Deep Learning for Hæmodynamics

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Geometry deformation

- The original geometry is a **patient–specific femoropopliteal bypass**, segmented from CT–scans [3]. We reverted the flow and considered it as a **bifurcation**.
- The **presence of a stenosis** is obtained by deforming the vessel boundary. We solve the **linear elasticity problem**:

$$\begin{cases} -\operatorname{div} \boldsymbol{\sigma}(\boldsymbol{u}) = 0 & \text{in } \Omega, \\ \boldsymbol{u} = \boldsymbol{0} & \text{on } \partial\Omega \backslash \Gamma_{\text{wall}}, \\ \boldsymbol{u} = \boldsymbol{\phi} & \text{on } \Gamma_{\text{wall}}, \end{cases}$$

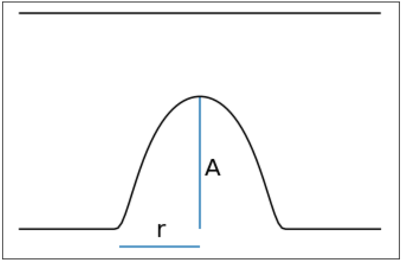
where $\boldsymbol{\sigma}(\boldsymbol{u}) = 2\mu \left(\frac{\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T}{2} \right) + \lambda \mathrm{Tr} \left(\frac{\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T}{2} \right) \boldsymbol{I}$. Let $\boldsymbol{c} \in \Gamma_{\text{wall}}$ (center) and $A, r \in \mathbb{R}^+$ (depth and length); then

$$\boldsymbol{\phi}(\boldsymbol{x}; A, r, \boldsymbol{c}) = -A \hat{\phi} \left(\frac{||\boldsymbol{x} - \boldsymbol{c}||_D}{r} \right) \boldsymbol{n}(\boldsymbol{c}),$$

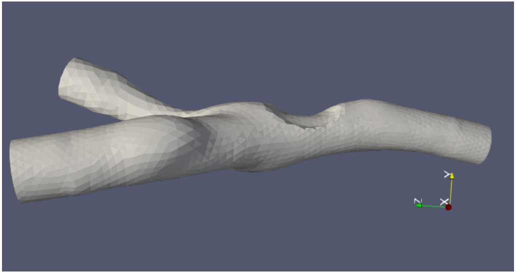
where $\hat{\phi}$ is the standard 1D mollifier.

Geometry deformation — Visualization

Simplified 2D–model of the stenosis



Application to the bifurcation 3D–model



Numerical simulation of hæmodynamics

- On the deformed geometry, we solve the **steady incompressible Navier–Stokes equations** to simulate blood flow.

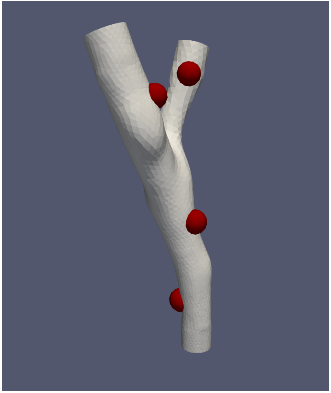
$$\begin{cases} \rho\,(\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u} - \operatorname{div}\boldsymbol{\sigma}(\boldsymbol{u},p) = 0 & \text{in } \Omega \\ \operatorname{div}\boldsymbol{u} = 0 & \text{in } \Omega \\ \boldsymbol{\sigma}(\boldsymbol{u},p)\cdot\boldsymbol{n} = \boldsymbol{0} & \text{on } \Gamma_{\text{out}}^{[1]} \cup \Gamma_{\text{out}}^{[2]} \\ \boldsymbol{u} = \boldsymbol{0} & \text{on } \Gamma_{\text{wall}} \\ \int_{\Gamma_{in}} \boldsymbol{u}\cdot\boldsymbol{n} = Q & \text{on } \Gamma_{\text{in}} \end{cases}$$

where $\boldsymbol{\sigma}(\boldsymbol{u},p) = -p\boldsymbol{I} + 2\mu\left(\frac{\nabla\boldsymbol{u}+(\nabla\boldsymbol{u})^T}{2}\right)$. Namely, we impose homogeneous Neumann BCs at the two outlets, an **inflow rate** Q at the inlet and homogeneous Dirichlet BCs on the wall.

- Concerning numerical discretization, we employ the **finite element method**, with P1–P1 elements and SUPG stabilization term.

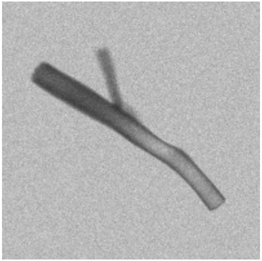
Postprocessing & Dataset construction

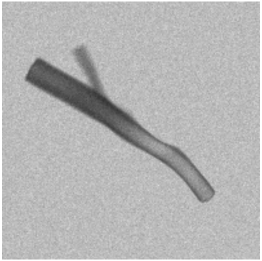
- The dataset is generated from **8'100 numerical solutions**, obtained by solving the NS equations for 9 different values of the inflow rate Q, 15 of the stenosis parameters A, r and considering 4 distinct locations for the stenosis center $\boldsymbol{c}$.
- For each simulation, we store:
  - The **velocity and pressure fields**
  - The characteristic **parameter values** (Q, A, r, $\boldsymbol{c}$)
  - The **Q–FFR** in the outlet branches
  - The **average WSS at the plaque throat**



Postprocessing & Dataset construction

- For each simulation, we take **2 BW snapshots** (200x200 pixels) at $\approx 30^\circ$ of distance and we add **Poisson random noise**. The snapshots show the **velocity heatmap** and they are as reminiscent as possible of **angiography images**.
- We further enrich the dataset by considering **5 different camera angles**. In total, the dataset is made of **40'500 images**. *Caveat: there may be views from which the stenosis is completely hidden.*





Network architecture

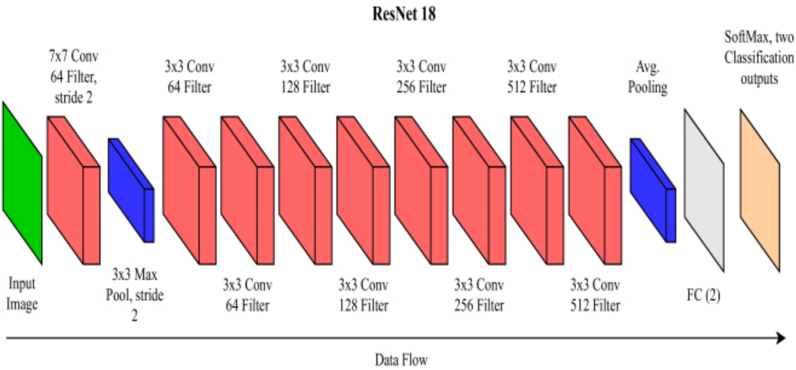


Figure: ResNet18 architecture used for training

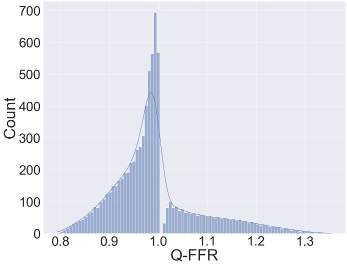
MI risk: function design

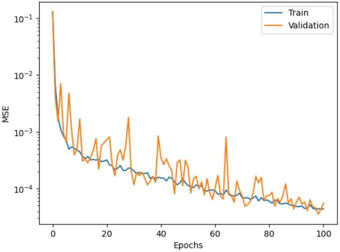
The main goal is to **predict the MI risk** associated to the presence of the stenosis. We designed it as follows:

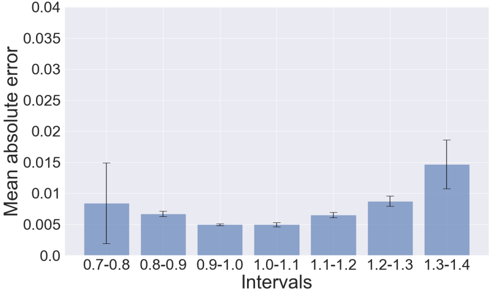
$$MI := \tanh\left(\sqrt{\frac{R^2}{2G}}\exp(A)\right) \qquad \in (0,1)$$

- A is the **diameter stenosis**.
- R is a **risk factor**, exponentially decreasing from the inlet to the outlets, hence **forcing the model to learn the stenosis location**.
- $\sqrt{\frac{1}{2G}}$ — with $G = \left(1 + \left|\frac{WSS}{WSS_0}\right|^2\right)^{-1}$ — is a common factor to many plaque growth models. Here WSS denotes an **average value of the WSS at plaque throat**, while WSS_0 is a reference value measured in absence of stenosis.
- We consider the Q–FFR, the diameter stenosis and the stenosis position as auxiliary tasks.

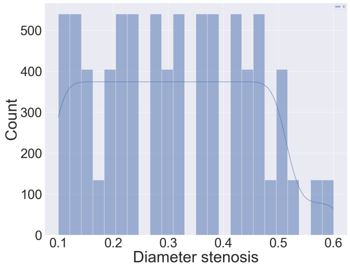
Single tasks results - Q–FFR

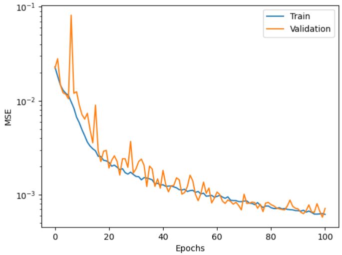


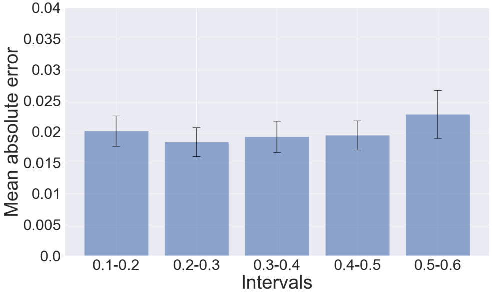




Single tasks results - Diameter stenosis







Single tasks results - Stenosis position

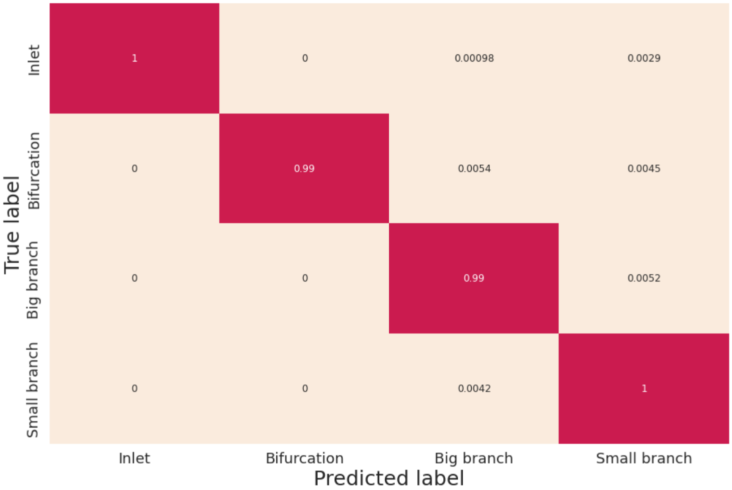
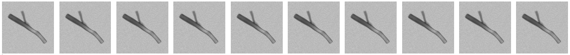


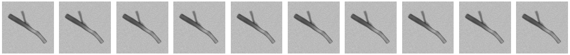
Figure: Confusion matrix for classification of the stenosis position

Scenarios for MI risk prediction

To perform MI risk prediction, we consider **three different scenarios**, based on the type and amount of data at disposal:

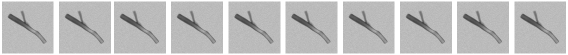
- **Scenario 1:** 

100% all labels

- **Scenario 2:** 

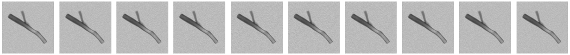
20% MI labels

100% Q–FFR labels

- **Scenario 3:** 

20% all labels

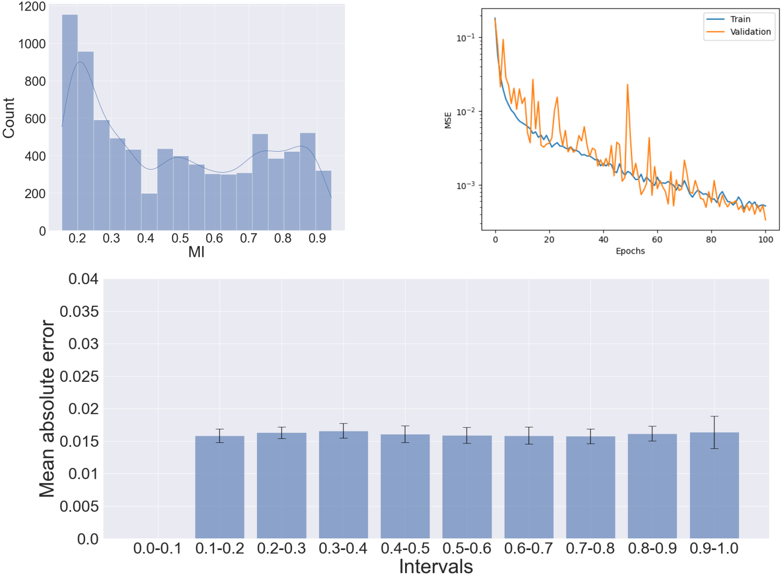
Single Task Learning

Scenario: 

100% all labels

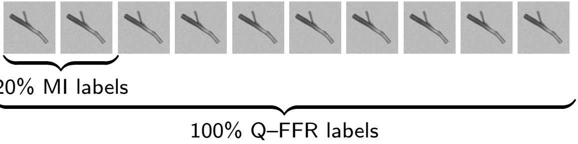
- The goal is to get a baseline model trained on MI predictions with 100 % of the data starting from random initialization.
- In the next scenarios, the goal is to obtain comparable results with respect to this scenario by leveraging the correct inductive bias or by domain specific feature sharing in multitask learning.

Single Task Learning



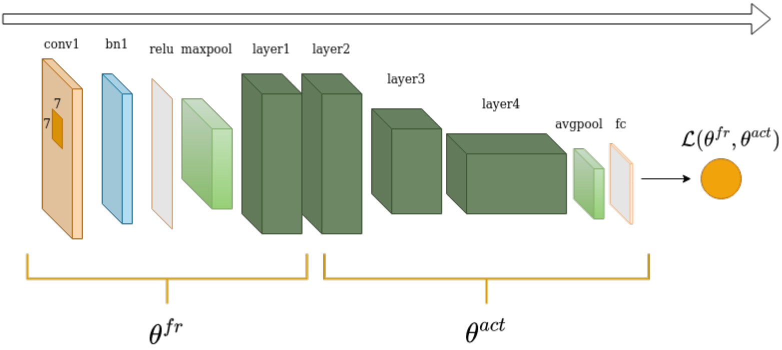
Transfer Learning

Scenario:



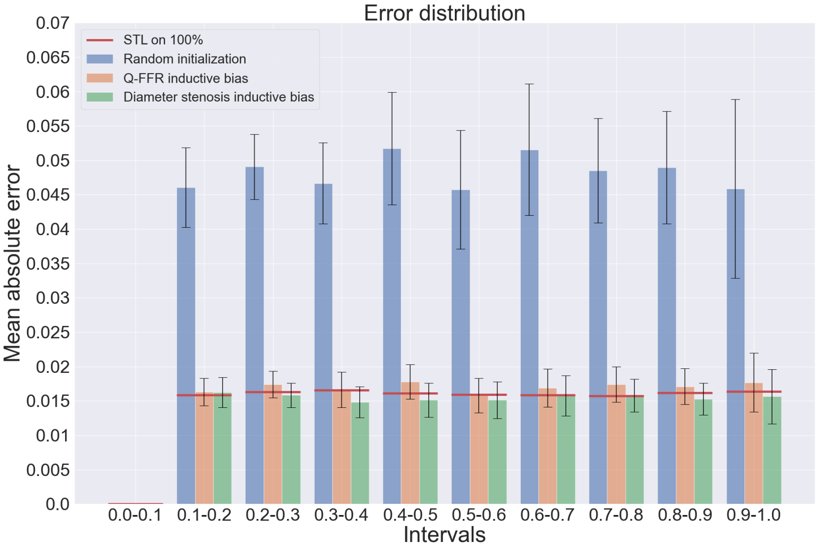
- The goal is to compare the performances of two models trained on MI predictions with 20 % of the data:
  - Starting from **random initialization**.
  - Using as inductive bias a **model pre–trained with Q–FFR predictions** on the whole dataset [4].
- The diameter stenosis, which can be estimated from angiography images, was also tested as inductive bias.

Transfer Learning



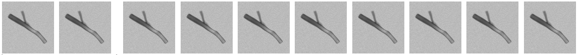
$$\theta^{fr}_{t+1} = \theta^{fr}_t - \alpha^{fr}_t \nabla_{\theta^{fr}_t} \mathcal{L}(\theta^{fr}_t, \theta^{act}_t) \qquad \theta^{act}_{t+1} = \theta^{act}_t - \alpha^{act}_t \nabla_{\theta^{act}_t} \mathcal{L}(\theta^{fr}_t, \theta^{act}_t)$$

Transfer Learning



Multitask Learning

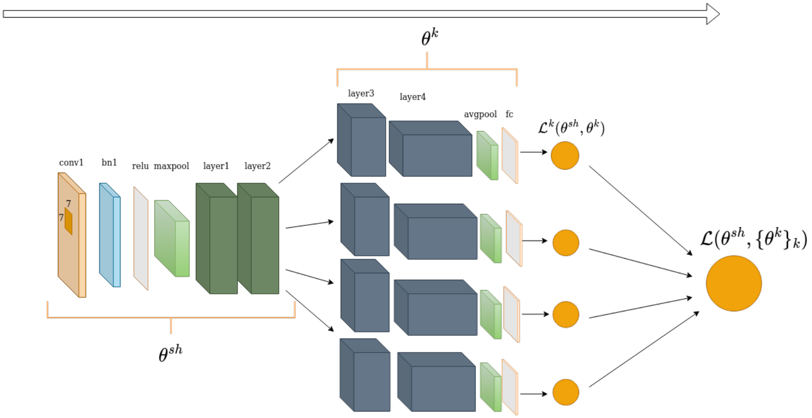
Scenario:



20% with all labels

- The goal is to compare the performances of models trained on MI predictions with 20 % of the data:
  - Using a standard **single task learning** algorithm.
  - Using a **multitask learning** algorithm to leverage **domain specific feature sharing**.
- The tasks to be included are an additional hyper–parameter: other than the Q–FFR, we may also consider the stenosis diameter and position as auxiliary tasks.

Multitask Learning



$$\theta^{sh}_{t+1} = \theta^{sh}_t - \alpha_t \sum_k w^k \nabla_{\theta^{sh}_t} \mathcal{L}^k(\theta^{sh}_t, \theta^k_t) \qquad \theta^k_{t+1} = \theta^k_t - \alpha_t \nabla_{\theta^k_t} \mathcal{L}^k(\theta^{sh}_t, \theta^k_t)$$

Multitask Learning

- In standard Multitask Learning (MTL), the loss function to be optimized over the network parameters $\mathcal{L}$ is defined as

$$\mathcal{L}(\theta^{sh}, \theta^1, \cdots, \theta^K) = \sum_{k=1}^{K} w^k \mathcal{L}^k(\theta^{sh}, \theta^k) \quad \text{with} \quad \sum_{k=1}^{K} w^k = 1,$$

  where K is the number of tasks and $\mathcal{L}_k$ is the individual loss on the k–th task.
- In a multi-objective optimization fashion, the task-specific parameters are always updated with their gradient (with weight 1) and the aim is to properly choose the w^k for the update of the shared parameters, always with $\sum_k w^k = 1$. This second approach is more suitable for the considered algorithms where usually one looks at the correlations between the gradients of the single tasks.

Multitask Learning

- Assuming no hierarchical structure exists among the tasks:
  - **Weighted Dynamical Average (WDA):** The weights are updated to ensure that the progress in all of them is the same [2]

$$w^k = w^k(t) = \frac{e^{\lambda_k/T}}{\sum_j e^{\lambda_j/T}}, \quad \text{with} \quad \lambda_k = \frac{\mathcal{L}^k(\theta^{sh}_{t-1}, \theta^k_{t-1})}{\mathcal{L}^k(\theta^{sh}_{t-2}, \theta^k_{t-2})} \tag{1}$$

- Assuming the existence of one main task:
  - **Adaptive Auxiliary Tasks (OL-AUX-N):** It updates the weights of the auxiliary ones with the update rule [1]

$$\Delta w^{aux,k} = \frac{\alpha}{N} \sum_{j=0}^{N-1} \left(\nabla_{\theta^{sh}_{t-j}} \mathcal{L}^{\text{main}}\right)^T \left(\nabla_{\theta^{sh}_{t-j}} \mathcal{L}^{\text{aux, k}}\right)$$

  - **Cosine similarity:** It adds $\nabla_{\theta^{sh}_t} \mathcal{L}^{\text{k}}$ to the update of θ^{sh} only if $\cos\left(\nabla_{\theta^{sh}_t} \mathcal{L}^{\text{main}}, \nabla_{\theta^{sh}_t} \mathcal{L}^{\text{k}}\right) \geq 0$.
  - When necessary, one can apply a softmax on the w^k to enforce the constraint $\sum_k w^k = 1$.

Multitask Learning - Gradients correlations

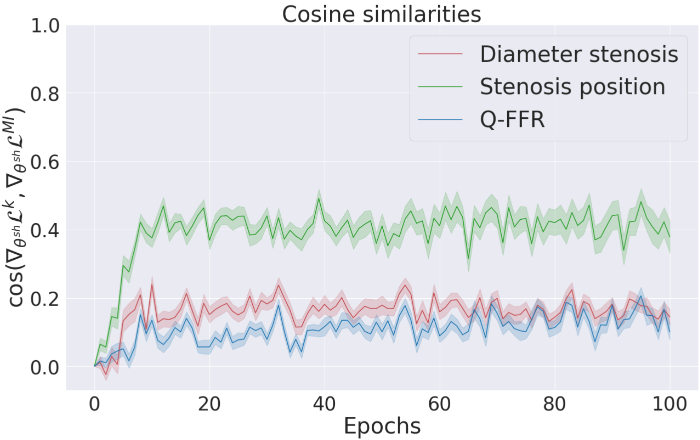
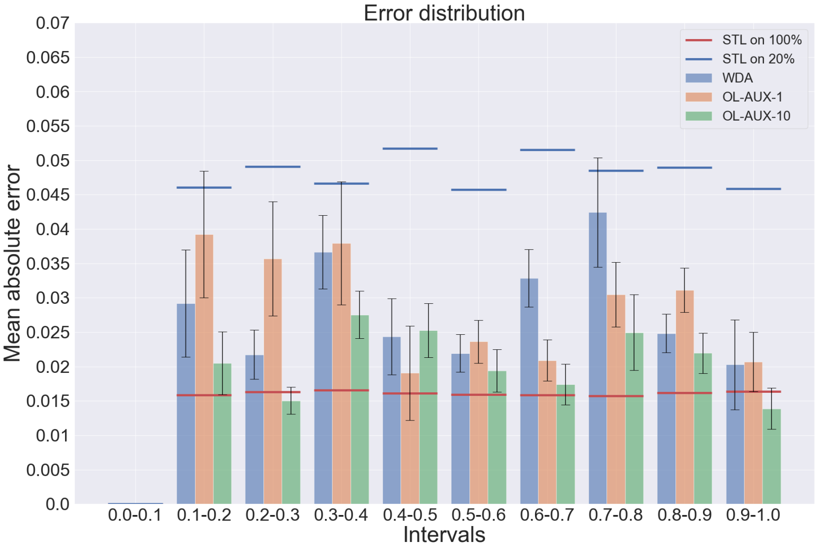
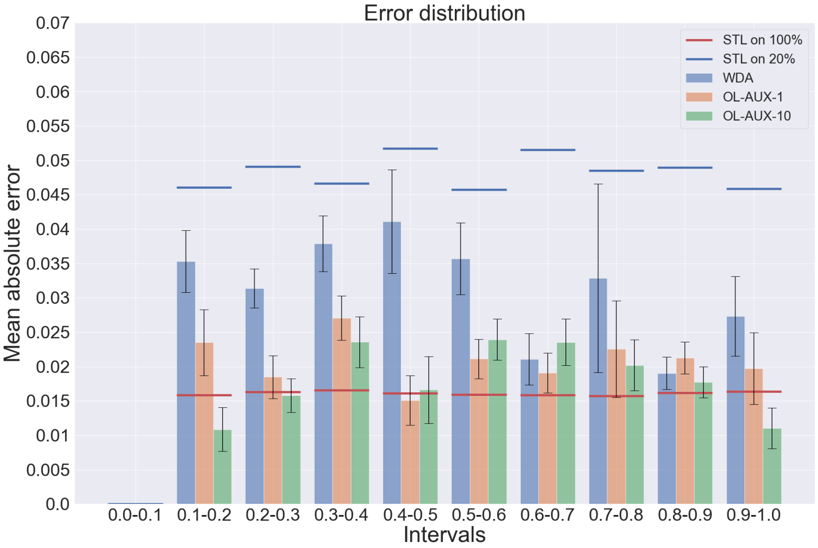


Figure: Gradients correlations with the MI gradients during training

Multitask Learning - All auxiliary tasks



Multitask Learning - Q–FFR only auxiliary task



Summary

- **Data**: 40'500 couples of BW noisy images of the velocity field in a bifurcation with stenosis.
- **Goal**: predicting the MI risk in different scenarios.
- **Results**:
  - **Single Task Learning**: if the MI labels are available on the whole dataset, ResNet18 predicts MI risk with absolute errors of roughly 0.015.
  - **Transfer Learning:** using a model trained on the whole dataset with Q–FFR predictions as inductive bias, we retain performances comparable to STL in Scenario 1.
  - **Multitask Learning:** with the OL–AUX–10 method, using the domain–specific feature sharing on few data, we attain results comparable to STL in Scenario 1. Best results are obtained using only Q–FFR as auxiliary task.

References

[1] Xingyu Lin et al. "Adaptive auxiliary task weighting for reinforcement learning". In: *Advances in neural information processing systems* 32 (2019).

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[3] Emilie Marchandise et al. "Quality open source mesh generation for cardiovascular flow simulations". In: *Modeling of Physiological Flows*. Springer, 2012, pp. 395–414.

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