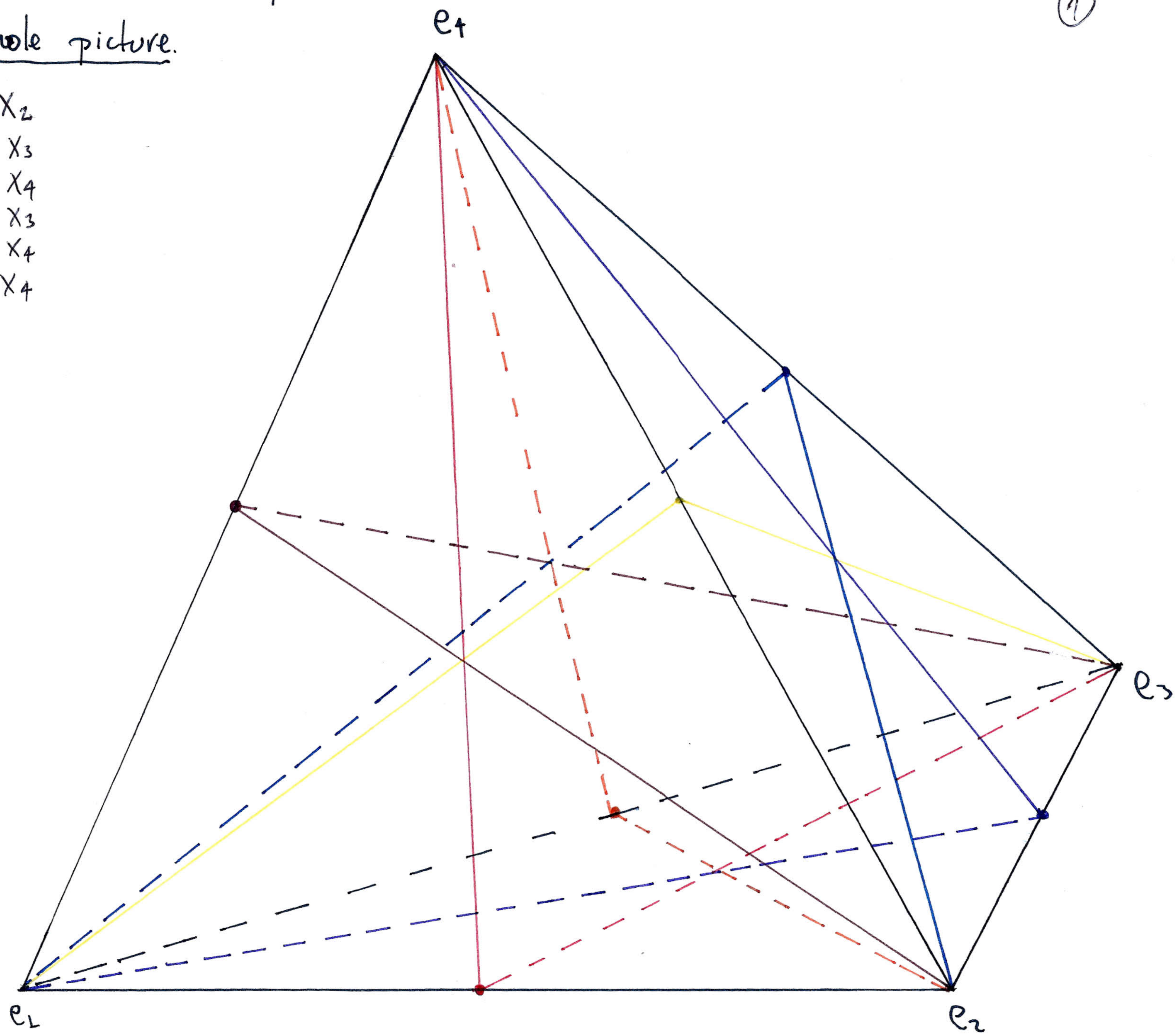
José Alejandro Samper

1

The whole picture.

- $x_1 = x_2$
- $x_1 = x_3$
- $x_1 = x_4$
- $x_2 = x_3$
- $x_2 = x_4$
- $x_3 = x_4$



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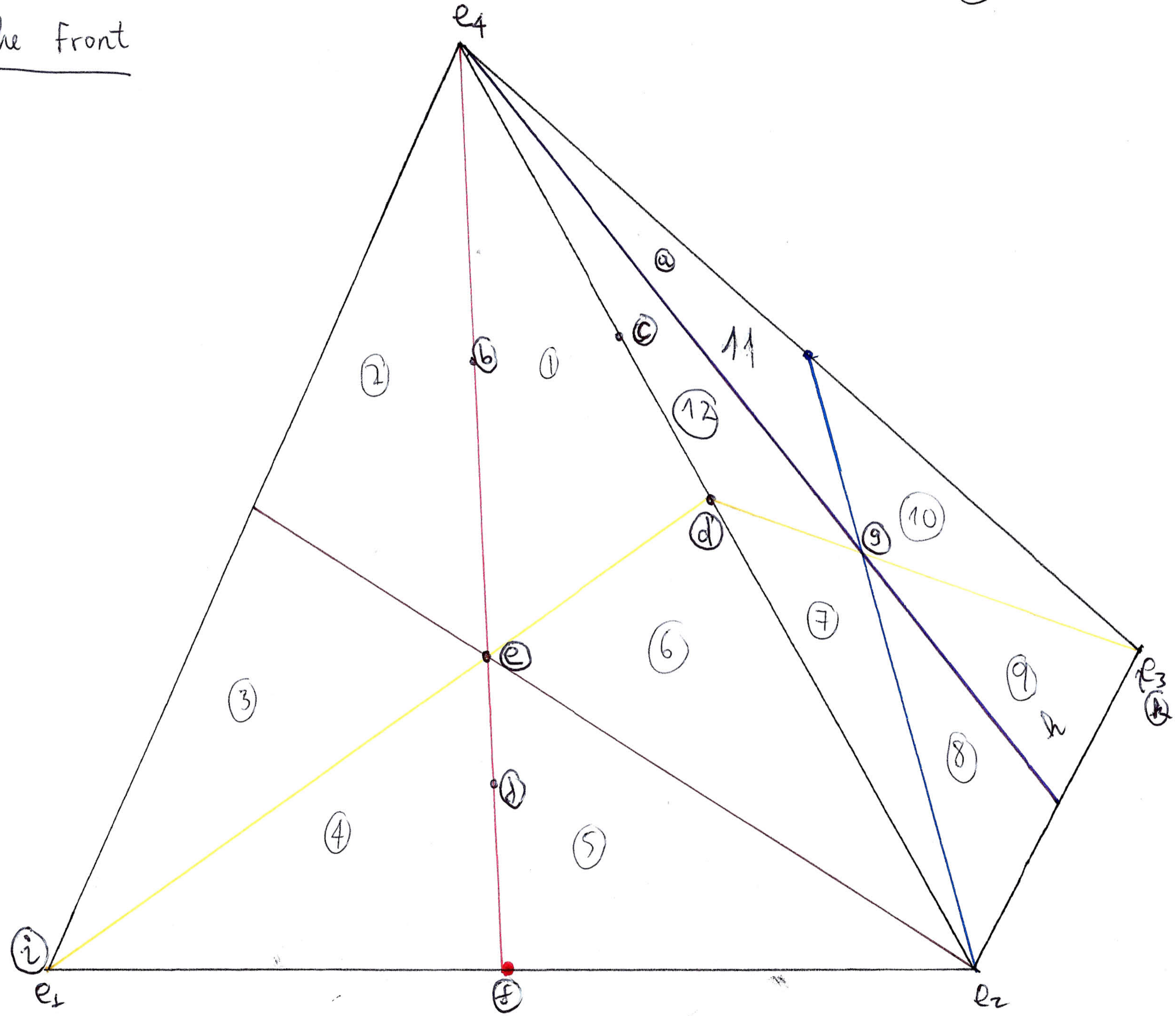
Picture of the Front



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(b)

Region	Permutation 1 2 3 4	
1	3 1 2 4	$x_3 < x_1 < x_2 < x_4$
2	3 2 1 4	$x_3 < x_2 < x_1 < x_4$
3	3 2 4 1	$x_3 < x_2 < x_4 < x_1$
4	3 4 2 1	$x_3 < x_4 < x_2 < x_1$
5	3 4 1 2	$x_3 < x_4 < x_1 < x_2$
6	3 1 4 2	$x_3 < x_1 < x_4 < x_2$
7	1 3 4 2	$x_1 < x_3 < x_4 < x_2$
8	1 4 3 2	$x_1 < x_4 < x_3 < x_2$
9	1 4 2 3	$x_1 < x_4 < x_2 < x_3$
10	1 2 4 3	$x_1 < x_2 < x_4 < x_3$
11	1 2 3 4	
12	1 3 2 4	

(c) For $i \neq j \in \{1,2,3,4\}$ we put $x_i > x_j$ to be the positive side of $x_i = x_j$ if $i > j$

Face	Sign vector
a	$(+,+,+,+,+,+)$
b	$(0,-,+,-,+,+)$
c	$(+,0,+,-,+,+)$
d	$(+,0,+,-,0,+)$
e	$(0,-,0,-,0,+)$
f	$(0,-,-,-,-,0)$
g	$(+,+,+,0,0,0)$
h	$(+,+,+,+,-,-)$
k	$(0,+,0,+,0,-)$
i	$(-,\cancel{0},-,0,0,0)$
j	$(0,-,-,-,-,+)$

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① Assume that A is a hyperplane arrangement with signs defined for each hyperplane of A. Then there is a face that corresponds to a given sign vector if and only if there is one vector in the groundspace that satisfies the inequalites-equalities implied by the vector.

With this in mind we can ~~easy~~ describe the faces of A_{n-1} very easy. All the inequalities-equalities are of the form $x_i > x_j$ or $x_i = x_j$, thus what we get is a coherent ordering of the variables. This induces an ordered partion ~~naturally~~ in the following way:

1) $i \sim j$ if $x_i = x_j$

2) $[i] < [j]$ if $x_i < x_j$

This injects the faces of A_{n-1} in the ordered partitions, ~~and the surjectivity is tri~~ The surjectivity follows also from the first fact. For $[i_1] < [i_2] < \dots < [i_j]$ a partition the define $x_u = x_j$ if $u \sim j$ and $x_u < x_j$ if $[u] < [j]$. And this gives a face.

Example: We use (a) with the face d. the sign vector gives: $x_4 > x_1$, $x_2 > x_3$, $x_2 = x_4$, $x_4 > x_3$. $x_2 > x_1$, $x_3 = x_1$,

that is: $x_1 = x_3 < x_2 = x_4$, thus we get the ordered partition: $\{1, 3\} < \{2, 4\}$.