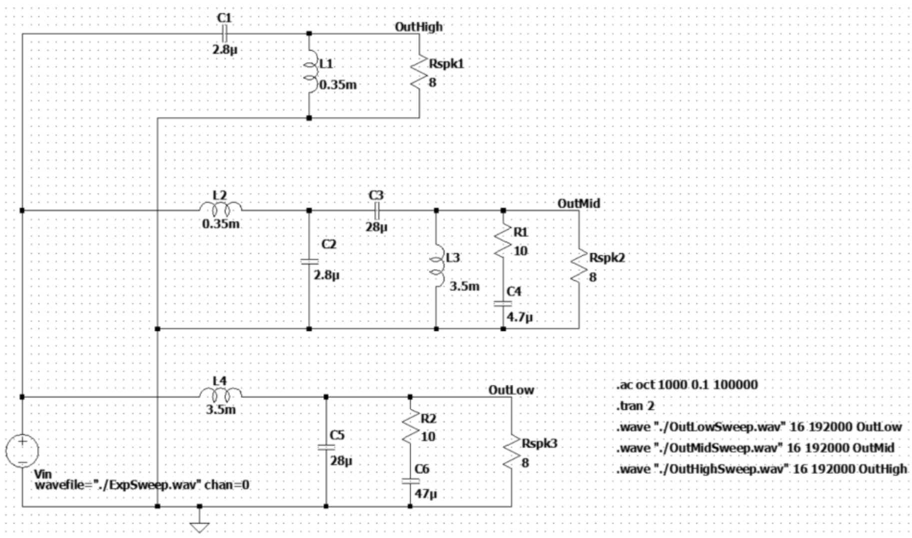
SASP Homework 4

Federico Ferreri 10666908
Emma Coletta 10683341

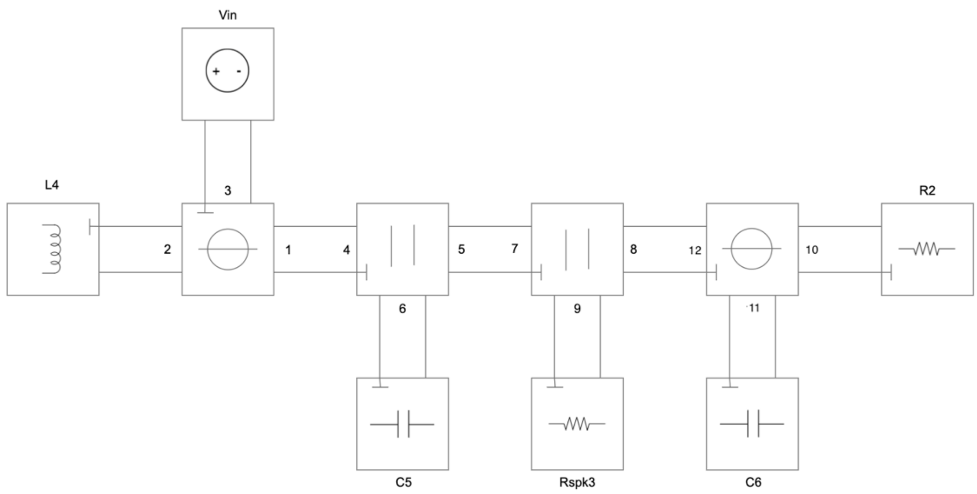
WDF Design

We opted to draw the WDF (Wave Digital Filter) schemes of the reference circuit as three separated connection trees, each representing a different type of filter subcircuit. We exclusively utilized 3-port series or parallel adaptors. In numbering the ports of the series adaptors, we considered the adapted port as the last port. Conversely, for the parallel adaptors, we treated the adapted port as the first port.

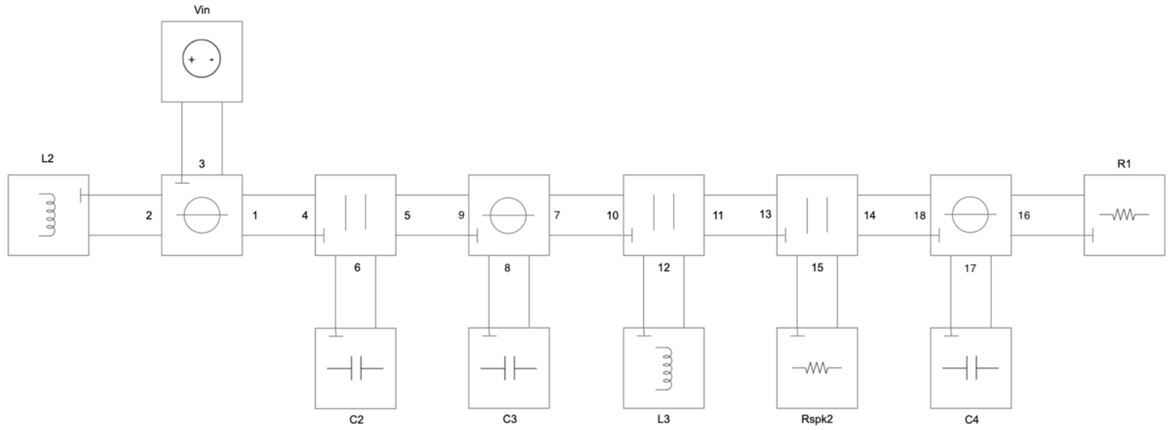
The reference circuit:



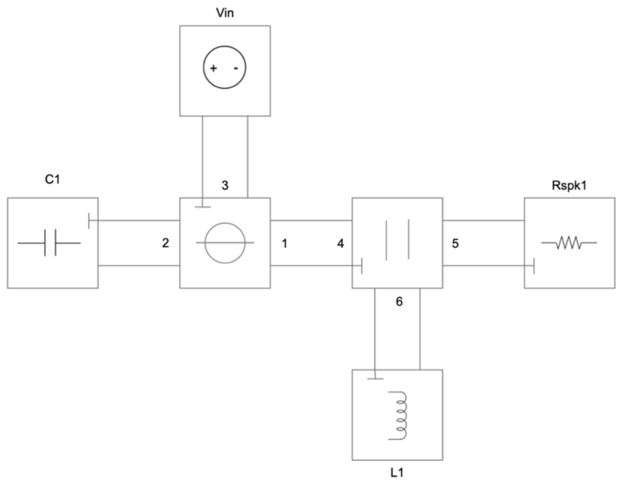
Low-pass filter subcircuit:



Band-pass filter subcircuit:



High-pass filter subcircuit:



Free Parameters

First, we establish the values of the free parameters for the resistive and dynamic elements. Then, we determine the values of all the remaining free parameters based on their adaptor type.

Low:

```
Z2_l = (2*L4)/Ts;
Z6_l = Ts/(2*C5);
Z9_l = RspkLow;
Z10_l = R2;
Z11_l = Ts/(2*C6);
Z12_l = Z11_l + Z10_l;
Z8_l = Z12_l;
Z7_l = (Z8_l * Z9_l)/(Z8_l + Z9_l);
Z5_l = Z7_l;
Z4_l = (Z5_l * Z6_l)/(Z5_l + Z6_l);
Z1_l = Z4_l;
Z3_l = Z1_l + Z2_l;
```

Mid:

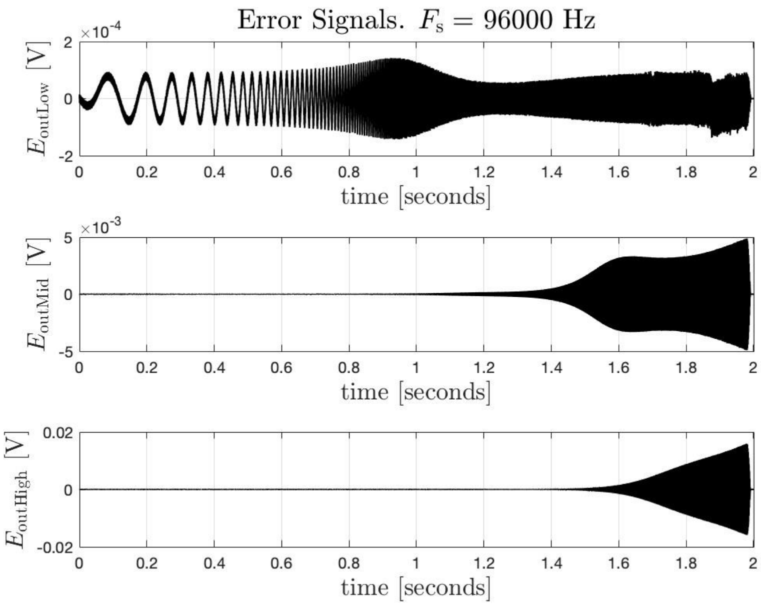
```
Z2_m = (2*L2)/Ts;
Z6_m = Ts/(2*C2);
Z8_m = Ts/(2*C3);
Z12_m = (2*L3)/Ts;
Z15_m = RspkMid;
Z17_m = Ts/(2*C4);
Z16_m = R1;
Z18_m = Z17_m + Z16_m;
Z14_m = Z18_m;
Z13_m = (Z14_m*Z15_m)/(Z14_m+Z15_m);
Z11_m = Z13_m;
Z10_m = (Z11_m*Z12_m)/(Z11_m+Z12_m);
Z7_m = Z10_m;
Z9_m = Z8_m + Z7_m;
Z5_m = Z9_m;
Z4_m = (Z5_m*Z6_m)/(Z5_m+Z6_m);
Z1_m = Z4_m;
Z3_m = Z1_m + Z2_m;
```

High:

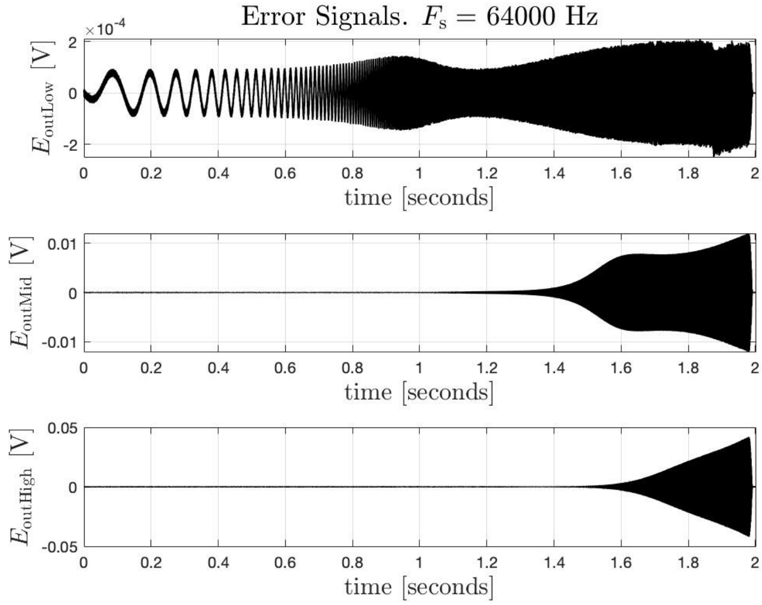
```
Z2_h = Ts/(2*C1);
Z5_h = RspkHigh;
Z6_h = (2*L1)/Ts;
Z4_h = (Z5_h*Z6_h)/(Z5_h+Z6_h);
Z1_h = Z4_h;
Z3_h = Z1_h + Z2_h;
```

Subplots of the Error Signals

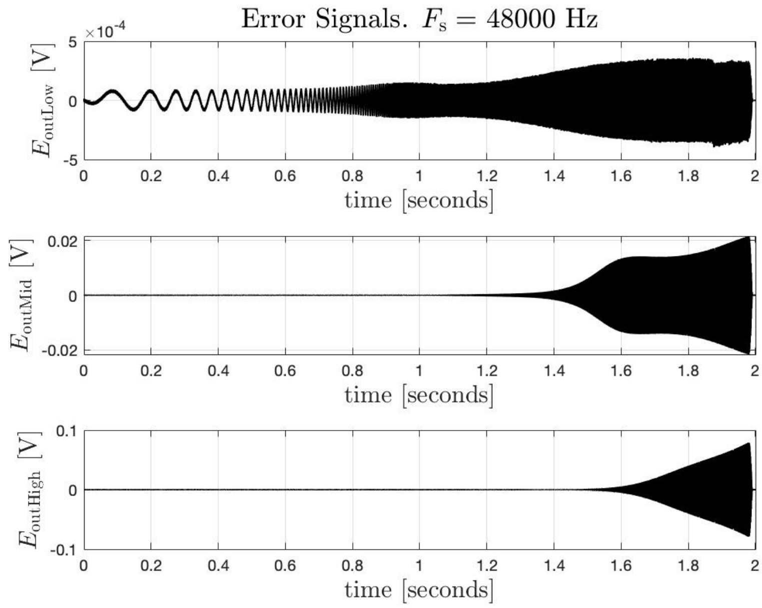
Subplot with downsampling factor equals to 2:



Subplot with downsampling factor equals to 3:



Subplot with downsampling factor equals to 4:



Questions

1. The error subplots for the three types of filters reveal significant variations in the amplitude peaks. This trend is consistent across different downsampling factors. Specifically, the low-pass filter error remains on the lowest order of magnitude, at 10^{-4}, in all three downsampling cases. The band-pass filter error is on the order of 10^{-3} for the downsampling factor of 2 and 10^{-2} for the other downsampling factors. Lastly, the high-pass filter error is on the highest order of magnitude, reaching 10^{-2} for the downsampling factor of 2 and 3, and 10^{-1} for the remaining case. These observations suggest that as the frequency increases, the discrepancy between the Wave Digital Filter (WDF) and the

LTspice implementation becomes higher. The magnitude error in the WDF output grows larger compared to the reference LTspice implementation as the frequency rises. Consequently, the output of the high-pass filter is the least accurate among the three WDF output signals. The observed behavior can be attributed to the utilization of the trapezoidal rule for approximating the time derivatives of dynamic elements within the WDF. This approximation technique introduces a warping mapping effect in the frequency domain:

$$\omega = \frac{2}{T_{\mathsf{s}}}\tan\left(\tilde{\omega}\frac{T_{\mathsf{s}}}{2}\right)$$

ω is the "continuous-time frequency", and $\tilde{\omega}$ is the "discrete-time frequency". ω is close to $\tilde{\omega}$ at low frequencies, while they differ more and more at high frequencies. The higher the sampling frequency Fs = 1/Ts, the more the difference between ω and $\tilde{\omega}$ becomes negligible in the whole frequency range of interest.

2. The higher the sampling frequency Fs, the lower the error. This happens due to the trapezoidal rule that introduces a warping mapping as explained in the previous answer.

3. When a single diode is added in parallel to the tweeter resistor Rh in the WD structure, it introduces changes to the neighboring junctions. These neighboring junctions are affected because the addition of the diode alters the flow of signals and introduces nonlinearity into the system. Its presence adds non-adaptable elements to the circuit, along with the voltage source. In this scenario, the WD structure becomes unsolvable without an iterative solver because to solve it, the values at the junctions need to be adapted or adjusted based on the surrounding circuit elements and their relationships but this way leads to adapt the same junction over two ports that is not possible. This means that the same junction would need to be adjusted separately for both ports but there are not enough free parameters to adapt it for multiple ports separately. If the ideal voltage source is replaced with a resistive voltage source, the situation changes. The resistive voltage source becomes adaptable, and the root of the WD structure would be only the diode element. However, solving the closed-form solution for b[k] at every time instance using the appropriate formula would be necessary. This calculation would significantly increase the complexity of the system due to the iterative nature of solving for b[k] at each time step.

$$b[k] = a[k] + 2Z[k]I_{\mathsf{s}} - 2\eta V_{\mathsf{th}}\mathrm{W}\left(\frac{Z[k]I_{\mathsf{s}}}{\eta V_{\mathsf{th}}}e^{\frac{Z[k]I_{\mathsf{s}}+a[k]}{\eta V_{\mathsf{th}}}}\right)$$

4. In the continuous-time domain: $v(t) = L \cdot \frac{di(t)}{dt}$;

   In the Laplace domain: $V(s) = s \cdot LI(s)$;

   Backward Euler: $s = \frac{1-z^{-1}}{T_S}$;

   $I(s) = \frac{V(s)}{s \cdot L}$;

   $\frac{1}{s} = \frac{T_S}{1-z^{-1}}$ → in the Z domain: $I(z) = \frac{T_S \cdot V(z)}{(1-z^{-1}) \cdot L}$;

$I(z) = \frac{z \cdot T_S \cdot V(z)}{(z-1) \cdot L} \quad \rightarrow \quad (z-1) \cdot I(z) = z \cdot V(z) \cdot \frac{T_S}{L} \quad \rightarrow \quad zI(z) - I(z) = z \cdot V(z) \cdot \frac{T_S}{L};$

In the discrete-time domain: $I[k] - I[k-1] = V[k]\frac{T_S}{L};$

$G_e[k] = \frac{T_S}{L};$

$\rightarrow \quad I[k] = V[k]G_e[k] + I_e[k];$

$I_e[k] = I[k-1];$

Substitution with the Kirchhoff-to-Wave transformation in the discrete-time domain:

$v[k] = \frac{a[k]+b[k]}{2};$

$$\rightarrow \quad \frac{a[k]-b[k]}{2Z[k]} = \frac{a[k]+b[k]}{2}G_e[k] + I_e[k];$$

$i[k] = \frac{a[k]-b[k]}{2Z[k]};$

Solving for b[k], we get the scuttering relation:

$b[k] = \frac{1-Z[k]G_e[k]}{1+Z[k]G_e[k]}a[k] - \frac{2Z[k]}{1+Z[k]G_e[k]}I_e[k];$

Adaptation case:

$b[k] = \frac{-I_e[k]}{G_e[k]} = -I_e[k]R_e[k];$

$Z[k] = \frac{1}{G_e[k]} = R_e[k] = \frac{L}{T_s};$

Wave Mapping in Case of Adaptation (Table 2):

$b[k] = -I[k-1]R_e[k] = \frac{b[k-1]-a[k-1]}{2};$

Adaptation Condition (Table 2): $\frac{L}{T_s};$