Example

December 4, 2015

Data:

- $N = 180$
- $Z = 5$
- $m_z = \{50, 60, 70, 20, 40\}$
- $a_z = \{30, 40, 45, 35, 30\}$
- *Tollerance* $T = 25$

We calculate $r_z = \frac{Nm_z}{\sum\limits_{z=1} m_z}$ and $T(r_z) = r_z - \frac{r_z * T}{100}$ the values are

z	1	2	3	4	5
m_z	50	60	70	20	40
r_z	38	45	53	15	30
$T(r_z)$	28	34	39	11	23
a_z	30	40	45	35	30

Table 1: Initial condition

Suppose that 10 taxis from zone 1 and 3 change status from *Available* to *Busy* or *Out of Service*, now we have $N = 160$ and this condition

z	1	2	3	4	5
m_z	50	60	70	20	40
$r_z^{'}$	33	39	46	13	27
$T(r_z^{'})$	24	29	34	10	20
$a_z^{'}$	20	40	35	35	30

Table 2: After the requests

Now we can see that the zone 1 and 3 have a deficit of taxis. In the next figure you can see the initial graph

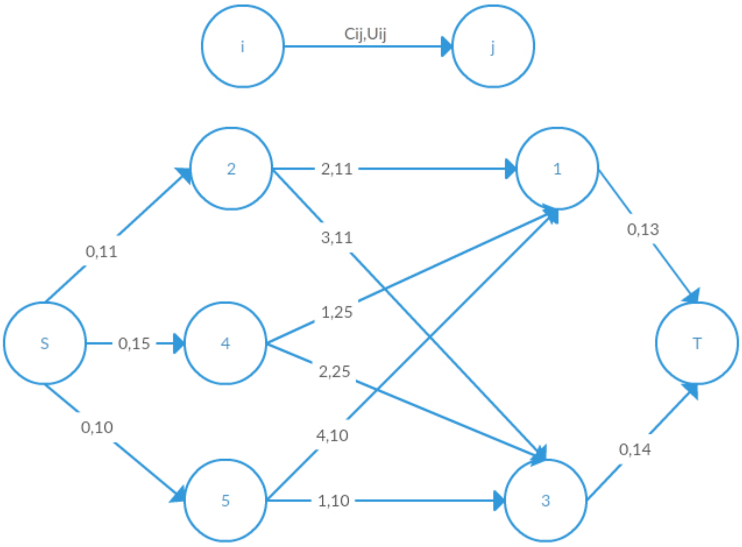

Figure 1: Our model in a graph

After that we must apply the maximum flow and draw the residual graph.

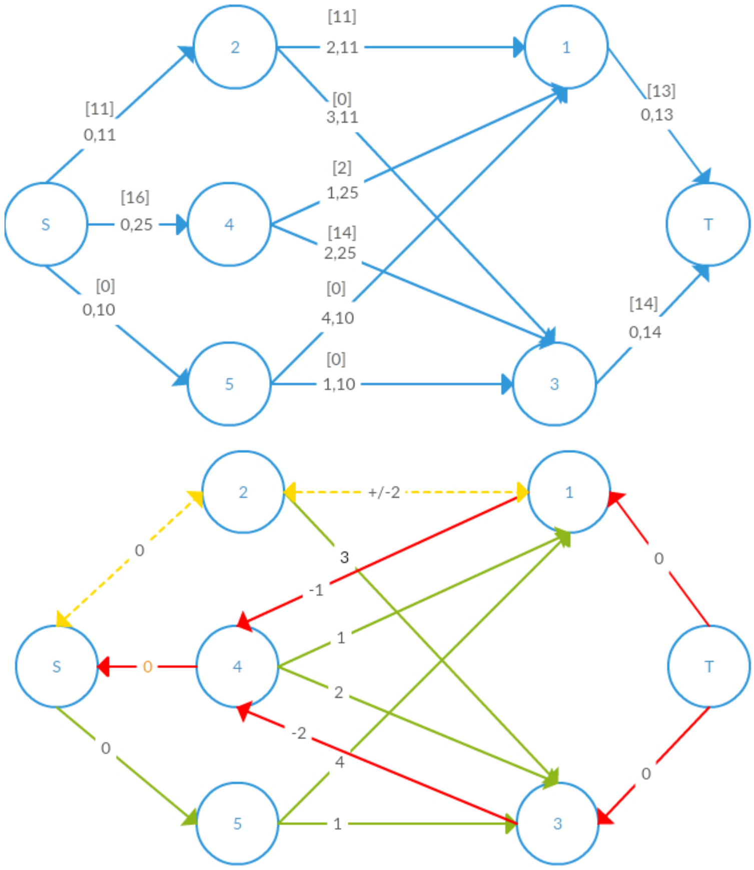

Figure 2: The maximum flow and its Residual Graph

Considering the residual graph in the figure 2 we see that there are several negative cycles in the residual graph ; we choose the path $S \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow S$ with total cost = -1

we must calculate $\theta = min\{$25-16; 25-2; 11; 11$\}$=9. Now we must add 9 units of flow to all the arcs that belong at $A^{+}(\bar{x})$ and subtract 9 unit of flow to all the arcs that belong at $A^{-}(\bar{x})$.

We obtain the next situtation

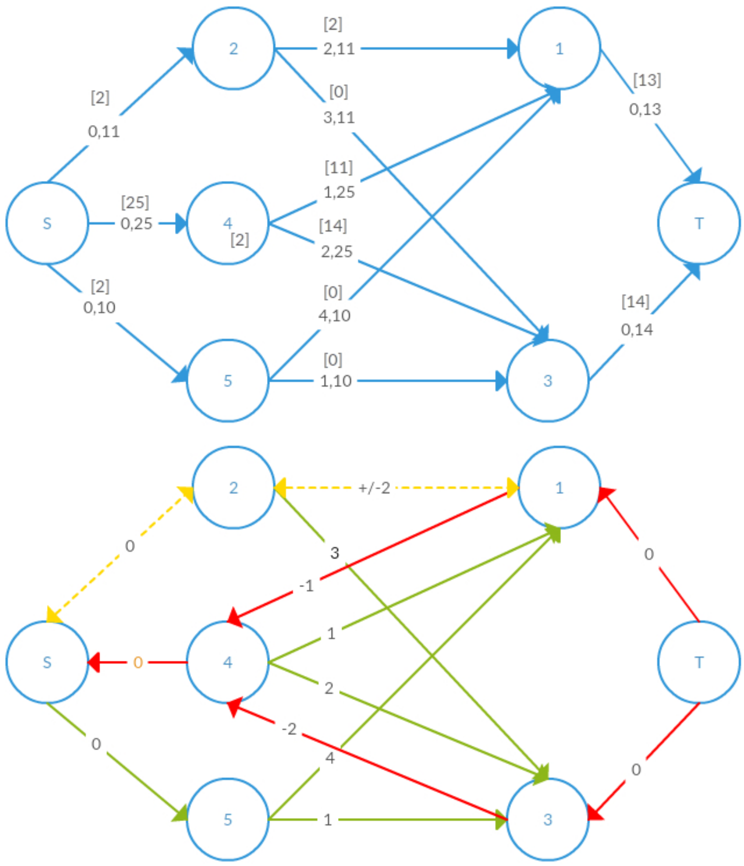

Figure 3: The Capacity and Residual graph after one iteration

Repeating the procedure we choose the path $S \rightarrow 5 \rightarrow 3 \rightarrow 4 \rightarrow S$ from the previous residual grpah and the cycle has $\theta = min\{$10-0,10-0,14,25$\}$=10.

Now we must add 10 units of flow to all the arcs that belong at $A^{+}(\bar{x})$ and subtract 10 unit of flow to all the arcs that belong at $A^{-}(\bar{x})$.

We obtain the next situation.

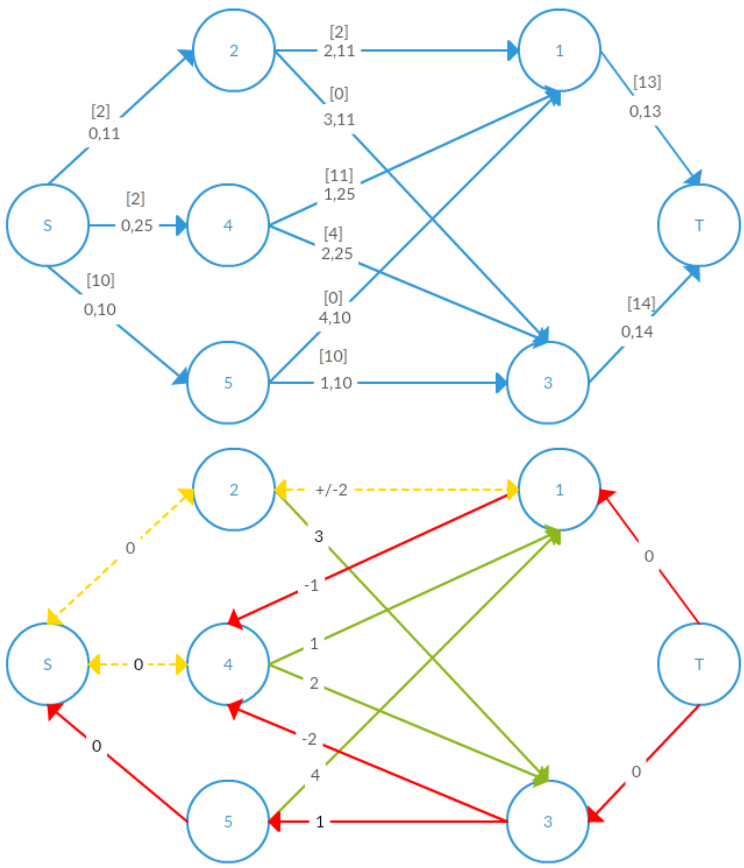

Figure 4: The Capacity and Residual graph after two iterations

Now we notice that there is another negative cycle in the path $S \to 4 \to 1 \to 2 \to S$, this because the previous step has permitted to release some flow from S to 4. We calculate $\theta = min\{$25-17; 25-13; 2; 2$\}$=2.

The algorithm adds 2 units of flow to all the arcs that belong at $A^+(\bar{x})$ and subtract 2 unit of flow to all the arcs that belong at $A^-(\bar{x})$, and we obtain the next final situation; in fact there aren't negative cycle yet, so we have obtain the optimal solution. You can see it in the next figure. The System must notify:

- 13 taxis from the zone 4 that their new area of competence is changed from 4 to 1
- 4 taxis from the zone 4 that their new area of competence is changed from 4 to 3
- 10 taxis from zone 5 that their new area of competence is changed from 5 to 3

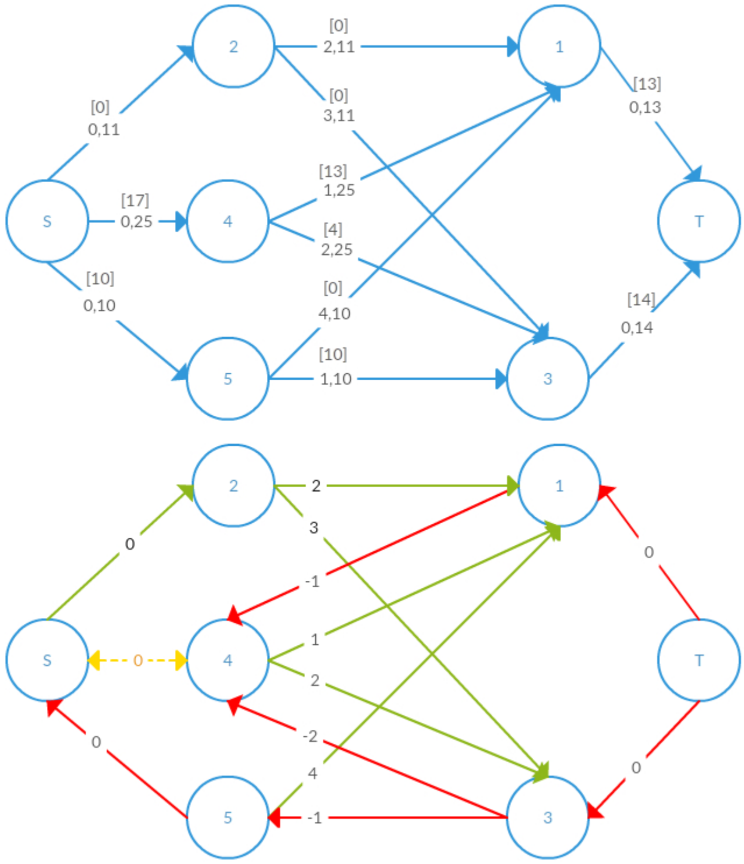

Figure 5: The final Capacity and Residual graph

z	1	2	3	4	5
m_z	50	60	70	20	40
$r_z^{'}$	33	39	46	13	27
$T(r_z^{'})$	24	29	34	10	20
$\ddot{a}_z$	33	40	49	18	20

Table 3: Final situation

We notice that this algorithm can «take» all $a_z - T(r_z)$ from one zone respecting the constrains, but if reach a request for example from the zone 5 we have already an incorrect taxis' distribution. To optimize the algorithm we can introduce a new, value smaller than T, that can be used to calculate the capacity of the arcs (s, i).