ES3

$$g(x) = \alpha x + \beta$$

x	-5	-3	-2	0	1	4
y	5	-3	-2	-1	-1	4

- **Base V_n**

$$\{1, x\} \qquad A = \begin{bmatrix} 1 & -5 \\ 1 & -3 \\ 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 4 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -5 & -3 & -2 & 0 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -5 \\ 1 & -3 \\ 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ -5 & 55 \end{bmatrix}$$

$$A^T \cdot y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -5 & -3 & -2 & 0 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -3 \\ -2 \\ -1 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

SBAGLIATO!

$$\begin{bmatrix} 6 & -5 \\ -5 & 55 \end{bmatrix} \cdot \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

$$\begin{cases} 6\alpha - 5\beta = 2 \\ -5\alpha + 55\beta = 3 \end{cases} \quad \left[\begin{array}{cc|c} 6 & -5 & 2 \\ -5 & 55 & 3 \end{array}\right] \quad \left[\begin{array}{cc|c} 0 & 1 & \frac{28}{305} \\ 1 & 0 & \frac{25}{61} \end{array}\right] \quad \begin{cases} \beta = \frac{28}{305} \\ \alpha = \frac{25}{61} \end{cases}$$

$\boxed{ES4}$

$$\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

- <u>Controllo se è diagonalizzabile</u>

$A \neq A^T$ non siamo sicuri che sia diagonalizza

$$\det(A-\lambda I) = \det \begin{bmatrix} 2-\lambda & 0 & 4 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{bmatrix} = 1\cdot(0-4+4\lambda)+2-\lambda\,(2-3\lambda+\lambda^2)$$

$$= -4+4\lambda+\left(2-\lambda\cdot(\lambda^2-3\lambda+2)\right)$$

$$= -4+4\lambda+2\lambda^2-6\lambda+4-\lambda^3+3\lambda^2-2\lambda$$

$$= -\lambda^3+5\lambda^2-4\lambda$$

$$-\lambda^3+5\lambda^2-4\lambda=0$$

$$-\lambda(\lambda^2-3\lambda+4)=0$$

$$\lambda=0 \quad \lambda=1 \quad \lambda=4$$

- <u>Autospazio di $\lambda=0$</u>

$$A-0I = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \quad \left[\begin{array}{ccc|c} 2 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right]$$

$$\begin{cases} 2v_1+4v_3=0 \\ v_2=0 \\ 1v_1+2v_3=0 \end{cases} \quad \begin{cases} v_1=-2v_3 \\ v_2=0 \\ 0=0 \end{cases} \quad \begin{bmatrix} -2v_3 \\ 0 \\ v_3 \end{bmatrix} \sim \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

- **Autospazio di $\lambda=1$**

$$A-1I=\begin{bmatrix}1&0&4\\0&0&0\\1&0&1\end{bmatrix}\quad\left[\begin{array}{ccc|c}1&0&4&0\\0&0&0&0\\1&0&1&0\end{array}\right]\quad\begin{cases}-v_3+4v_3=0\\0=0\\v_1=-v_3\end{cases}\quad\begin{cases}v_3=0\\0=0\\v_1=0\end{cases}\quad\begin{bmatrix}0\\v_2\\0\end{bmatrix}\sim\begin{bmatrix}0\\1\\0\end{bmatrix}$$

- **Autospazio di $\lambda=4$**

$$A-4I=\begin{bmatrix}-2&0&4\\0&-3&0\\1&0&-2\end{bmatrix}\quad\left[\begin{array}{ccc|c}-2&0&4&0\\0&-3&0&0\\1&0&-2&0\end{array}\right]\quad\begin{cases}-2v_1+4v_3=0\\v_2=0\\v_1-2v_3=0\end{cases}\quad\begin{cases}v_1=2v_3\\v_2=0\\0=0\end{cases}\quad\begin{bmatrix}2v_3\\0\\v_3\end{bmatrix}\sim\begin{bmatrix}2\\0\\1\end{bmatrix}$$

- **Diagonalizzazione**

$$A=X\cdot\Lambda\cdot X^{-1}$$

$$A=\underbrace{\begin{bmatrix}-2&0&2\\0&1&0\\1&0&1\end{bmatrix}}_{X}\cdot\underbrace{\begin{bmatrix}0&0&0\\0&1&0\\0&0&4\end{bmatrix}}_{\Lambda}\cdot X^{-1}$$