Deep Learning for Haemodynamics

Emmanuel Abbe, Federico Betti, Annalisa Buffa,
Simone Deparis, Ortal Yona Senouf, Riccardo Tenderini

Dataset generation - Geometry deformation

- The initial geometry is a **bifurcation** segmented from CT-scans [3].
- The **presence of a stenosis** was simulated by deforming the vessel boundary. The deformation is imposed by means of the diameter stenosis and the elongation of the stenosis along the flow direction. We solve the following linear elasticity problem:

$$\begin{cases} -\operatorname{div} \boldsymbol{\sigma}(\boldsymbol{u}) = 0 & \text{in } \Omega, \\ \boldsymbol{u} = \boldsymbol{0} & \text{on } \partial\Omega \backslash \Gamma_{\text{wall}}, \\ \boldsymbol{u} = \boldsymbol{\phi} & \text{on } \Gamma_{\text{wall}}, \end{cases} \tag{1}$$

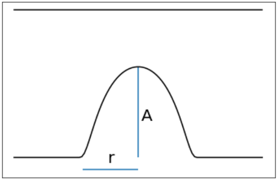
where $\boldsymbol{\sigma}(\boldsymbol{u}) = 2\mu\boldsymbol{\varepsilon}(\boldsymbol{u}) + \lambda \mathrm{Tr}(\boldsymbol{\varepsilon}(\boldsymbol{u}))\boldsymbol{I}$, $\boldsymbol{\varepsilon}(\boldsymbol{u}) = \frac{\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T}{2}$.
Chosen a point $\boldsymbol{c} \in \Gamma_{\text{wall}}$ and $A, r \in \mathbb{R}^+$, we take as boundary datum

$$\boldsymbol{\phi}(\boldsymbol{x}; A, r, \boldsymbol{c}) = -A\,\hat{\phi}\left(\frac{||\boldsymbol{x} - \boldsymbol{c}||_{\boldsymbol{D}}}{r}\right)\boldsymbol{n}(\boldsymbol{c}), \tag{2}$$

where $\hat{\phi}$ is the standard one-dimensional mollifier.

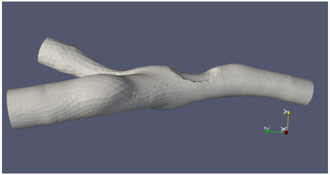
Dataset generation - Geometry deformation

Simplified 2D-model of a stenosis.



$\Downarrow$

Application to 3D-model.



Dataset generation - Haemodynamics

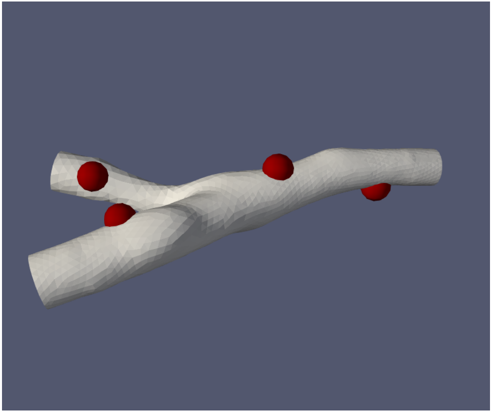
On the deformed geometry, we solve the **steady incompressible Navier-Stokes equations** to simulate blood dynamics. They read as

$$\begin{cases} \rho\,(\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u} - \operatorname{div}\boldsymbol{\sigma}(\boldsymbol{u},p) = 0 & \text{in } \Omega, \\ \operatorname{div}\boldsymbol{u} = 0 & \text{in } \Omega, \\ \boldsymbol{\sigma}(\boldsymbol{u},p)\cdot\boldsymbol{n} = \boldsymbol{0} & \text{on } \Gamma_{\text{out}}^{[1]} \cup \Gamma_{\text{out}}^{[2]}, \\ \boldsymbol{u} = \boldsymbol{0} & \text{on } \Gamma_{\text{wall}}, \\ \int_{\Gamma_{in}} \boldsymbol{u}\cdot\boldsymbol{n} = Q, \end{cases} \tag{3}$$

where $\boldsymbol{\sigma}(\boldsymbol{u},p) = -p\boldsymbol{I} + 2\mu\boldsymbol{\varepsilon}(\boldsymbol{u})$. That is, together with the flow conservation, we impose homogeneous Neumann (parola che sa Riccardo e poi da mettere in grassetto) at the outlets and homogeneous Dirichlet (**rigid vessel**) on the vessel wall. Finally, we prescribe an **inflow rate Q**.

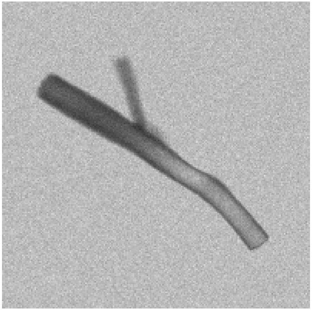
Training dataset

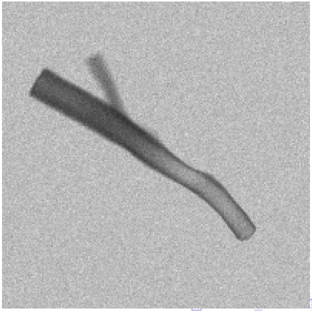
The generated dataset consists of **8100 finite element solutions** for 9 different values of the inflow rate Q, 15 of the two aforementioned stenosis parameters A and r and with 4 possible locations for the stenosis.



Training dataset

- For each simulation, **2 BW snapshots** (200x200 pixels) at 30 degrees of distance are taken and Poisson random noise is added. The snapshots show the **velocity heatmap** in the vessel and they are as reminiscent as possible of angiography images.
- We enriched the dataset by repeating these pipeline from 5 different camera angles. There may be views from which the stenosis is completely hidden.
- In total, the dataset is made of **40 thousand images**.





Network architecture

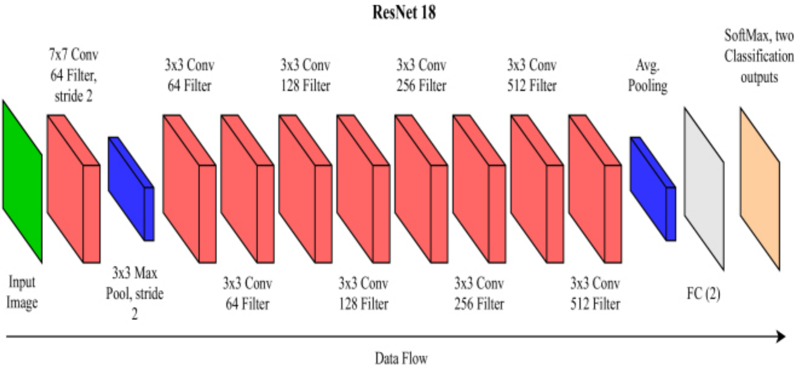


Figure: ResNet18 architecture used for training

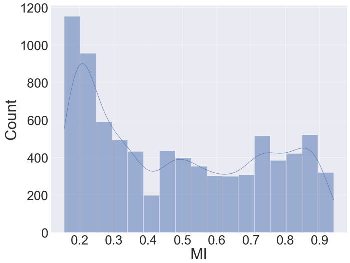
Single tasks results

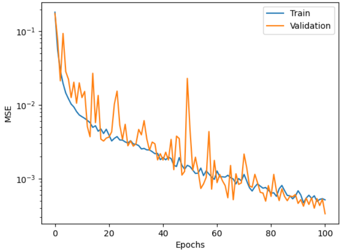
- The main goal is to **predict the MI risk** associated to the presence of the stenosis. We designed it as a three factors product:

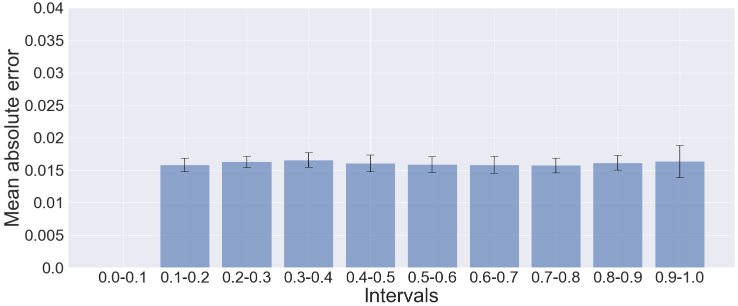
$$MI = \tanh\left(\sqrt{\frac{r^2}{2K}}\exp(A)\right). \tag{4}$$

  - A is the **diameter stenosis**.
  - r is a **risk factor** exponentially decreasing from the inlet to the outlets which **forces the model to learn the position of the stenosis**.
  - $\sqrt{\frac{1}{2K}}$, with $K = \left(1 + |\frac{WSS}{WSS_0}|^2\right)^{-1}$, is a commonly used factor in the modeling of plaque growth. WSS denotes an **average value of the Wall Shear Stress at the throat of the plaque** and WSS_0 is a reference value measured in the absence of stenosis.
  - The tanh clips all values into $(0, 1)$.
- To help in the MI risk prediction, we consider the Q-FFR, the diameter stenosis and the stenosis position as auxiliary tasks.

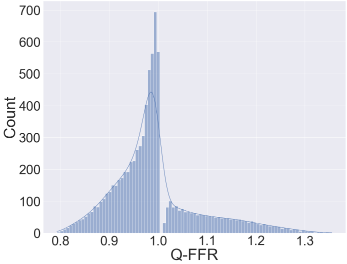
Single tasks results - MI

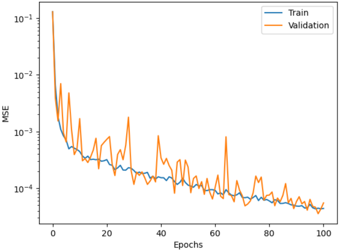


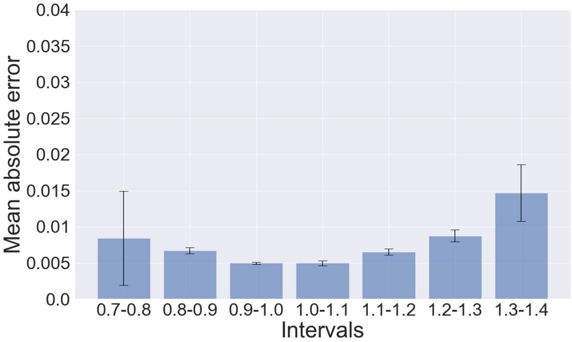




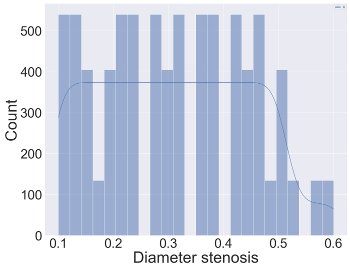
Single tasks results - Q–FFR

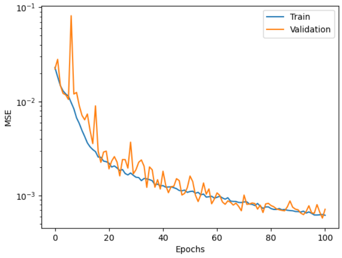


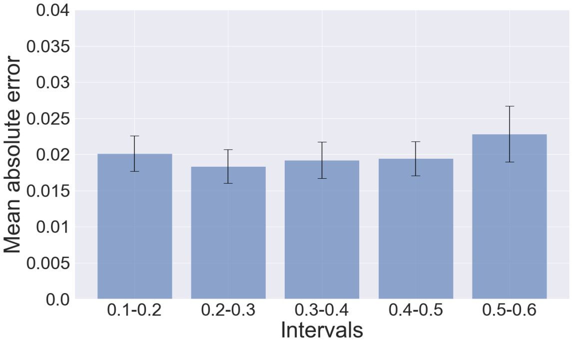




Single tasks results - Diameter stenosis







Single tasks results - Stenosis position

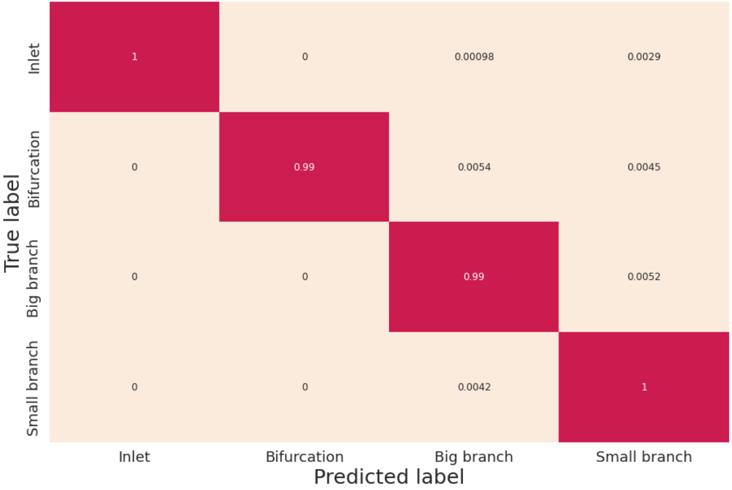
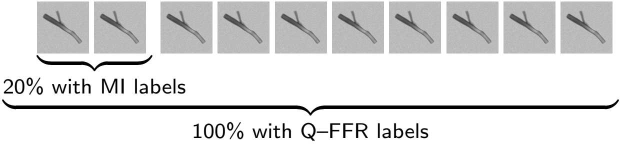


Figure: Confusion matrix for classification of the stenosis position

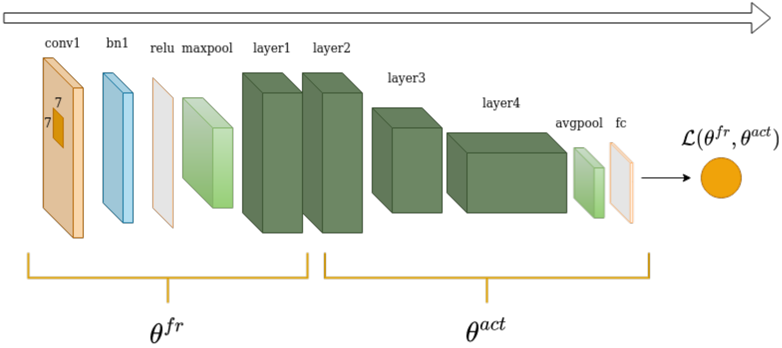
Transfer learning

- The scenario is the following:



- The goal is to compare the performances of two models trained on MI predictions with 20 % of the data:
  - Starting from **random initialization**.
  - Using as **inductive bias** a **model pre–trained with Q–FFR predictions** on the whole dataset [4].
- The diameter stenosis, which can be estimated from angiography images, was also tested as inductive bias.

Transfer learning



$$\theta_{t+1}^{fr} = \theta_t^{fr} - \alpha_t^{fr} \nabla_{\theta_t^{fr}} \mathcal{L}(\theta_t^{fr}, \theta_t^{act}) \qquad \theta_{t+1}^{act} = \theta_t^{act} - \alpha_t^{act} \nabla_{\theta_t^{act}} \mathcal{L}(\theta_t^{fr}, \theta_t^{act})$$

Transfer learning

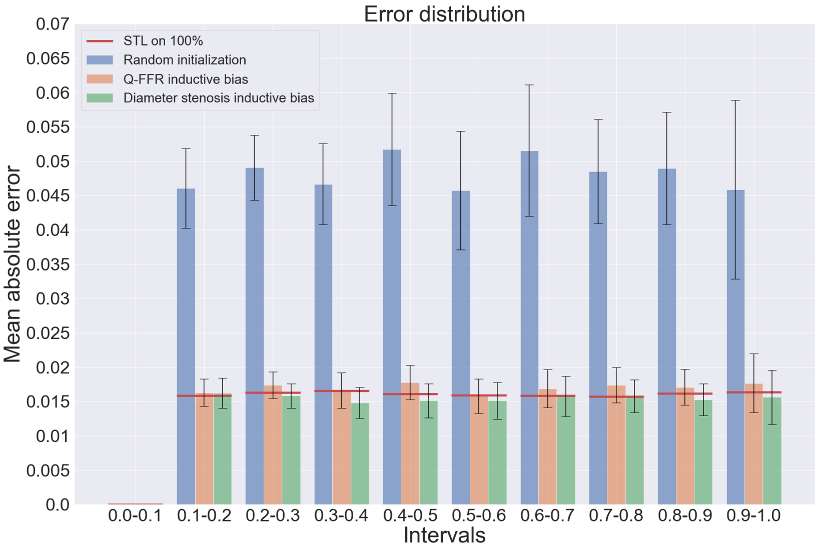
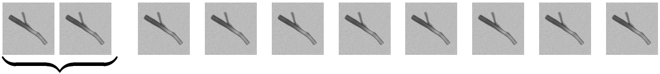


Figure: Error distribution with/without inductive bias from easier tasks

Multitask learning

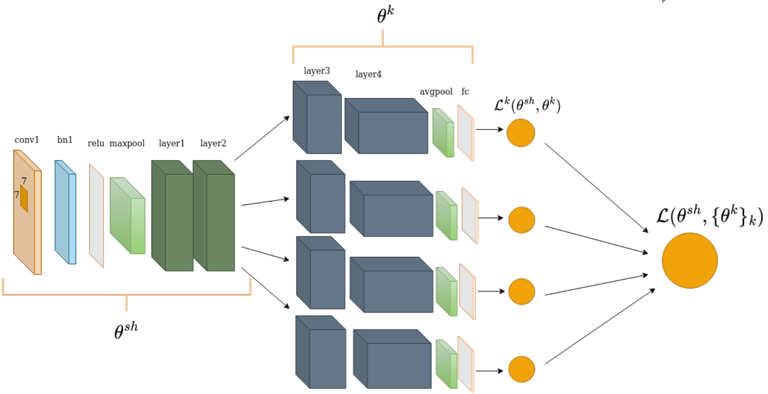
- The scenario is the following:



20% with all labels

- The goal is to compare the performances of models trained on MI predictions with 20 % of the data:
  - Using a standard **single task learning** algorithm.
  - Using a **multitask learning** algorithm to leverage **domain specific feature sharing**.
- The tasks to be included are an additional hyper-parameter: by construction of the MI function we may include also the diameter stenosis and the stenosis position as auxiliary tasks.

Multitask learning



$$\theta_{t+1}^{sh} = \theta_t^{sh} - \alpha_t \sum_k w^k \nabla_{\theta_t^{sh}} \mathcal{L}^k(\theta_t^{sh}, \theta_t^k) \qquad \theta_{t+1}^k = \theta_t^k - \alpha_t \nabla_{\theta_t^k} \mathcal{L}^k(\theta_t^{sh}, \theta_t^k)$$

Multitask learning

- In MTL, the goal is to properly choose the w^k for the update of the shared parameters, under the constraint that $\sum_k w^k = 1$.
- Usually, when there is not a clear hierarchy between the tasks, the goal is to make sure that the progress in all tasks is the same.
- **Weighted Dynamical Average (WDA): [2]**

$$w^k = w^k(t) = \frac{e^{\lambda_k/T}}{\sum_j e^{\lambda_j/T}}, \ \text{with} \ \lambda_k = \frac{\mathcal{L}^k(\theta^{sh}_{t-1}, \theta^k_{t-1})}{\mathcal{L}^k(\theta^{sh}_{t-2}, \theta^k_{t-2})} \tag{5}$$

Multitask learning

- In a different fashion, to prioritize one particular task, one can adapt the w^k prioritizing the performance on one main task. For us, the MI is such main task.
- **Adaptive Auxiliary Tasks (OL-AUX-N): [1]** Learn w^k at training time with the update rule

$$\Delta w^{aux,k} = \frac{\alpha}{N} \sum_{j=0}^{N-1} \nabla_{\theta^{sh}_{t-j}} \mathcal{L}^{\text{main}}(\theta^{sh}_{t-j}, \theta^{main}_{t-j})^T \nabla_{\theta^{sh}_{t-j}} \mathcal{L}^{\text{aux, k}}(\theta^{sh}_{t-j}, \theta^{aux,k}_{t-j})$$

- **Cosine similarity:** Add $\nabla_{\theta^{sh}_t} \mathcal{L}^{\text{k}}(\theta^{sh}_t, \theta^{aux,k}_t)$ to the update of θ^{sh} only if $\nabla_{\theta^{sh}_t} \mathcal{L}^{\text{main}}(\theta^{sh}_t, \theta^{aux,k}_t)^T \nabla_{\theta^{sh}_t} \mathcal{L}^{\text{k}}(\theta^{sh}_t, \theta^{aux,k}_t) \geq 0$.

In both cases, one can apply a softmax on the w^k to enforce the constraint $\sum_k w^k = 1$.

Multitask learning - Task gradients correlation

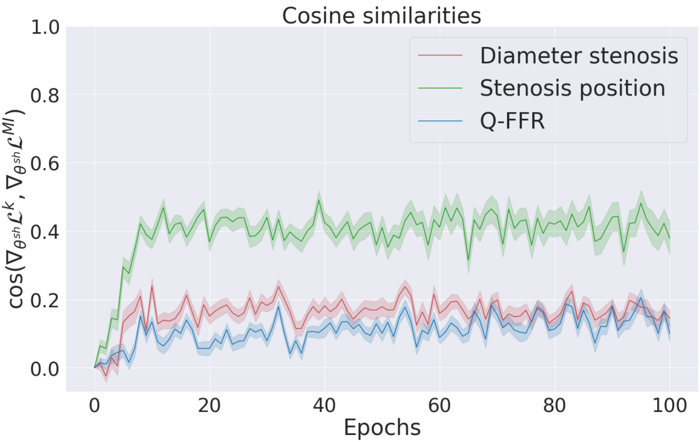


Figure: Gradients correlations with the MI gradients during training

Multitask Learning - All auxiliary tasks

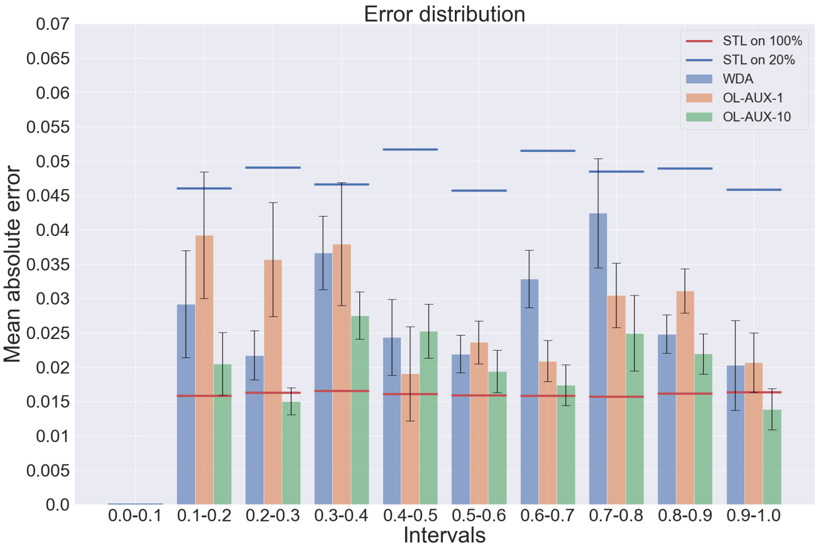


Figure: Error distribution of the MI for different algorithms

Multitask learning - Q–FFR only auxiliary task

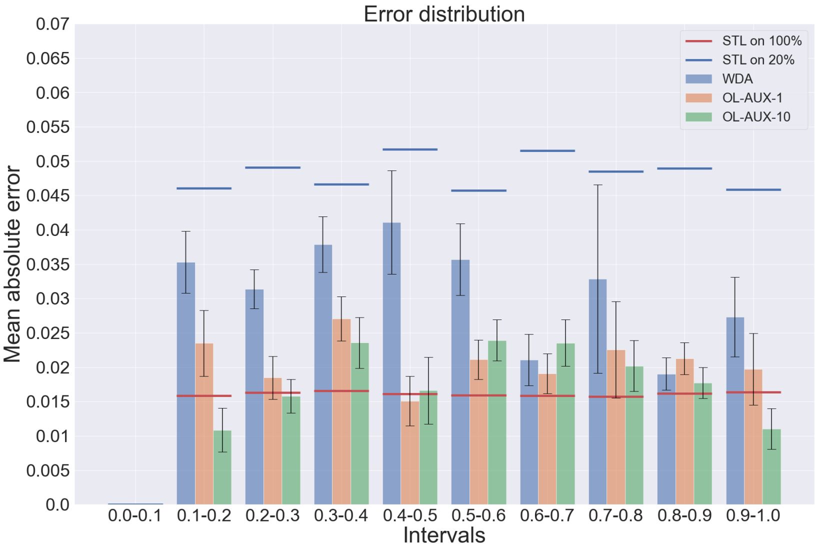


Figure: Error distribution of the MI for different algorithms

Summary

- We started from a dataset of 40 thousand couples of BW images of the velocity field on a bifurcation with stenosis.
- **Goal**: Predicting the MI risk associated with the presence of a stenosis.
- **Methods and results**:
  - **Transfer learning:** A model for MI risk trained on few data with the inductive bias of a model trained on the whole dataset with Q–FFR predictions performs as well as a model trained for MI on the whole dataset.
  - **Multitask learning:** Using the domain-specific feature sharing induced by multitask learning on few data, we reach comparable results with respect to the single task learning model on the whole dataset. The Q–FFR on its own as an auxiliary task outperforms a combination of Q–FFR, diameter stenosis and stenosis position.

References

[1] Xingyu Lin et al. "Adaptive auxiliary task weighting for reinforcement learning". In: *Advances in neural information processing systems* 32 (2019).

[2] Shikun Liu, Edward Johns, and Andrew J Davison. "End-to-end multi-task learning with attention". In: *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*. 2019, pp. 1871–1880.

[3] Emilie Marchandise et al. "Quality open source mesh generation for cardiovascular flow simulations". In: *Modeling of Physiological Flows*. Springer, 2012, pp. 395–414.

[4] Sinno Jialin Pan and Qiang Yang. "A survey on transfer learning". In: *IEEE Transactions on knowledge and data engineering* 22.10 (2009), pp. 1345–1359.