Description Logics with Pointwise Circumscription

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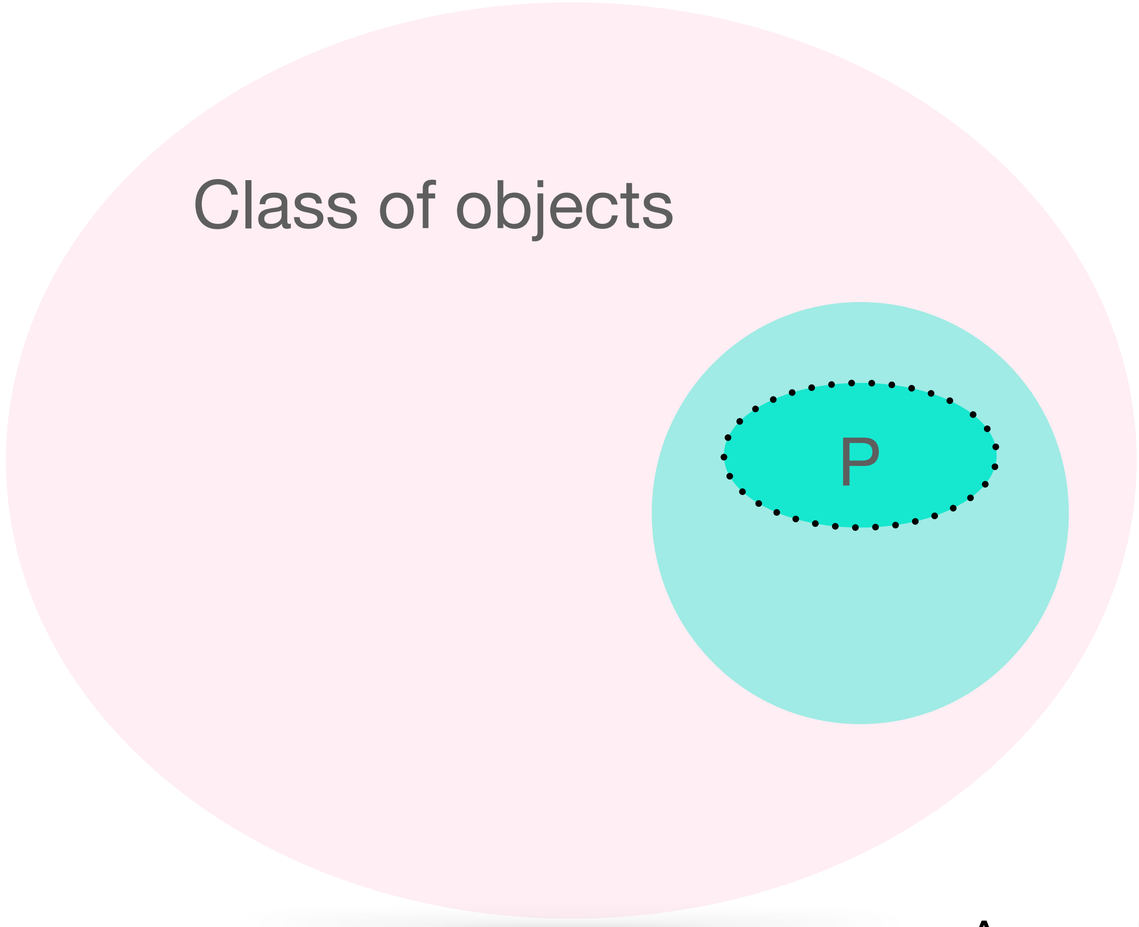
Highlights

- **Non-monotonic** extension of DLs based on **pointwise circumscription**
    - Less expressive form of circumscription — still powerful
- **Complexity** of reasoning in **ALCIO** under pointwise circumscription
    - Concept satisfiability w.r.t. KBs of modal depth ≤ 1 is NExp-complete
    - Undecidability of reasoning w.r.t general KBs

What is circumscription?

Minimizing extensions of predicates



Circumscribe the extension of P

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Minimize P

↓

Possibly removing infinitely many tuples

A model is minimal if P cannot be further minimized

Example

Pizza margherita

$$Pizza(margherita) \qquad Vegetarian(tomato) \qquad Vegetarian(mozzarella)$$

$$has\_ingredient(margherita, tomato) \qquad has\_ingredient(margherita, mozzarella)$$

$$Pizza \sqcap \forall has\_ingredient\,.\,Vegetarian \sqsubseteq Vegetarian\_Pizza$$

Is pizza margherita a vegetarian pizza?

Classical semantics does not rule out the existence of non vegetarian ingredients

Minimizing $has\_ingredient$ we derive that pizza margherita is vegetarian

Motivations

State-of-art

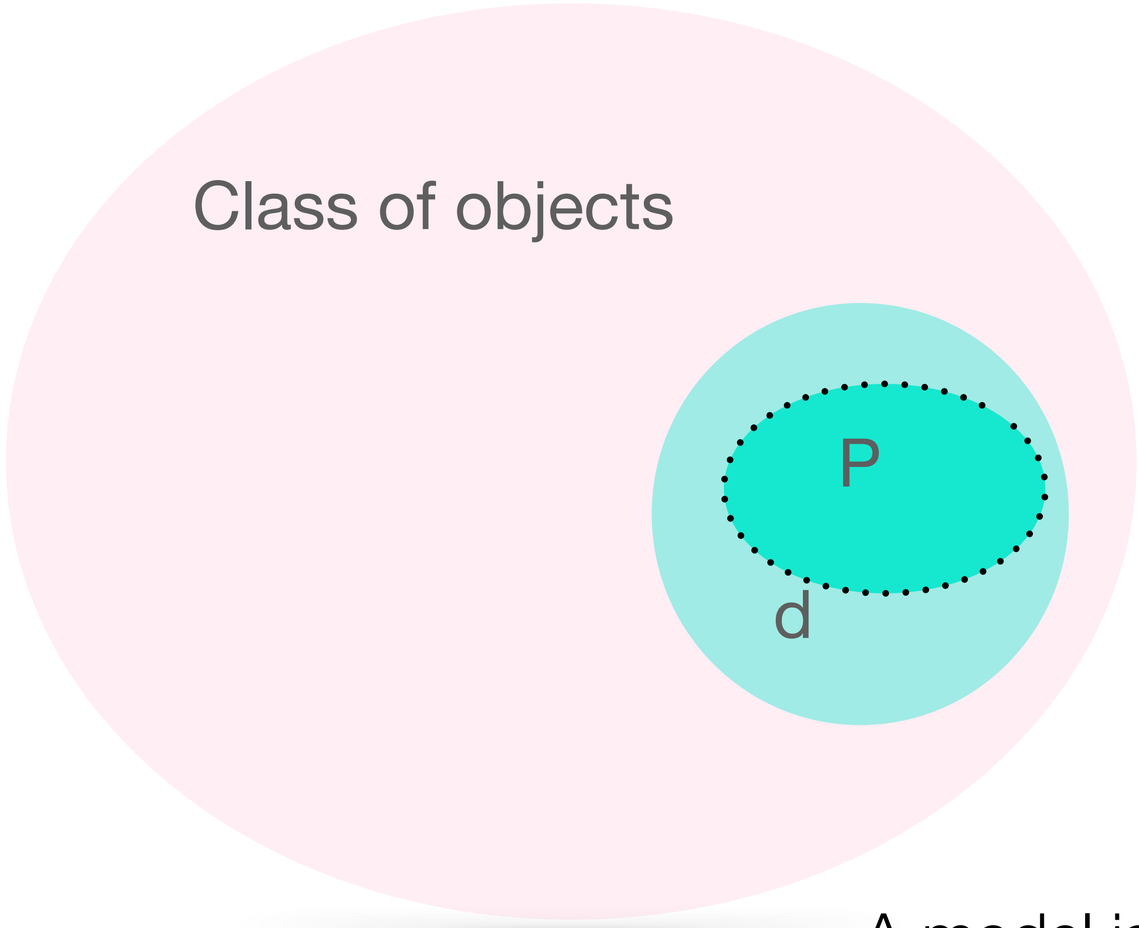
- Wide literature on **circumscribed DLs**
  - Concepts and roles can be minimized, fixed or vary
- In ALCIO, concept satisfiability is NExp^{NP}-complete with only varying roles
- **Undecidability** is encountered already in ALC if **roles** are **minimized** or **fixed**

Pointwise circumscription allows for a simple and intuitive form of minimization

Decidability in a fragment of ALCIO, allowing **minimized** or **fixed roles**

What is Pointwise Circumscription?

a sound approximation of circumscription



Minimize predicates at single tuples

Can we minimize P at d?

↓

Iterate over other tuples in P

A model is minimal if no tuple can be removed from P

Challenges

- Pointwise circumscription (pwc) is inherently local
    - Cycles are not detect — pwc *not always* coincides with circumscription
- We can use of the **mosaic technique** — no nested quantifiers
- Standard normalization techniques cannot be applied
- Nesting of quantifiers makes reasoning harder

Many KR examples call for simple TBoxes, often acyclic

Positive Results

The mosaic technique

Checking the existence of minimal models can be reduced to checking existence of finite family of **minimal fragments** of models.

Our algorithm

- Encode a system of inequalities
- Variables correspond to minimal fragments
- A solution tells which fragments are needed to build a minimal model and how many copies we need

KBs with no nested quantifiers

We want to check satisfiability of complex concepts

Constraints for Concept Satisfiability

A new tool

A constraint is a pair (C, D) — " if C holds, then D"

Act as filters over the set of minimal models

Simulate normalization with constraints

$$C_0 = \exists R \,.\, \exists R \,.\, B$$

We produce the constraints

$$(D_0, \exists R \,.\, D_1) \qquad (D_1, \exists R \,.\, B)$$

$\rightarrow$ Check satisfiability of D_0

Theorem

Under pointwise circumscription, concept satisfiability w.r.t. KBs in ALCIO with modal depth ≤ 1 is in **NExp**, if roles are either **minimized** or **fixed**.

Negative results

General TBoxes

- Normalization techniques are not applicable
- Constraints cannot be used for normalizing TBoxes
- Modal depth cannot be reduced

Theorem

Under pointwise circumscription, concept satisfiability w.r.t. KBs ALCIO with arbitrary depth is **undecidable**.

Conclusions

- We introduced pointwise circumscription (pwc) for DLs
- We studied the complexity of reasoning in ALCIO

Future work

- Constraints are interesting on their own
- Complexity of other DLs with pwc
- Characterize fragments where circumscription and pwc coincide

Thank you for the attention!