Assignment

Homework HL4

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Musical Acoustics

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In the following report we describe the process of implementing the model of a *trumpet* in COMSOL Multiphysics, with the goal of simulating its acoustic response and computing its **input impedance**, **radiated sound pressure** and **directivity pattern**. The components of the trumpet (i.e. the *tube*, the *bell* and the *mouthpiece*) will be progressively added and combined together, starting from a simple tube. The employed parameters can be found in the following table.

r_T	r_S	N	L_t	L_{space}	$\lambda_{\max}$
$0.6\ [cm]$	$2\ [m]$	255	$1.7\ [m]$	$20\ [mm]$	$\frac{c_0}{f_{\max}} = 0.08575\ [m]$
$f_{\min}$	$f_{\max}$	c_0	L_h	m	S_t
$50\ [Hz]$	$4000\ [Hz]$	$343\ [m/s]$	$0.2\ [m]$	28	$\pi r_T^2 = 1.13 \cdot 10^{-4}\ \left[m^2\right]$

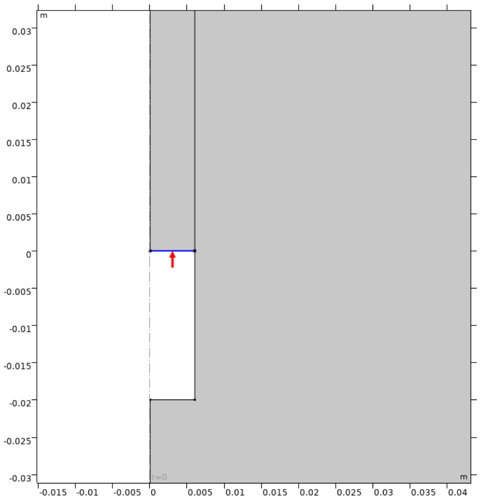
Table 1

Exercise 1 - *Simple Tube*

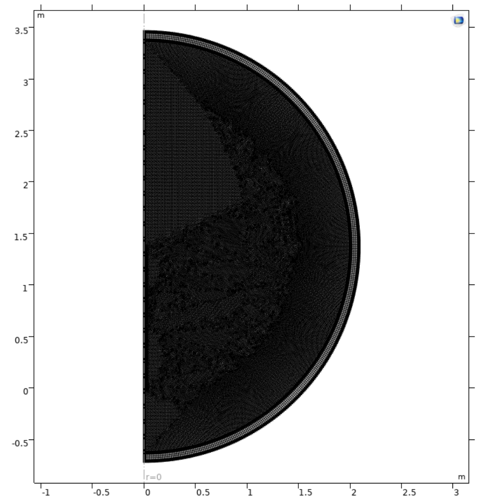
The model was built in a **2D axisymmetric environment** in order to ease the computational load. In this domain, we can model the tube as a *slender rectangle* (r_T, L_T). In order to simulate a **free field condition**, the tube is surrounded by an *air semicircle* (i.e. a sphere when translated in the tridimensional domain) centered at the upper end of the tube (r_S). The input opening of the tube is left free from the virtual air domain by subtracting a small rectangle $\left(r_T, L_{\text{space}}\right)$ from the two circles. Now, we want to **truncate the computational region** and thus we need to add an *absorbing layer* for wave equations, i.e. a **perfectly matched layer** (**PML**). To do so, we add another *air semicircle* $(r_{\text{PML}} = r_S + \lambda_{\max}\ [m])$ centered at the origin (which coincides with the upper end of the tube) and set a '*Perfectly Matched Layer*' condition from the '*Definitions*' tab after applying a boolean difference to the two semicircles, so that the PML can be applied to the external circular crown. At this point, we impose an **interior sound hard boundary** condition on the tube wall and we insert a '**circular *port***' $\left(n = 0,\ A^{in} = 1.1\ [Pa],\ \phi = 0\right)$ in correspondence with the open end of the tube.

Concerning the mesh, we opted for a *free triangular mesh* for the air domain (exluding the PML), taking care to satisfy the five points per wavelength condition, while for the PML a *mapped distribution* of five elements was employed.

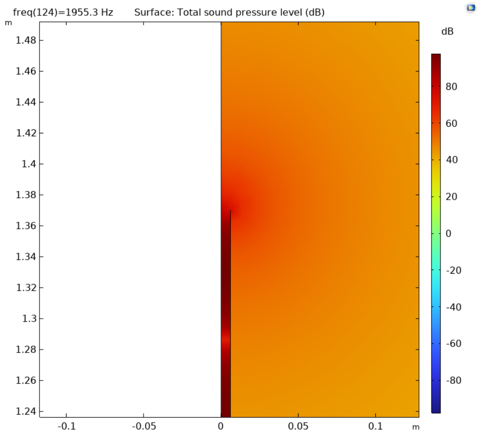
We are now ready for the simulation. The ratio between the input pressure $P(\omega)$ and the input velocity $U(\omega)$ represents the input impedance of the tube, and was evaluated in the range $[f_{\min}, f_{\max}]\ [Hz] = [50,\ 4000]\ [Hz]$ in $N = 255$ points by means of a *frequency domain study*. The result is reported in Figure 2.



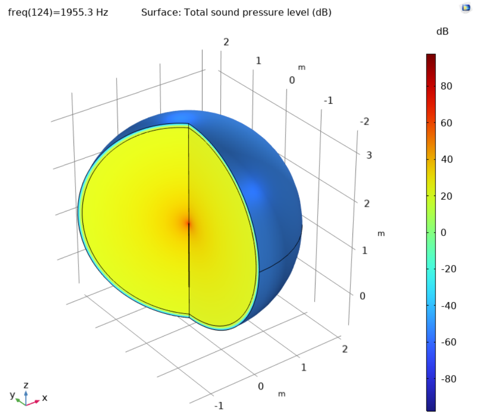
(a) Tube opening port.



(b) Domain mesh as a whole.



(c) Sound Pressure Level at the tube's end.



(d) Sound Pressure Level 3D field.

Figure 1

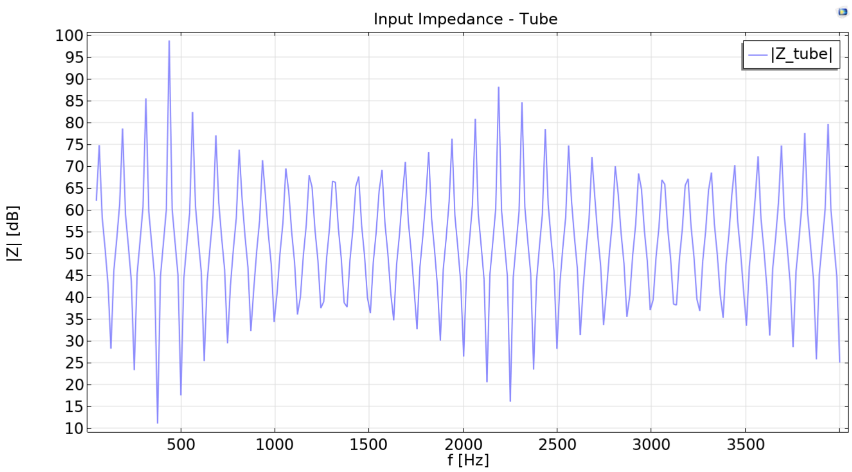


Figure 2

In order to plot the **directivity pattern** of the system we created a parametric 3D circumference in correspondence with the PML internal layer and employed a *maximum non-local coupling* to extract the maximum SPL measured on the internal boundary of the PML (this will be needed to *normalize* the plot). The pressure acoustic COMSOL physics provides a global variable named $'acpr.Lp_t' = L_t$ which returns the SPL value if properly evaluated. Hence, by evaluating the expression[1]:

$$\tilde{L} = L_t - L_{\max} \tag{1}$$

on the parametric curve and plotting it in polar coordinates we can obtain the normalized directivity pattern of the source. We decided to compare the directivity patterns of the same source at five different representative frequencies, in order to show how the radiation **directionality** varies with the source emission frequency.

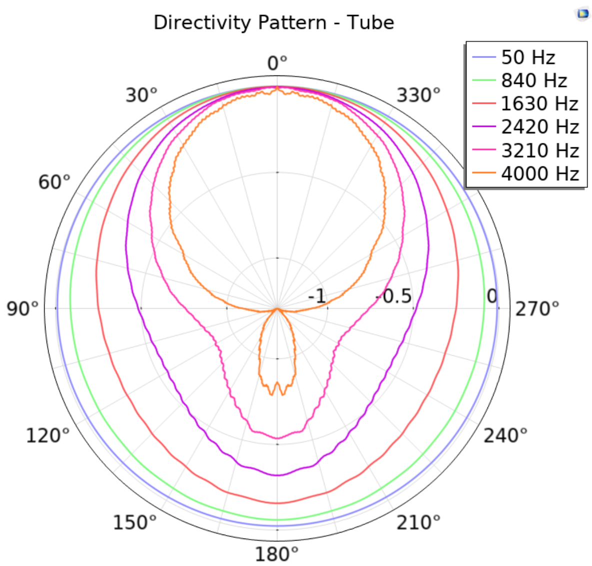


Figure 3

We clearly notice how at *low emission frequencies* the tube can be treated as an **omnidirectional sound source**, while with the increase of the emission frequency *the directionality of the sound source grows.*

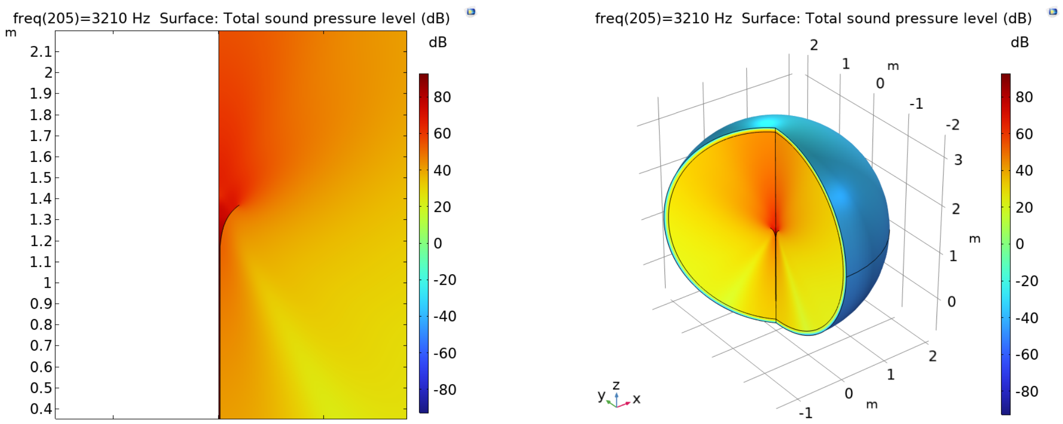
[1]Since we are working in the logarithmic domain (dB) the standard normalization procedure, consisting in a ratio, turns into a subtraction.

Exercise 2 - *Tube and Bell*

In order to study how the acoustic characteristics of the system vary when modifying its geometry, we will now attach an exponential bell at the end of the tube we designed in the previous section. From a geometrical point of view, the bell corresponds to a parametric curve of height L_h whose radius is described by the following expression:

$$r_B = \sqrt{\frac{S_t}{\pi} e^{mx}} \tag{2}$$

All the other parameter values, including the mesh settings, were kept unaltered from the previous study.



(a) **(b)**

Figure 4

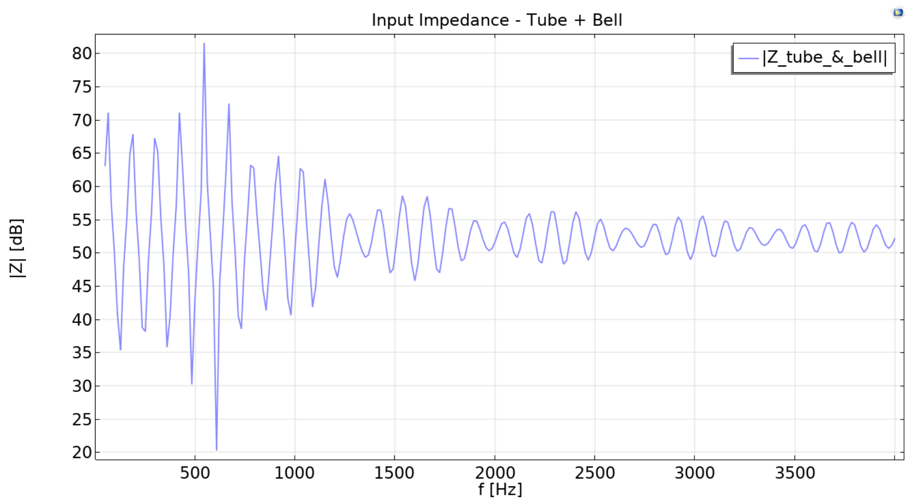


Figure 5

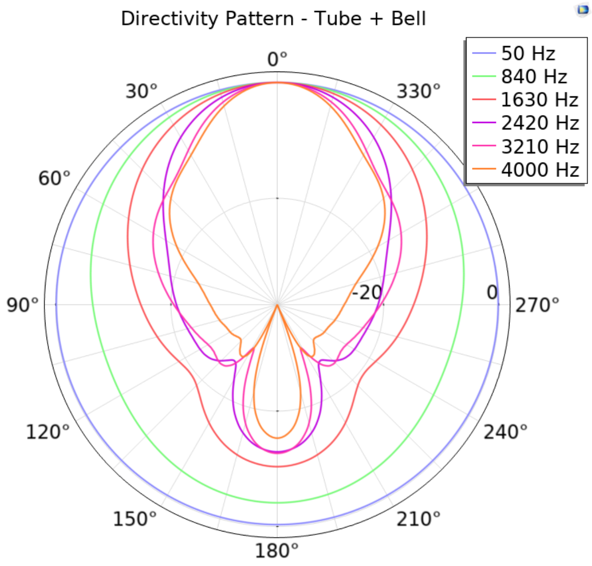


Figure 6

We can notice how, generally speaking, the peaks (positive and/or negative) of the tube input impedance are slightly higher than the ones of the model that includes the bell. In addition to that, for frequencies higher than ≈ 400 $[Hz]$ the eigenfrequencies appear to be shifted back and the peak magnitude decreases drastically, suggesting that the higher the frequency, the higher the damping. The directivity trend is the same as for the simple tube geometry.

Exercise 3 - *Mouthpiece*

We proceed now to study the acoustic behaviour of the mouthpiece alone. In order to model the piece, we employed a circle (r_M) and a trapezoid whose short side equals to 1/8 of the tube radius. We reduced the computational load by making the air domain smaller.

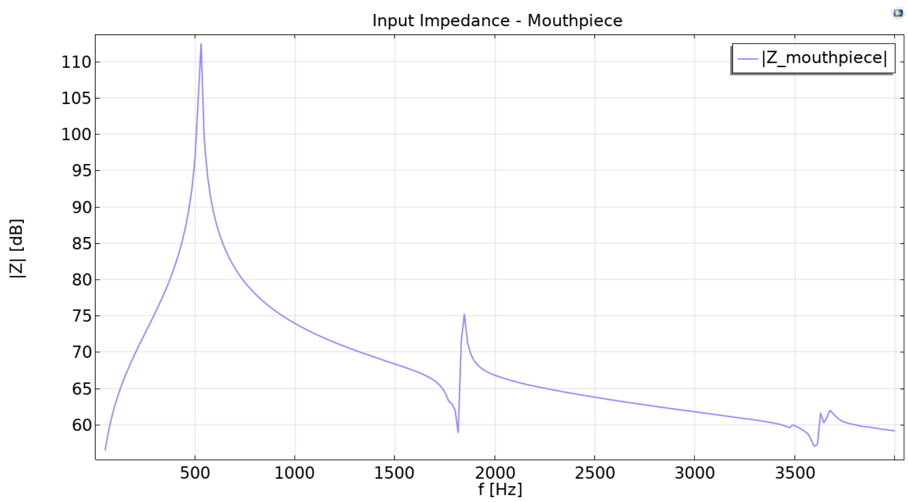


Figure 7

Exercise 4 - Complete Model

The combination of the three components (bore, bell and mouthpiece) results in a model intended to simulate the acoustic behaviour of a trumpet. The procedure is totally analogous to the one adopted in Exercise 1 and 2: given a pressure input at the port (i.e. at the input of the mouthpiece) we compute the input impedance by dividing the input pressure by the acoustic velocity. Just as before, the meshing parameters, boundary conditions and frequency study parameters are left unaltered. The 2D/3D SPL graphs are not reported since because of the dimension scale it's not possible to appreciate sensible differences with respect to the ones depicted in Figure 4. In addition, Figure 9 reports a comparison between the input impedances of each single component and the one of the complete model.

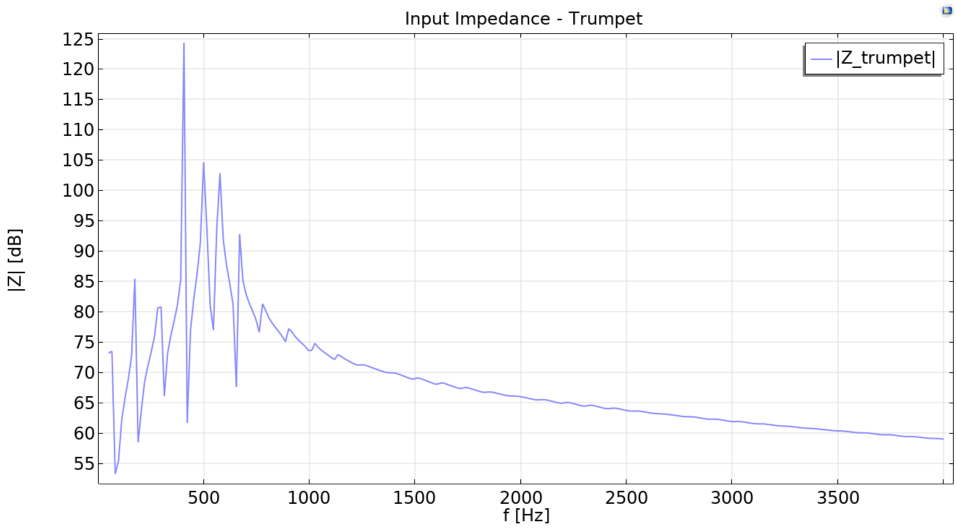


Figure 8

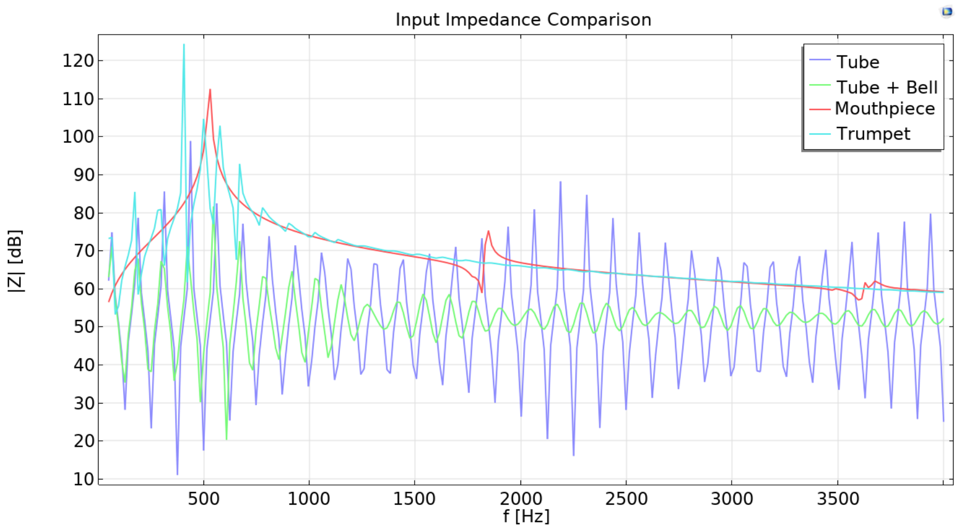


Figure 9

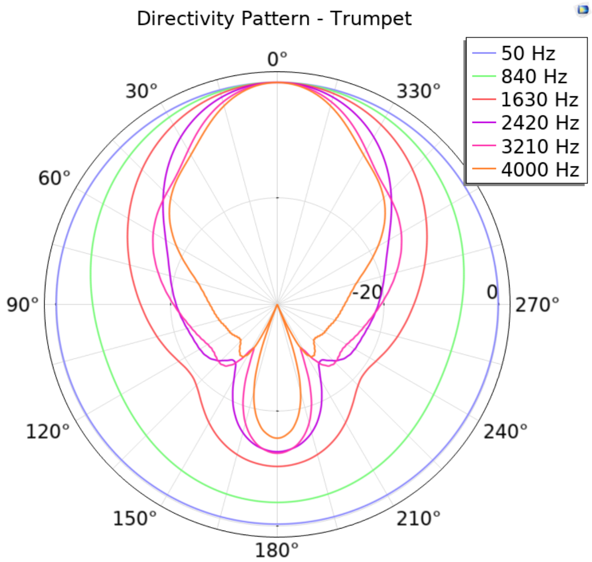


Figure 10

The directivity pattern trend confirms the conclusions we deduced before: at low frequencies, the trumpet can be considered an omnidirectional sound source, but its directionality grows with the increasing of the frequency and is maximum in the direction that corresponds to the main axis of the instrument bore.

Concerning the input impedance, it clearly appears to be a combination of the three components' respective impedances: the alternance of peaks and valleys (antiresonances/resonances) follows the single-peak profile of the mouthpiece impedance.

Conclusions

The simulations clearly show that it is possible to successfully study the acoustic behaviour of a system in a "modular" way, by dividing it in its different components and analyzing them separately. Moreover, we can observe that the bell of a wind instrument is an efficient tool to adapt the impedance of a tube and thus increase its radiating potential. In addition, the directivity of a sound source appears to be strongly dependant on frequency, and for a geometry analogous to the one we studied it is directly proportional, along the bore axis, to the emission frequency. Hence, at low frequencies, a sound source can effectively be treated as an omnidirectional sound source.