• Determina se esiste A^{-1}

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 3 & 2 \\ 1 & -4 & 1 \end{bmatrix} \qquad \left[\begin{array}{ccc|ccc} 0 & 1 & -1 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 1 & -4 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A \cdot E = \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 1 & -4 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 - R_1 = \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & -7 & -1 & 0 & -1 & 1 \end{array}\right]$$

$$R_3 + 7R_2 = \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -8 & 7 & -1 & 1 \end{array}\right]$$

$$-\frac{1}{8}R_3 = \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{7}{8} & \frac{1}{8} & -\frac{1}{8} \end{array}\right]$$

$$R_2 + R_3 = \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{7}{8} & \frac{1}{8} & -\frac{1}{8} \end{array}\right]$$

$$R_1 - 3R_2 = \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -\frac{3}{8} & \frac{5}{8} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{7}{8} & \frac{1}{8} & -\frac{1}{8} \end{array}\right]$$

$$R_1 - 2R_3 = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{11}{8} & \frac{3}{8} & \frac{3}{8} \\ 0 & 1 & 0 & \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} \\ 0 & 0 & 1 & -\frac{7}{8} & \frac{1}{8} & -\frac{1}{8} \end{array}\right] \longrightarrow A^{-1} = \frac{1}{8}\begin{bmatrix} 11 & 3 & 3 \\ 1 & 1 & -1 \\ -7 & 1 & -1 \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ h & 1 & 1 & h \end{pmatrix}$$

$$\det \begin{vmatrix} 2 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & h \end{vmatrix} = -1 \cdot \det \begin{pmatrix} 1 & 1 \\ 1 & h \end{pmatrix} + 0 + 0 = h - 1$$

$$A = \begin{pmatrix} -3 & 2h & -2 \\ -3 & 2+2h & -1 \\ h & 0 & 1 \end{pmatrix}$$

$rk(A) =$ ordine di A se e solo se $\det A \neq \emptyset$

$$\det A = -3 \begin{vmatrix} 2+2h & -1 \\ 0 & 1 \end{vmatrix} - 3 \begin{vmatrix} 2h & -2 \\ 0 & 1 \end{vmatrix} + h \begin{vmatrix} 2h & -2 \\ 2+2h & -1 \end{vmatrix} =$$

$$= -6 - 6h - (-3 \cdot 2h) + (h(-2h - (-4 - 4h)))$$

$$= -6 - 6h + 6h + 2h^2 + 4h = +2h^2 + 4h - 6$$

Devo trovare in che punti è uguale a $\emptyset$

$$h^2 + 2h - 3 = 0$$

$$h^2 - 1h + 3h - 3 = 0$$

$$h(h-1) + 3(h-1) = 0$$

$$(h+3)(h-1) = 0$$

$h = 1$

$h = -3$

$$\boxed{\text{QUINDI } rk(A) = 3 \text{ SE } H \neq 1 \text{ e } H \neq -3}$$