

# Entanglement-based Quantum LiDAR

Esame di metà tesi

Federico Collé



**UNIVERSITÀ  
DI TORINO**



Relatore: Prof. Leonardo Castellani

Co-relatori: Dr. Ivano Ruo Berchera  
Dr. Alessio Avella

# Overview

- Tecnologie quantistiche
- LiDAR
- Spontaneous Parametric Down Conversion
- Quantum Target Detection & Ranging
- Misure Phase-Insensitive
- Setup sperimentale & Analisi dati
- Passi successivi

# Tecnologie quantistiche

Tecnologie quantistiche: applicazioni tecnologiche legate a fenomeni prettamente quantistici.

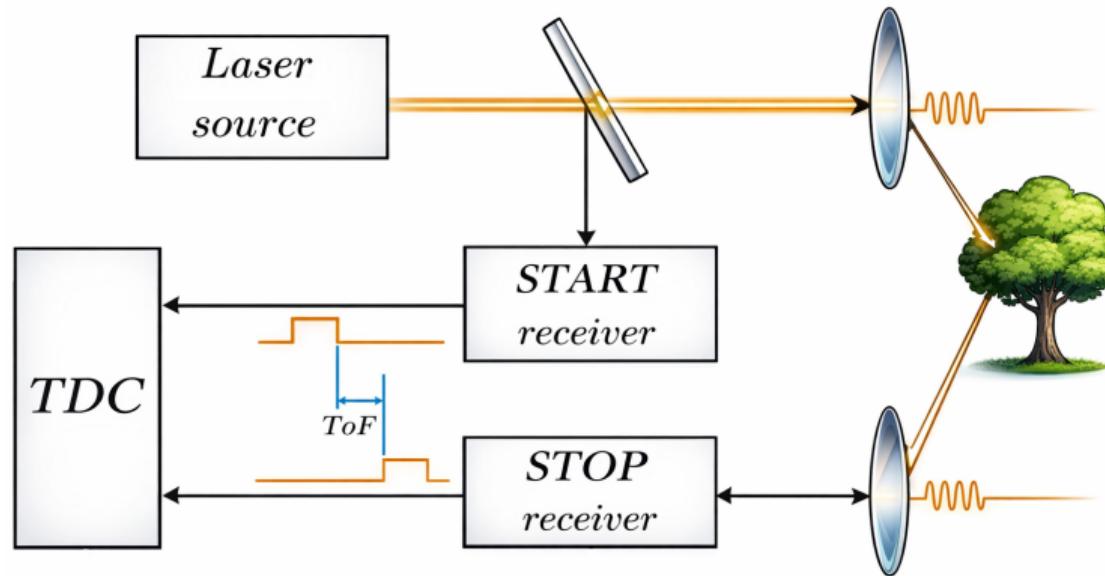
Alcuni esempi:

- Quantum computation
- Quantum communication
- Quantum simulation
- Quantum metrology & sensing  
→ **imaging**



# LiDAR

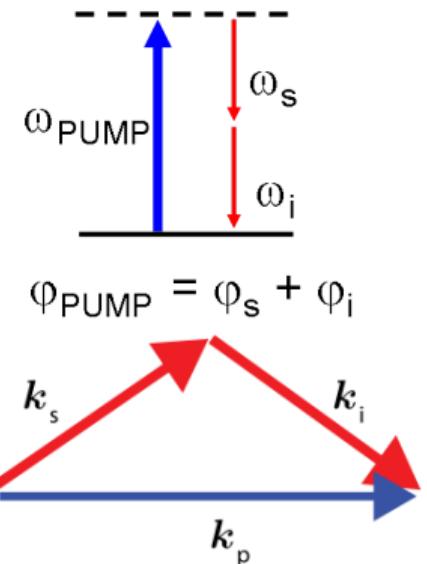
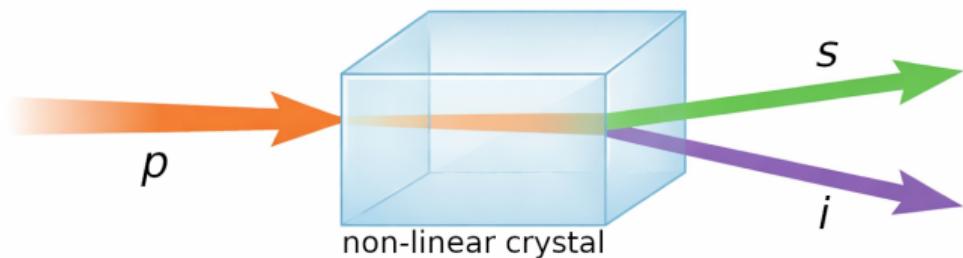
LIDAR = LIght Detection And **Ranging**.



È possibile migliorare il protocollo LiDAR sfruttando correlazioni quantistiche?

# SPDC

Spontaneous Parametric Down Conversion (SPDC):



# SPDC

$[\hat{n}_s - \hat{n}_i, \hat{H}] = 0 \rightarrow$  correlazione fra *signal* e *idler*.

# Quantum Target Detection (QTD)

Protocollo: **Quantum Illumination**

Vantaggio in situazione ostile:

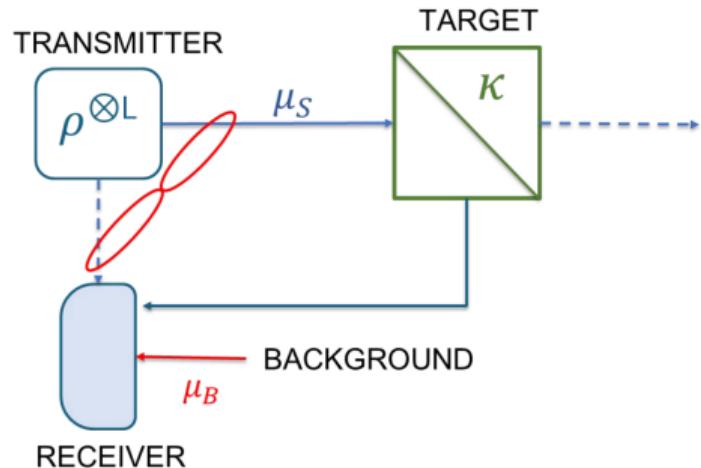
- Probe: stato  $\rho_T$ , numero medio di fotoni  $\mu_0 \ll 1$  (fotoni/bin)
- Background: stato  $\rho_B$ , numero medio di fotoni  $\mu_B \gg 1$
- Target: riflettività  $k \ll 1$

Target modellizzato da canale rumoroso e con perdite:  $\rho_k = \mathcal{E}_{k,\mu_B}(\rho_T)$

Binary hypothesis testing:

$$\mathcal{H}_0 : \rho_R = \rho_B$$

$$\mathcal{H}_1 : \rho_R = \rho_k$$



# Quantum Target Detection (QTD)

Misura eseguita con stato a multi-copia  $\rho_T^{\otimes L}$ .  
 $\mathcal{E}_{k,\mu_B}$  agisce indipendentemente su ogni copia.  
Per  $L \rightarrow \infty$  vale il **Quantum Chernoff Bound** (QCB):

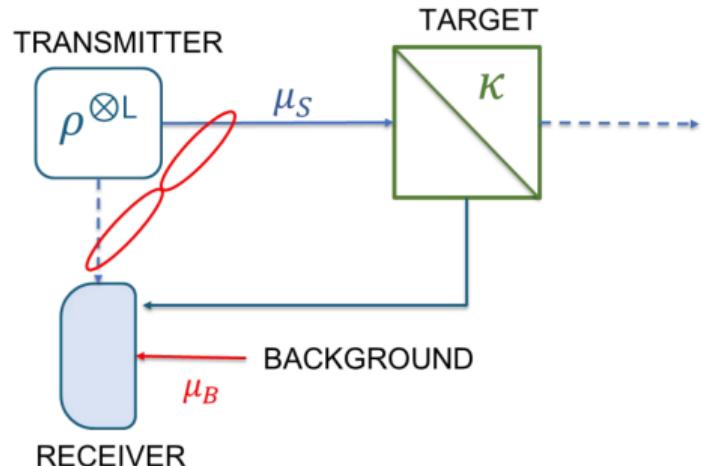
$$p_{err}(\rho_0^{\otimes L}, \rho_1^{\otimes L}) \leq \frac{1}{2} e^{-\xi_{QCB} L}$$

Con:

$$\xi_{QCB}(\mathcal{H}_0, \mathcal{H}_1) := \max_{\alpha \in [0,1]} C_\alpha(\rho_0, \rho_1)$$

$$C_\alpha(\rho_0, \rho_1) := -\log(\text{Tr}[\rho_0^\alpha \rho_1^{1-\alpha}])$$

$\xi_{QCB}$  = Chernoff information,  
 $C_\alpha$  =  $\alpha$ -information.

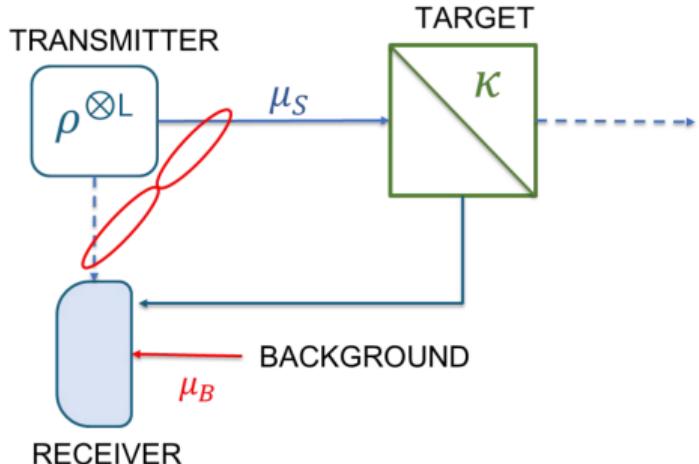


# Quantum Target Detection (QTD)

Con entangled ancilla mode si raggiunge un quantum advantage di **6 dB**.

Problemi del QTD:

- 6 dB solo per apparati estremamente complessi
- storage dell'idler richiede quantum memories
- c'è vantaggio solo per bassi *SNR*



S. Lloyd, Science 321, 1463 (2008)

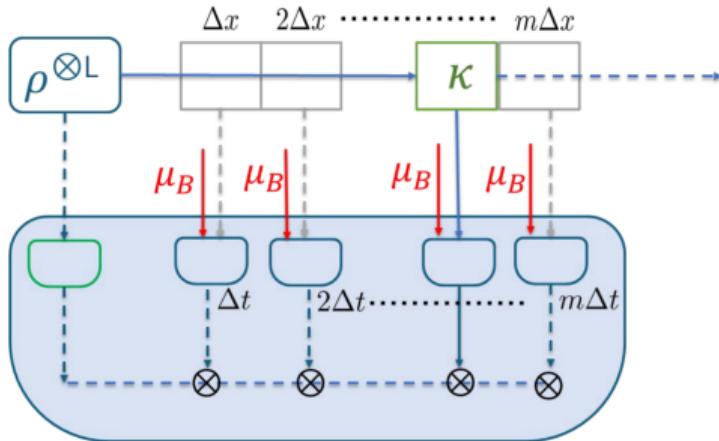
Tan et al., Phys. Rev. Lett. 101, 253601 (2008)

J. Shapiro, IEEE A&E Systems Magazine 35, 8 (2020)

Torromé & Barzanjeh, Prog. Quantum Electron. 93 (2024)

Sorelli et al., IEEE A&E Systems Magazine 37 (2021)

# Quantum Target Ranging (QTR)



$\rho_j$  è lo stato con target + rumore.

Ranging = stima del tempo di volo.  
Asse dei tempi discretizzato in  $m$  slot lunghi  $\Delta t$ :

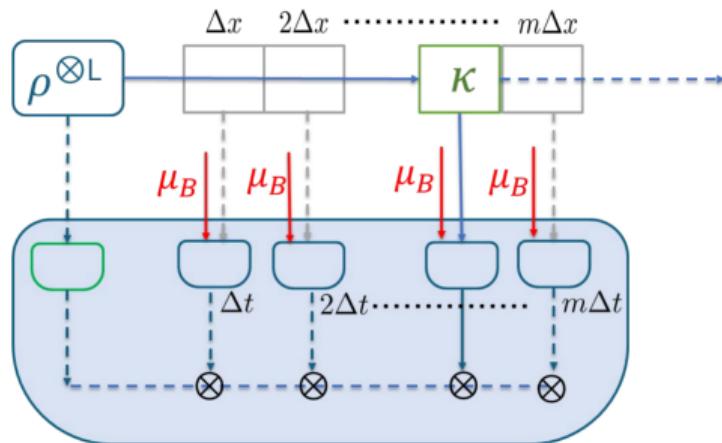
$$\Delta x = c \Delta t / 2 \quad x = j \Delta x$$

il target si trova in uno degli  $m$  slot.

Multi-hypothesis testing:

$$\mathcal{H}_j : \rho_R = \rho_j$$

# Quantum Target Ranging (QTR)



Per  $L \gg 1$ ,  $p_{\text{err}} \propto e^{-\xi^{(m)} L}$

**Asintoticamente**, QTR diventa problema binario:

$$\xi_{QCB}^{(m)} = \min_{i,j} \xi_{QCB}^{(2)}(\mathcal{H}_0, \mathcal{H}_1)$$

Se  $\rho_1 = \rho_k \otimes \rho_B \otimes \dots$  e  $\rho_2 = \rho_B \otimes \rho_k \otimes \dots$ , allora

$$\xi_{TR} = \max_{\alpha \in [0,1]} C_\alpha(\rho_1, \rho_2) = 2 C_{1/2}(\rho_k, \rho_B)$$

$$\xi_{TD} := \max_{\alpha \in [0,1]} C_\alpha(\rho_B, \rho_k) \rightarrow \xi_{TR} \geq \xi_{TD} \rightarrow \boxed{p_{\text{err}}^{\text{TR}} \leq p_{\text{err}}^{\text{TD}}}$$

## Misure Phase-Insensitive

In regime ottico è difficile e poco pratico preservare la fase.

→ **Misura phase-insensitive:** si misura solo il numero di fotoni.

Caso **quantistico:**  $\rho_Q = \rho_{\text{TMSV}}^{\otimes R}$

$$\rho_{\text{TMSV}} = (|TMSV\rangle\langle TMSV|)$$

$$|TMSV\rangle = \sum_n c_n^{\mu_0} |n, n\rangle_{s.i} \langle n, n|$$

Per lo slot  $j$  si ha distribuzione di conteggi:

$$P_j(\vec{n}) = \text{Tr}[\rho_j |\vec{n}\rangle\langle \vec{n}|], \quad |\vec{n}\rangle = \bigotimes_i |n_i\rangle$$

Si ottiene un set di distribuzioni di probabilità classiche:

$$\{P_1(\vec{n}), \dots, P_m(\vec{n})\}$$

# Misure Phase-Insensitive

Caso **classico**: probe è stato coerente.

$$\text{Vale } \xi_{\text{coh}} = 2\mu_B + k\mu_0 - 2\sqrt{\mu_B}\sqrt{\mu_B + k\mu_0}$$

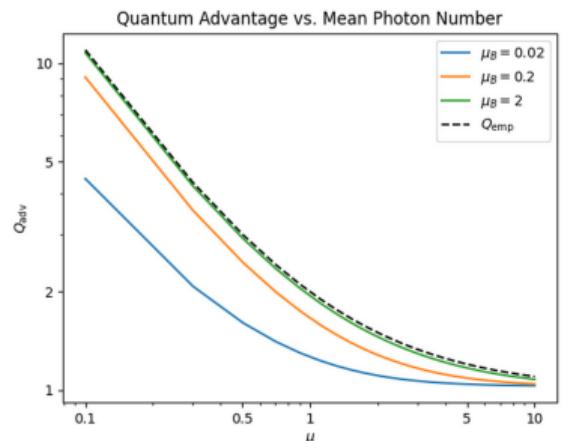
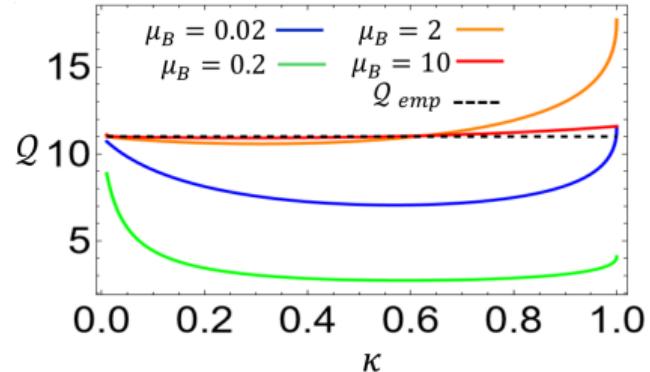
**Quantum advantage:**

$$Q := \frac{\xi_Q}{\xi_{\text{coh}}}$$

$$\text{Limite empirico: } Q_{\text{emp}} = 1 + \frac{1}{\mu_0}.$$

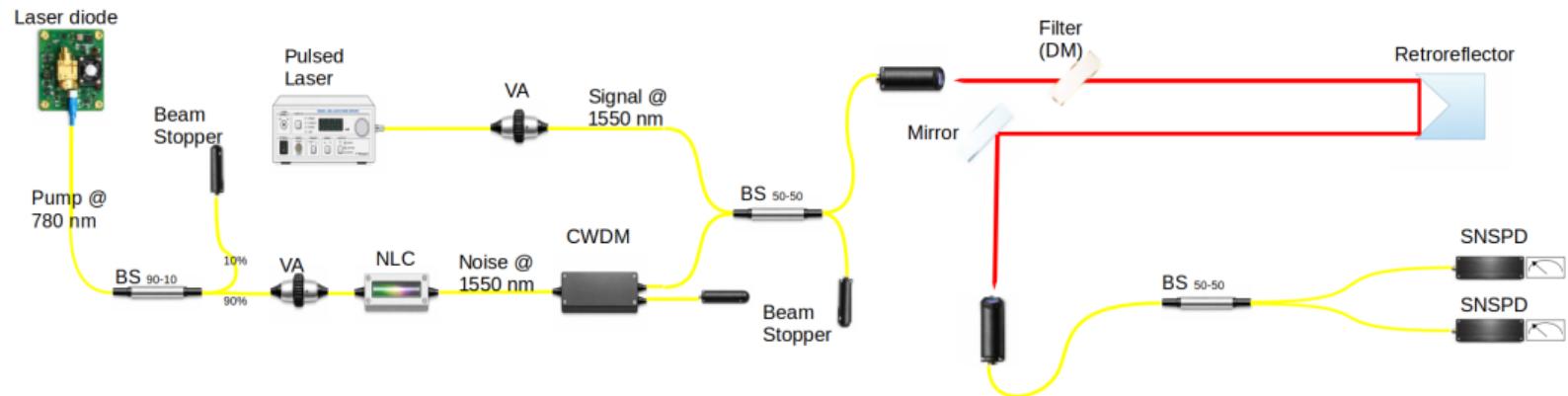
Lopaeva et al., Phys. Rev. Lett. 110, 153603 (2013)

Ortolano & Ruo Berchera, Phys. Rev. Res. 7, L022059 (2025)



# Setup sperimentale

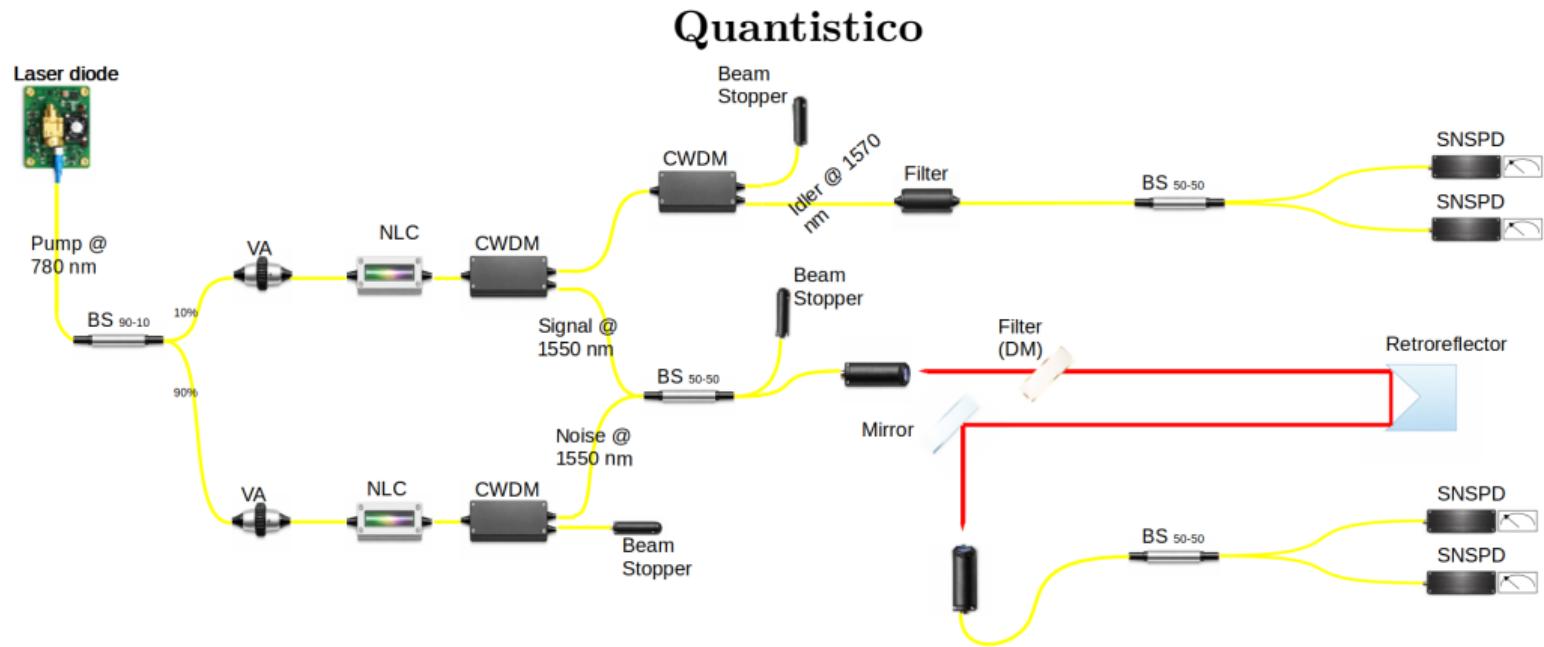
Classico



BS: Beam Splitter VA: Variable Attenuator NLC: Non Linear Crystal CWDM: Coarse Wavelength Division Multiplexing

DM: Dichroic Mirror SNSPD: Superconducting Nanowire Single Photon detector

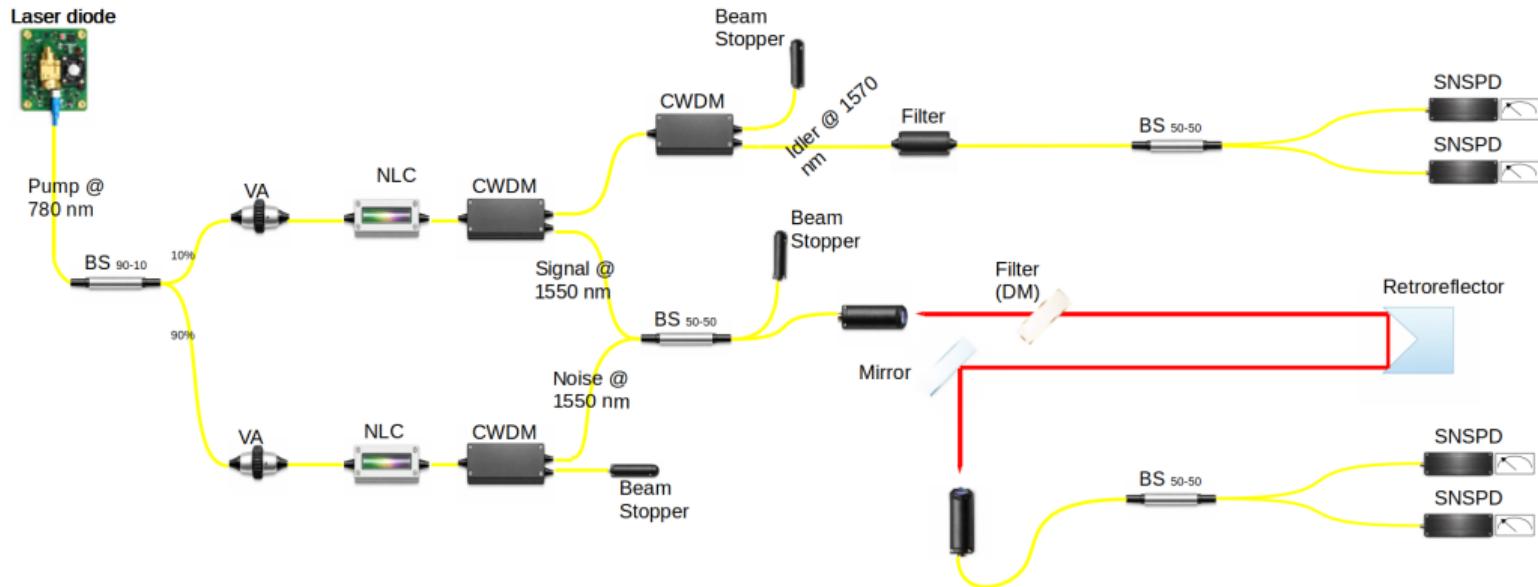
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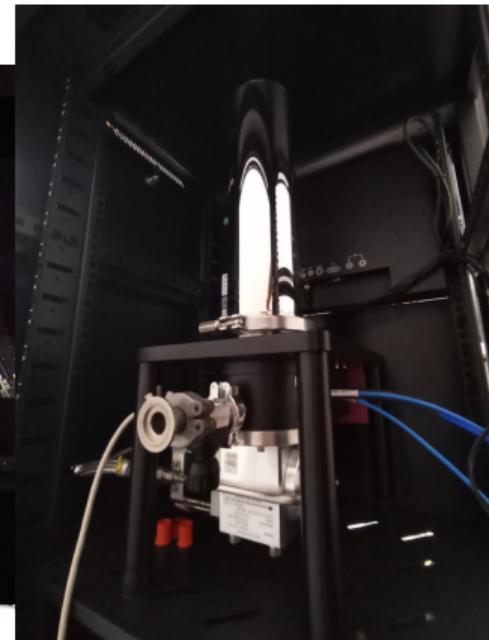
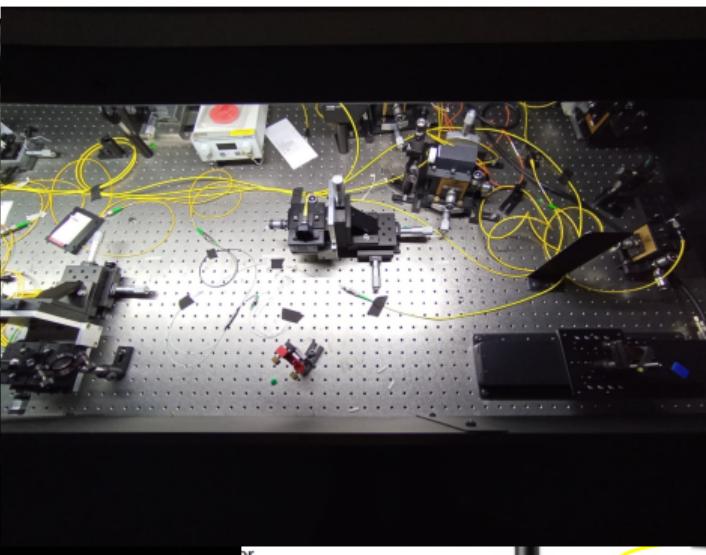
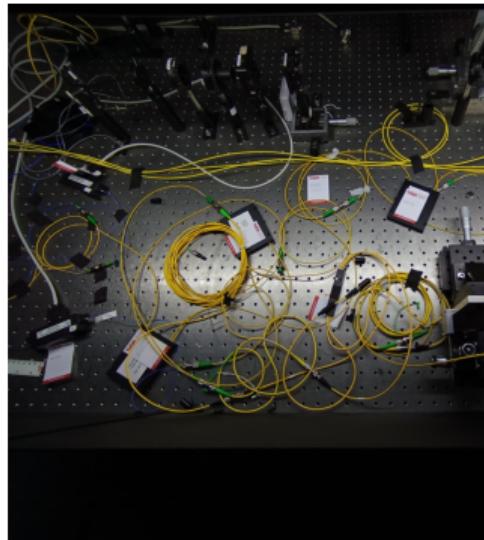
BS: Beam Splitter VA: Variable Attenuator NLC: Non Linear Crystal CWDM: Coarse Wavelength Division Multiplexing

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# Setup sperimentale



# Setup sperimentale

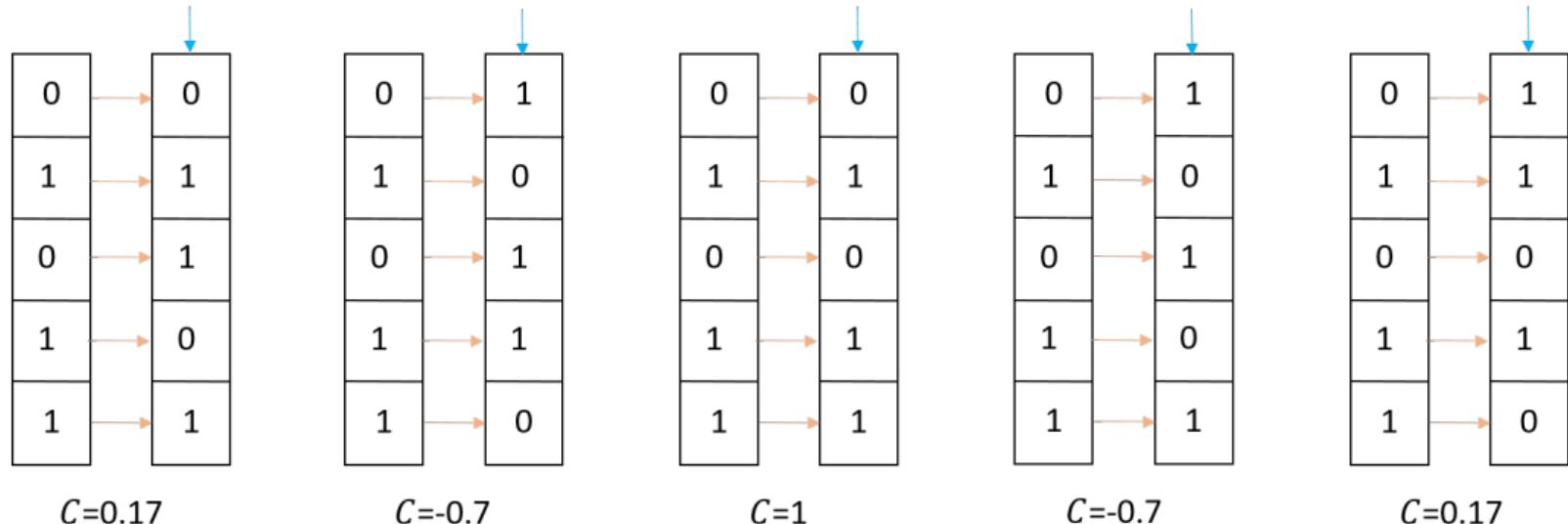


Stopper



## Analisi dati

SNSPD salvano il tempo di arrivo dei fotoni inserendolo .

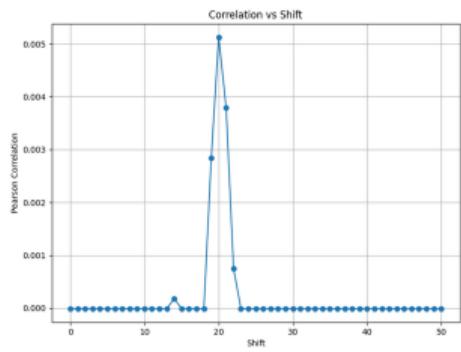


Le stringhe vengono shiftate fino a trovare il picco del coefficiente di Pearson,  
 $C = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$ .

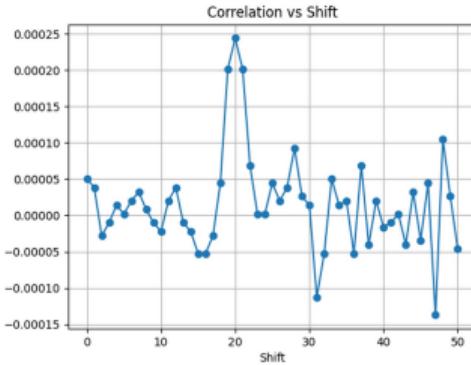
# Analisi dati

Picco inizialmente individuato in assenza di rumore.

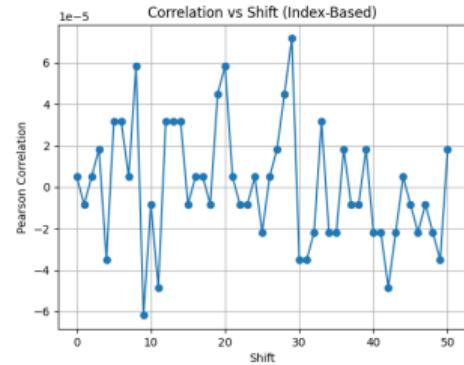
La posizione individuata è la differenza di tempo fra *signal* e *idler*.



$$\mu_B \sim 10^{-9}$$



$$\mu_B \sim 10^{-6}$$



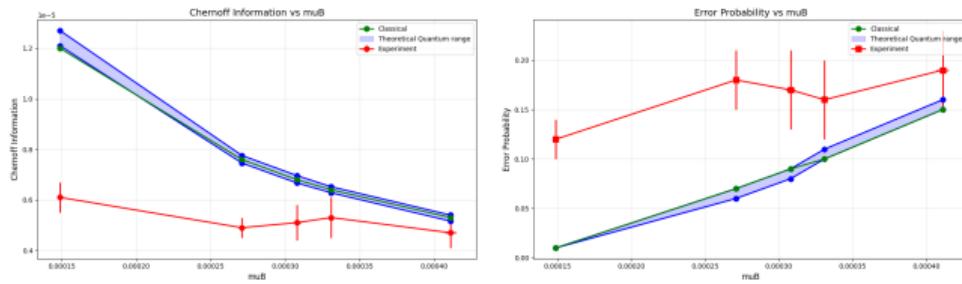
$$\mu_B \sim 10^{-4}$$

All'aumentare del rumore, individuare il picco è sempre più difficile.

→ su  $N$  misure,  $\#\text{err}$  trovano un picco diverso:

$$p_{\text{err,exp}} = \frac{N - \#\text{err}}{N}$$

# Risultati



## Passi successivi

DA SCRIVERE

Grazie per l'attenzione