	-1	0	1	0
Y	8	5	-7	-1	1

✓ Punti base ↗

✓ Spazio: polinomio di secondo grado

• Base per V_h

$$\{1, 2x, x^2\}$$

$$A = \begin{bmatrix} 1 & -4 & 4 \\ 1 & -2 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 4 & 4 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -2 & 0 & 2 & 4 \\ 4 & 1 & 0 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -4 & 4 \\ 1 & -2 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 10 \\ 0 & 40 & 0 \\ 10 & 0 & 34 \end{bmatrix}$$

$$A^T \cdot y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -4 & -2 & 0 & 2 & 4 \\ 4 & 1 & 0 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 5 \\ -7 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -40 \\ 40 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 10 \\ 0 & 40 & 0 \\ 10 & 0 & 34 \end{bmatrix} \cdot \begin{bmatrix} \gamma \\ \beta \\ \alpha \end{bmatrix} = \begin{bmatrix} 6 \\ -40 \\ 40 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2 \cdot R_1 \quad \left[\begin{array}{ccc|c} 5 & 0 & 10 & 6 \\ 0 & 40 & 0 & -40 \\ 0 & 0 & 14 & 28 \end{array} \right] \quad \begin{cases} \gamma = -\frac{14}{5} \\ \beta = -1 \\ \alpha = 2 \end{cases} \quad \begin{cases} \gamma = -2,8 \\ \beta = -1 \\ \alpha = 2 \end{cases}$$

$$g(x) = 2x^2 + -2x + 2,8$$

ESEMPIO:

$$g(x) = \alpha x^2 + \beta x + \gamma$$

$$\begin{array}{c|cccc} x & -1 & 0 & 0 & 1 \\ \hline y & 0 & 0 & -2 & 0 \end{array}$$

$$\{1, x, x^2\}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$A^T \cdot y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} \gamma \\ \beta \\ \alpha \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 4\gamma + 2\alpha = -2 \\ 2\beta = 0 \\ 2\gamma + 2\alpha = 0 \end{cases}$$

$$\begin{cases} \alpha = 1 \\ \beta = 0 \\ \gamma = -1 \end{cases}$$

$$g(x) = 1x^2 - 1$$

