Assignment

Homework HW3

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Musical Acoustics

POLITECNICO
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November 12, 2024

Guitar Sound Synthesis

Bridge impedance

In order to derive the bridge impedance in the range $[0 \quad 500][Hz]$, the guitar was modeled via the **two-mass system** depicted in Figure 1, that was then implemented in *Simulink/Simscape* (Figure 2). The given data are reported in Table 1.

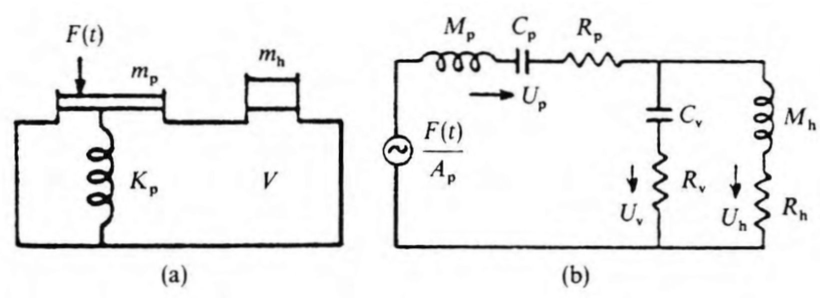


Figure 1

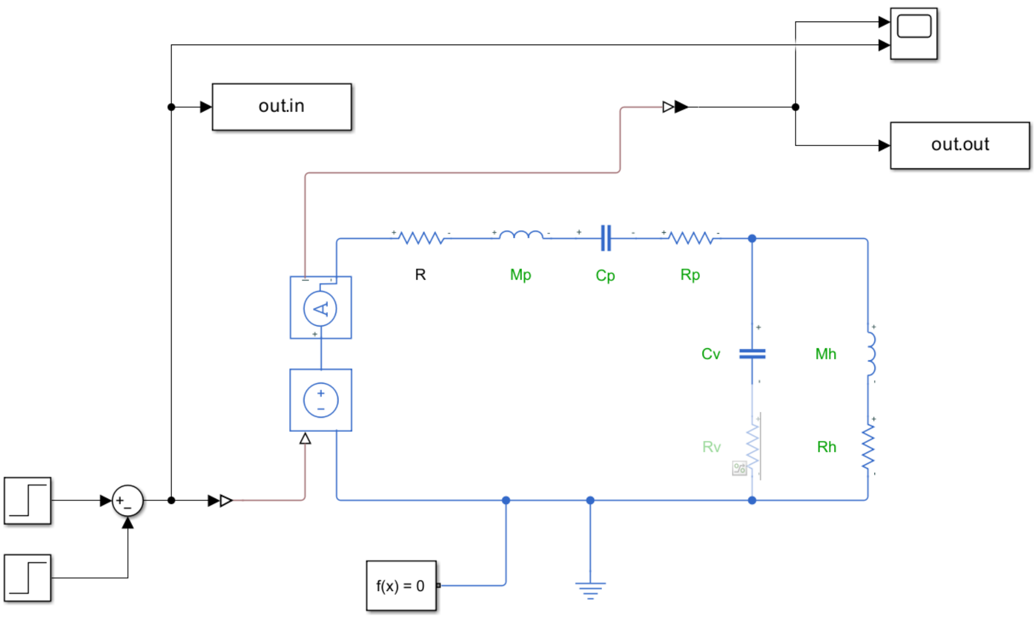


Figure 2

$k_p\ [N/m]$	$m_p\ [kg]$	$A_p\ [m^2]$	$r_p\ [Nm/kg/s]$	$m_h\ [kg]$	$A_h\ [m^2]$	$RH\ [N/m]$	$V\ [m^3]$
$1.41{\cdot}10^5$	$4.93{\cdot}10^{-2}$	$1.44{\cdot}10^{-2}$	32	$8.04{\cdot}10^{-4}$	$7.85{\cdot}10^{-3}$	30	$1.72{\cdot}10^{-2}$

Table 1

The *auxiliary resistance* $R = 5\ [k\ ]$ was employed in order to reduce noise, thus **improving the definition** of the peaks and dives of the resulting impedance (the lower the resistance, the narrower the widths of the impedance peaks and dives).

The air volume resistance R_v is assumed to be zero, and it is thus commented through in the electric equivalent. In order to compute the bridge impedance, the system is excited with a **unitary impulse**, whose width ε depends on the *sampling frequency* $F_s = 44.1\ [kHz] = {}^1/_{\varepsilon}$. In addition to that, it is necessary to pay attention to the time at which the impulse happens; if the input does not start at $t = 0$, this will result in a **uniformly distributed spectral content** spread all over the considered frequency band: this is obviously due to the employment of the Fourier transform on a signal that contains a step.

The input voltage and the current circulating in the first loop of the circuit (measured via an ammeter) respectively stand for the acoustic pressure p and the volume acoustic flow U in the acoustic analog; during the simulation, they are exported to the MatLab workspace, where the **bridge impedance** is computed in accordance with the following expression:

$$Z_B(\omega) = \frac{\mathcal{F}\{p(t)\}}{\mathcal{F}\{U(t)\}} = \frac{\mathcal{F}\{V(t)\}}{\mathcal{F}\{I(t)\}} \tag{1}$$

where the operator $\mathcal{F}\{\square\}$ represents the ***fast Fourier transform***. The impedance curve obtained via this procedure is reported in Figure 3.

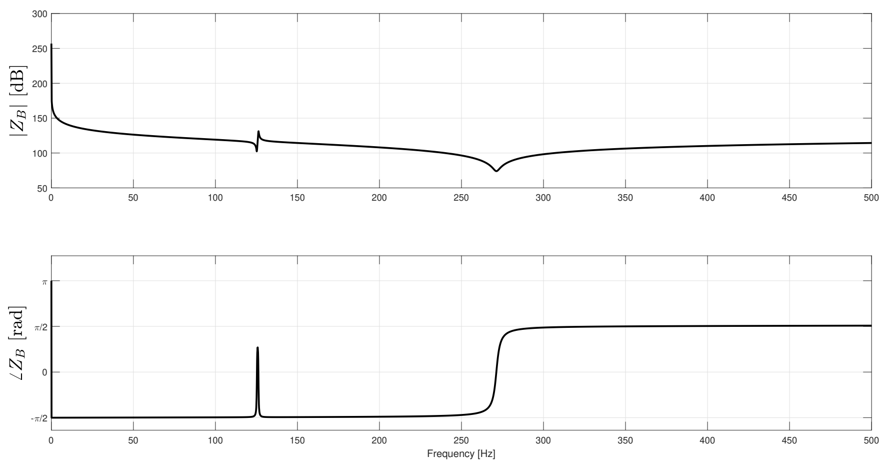


Figure 3

The impedance reflects the expected behaviour of the system, with **two resonances** separated by an **anti-resonance**: driving point FRFs with more than one resonance are always characterized by *one antiresonance between each couple of resonances*.

Transfer function: Plucking Point - String

We can model the transfer function between the excitation point and the bridge as:

$$H_{EB}(\omega) = \frac{F(\omega)}{X(\omega)} = \frac{1}{2}\left[1 + H_{E_2R_1}(\omega)\right] \frac{H_{E_1R_1}(\omega)}{1 - H_{\text{loop}}(\omega)} \frac{Z(\omega)}{\omega} \left[1 - R_b(\omega)\right] \tag{2}$$

where $F(\omega)$ is the force exerted on the bridge by the strings, the wave variable $X(\omega)$ is the acceleration at the excitation point and $Z(\omega)$ is the normalized bridge impedance:

$$Z(\omega) = \frac{Z_B(\omega)}{\max\{|Z_B(\omega)|\}} \tag{3}$$

In order to understand the meaning of the other components of the expression, let's consider the dual delay-line waveguide model depicted in Figure 4:

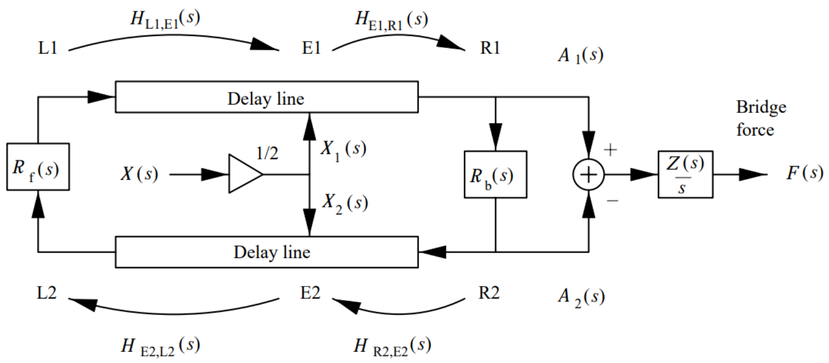


Figure 4: Dual delay-line waveguide model for a plucked string with output at the bridge.

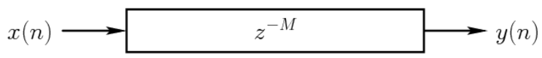


Figure 5: Block diagram representation for a standard integer M delay line.

The reflection filters $R_b(\omega)$ and $R_f(\omega)$ are related to the travelling of the waveforms through the delay lines and produce a phase inversion and slight frequency-dependant damping. Moreover, $H_{\text{loop}}(\omega)$ is the transfer function of the modeled string, $H_{E_1R_1}(\omega)$ is the transfer function from the plucking point to the bridge and $H_{E_2R_1}(\omega)$ is the transfer function from the plucking point to the bridge through the nut. The delay-lines are built in the z-domain, as illustrated in Figure 5.

The idea behind the adopted procedure is a *sampling of the wave travelling along the string in the frequency domain*, where the number of samples N_s is determined as a function of the string's fundamental frequency f_0. For a guitar tuned in standard E, the **fundamental frequencies** are the following (starting from the low E up to the high E string):

$$\underline{f}_0 = \begin{bmatrix} 82.41 & 110.00 & 146.83 & 196.00 & 246.94 & 329.63 \end{bmatrix}$$

Thus:

$$N_s = \left\lfloor \frac{F_s}{2 \cdot f_0} \right\rfloor = \begin{bmatrix} 267 & 200 & 150 & 112 & 89 & 66 \end{bmatrix} \tag{4}$$

At this point, in order to understand the computational procedure employed to synthesize the sound of each of the six strings, let us consider a generic string index i and a generic plucking position ratio $\beta = 1/g$ (i.e. the string is considered to be excited at *one g-th of its length*). We define the two parameters N_{left} and N_{right} as:

$$\begin{cases} N_{\text{left}} = \lfloor \beta \cdot N_{s_i} \rfloor \\ N_{\text{right}} = N_{s_i} - N_{\text{left}} \end{cases} \tag{5}$$

This corresponds to subdividing the travelling path of the wave along the string (i.e. the loop) in two sections separated by the excitation point and whose extremities are represented by the nut and the bridge, sampled in the frequency domain according to Equation 5. The reflection filters in correspondence with the nut and the bridge are respectively approximated as $R_f = R_b = -0.99$. Before proceeding, we need to define the z variable, in order to switch domains: $z = e^{j\omega T} = e^{j\omega F_s^{-1}}$. Now, by following the corresponding path in the waveguide model (refer to Figure 4) we can build the necessary transfer functions:

$$H_{E_2R_1}(\omega) = z^{-2N_{\text{left}}} \cdot z^{-N_{\text{right}}} \cdot R_f$$

$$H_{E_1R_1}(\omega) = z^{-N_{\text{right}}}$$

$$H_{\text{loop}}(\omega) = z^{-2N_{\text{left}}} \cdot z^{-2N_{\text{right}}} \cdot R_b \cdot R_f$$

Now we have all that we need to compute the transfer function $H_{EB}(\omega)$ (Equation 2). By fixing $\beta = 1/5 = 0.20$ and repeating the procedure for each of the six strings ($i = 1, \dots, 6$) we obtain the following FRFs:

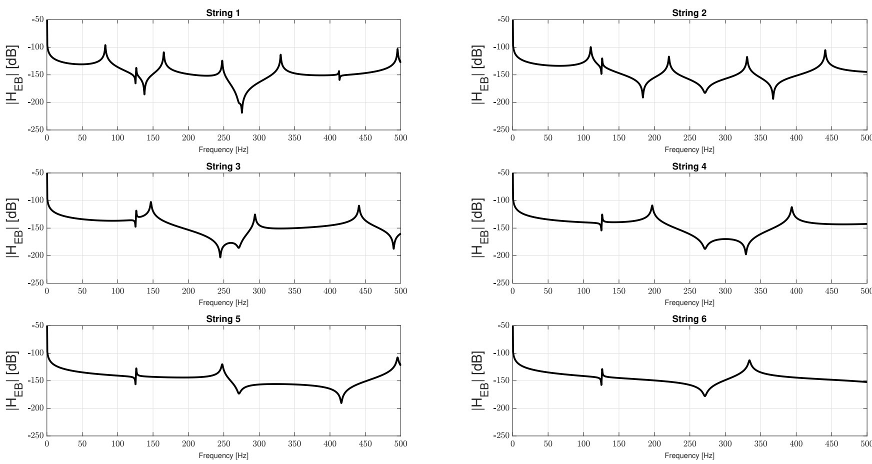
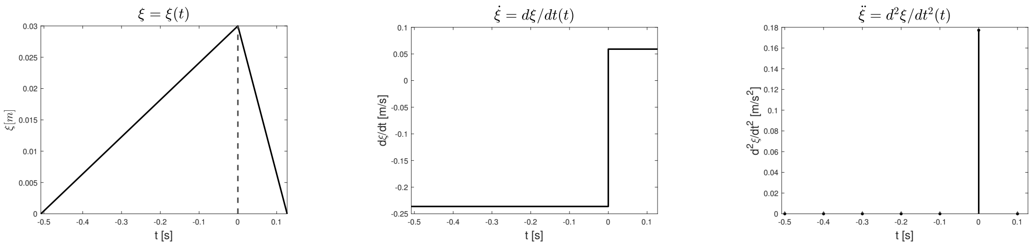


Figure 6

Time response

Our task is to compute the time response of the system to a plucking at $t_0 = 0$ $[s]$ given the maximum displacement $\xi_{\max} = 3 \cdot 10^{-3}$ $[m]$. Since we adopted acceleration as the wave variable, an ideal pluck can be modeled as an impulse. In order to derive the initial condition a_0 relative to the acceleration corresponding to an initial transverse displacement in the plucking point $\xi_{\max}$, we start from the waveform of the string at $t = 0$ $[s]$, which also corresponds to the displacement in time of the excitation point, given the dynamics of plucked strings (Figure 7a). Obviously, the graphs represented in Figure 7 are purely illustrative.



(a) **(b)** **(c)**

Figure 7

The amplitude of the Dirac's delta in Figure 7c, given by the difference between the slopes of the two segments of the waveform, corresponds to the magnitude of the acceleration of the excitation point at the instant t_0: $a_0 = 0.0177$ $[m/s^2]$. In the examined case, an ideal pluck can thus be modeled as an impulse with a_0 amplitude. In order to compute the time response (in terms of force) of the bridge, we need to consider the Fourier transform of such an impulse, which will be dubbed $X(\omega)$, and multiply it by the transfer function $H_{EB}(\omega)$. The inverse Fourier transform of the resulting frequency domain function will be the time response we were looking for:

$$X(\omega) = \mathcal{F}\{a_0 \cdot \delta(t)\} \tag{6}$$

$$F(\omega) = H_{EB}(\omega) \cdot X(\omega) = \mathcal{F}\{f(t)\} \tag{7}$$

$$\Longrightarrow f(t) = \mathcal{F}^{-1}\{H_{EB}(\omega) \cdot \mathcal{F}\{a_0 \cdot \delta(t)\}\} \tag{8}$$

The described procedure was employed for each of the six strings of the guitar, assuming a string length corresponding to that of a 25 $[in] = 6.35 \cdot 10^{-1}$ $[m]$ scale guitar plucked at 1/5 of its length.

The resulting signals are reported in Figure 8.

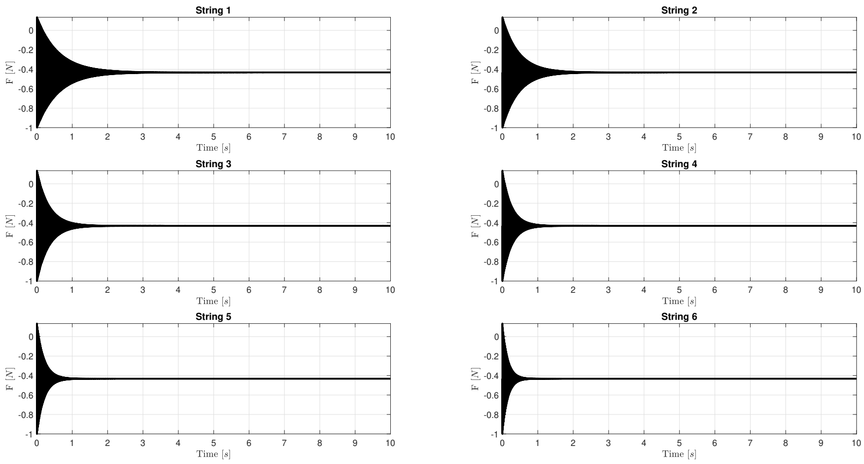


Figure 8: Force time responses of the six guitar strings.

The signal trends are coherent with the expected behaviour of the force at the bridge, however their amplitude seems to be smaller than it should and, in addition to that, the oscillations are not centered around zero, resulting in a constant negative force once the steady state is reached: this is non-physical, but we were not able to identify the cause of the phenomenon. Presumably, it has something to do with the FFT MatLab algorithm and the subsequent domain switches.

Conclusions

This study provided an interesting insight in the process of simulating the behaviour of a musical instrument via digital means. Particularly, it was possible to observe how to synthesize the sound of a guitar by choosing the appropriate model. An idea to dive even deeper in the matter could be to adopt a more complex model, beyond the use of simple delay lines.