Assignment

Homework HL2

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Musical Acoustics

POLITECNICO
MILANO 1863

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Design and Analysis of Spherical Helmholtz Resonators

Resonator Design

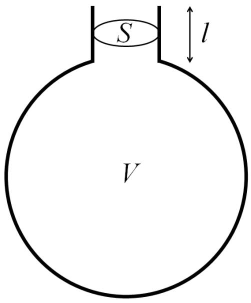


Figure 1: Helmholtz resonator schematic representation.

In order to model a spherical **Helmholtz resonator** that meets the given requirements (i.e. $f_1 = 300\ [Hz]$ and $d_1 = 4\ [cm]\,, l_1 = 1\ [cm]$), we started from the analytical expression of the natural frequency for such a resonator. By imposing $f_1 = 300\ [Hz]$, it is possible to extract the corresponding volume for the air chamber and consequently the sphere diameter:

$$f_1 = \frac{c}{2\pi}\sqrt{\frac{S}{Vl}} \implies D = 2 \cdot \sqrt[3]{\frac{c^2}{16\pi^3} \cdot \frac{3S}{f_1^2 l_p}} = 0.14059\ [m] \tag{1}$$

The parameter dubbed l_p corresponds to the the *ideal* cylindrical open pipe corresponding to the *real* pipe:

$$l_p = l + \Delta = l + \alpha \cdot r = 2.86\ [cm] \tag{2}$$

where r is the radius of the cylindrical pipe. This allows us to include **air losses** in the computation. In the examined case, the corrective parameter α was determined to be $\alpha = 0.93$ via an **empirical process** that involved COMSOL Multiphysics: by trying out different values of α, we chose the one which resulted in an admittance peak at $f = 300\,[Hz]$. At this point, we built the model in COMSOL (viscous gas condition, 2D axial symmetric environment - *Figure 2*).

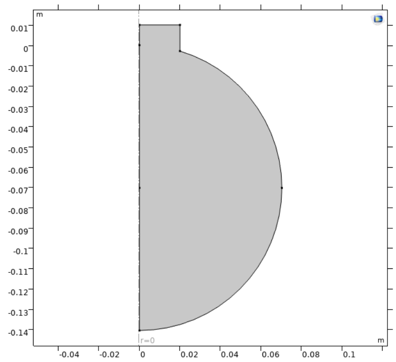


Figure 2: Comsol basic model.

The basic geometry was creating by applying a boolean union to a properly dimensioned circle and rectangle.

Meshing and Frequency Sweep Study

It is important to choose an appropriate mesh in order to perform a frequency sweep analysis of the input impedance. To do so, we define an *extremely fine* **edge mesh** on the external boundary of the resonator and then we apply an *extremely fine* **free triangular mesh** to the whole domain. At this point, to **avoid spatial aliasing**, we need to impose a maximum element size that satisfies the condition $l_{min} \lessapprox c/8f_{max} = 0.003\,[m]$, where f_{max} is the maximum frequency of the study, i.e. $10\,[kHz]$. The final result is reported in Figure 3.

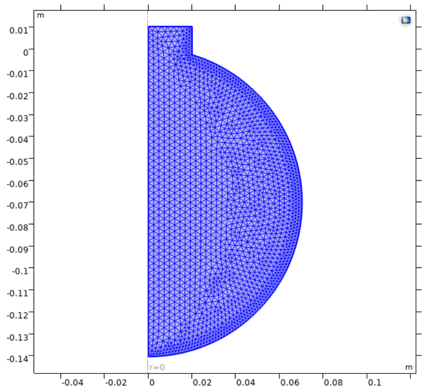


Figure 3: Mesh employed for the frequency study.

After setting a *pressure boundary condition* of $1000\,[Pa]$ in correspondence with the open end of the pipe, we moved on to the frequency sweep analysis, which was performed in the range $[10, 10000]\,[Hz]$ with a step of $10\,[Hz]$. By using two **probes** located on the upper end of the pipe we managed to extract the simulation values for the ***total acoustic pressure*** and ***velocity***. The data were then imported in MatLab, where we computed the acoustic impedance as $Z(\omega) = p(\omega)/v(\omega)$ and plotted it: the result is depicted in Figure 4.

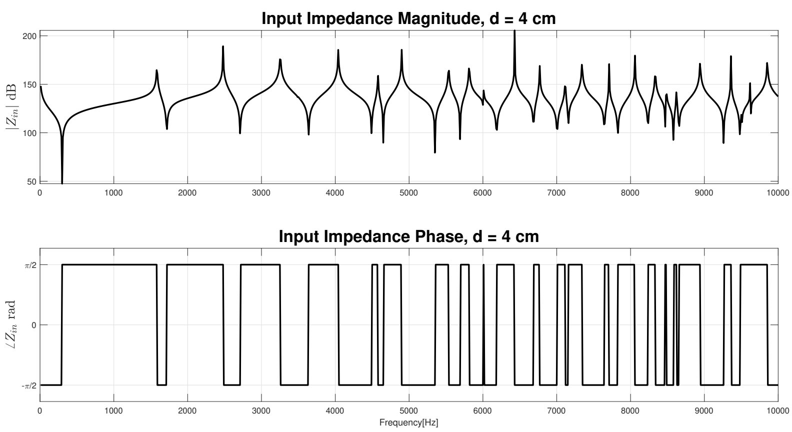
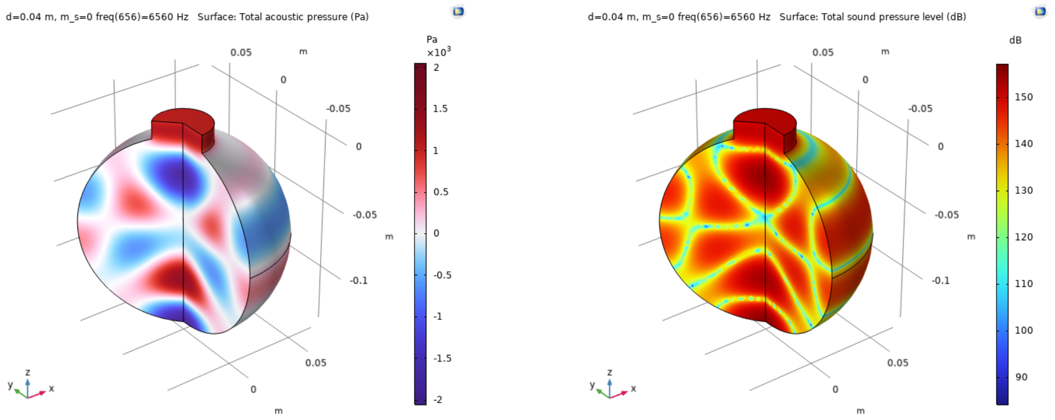


Figure 4



(a) **(b)**

Figure 5: Acoustic pressure and sound pressure level simulation results for $f = 6560\ [Hz]$.

We can notice that approximately after 4 $[kHz]$ the trend of the impedance graph changes: this is due to the **influence of the modes of vibration of the sphere** and their combination with the ones of the input pipe.

Diameter Sweep

At this point, we created a *sweep parameter analysis*, running the simulation for three different values of the pipe diameter $d_1 = [1, 2, 8]\,[cm]$. The employed procedure is analogous to the one described in the previous section, and the results are reported in the following figures.

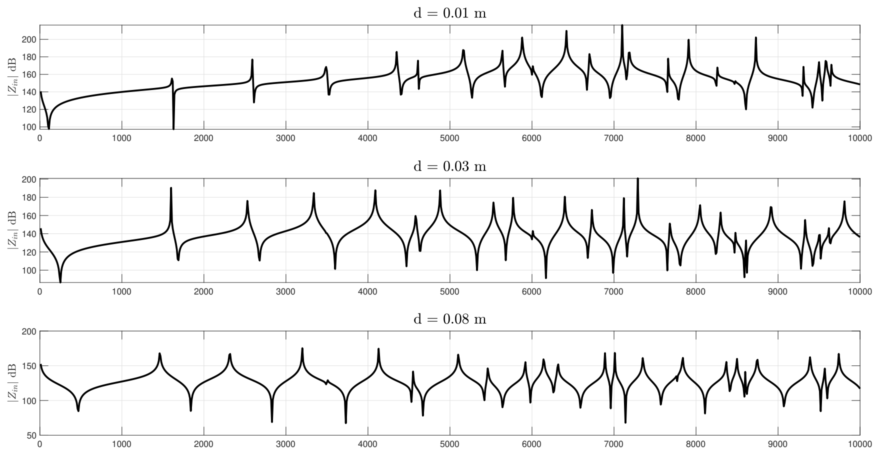


Figure 6

Notice that as the pipe dimensions increase *the resonance and anti-resonance frequencies are subject to variations*. Just as in the case of $d = 0.04\ [m]$, after $f \approx 4\ [kHz]$ the influence of the sphere modes is much more relevant and noticeable.

Modal Analysis

In order to perform a modal analysis and to choose the best resonator we must set up a **parametric sweep over all combinations** of the two parameters, i.e. the diameter $d = [1, 3, 4, 8]$ $[cm]$ and the modal index $m = [0, 1, 2, 3]$. By following the same procedure adopted in the previous sections, we exported the probes data in MatLab and computed the impedance curves for each given diameter, highlighting the influence of the different modal components (Figures 7 - 10). The red curves represent the sum of the four modal components.

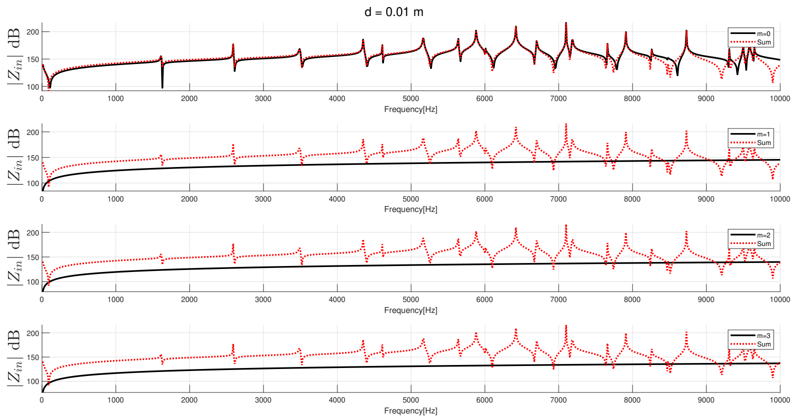


Figure 7: Influence of the first four modal components on the input impedance for the resonator characterised by $d = 0.01$ $[m]$.

The resonator which features the **lowest number of resonances in the desired frequency range** is the first one, for which $d = 1$ $[cm]$. In the examined frequency range, it is also the resonator for which *the higher order modes influence the impedance the least.*

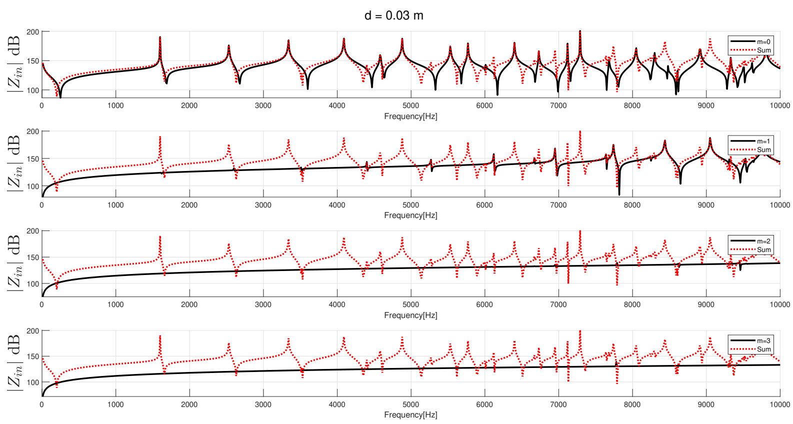


Figure 8: Influence of the first four modal components on the input impedance for the resonator characterised by $d = 0.03$ $[m]$.

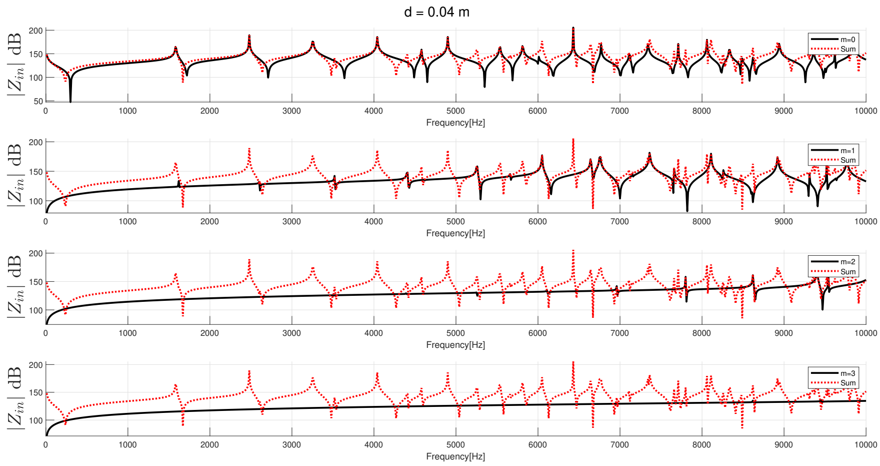


Figure 9: Influence of the first four modal components on the input impedance for the resonator characterised by $d = 0.04\ [m]$.

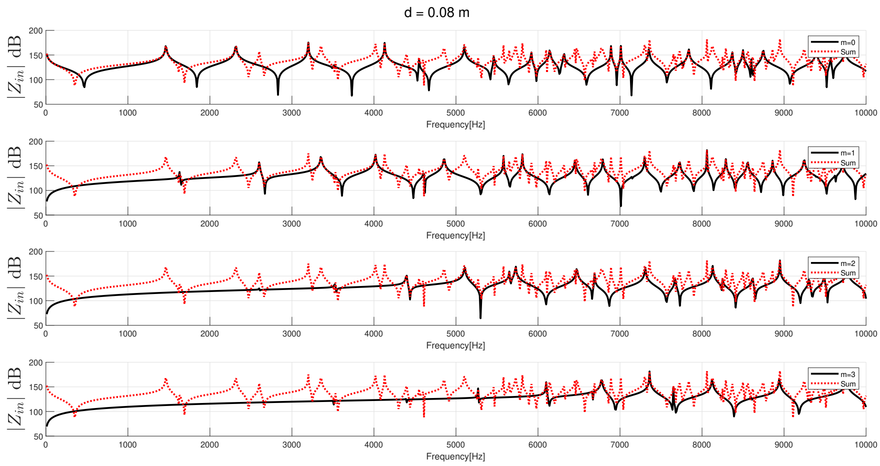


Figure 10: Influence of the first four modal components on the input impedance for the resonator characterised by $d = 0.08\ [m]$.

Coupled Resonator

We now consider another resonator coupled at the closed end of the previous one, with $D_2 = 15.86\ [cm]$, $d_2 = 4\ [cm]$ and $l_2 = 2l_1$. The COMSOL model was built via a procedure analogous to the one described for the first resonator, and that means by also applying the *proper empirical corrective factors* to the pipe lengths ($\alpha_1 = 0.93 = \alpha_2/2$, see Equation 2) and appropriately choosing the *meshing parameters*. Refer to Figure 11 for a more detailed description of the geometric structure of the object.

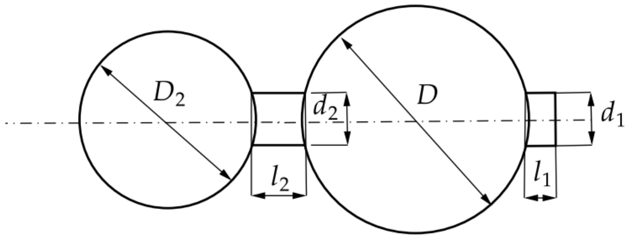


Figure 11: Coupled resonator structure.

The same procedure employed in the previous analysis involving the two COMSOL probes was applied for the complex resonator. The results of the frequency study and the subsequent MatLab computation of the impedance curve are reported in the following figure.

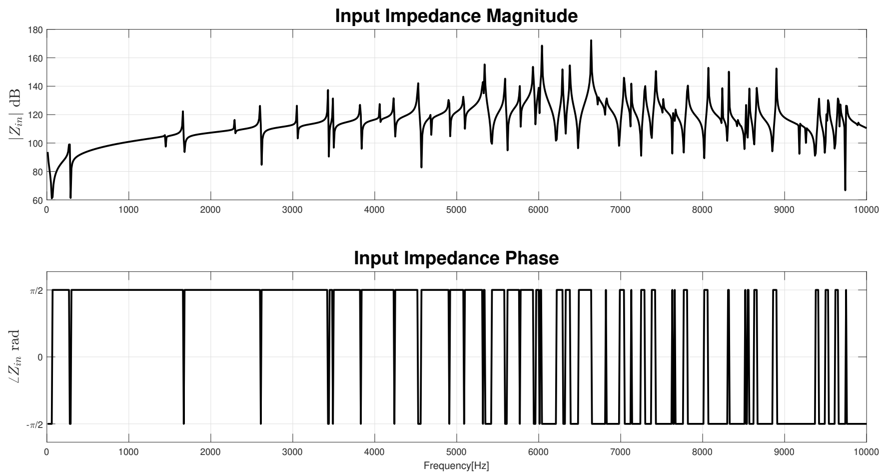


Figure 12

This is the impedance curve of a system that consists of **two different components interacting with each other**: this is evident because of the **recurring pattern** composed of *two negative peaks* (resonances) separated by a *peak* (anti-resonance). This pattern can be distinctly observed up to $\approx 3500[Hz]$: from this point on, the same phenomenon we observed for the single resonator (i.e. *distortion due to the interaction between the vibrational modes of the pipe and the ones of the sphere*) comes into play.

Electric Analog

The electric analog of a Helmholtz resonator consists of a *tension source* (which represents the acoustic pressure) and an *inductor* (representing the input pipe) in series with a *capacitance* (which stands for the air volume). In addition, a *resistance* is required to simulate the air resistance losses. The equivalent inductance and capacitance can be analytically expressed as functions of the physical parameters of the resonators via the following formulas:

$$L_{eq} = \frac{\rho \cdot l_p}{S} \ [kg/m^4] \tag{3}$$

$$C_{eq} = \frac{V}{\rho c^2} \ [m^5/N] \tag{4}$$

where $\rho = 1.2 \ [kg/m^3]$ and $c = 343 \ [m/s]$ are respectively the air density and the sound velocity in air. The coupling of two resonators results in **two circuit loops** connected via their shared element, i.e. the capacitance representing the first air chamber. In theory, the coupling of two elements (i.e. the pipe and the spherical air chamber) should result in the presence of a **coupling resistance** $R = \rho c/S$ in the electric analog: this is due to the air losses that characterise the *non-ideal scenario*. However, a problem arises: the acoustic-electric analogy is valid under certain assumption, one of which is that the system is actually vibrating, meaning that **the input signal must reach and excite the system without dying out**. The two resistances that can be derived via the aforementioned expression are in the *megaohms* range, and this results in a ***non-vibrating system*** when running the simulation. For this reason, when dealing with **lumped-elements approximations** of such circuits the coupling resistances are set to be *much lower than the theoretical ones*. **The lower the resistances**, **the narrower the widths of the impedance peaks and dives**. In the examined case, we opted for a single resistance $R = 5 \ [k\Omega]$. Its value was *chosen empirically* in order to obtain an output somewhat similar to the one derived from the previously described COMSOL simulation. The complete electric analog for the coupled resonator as was implemented in Simulink is illustrated in Figure 13 (refer to the provided MatLab file for the values of the equivalent electric parameters L_1, C_1, L_2, C_2).

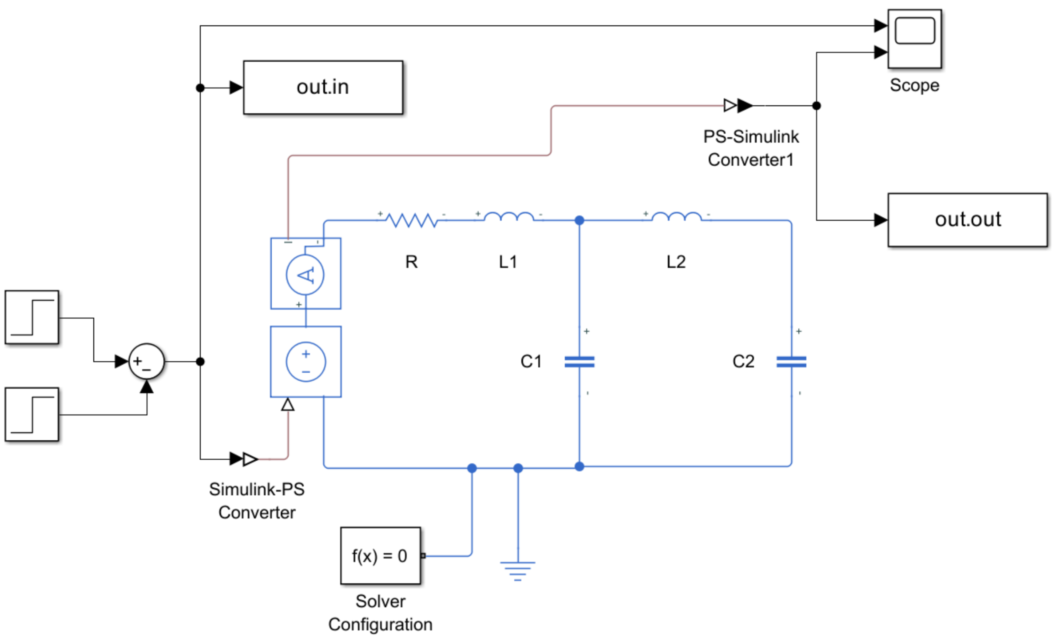


Figure 13

In order to obtain the frequency response of the system we need to excite the system with an **impulsive signal**: this can be achieved achieved by subtracting ***two slightly out of phase step signals***. The width of the impulse was set, on the basis of the **Nyquist theorem**, at $1/2f_{max} = 1/20000$. The input impulse controls the voltage source (which represents the acoustic pressure in the acoustic analog) and it is exported to the MatLab workspace in order for it to be available for further computations. The same thing applies to the current, measured via an *ammeter*, which represents the volume acoustic flow in the acoustic analog. Once the two signals are available in the workspace, the input impedance is computed as $Z(\omega) = {}^{p(\omega)}/_{U(\omega)}$, just as was done before. The input signal and the ammeter reading are reported in Figure 14, while the impedance curve is depicted in Figure 15.

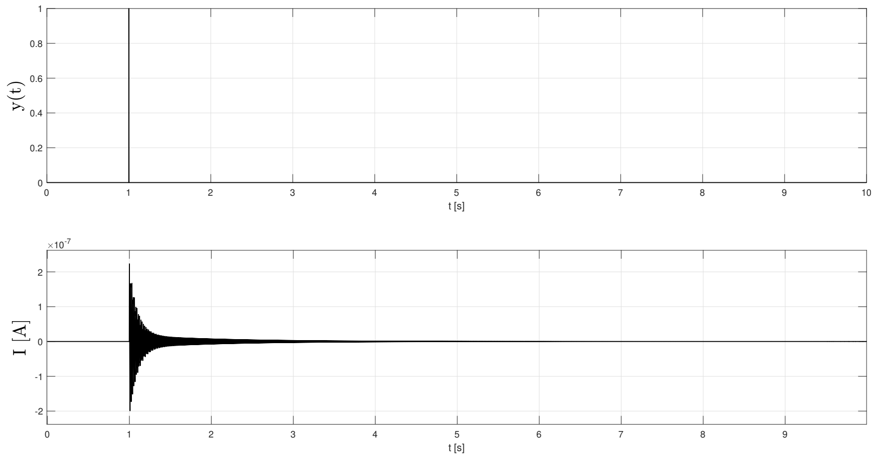


Figure 14

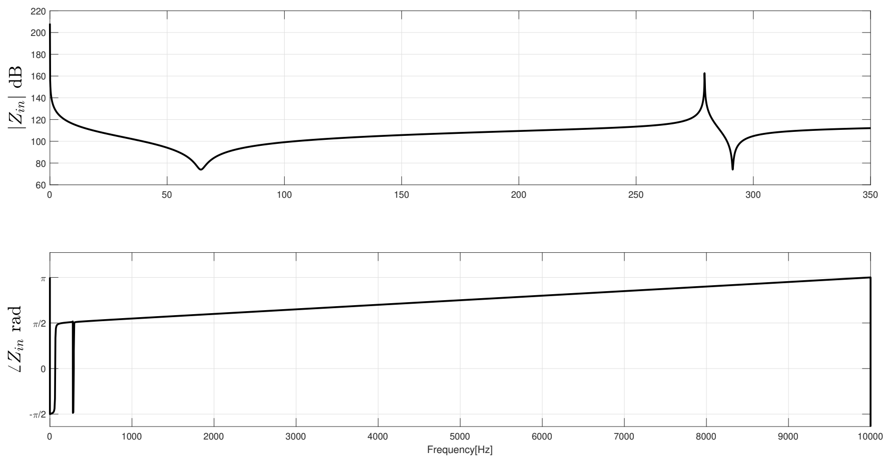


Figure 15

We can immediately notice how **the resonance frequencies of the coupled system are not the analogous frequencies of the single components**, but the interaction between the two resonators generates a more complex behaviour. In addition, as stated before, the adherence between the impedance curve obtained via the COMSOL computation and the one that results from the electric analog Simulink simulation depends on the evaluation of the system's damping, i.e. on the chosen resistance $\boldsymbol{R}$, which merely *governs the width of the impedance peaks and valleys.* Nonetheless, **the resonance frequencies are coincident**, and that shows the validity of the lumped-elements electric analog model adopted.

Conclusions

The study described in the previous pages made it possible to gain further insights on the behaviour of simple and complex acoustic systems, with a strong focus on how such systems interact with each other when coupled. It also allowed us to gain familiarity with advanced technical tools (such as COMSOL Multiphysics and MatLab Simulink/Simscape) and complex procedures. We were able to observe how a pipe and a spherical air chamber interact from an acoustic perspective when set up as a Helmholtz resonator and how two of such resonators can be coupled to obtain a system that behaves differently from the mere sum of its parts. In addition, the last section showed how the electric analog approach can be an extremely valid way to study the behaviour of such systems if employed with the right degree of caution along with the appropriate theoretic assumptions.