Some Parallel Computing Concepts

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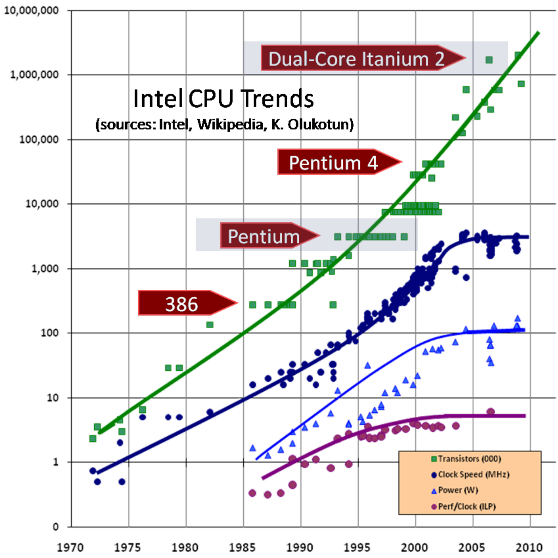
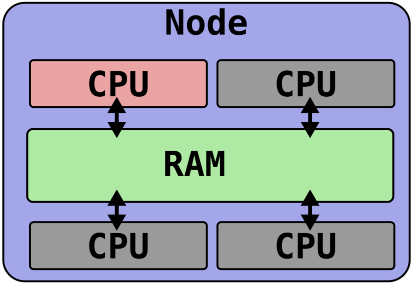
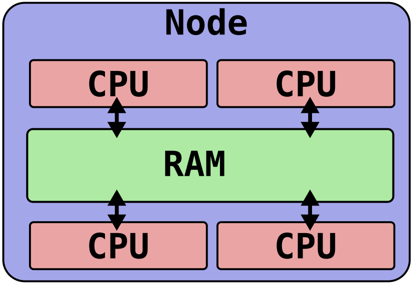
Free Lunch is over



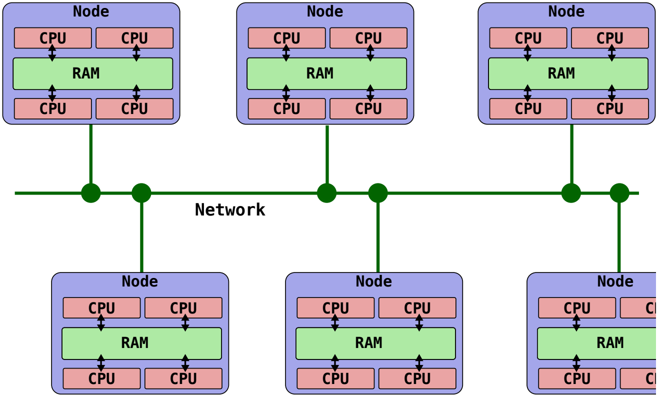
Single core



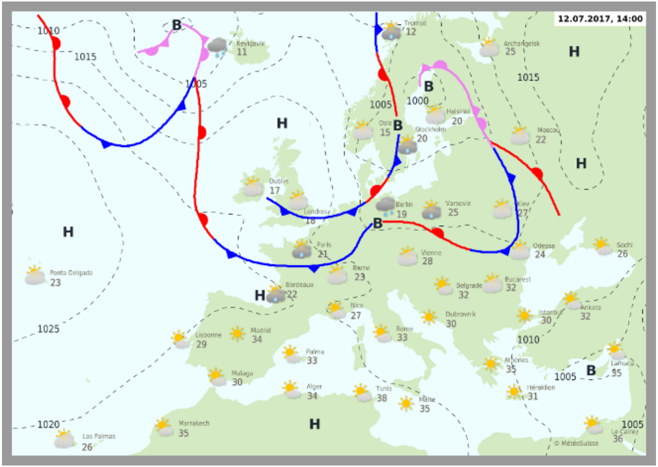
Shared memory (multithread, openmp)



Distributed memory (cluster, mpi)



Sequential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Malaga 30
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Marrakech 35
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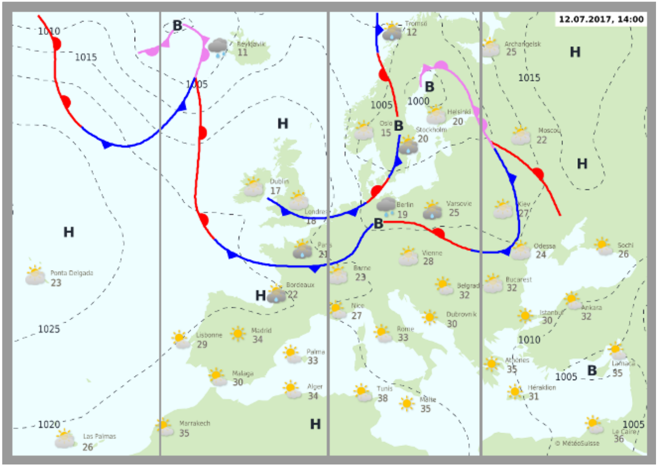
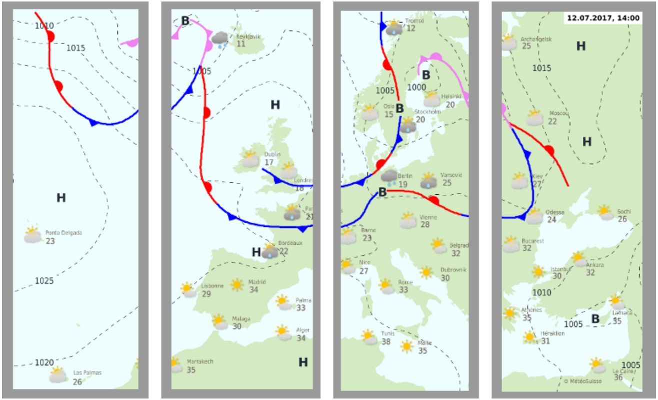
Parallel: Shared Memory



Parallel: Distributed Memory



Single Worker vs Multiple Worker

- Suppose 1 worker can dig a hole in 2 hours
- How much does it take for 2 workers to dig the same hole ?
- Can 60 workers dig the same hole in 2 minutes ?

The Speed-Up

We define the speed-up with p workers as:

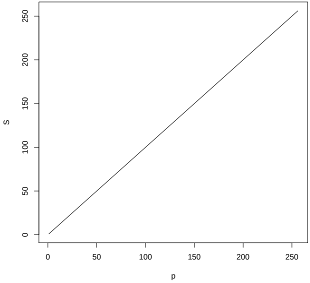
$$S(p) = \frac{T_1}{T_p} \tag{1}$$

where T_i is the time required for i workers.

Ideal Speed-Up

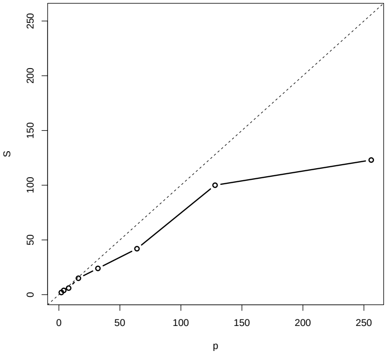
Ideally, p workers should be able to complete a task p times faster than one single worker:

$$T_p = \frac{T_1}{p} \quad \rightarrow \quad S(p) = \frac{T_1}{T_p} = p \tag{2}$$



Measured Speed Up

But of course the reality can be far from ideal. . .



Absolute vs. Relative Speed-Up

Attention

When applied to computer programs. We will take the best sequential implementation (T_s) instead of a parallel implementation with just one processor (T_1):

$$S(p) = \frac{T_s}{T_p} < \frac{T_1}{T_p} \tag{3}$$

Better speed-up: larger domain

Embarrassingly parallel

- Suppose 1 worker can dig a hole in 2 hours
- Then, 60 workers can dig 60 holes in 2 hours
- In average, that's 1 hole per 2 minutes !

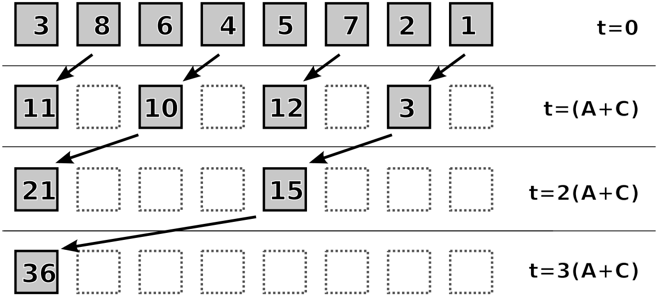
Definition

Suppose we want to sum an large array of n numbers in parallel. Sequentially, it will take:

$$T_s = (n-1) \cdot A \tag{4}$$

where n is the array size and a the constant duration of a single addition. (approximation).

First Parallel implementation (1)



First Parallel implementation (2)

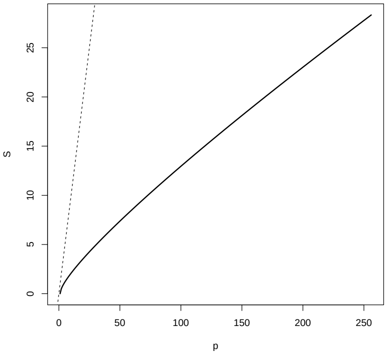
Suppose we have 1 number per processor ($p = n$), then the parallel time will be:

$$T_p = (A + C) \cdot \log_2(n) \tag{5}$$

where $A + C$ is sum of the duration of a single addition, plus a single communication time.

First Parallel Implementation Speed-Up

$$S(p) = \frac{T_s}{T_p} = \frac{(p-1)A}{(A+C)\cdot \log_2(p)} = \frac{p-1}{\left(1+\frac{C}{A}\right)\log_2 p} \qquad (6)$$



Better parallel implementation

Suppose now we have less processors than numbers to sum ($p << n$), then the parallel time will be:

$$T_p = \left(\frac{n}{p} - 1\right) A + (C + A)\log_2(p) \tag{7}$$

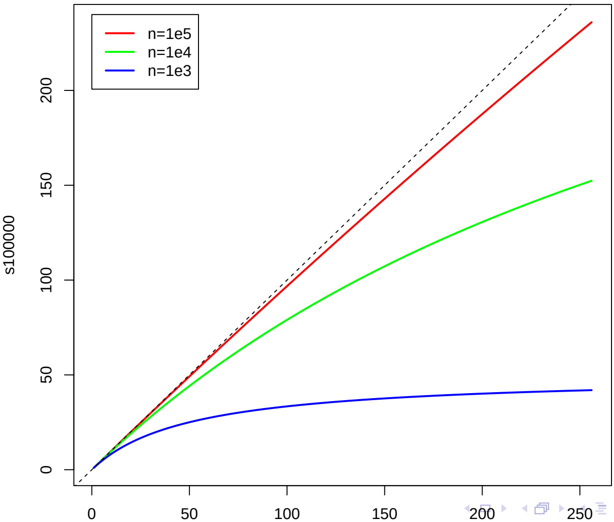
Better implementation Speed-up (1)

$$S(p) = \frac{p}{\frac{n-p}{n-1} + \left(1 + \frac{C}{A}\right) \frac{p}{n-1} \log_2(p)} \tag{8}$$

Since n is very large $\frac{n-p}{n-1} \approx 1$ and $\frac{p}{n-1} \approx \frac{p}{n}$. Therefore:

$$S(p) \approx \frac{p}{1 + \left(1 + \frac{C}{A}\right) \frac{p}{n} \log_2(p)} \tag{9}$$

Better implementation Speed-up (2)



Problem definition

A given parallel problem has sequential parts. For instance:

- A single processor reads a configuration file and makes data available to other processors.
- A single processor must collect all data to write results on the disk.
- Sequential post/pre processing

Amdahl's Law derivation (1)

Let W be the total amount of work and R the "computing power" of a single processor. The sequential time will be:

$$T_s = \frac{W}{R} \tag{10}$$

Let α be the fraction of the parallel implementation running sequentially:

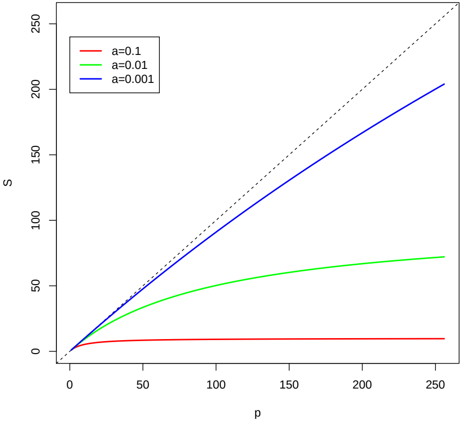
$$T_p = \frac{\alpha W}{R} + \frac{(1-\alpha)W}{pR} \tag{11}$$

Amdahl's Law derivation (2)

Therefore the speed-up is:

$$S(p) = \frac{T_s}{T_p} = \frac{\frac{W}{R}}{\frac{\alpha W}{R} + \frac{(1-\alpha)W}{pR}} = \frac{1}{\alpha + \frac{(1-\alpha)}{p}} \leq \frac{1}{\alpha} \qquad (12)$$

Amdahl's Law



Problem definition

Suppose now that the amount of work varies with the number of available processors. Remember that $T = \frac{W}{R}$:

$$W_p = \beta T_p R + p(1 - \beta) T_p R \tag{13}$$

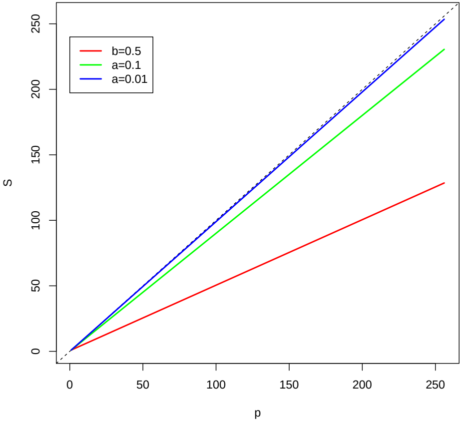
where βT_p is the amount of time spent in the sequential part of the code.

Law derivation

Since $T_s = \frac{W}{R}$ the speed-up is now:

$$S(p) = \frac{T_s}{T_p} = \beta + p(1-\beta) = O(p) \tag{14}$$

Gustafson's Law



Efficiency

We define the parallel efficiency with p workers as:

$$E(p) = S(p)/p = \frac{T_1}{pT_p} \tag{15}$$

Efficiency: Amdahl's Law