

Entanglement-based Quantum LiDAR

Esame di metà tesi

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Overview

- Tecnologie quantistiche
- LiDAR
- Spontaneous Parametric Down Conversion
- Quantum Target Detection & Ranging
- Misure Phase-Insensitive
- Setup sperimentale & Analisi dati
- Passi successivi

Tecnologie quantistiche

Tecnologie quantistiche: applicazioni tecnologiche legate a fenomeni prettamente quantistici.

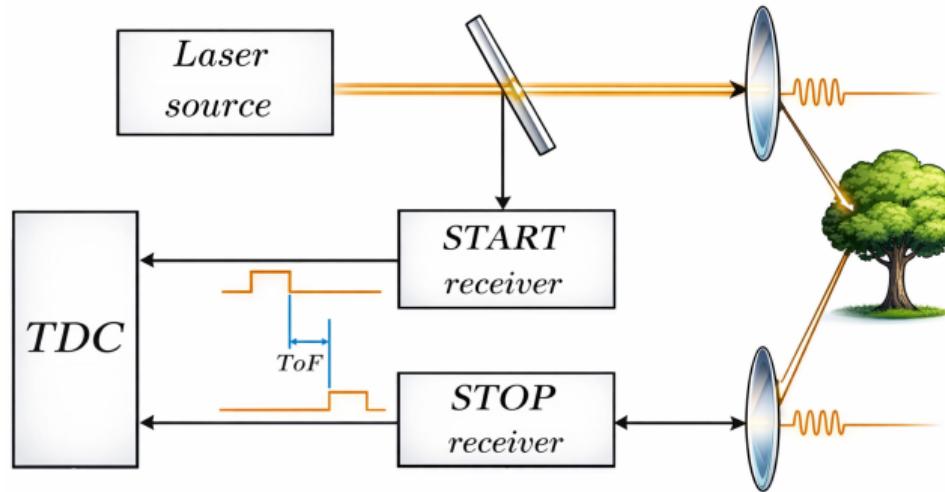
Alcuni esempi:

- Quantum computation
- Quantum communication
- Quantum simulation
- Quantum metrology & sensing
→ **imaging**



LiDAR

LIDAR = LIght Detection And **Ranging**.



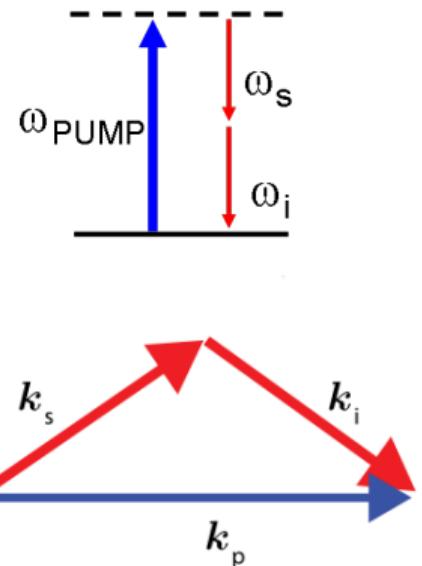
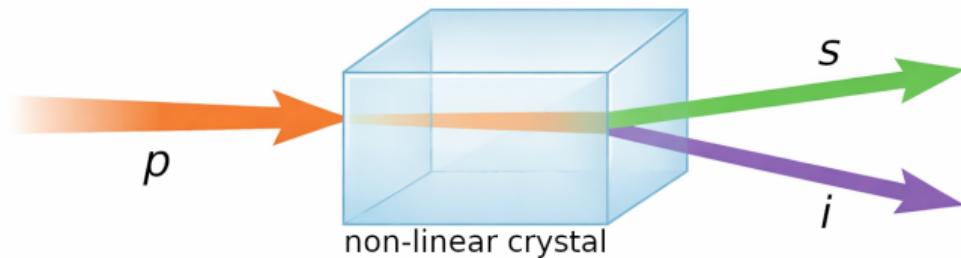
Performance del protocollo misurata con probabilità di errore p_{err} in

- *Detection*: target presente o assente
- *Ranging*: in quale time-bin è arrivato il ritorno

→ È possibile migliorare il protocollo LiDAR sfruttando correlazioni quantistiche?

SPDC

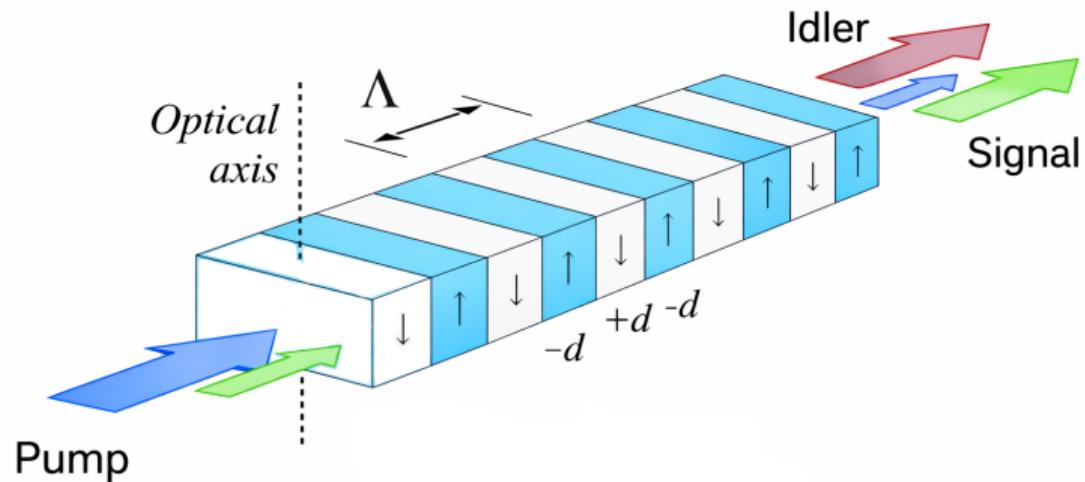
Spontaneous Parametric Down Conversion (SPDC):



$$[\hat{n}_s - \hat{n}_i, \hat{H}] = 0 \rightarrow \text{correlazione fra } \textit{signal} \text{ e } \textit{idler}.$$

SPDC

SPDC processo poco efficiente → Quasi Phase Matching (QPM) :
Cristallo non lineare a poli periodici, $\chi^{(2)} = (-1)^n \chi_0^{(2)}$.



$$\kappa_{pump} \simeq \kappa_s + \kappa_i + \kappa_\Lambda$$

$$\kappa_\Lambda = \frac{2\pi}{\Lambda}$$

Quantum Target Detection (QTD)

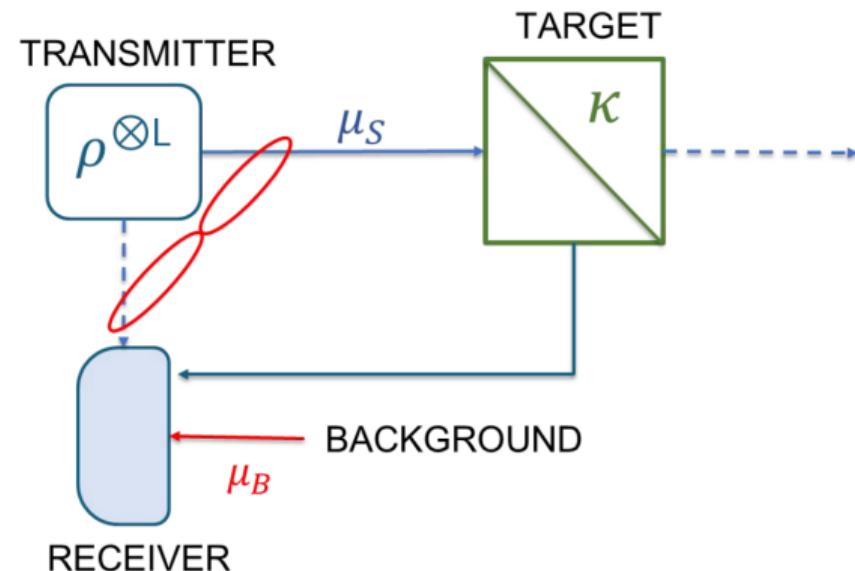
Protocollo: **Quantum Illumination**

Vantaggio in situazione ostile:

- Probe: stato ρ_T , numero medio di fotoni $\mu_0 \ll 1$ (fotoni/bin)
- Background: stato ρ_B , numero medio di fotoni $\mu_B \gg 1$
- Target: riflettività $k \ll 1$

Target modellizzato da canale rumoroso e con perdite: $\rho_k = \mathcal{E}_{k,\mu_B}(\rho_T)$

Binary hypothesis testing:



$$\mathcal{H}_0 : \rho_R = \rho_B$$

$$\mathcal{H}_1 : \rho_R = \rho_k$$

Quantum Target Detection (QTD)

Misura eseguita con stato a multi-copia $\rho_T^{\otimes L}$.

\mathcal{E}_{k,μ_B} agisce indipendentemente su ogni copia.

Per $L \rightarrow \infty$ vale il **Quantum Chernoff**

Bound (QCB):

$$p_{err}(\rho_0^{\otimes L}, \rho_1^{\otimes L}) \leq \frac{1}{2} e^{-\xi_{QCB} L}$$

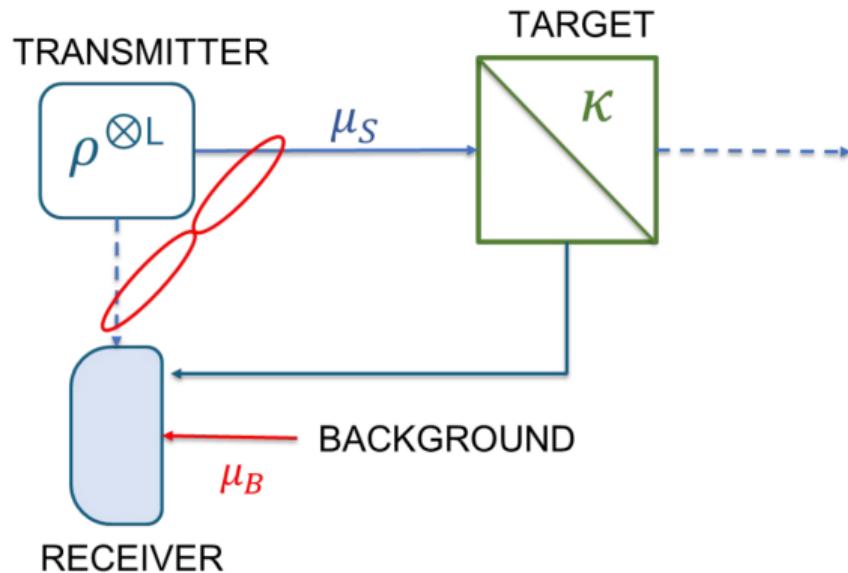
Con:

$$\xi_{QCB}(\mathcal{H}_0, \mathcal{H}_1) := \max_{\alpha \in [0,1]} C_\alpha(\rho_0, \rho_1)$$

$$C_\alpha(\rho_0, \rho_1) := -\log(\text{Tr}[\rho_0^\alpha \rho_1^{1-\alpha}])$$

ξ_{QCB} = Chernoff information,

C_α = α -information.



Quantum Target Detection (QTD)

Con entangled ancilla mode si raggiunge un quantum advantage di **6 dB**.

Problemi del QTD:

- 6 dB solo per apparati estremamente complessi
- storage dell'idler richiede quantum memories
- c'è vantaggio solo per bassi *SNR*

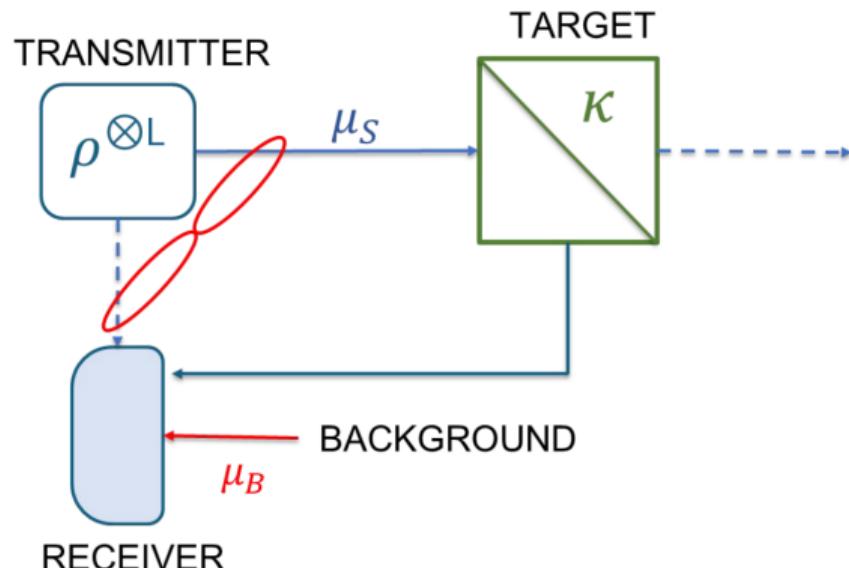
S. Lloyd, Science 321, 1463 (2008)

Tan et al., Phys. Rev. Lett. 101, 253601 (2008)

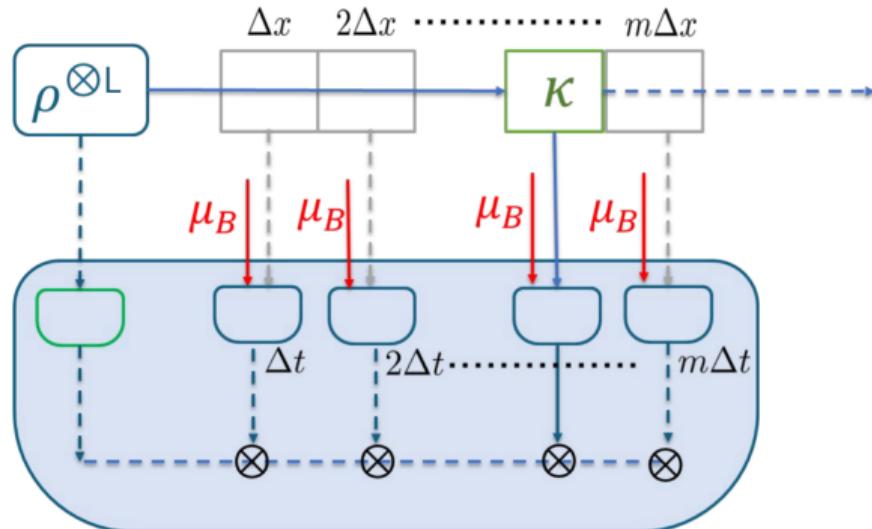
J. Shapiro, IEEE A&E Systems Magazine 35, 8 (2020)

Torromé & Barzanjeh, Prog. Quantum Electron. 93 (2024)

Sorelli et al., IEEE A&E Systems Magazine 37 (2021)



Quantum Target Ranging (QTR)



Ranging = stima del tempo di volo.
Asse dei tempi discretizzato in m slot lunghi Δt :

$$\Delta x = c \Delta t / 2 \quad x = j \Delta x$$

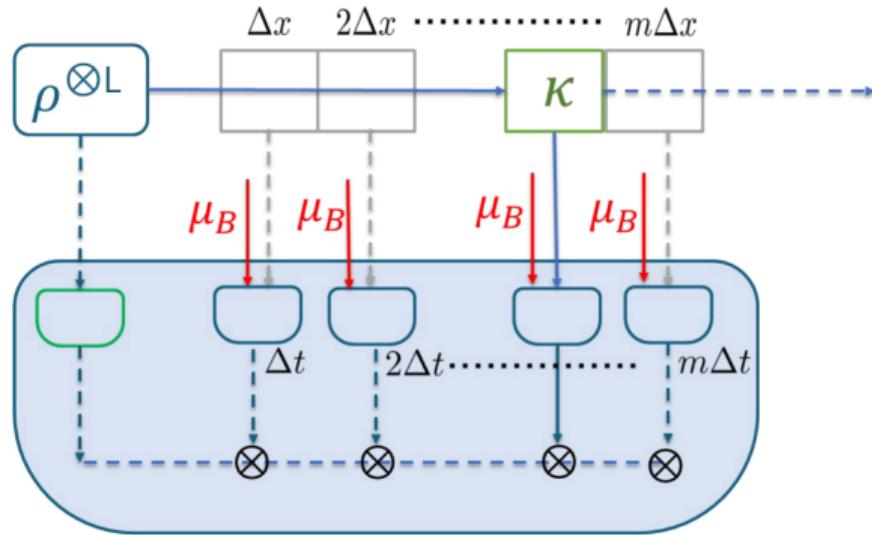
il target si trova in uno degli m slot.

Multi-hypothesis testing:

$$\mathcal{H}_j : \rho_R = \rho_j$$

ρ_j è lo stato con target + rumore.

Quantum Target Ranging (QTR)



Per $L \gg 1$, $p_{\text{err}} \propto e^{-\xi^{(m)} L}$

Asintoticamente, QTR diventa problema binario:

$$\xi_{QCB}^{(m)} = \min_{i,j} \xi_{QCB}^{(2)}(\mathcal{H}_0, \mathcal{H}_1)$$

Se

$\rho_1 = \rho_k \otimes \rho_B \otimes \dots$ e $\rho_2 = \rho_B \otimes \rho_k \otimes \dots$, allora

$$\xi_{TR} = \max_{\alpha \in [0,1]} C_\alpha(\rho_1, \rho_2) = 2 C_{1/2}(\rho_k, \rho_B)$$

$$\xi_{TD} := \max_{\alpha \in [0,1]} C_\alpha(\rho_B, \rho_k) \rightarrow \xi_{TR} \geq \xi_{TD} \rightarrow p_{\text{err}}^{\text{TR}} \leq p_{\text{err}}^{\text{TD}}$$

Misure Phase-Insensitive

In regime ottico è difficile e poco pratico preservare la fase.

→ **Misura phase-insensitive:** si misura solo il numero di fotoni.

Caso quantistico:

$$\rho_Q = \rho_{\text{TMSV}}^{\otimes L} \quad \rho_{\text{TMSV}} = (|TMSV\rangle\langle TMSV|) \quad |TMSV\rangle = \sum_n c_n^{\mu_0} |n, n\rangle_{s.i} \langle n, n|$$

Per lo slot j si ha distribuzione di conteggi:

$$P_j(\vec{n}) = \text{Tr}[\rho_j |\vec{n}\rangle\langle \vec{n}|], \quad |\vec{n}\rangle = \bigotimes_i |n_i\rangle$$

Si ottiene un set di distribuzioni di probabilità classiche:

$$\{P_1(\vec{n}), \dots, P_m(\vec{n})\}$$

Misure Phase-Insensitive

Caso **classico**: probe è stato coerente.

$$\text{Vale } \xi_{\text{coh}} = 2\mu_B + k\mu_0 - 2\sqrt{\mu_B}\sqrt{\mu_B + k\mu_0}$$

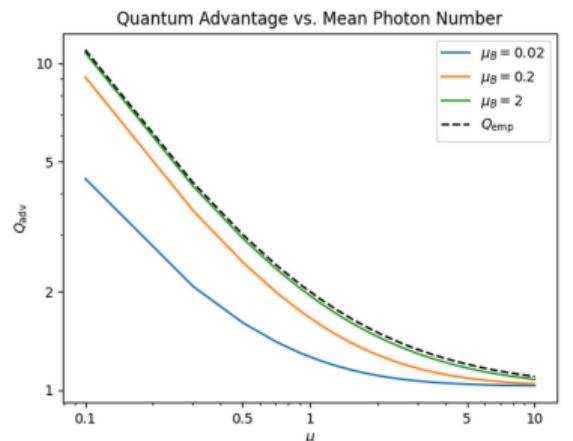
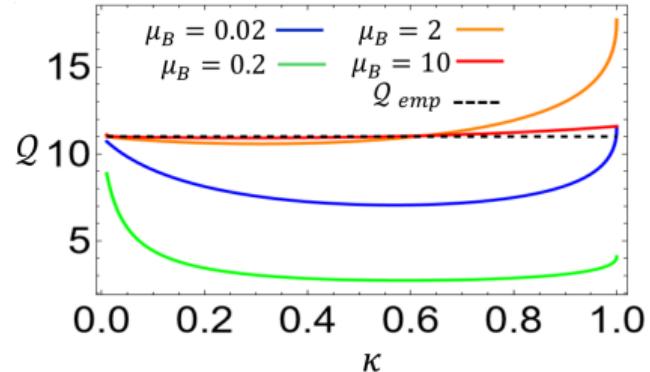
Quantum advantage:

$$Q := \frac{\xi_Q}{\xi_{\text{coh}}}$$

$$\text{Limite empirico: } Q_{\text{emp}} = 1 + \frac{1}{\mu_0}.$$

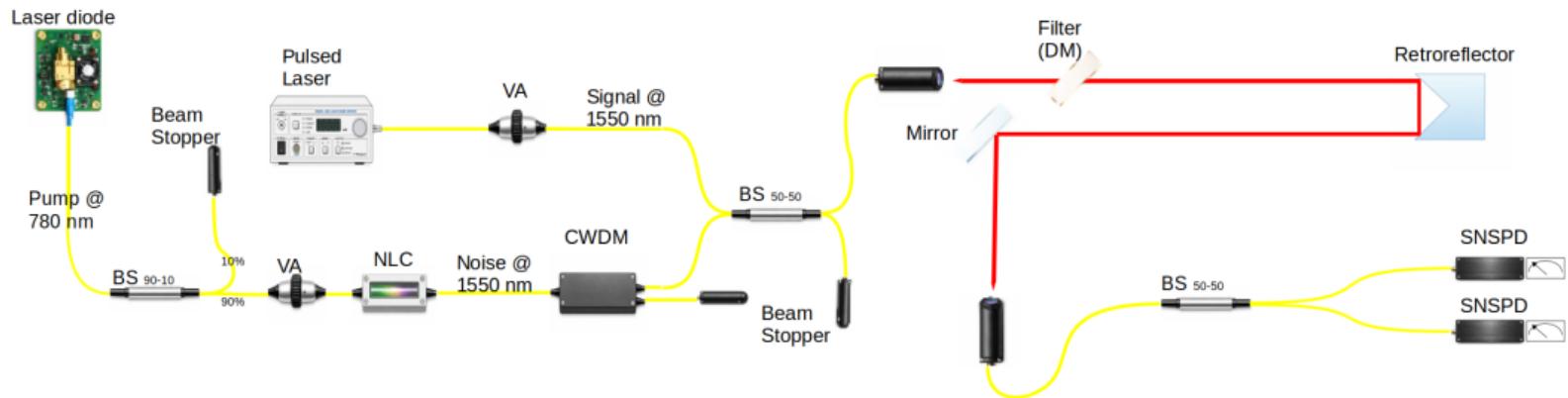
Lopaeva et al., Phys. Rev. Lett. 110, 153603 (2013)

Ortolano & Ruo Berchera, Phys. Rev. Res. 7, L022059 (2025)



Setup sperimentale

Classico

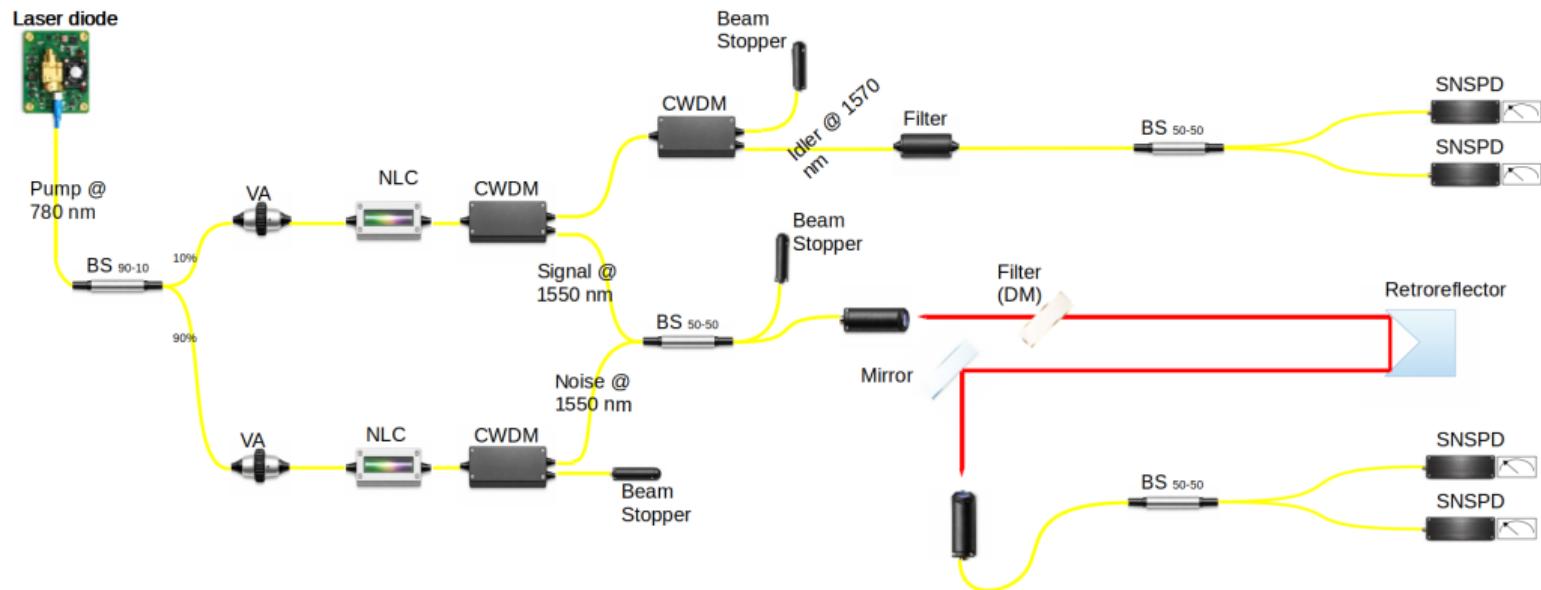


BS: Beam Splitter VA: Variable Attenuator NLC: Non Linear Crystal CWDM: Coarse Wavelength Division Multiplexing

DM: Dichroic Mirror SNSPD: Superconducting Nanowire Single Photon Detector

Setup sperimentale

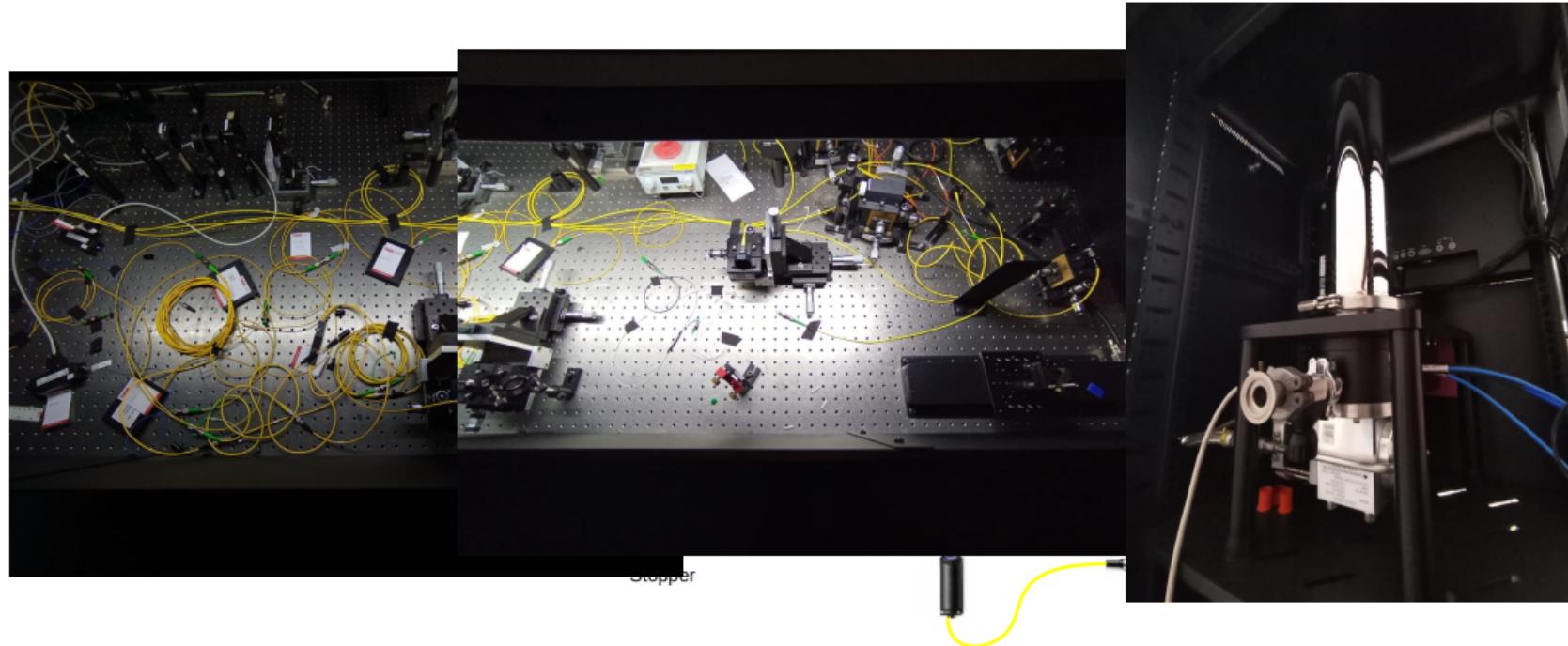
Quantistico



BS: Beam Splitter VA: Variable Attenuator NLC: Non Linear Crystal CWDM: Coarse Wavelength Division Multiplexing

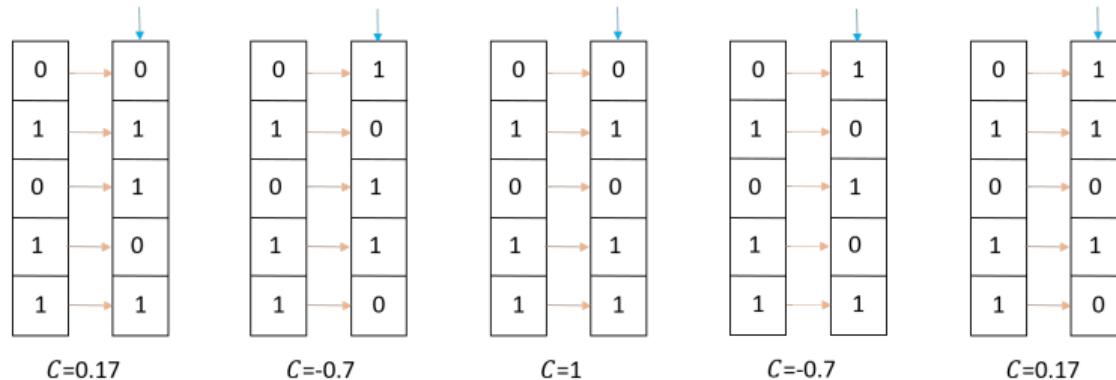
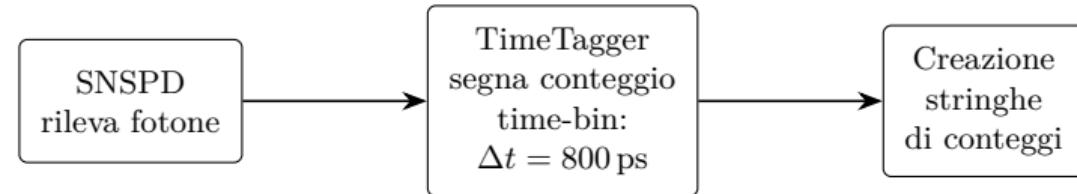
Setup sperimentale

Quantistico



BS: Beam Splitter VA: Variable Attenuator NLC: Non Linear Crystal CWDM: Coarse Wavelength Division Multiplexing

Analisi dati

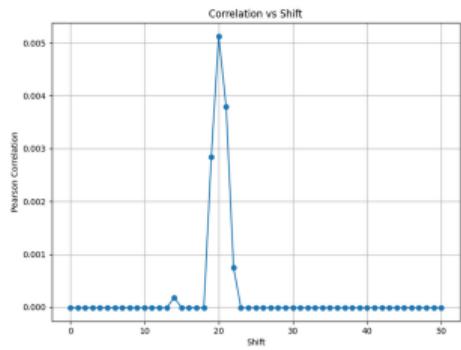


Si calcola coefficiente di Pearson $C_P = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$ shiftando le stringhe per trovare il picco.

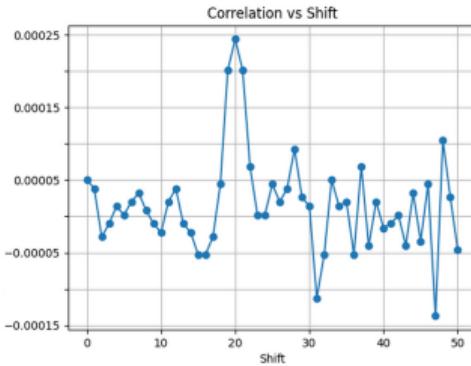
Analisi dati

Picco inizialmente individuato in assenza di rumore.

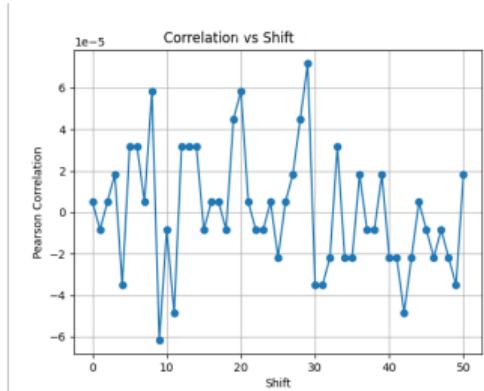
La posizione individuata è la differenza di tempo fra *signal* e *idler*.



$$\mu_B \sim 10^{-9}$$



$$\mu_B \sim 10^{-6}$$



$$\mu_B \sim 10^{-4}$$

All'aumentare del rumore, individuare il picco è sempre più difficile.

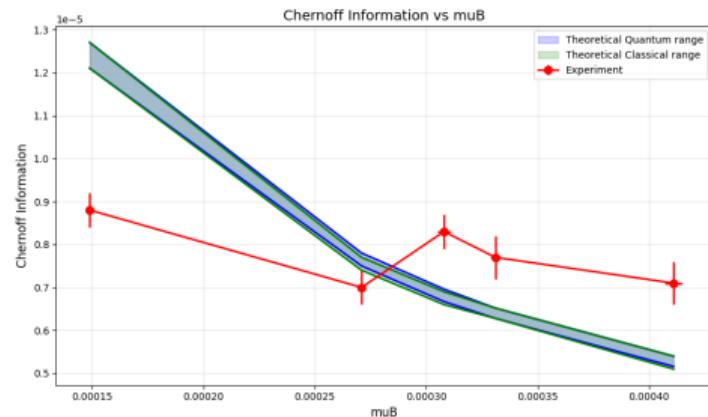
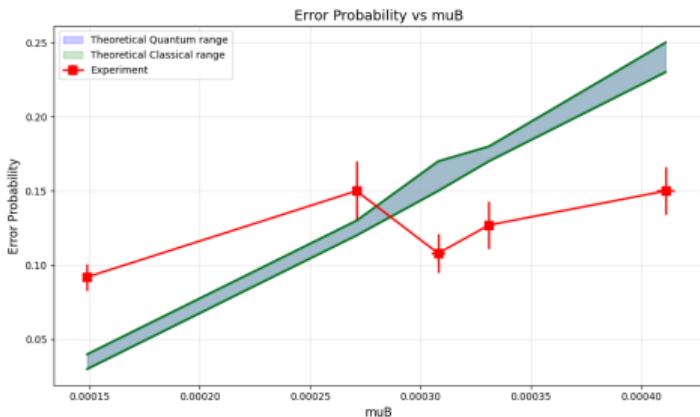
→ su N misure, $\#\text{err}$ trovano un picco diverso:

$$p_{\text{err,exp}} = \frac{N - \#\text{err}}{N}$$

Risultati

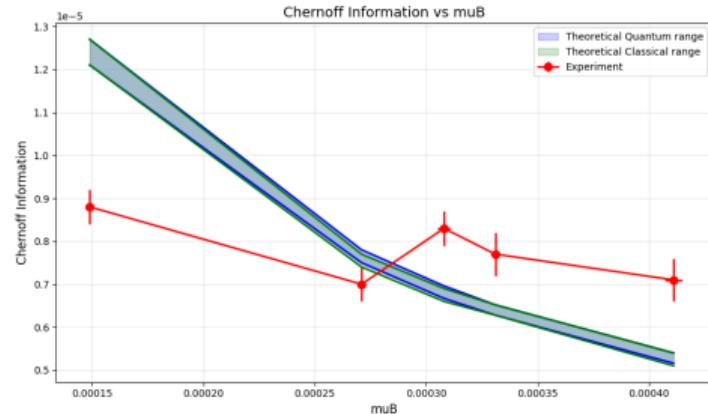
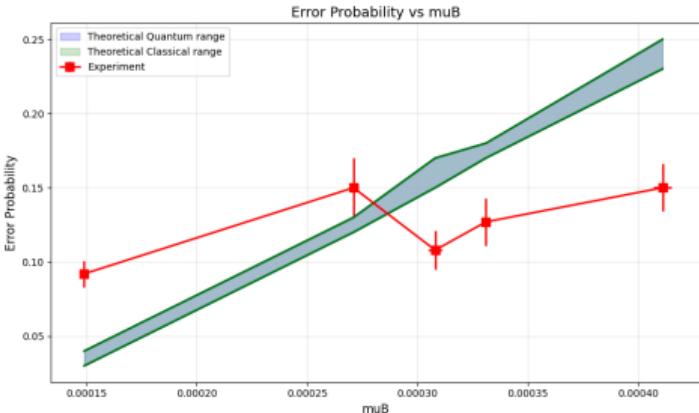
Si è studiato l'andamento (protocollo quantistico) di $p_{err,exp}$ e di ξ_{TR} con μ_B .
Parametri:

$$\mu = (9.83 \pm 7) \cdot 10^{-5} \quad \kappa = (1.74 \pm 0.05) \cdot 10^{-4} \quad \eta_{idler} = (9.4 \pm 4) \cdot 10^{-4}$$



Differenza risultato sperimentale - teoria dovuta a ...

Risultati



Zona esplorata presenta $Q \simeq 1$:

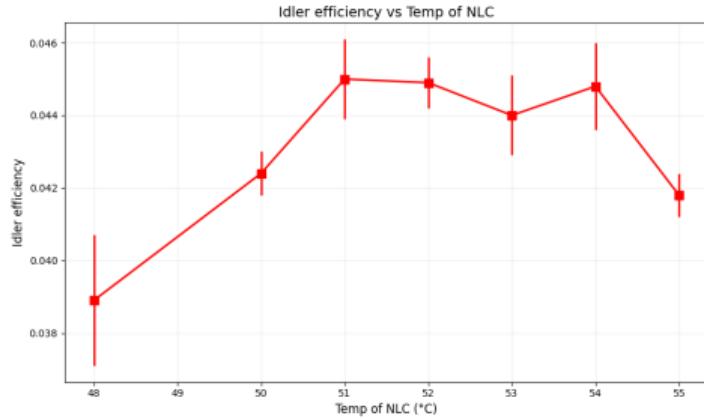
- $\eta_{idler} \ll 1$
- μ e μ_B mantenuti bassi per non surriscaldare gli SNSPD

$$\mu = (9.83 \pm 7) \cdot 10^{-5} \quad \mu_B \in [1.49, 4.11] \cdot 10^{-4}$$

$$\kappa = (1.74 \pm 0.05) \cdot 10^{-4} \quad \eta_{idler} = (9.4 \pm 4) \cdot 10^{-4}$$

Passi successivi

- Aumentare η_{idler} : controllo accoppiamento fibre ed efficienza SNSPD
- Studio teorico a bassi μ e μ_B per trovare zone ad alto Q
- Misure su protocolli classico e quantistico e confronto
- Covert sensing?



Grazie per l'attenzione