4. Use Fourier-Motzkin elimination on P to eliminate x_1. P is the polygon decribed by

$$\begin{bmatrix} -1 & -4 \\ -2 & -1 \\ 1 & -2 \\ 1 & 0 \\ 2 & 1 \\ -2 & 6 \\ -6 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} -9 \\ -4 \\ 0 \\ 4 \\ 11 \\ 17 \\ -6 \end{bmatrix}.$$

We multiply the matrix out and obtain the following inequalities:

$$\begin{cases} -x_1 - 4x_2 \leq -9 & \Rightarrow x_1 \geq 9 - 4x_2 \\ -2x_1 - x_2 \leq -4 & \Rightarrow x_1 \geq \frac{x_2}{2} - 2 \\ x_1 - 2x_2 \leq 0 & \Rightarrow x_1 \leq 2x_2 \\ x_1 \leq 4 & \\ 2x_1 + x_2 \leq 11 & \Rightarrow x_1 \leq \frac{11}{2} - \frac{x_2}{2} \\ -2x_1 + 6x_2 \leq 17 & \Rightarrow x_1 \geq 3x_2 - 17 \\ -6x_1 - x_2 \leq -6 & \Rightarrow x_1 \geq 1 - \frac{x_2}{6} \end{cases}$$

The first thing to notice is that $x_1 \leq 4$, so we already have an upper bound for x_1. We now proceed to completely eliminate $x_1$using the above inequalities. That is, we're looking for a lower bound and upper bound for x_1 in terms of only x_2.

Now we have $\begin{cases} 9 - 4x_2 & & \\ 3x_2 - 17 \leq x_1 \leq & 2x_2 \\ 1 - \frac{x_2}{6} & 4 \\ \frac{x_2}{2} - 2 & \frac{11}{2} - \frac{x_2}{2} \end{cases}$

Next, we characterize $proj_1(P)$. $proj_1(P)$ is given by the projection of the polytope onto the x_2 axis (then x_1 is 0). Now we solve all possible inequalities:

- $9 - 4x_2 \leq 2x_2 \Rightarrow x_2 \geq \frac{3}{2}$
- $9 - 4x_2 \leq 4 \Rightarrow x_2 \geq \frac{5}{4}$
- $9 - 4x_2 \leq \frac{11}{2} - \frac{x_2}{2} \Rightarrow x_2 \geq 1$
- $3x_2 - 17 \leq 2x_2 \Rightarrow x_2 \leq 17$
- $3x_2 - 17 \leq 4 \Rightarrow x_2 \leq 7$
- $3x_2 - 17 \leq \frac{11}{2} - \frac{x_2}{2} \Rightarrow x_2 \leq 7$
- $1 - \frac{x_2}{6} \leq 2x_2 \Rightarrow x_2 \geq \frac{6}{13}$
- $1 - \frac{x_2}{6} \leq 4 \Rightarrow x_2 \geq -18$
- $1 - \frac{x_2}{6} \leq \frac{11}{2} - \frac{x_2}{2} \Rightarrow x_2 \geq \frac{3}{2}$
- $\frac{x_2}{2} - 2 \leq 2x_2 \Rightarrow x_2 \leq 4$
- $\frac{x_2}{2} - 2 \leq 4 \Rightarrow x_2 \leq 12$
- $\frac{x_2}{2} - 2 \leq \frac{11}{2} - \frac{x_2}{2} \Rightarrow x_2 \leq \frac{15}{2}$

Thus, we have $proj_1(P) = \left\{ x_2 | \, \frac{3}{2} \leq x_2 \leq 4 \right\}$