① If w_0 is an element of a Coxeter group such Michael Jackanich that $sw_0 < w_0$ for every simple reflection $s \in S$, prove that $w \le w_0, \forall w \in W$.

Pf (By induction)

Show w is a subword of w_0

$$\ell(e) = \ell(w_0)$$

So $e \le w_0$

I.H.

Assume true for any $u \in W$ with $\ell(u) < n$
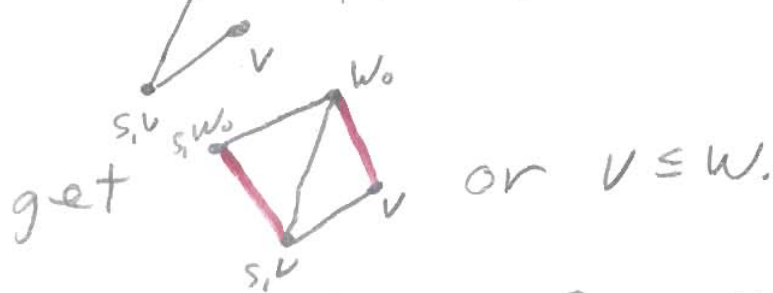
that $u \le w_0$

Let $v \in W$ s.t. $\ell(v) = n$.

Then $v = s_1 \cdots s_n$, but $s_1 v = s_2 \cdots s_n$

So $\ell(s_1 v) = n-1 < n$

Therefore $s_1 v \le w_0$ by the induction hypothesis

So we have $s_1 v \le v$, $s_1 v \le w_0$, and $s_1 w_0 \le w_0$

then by the lifting property we

get ⬦ or $v \le W$.

Therefore $w \le w_0$ for all $w \in W$

Worked with David Bangor.