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Control of Vehicle Platoons for Highway Safety and Efficient Utility: Consensus with Communications and Vehicle Dynamics*

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Abstract Platoon formation of highway vehicles is a critical foundation for autonomous or semi-autonomous vehicle control for enhanced safety, improved highway utility, increased fuel economy, and reduced emission toward intelligent transportation systems. Platoon control encounters great challenges from vehicle control, communications, team coordination, and uncertainties. This paper introduces a new method for coordinated control of platoons by using integrated network consensus decisions and vehicle control. To achieve suitable coordination of the team vehicles based on terrain and environmental conditions, the emerging technology of network consensus control is modified to a weighted and constrained consensus seeking framework. Algorithms are introduced and their convergence properties are established. The methodology employs neighborhood information through on-board sensors and V2V or V2I communications, but achieves global coordination of the entire platoon. The ability of the methods in terms of robustness, disturbance rejection, noise attenuation, and cyber-physical interaction is analyzed and demonstrated with simulated case studies.

Keywords Platoon control, safety, collision avoidance, consensus control, networked systems, vehicle dynamics, communications.

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1 Introduction

Highway vehicle control is a critical task in developing intelligent transportation systems. Platoon formation has been identified as one promising strategy for enhanced safety, improved highway utility, increased fuel economy, and reduced emission toward autonomous or semi-autonomous vehicle control. The goal of longitudinal platoon control is to ensure that all the vehicles move in the same lane at the same speed with desired inter-vehicle distances. Platoon control adjusts vehicle spatial distribution such that roadway utilization is maximized while the risk of collision is minimized or avoided. In this study, platoon control will be realized in the framework of weighted and constrained consensus control with switching network topologies.

Platoon control has been studied in the contexts of intelligent highway control and automated highway systems for many years with numerous methodologies and demonstration systems [3, 15]. Many control methodologies have been applied, including PID controllers, state feedback, adaptive control, state observers, among others, with safety, string stability, and team coordination as the most common objectives [1, 11, 18]. On the other hand, an in-depth analysis of interaction among observation random noises and platoon formation, rigorous convergence analysis under random observation and network uncertainties, quantitative characterization of benefits of using communications in assisting platoon control remain major open issues.

This paper aims to introduce a new framework for vehicle coordination and control, based on the emerging technology of network consensus control under a stochastic framework. Algorithms are introduced and their convergence properties are established. In this paper, platoon formation is formulated as a weighted and constrained consensus control problem. Consensus control aims to coordinate all subsystems such that their formation converges to a desired distribution pattern. In vehicle applications the desired pattern is that the *weighted distances* between consecutive vehicles are equal. Consensus control is an emerging field in networked control and remains an active research field [2, 4–6, 16]. More recently, switching topologies and communication noises are taken into considerations in consensus control [9, 10, 21]. Most consensus controls are un-weighted and unconstrained. In our recent work [21, 22], within constrained consensus control, a Markov model is used to treat a much larger class of systems, where the network graph is modulated by a discrete-time Markov chain. Our work also provides convergence and rates of convergence for the corresponding recursive algorithms. In addition, communication delays are considered. Some of the useful features of [21, 22] are extended to the weighted and constrained consensus methodology for platoon control in this paper. In addition, the technique of post-iterate averaging is employed to enhance the platoon control performance. For detailed information on post-iterate averaging, see [14, 17] for its original introduction and [8, Chapter 11] for its extension to more general systems. With the iterate averaging, our algorithms provide the best convergence rate in terms of the best scaling factor and the smallest asymptotic covariance. Most significantly, they achieve asymptotically the well-known Cramér-Rao lower bounds [13], hence are best over all algorithms. This fast convergence feature is highly desirable for fast team formation.

The main features that distinguish this paper from the existing vehicle platoon control include: (1) Local control to achieve a global deployment. Although a desired coordination is achieved for the entire team, each vehicle only needs to communicate with its neighboring members by on-board sensors, V2V (vehicle-to-vehicle) or V2I (vehicle-to-infrastructure) communications. As such communication costs and complexity remain minimal. (2) Scalability. Expanding and reduction of the team members do not complicate control strategies. (3) Robustness. Fluctuations in vehicle positions, addition or reduction of the vehicles can be readily accommodated. (4) Constrained control. Vehicle platoons need to be confined to a desired platoon length. This introduces naturally a constraint on the platoon control, which is a unique feature beyond standard consensus control. Algorithms of this paper ensure that such a constraint is respected in each step of iteration. (5) Weighted consensus. Vehicles are subject to road, traffic, and weather conditions. Different vehicles have different safety distance requirements. For example, a heavy truck needs much more distance than a small car. As such, we introduce a weighted consensus control problem. This changes the control algorithms and iteration direction matrices. (6) Convergence under random noises. Due to sensor and communication uncertainties, observations are corrupted by noises. It can be shown that in platoon stability and convergence, noise effects are dominantly determining how fast a platoon can be formed. This paper presents rigorous analysis of platoon convergence and improvement by post-iterative averaging. (7) Time-varying topologies. When vehicles join or leave a platoon, or communication links are initiated or terminated, network topologies change. The algorithms of this paper work under switching topologies, a substantial potential to be explored to accommodate highly dynamic nature of platoon control. (8) Asymptotically optimal convergence rates. Achievable convergence rates under stochastic noises are bounded below by the Cramer-Rao (CR) lower bound. It is shown that our algorithms can achieve asymptotically the CR lower bound. In this sense they are asymptotically optimal. (9) Interaction between vehicle physical space and communications/decision cyber space. At present, on-board sensors are used in vehicle distance measurements. This paper employs convergence rates as a performance measure to evaluate additional benefits of different communication topologies in improving platoon formation, robustness, and safety. The framework has a far-reaching implication for integrated vehicle and cyber-space design to utilize communication resources wisely for platoon control performance improvement.

The rest of the paper is organized into the following sections. Section 2 introduces the basic platoon formation problem. Section 3 describes how a typical vehicle coordination problem can be formulated as a weighted and constrained consensus control problem. Algorithms for weighted and constrained consensus control are presented in Section 4. Their convergence properties and convergence rates are established. Section 5 further enhances the algorithms by post-iterate averaging. It is shown that consensus control for vehicles are subject to noises and their effect can be attenuated by the post-iterate averaging. Optimality of such modified algorithms are established. Using the CR lower bound as a performance measure, Section 6 demonstrates how much a specific communication network can improve platoon formation. Section 7 explores interaction between coordination decisions on the cyber space and vehicle control on the physical space. Finally, Section 8 points out directions of further studies.

2 Preliminaries on Platoon Formation

We concentrate on longitudinal platoon control. This represents the case of $r+1$ vehicles driving in the same lane, forming a platoon. The leading vehicle serves as a reference, whose position p_0 is used as the origin of the line coordinate (hence, $p_0(t) \equiv 0$), and its speed v_0 is the reference speed for all remaining vehicles in the platoon to follow. Coordination of vehicle control is to sustain a platoon formation, avoid collision, adjust the formation according to weather and road conditions, converge fast to a new formation after disturbances, reconfigure a formation after vehicle addition and departure.

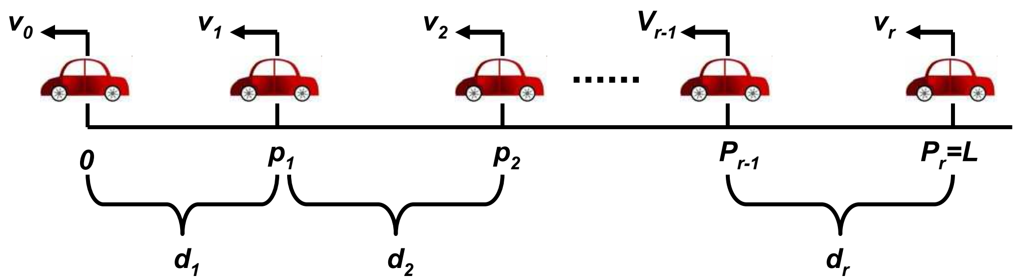

Figure 1: Platoon coordinates

In a platoon formation, see Figure 1, each vehicle's position is defined by the central point of its length and denoted by $p_j(t)$, $j = 1, \ldots, r$, which is the distance of the jth vehicle to the leading vehicle. The vehicle velocity will be denoted by $v_j(t) = dp_j(t)/dt \geq 0$, $j = 1, \ldots, r$. Let the inter-vehicle distances be defined as

$$d_j(t) = p_j(t) - p_{j-1}(t), j = 1, \ldots, r.$$

The leading vehicle's speed $v_0(t)$ is the speed target for all the other vehicles in the platoon to follow. Also, a desired distance β between consecutive vehicles is a goal that balances efficiency and safety. In principle, β is a function of weather, road condition, platoon traveling speed, terrain composition (uphill or downhill), and road curvatures. If one does not consider differences among vehicles and terrain conditions, the following un-weighted and unconstrained consensus problem follows.

Definition 1 A platoon is said to be in (un-weighted and unconstrained) consensus if

$$v_j(t) = v_0(t), \text{ and } d_j(t) = \beta(t), j = 1, \ldots, r.$$

Denote $d(t) = [d_1(t), \ldots, d_r(t)]'$ and $v(t) = [v_1(t), \ldots, v_r(t)]'$, and consensus errors

$$e(t) = \begin{bmatrix} e_1(t) \\ \vdots \\ e_r(t) \end{bmatrix} = d(t) - \beta \mathbb{1}; \varepsilon(t) = \begin{bmatrix} \varepsilon_1(t) \\ \vdots \\ \varepsilon_r(t) \end{bmatrix} = v(t) - v_0(t)\mathbb{1}$$

where $\mathbb{1} = [1, \ldots, 1]'$. Starting at $t = 0$ with initial condition $e(0)$ and $\varepsilon(0)$, the goal of consensus control is to achieve convergence

$$e(t) \to 0; \varepsilon(t) \to 0, t \to \infty$$

either strongly or in mean squares. It is noted that to accommodate the time-varying environment, convergence speeds are of interest also. Since the speed convergence follows from the distance convergence, this paper will focus on the distance consensus.

The basic scheme of platoon formation employs a sensor-based network topology, in which a vehicle uses sensors to measure its own speed and relative distance to the vehicle ahead of it. As a result, v_{j-1}, v_j, d_j are available to the jth vehicle in its control strategies. This sensor-based inter-vehicle information can be represented by a string topology, shown in Figure 2.

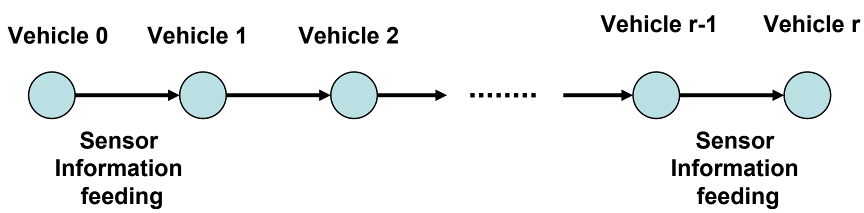

Figure 2: Sensor-based inter-vehicle communication networks

On the other hand, inter-vehicle wireless communications allow enhanced information exchange between vehicles. Figure 3 represents a more advanced inter-vehicle communication, in which the jth vehicle receives not only the parameters from the $(j-1)$th vehicle by sensors, but also the information from $(j-2)$th vehicle via wireless communications. Benefits and limitations of communication networks on platoon formation will be studied in this paper.

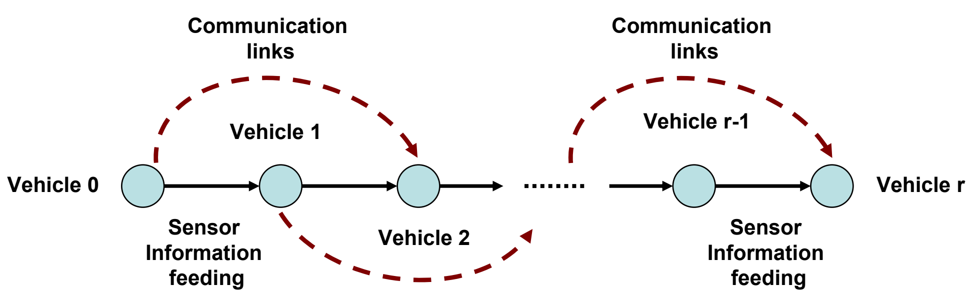

Figure 3: Information network topologies using inter-vehicle communications

3 Weighted and Constrained Consensus Control for Platoon Coordination

The basic consensus control formulation follows [21] in which an un-weighted but constrained consensus control strategy under Markovian switching network topology was introduced. The strategy is extended here to include weighted consensus and vehicle dynamics. This weighted and constrained consensus control problem has applications to UAV control problems [19], and smart grids [20], among others.

A team consists of r vehicles. They are to be deployed along a highway segment of total length L. At time t, we should denote the total length of the surveillance range as $L(t)$. In algorithm development, L is treated as a constant. Its changes will be viewed as a disturbance to the consensus control problem. d_j is the distance between vehicle j and vehicle $j-1$. We have the constraint

$$\sum_{j=1}^{r} d_j(t) = L. \tag{1}$$

Due to vehicle weights and types and terrain conditions, desired inter-vehicle distances vary with j. Each inter-vehicle distance has a weighting factor γ^j. The goal of platoon control is to achieve consensus on weighted distance d_j/γ^j, namely

$$\frac{d_j(t)}{\gamma^j} \to \beta, \quad j = 1, \ldots, r$$

for some constant β. The convergence is either with probability one (w.p.1.) or in means squares (MS). For notational convenience in algorithm development, we use $x_j(t) = d_j(t)$ and denote the state vector $x(t) = [x_1(t), \ldots, x_r(t)]'$. The weighting coefficients are $\gamma = [\gamma^1, \ldots, \gamma^r]'$, and the state scaling matrix $\Psi = \mathrm{diag}[1/\gamma^1, \ldots, 1/\gamma^r]$, where v' is the transpose of a vector or a matrix v. Let $\mathbb{1}$ be the column vector of all 1s. Together with the constraint (1), the target of the constrained and weighted consensus control is

$$\Psi x(t) \to \beta \mathbb{1},$$

subject to $\mathbb{1}' x(t) = L$. It follows from $\gamma' \Psi = \mathbb{1}'$ that

$$\beta = \frac{L}{\gamma' \mathbb{1}} = \frac{L}{\gamma^1 + \cdots + \gamma^r}.$$

The vehicles are linked by an information network, represented by a directed graph $\mathcal{G}$. $(i, j) \in \mathcal{G}$ indicates estimation of the state d_j by the vehicle i via a communication link. For node i, $(i, j) \in \mathcal{G}$ is a departing edge and $(l, i) \in \mathcal{G}$ is an entering edge. The total number of communication links in $\mathcal{G}$ is l_s. From its physical meaning, node i can always observe its own state, which will not be considered as a link in $\mathcal{G}$.

For a selected time interval T,* the consensus control is performed at the discrete-time steps $nT, n = 1, 2, \ldots$. At the control step n, the value of x will be denoted by $x_n = [x_n^1, \ldots, x_n^r]'$.

Vehicle platoon control updates x_n to x_{n+1} by the amount u_n

$$x_{n+1} = x_n + u_n \tag{2}$$

*For conciseness, we assume synchronous sampling schemes in this paper. However, the results of this paper can be extended to irregular sampling or random sampling schemes, under appropriate constraints on the sampling intervals.

with $u_n = [u_n^1, \dots, u_n^r]'$. In platoon control, a distance adjustment a_n^{ij} (called link control) of vehicle i at the nth step based on the weighted separations of vehicles i and j to the vehicles in front of them respectively is the decision variable. The control u_n^i is determined by the link control a_n^{ij} as

$$u_n^i = -\sum_{(i,j)\in\mathcal{G}} a_n^{ij} + \sum_{(j,i)\in\mathcal{G}} a_n^{ji}. \tag{3}$$

The most relevant implication in this control scheme is that for all n,

$$\sum_{i=1}^{r} x_n^i = \sum_{i=1}^{r} x_0^i = L \tag{4}$$

that is, the constraint (1) is always satisfied. Consensus control seeks control algorithms such that $\Psi x_n \to \beta \mathbb{1}$ under the constraint (4).

A link $(i,j) \in \mathcal{G}$ entails an estimate $\widehat{x}_n^{ij}$ of x_n^j by node i with observation noise ζ_n^{ij}. That is,

$$\widehat{x}_n^{ij} = x_n^j + \zeta_n^{ij}. \tag{5}$$

Let $\widetilde{x}_n$ and ζ_n be the l_s-dimensional vectors that contain all $\widehat{x}_n^{ij}$ and ζ_n^{ij} in a selected order, respectively. Then, (5) can be written as

$$\widetilde{x}_n = H_1 x_n + \zeta_n \tag{6}$$

where H_1 is an $l_s \times r$ matrix whose rows are elementary vectors such that if the ℓth element of $\widetilde{x}_n$ is $\widehat{x}^{ij}$ then the ℓth row in H_1 is the row vector of all zeros except for a "1" at the jth position. Each link in $\mathcal{G}$ provides information $\delta_n^{ij} = x_n^i/\gamma^i - \widehat{x}_n^{ij}/\gamma^j$, an estimated difference between the weighted x_n^i and x_n^j. This information may be represented by a vector δ_n of size l_s containing all δ_n^{ij} in the same order as $\widetilde{x}_n$. δ_n can be written as

$$\delta_n = H_2 \Psi x_n - \widetilde{\Psi}\widetilde{x}_n = H_2 \Psi x_n - \widetilde{\Psi} H_1 x_n - \widetilde{\Psi}\zeta_n = H x_n - \widetilde{\Psi}\zeta_n, \tag{7}$$

where the link scaling matrix $\widetilde{\Psi}$ is the $l_s \times l_s$ diagonal matrix whose k-th diagonal element is $1/\gamma^j$ if the k-th element of $\widetilde{x}_n$ is $\widehat{x}_n^{ij}$; H_2 is an $l_s \times r$ matrix whose rows are elementary vectors such that if the ℓth element of $\widetilde{x}(k)$ is $\widehat{x}^{ij}$ then the ℓth row in H_2 is the row vector of all zeros except for a "1" at the ith position, and $H = H_2\Psi - \widetilde{\Psi}H_1$.

Due to network constraints, the information δ_n^{ij} can only be used by nodes i and j. When the platoon control is linear, time invariant, and memoryless, we have $a_n^{ij} = \mu_n g_{ij} \delta_n^{ij}$ where g_{ij} is the link control gain and μ_n is a global time-varying scaling factor which will be used in state updating algorithms as the recursive step size. Selections of the link gains and μ_n are to ensure convergence of the consensus control. Their further impact on convergence rates will become clearer later. Let G be the $l_s \times l_s$ diagonal matrix that has g_{ij} as its diagonal element. In this case, the control becomes $u_n = -\mu_n J' G \delta_n$, where $J = H_2 - H_1$. For convergence analysis, we note that μ_n is a global control variable and we may represent u_n equivalently as

$$u_n = -\mu_n J' G (H x_n - \widetilde{\Psi}\zeta_n) = \mu_n (M x_n + W \zeta_n), \tag{8}$$

with $M = -J'GH$ and $W = J'G\widetilde{\Psi}$. This, together with (2), leads to

$$x_{n+1} = x_n + \mu_n(Mx_n + W\zeta_n). \tag{9}$$

It can be directly verified that $\widetilde{\Psi}H_1\Psi^{-1} = H_1$, $H\Psi^{-1} = J$, $J\mathbb{1} = 0$, $\Psi^{-1}\mathbb{1} = \gamma$. These imply that $\mathbb{1}'M = 0$, $\mathbb{1}'W = 0$, $M\Psi^{-1}\mathbb{1} = M\gamma = 0$. Note that for simplicity, we presented the problem using the simplest setup. In the above, the noise $W\zeta_n$ is additive. Much more general noise types can be treated as demonstrated in [21]. The following assumption is imposed on the network.

(A0) (1) All link gains are positive, $g_{ij} > 0$.
(2) $\mathcal{G}$ is strongly connected.†

We now use an example to illustrate the above concepts.

Example 1 A team of four vehicles has an assigned total length L. Vehicle i controls the distance d_i, $i = 1, 2, 3, 4$. Then the condition $d_1 + d_2 + d_3 + d_4 = L$ is imposed as a constraint. The information topology is that in addition to observing their own controlled variables, vehicle 1 observes also d_2, vehicle 2 observes also d_1 and d_3, vehicle 3 observes d_2 and d_4. the controller for d_4 observes d_3 also. The total length $L = 53.9$ m. The weighting factors are $\gamma^1 = 12$, $\gamma^2 = 15$, $\gamma^3 = 20$, and $\gamma^4 = 28$. As a result,

$$\mathcal{G} = \{(1,2), (2,1), (2,3), (3,2), (3,4), (4,3)\}$$

$x = [d_1, d_2, d_3, d_4]'$, $\gamma = [12, 15, 20, 28]'$, $\Psi = \text{diag}[1/12, 1/15, 1/20, 1/28]$.

Since $L = 53.9$, we have

$$\beta = \frac{L}{\gamma^1 + \gamma^2 + \gamma^3 + \gamma^4} = 0.7187,$$

and the weighted consensus is $\Psi x = 0.7187\mathbb{1}$ or

$$x = 0.7187\Psi^{-1}\mathbb{1} = [8.624, 10.781, 14.374, 20.124]'.$$

By choosing the order for the links as $(1,2), (2,1), (2,3), (3,2), (3,4), (4,3)$, we have

$$\widetilde{x} = [\widehat{x}^{12}, \widehat{x}^{21}, \widehat{x}^{23}, \widehat{x}^{32}, \widehat{x}^{34}, \widehat{x}^{43}]'$$

and

$$H_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}; H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

†A directed graph is called strongly connected if there is a path from each node in the graph to every other node. This assumption is stronger than what is absolutely needed for convergence. However, it makes practical sense that the local information can be propagated to other subsystems through the network.

It follows that $\widetilde{\Psi} = \text{diag}[1/15, 1/12, 1/20, 1/15, 1/28, 1/20]$ and

$$\begin{aligned} H &= H_2\Psi - \widetilde{\Psi}H_1 \\ &= \begin{bmatrix} 1/12 & 0 & 0 & 0 \\ 0 & 1/15 & 0 & 0 \\ 0 & 1/15 & 0 & 0 \\ 0 & 0 & 1/20 & 0 \\ 0 & 0 & 1/20 & 0 \\ 0 & 0 & 0 & 1/28 \end{bmatrix} - \begin{bmatrix} 0 & 1/15 & 0 & 0 \\ 1/12 & 0 & 0 & 0 \\ 0 & 0 & 1/20 & 0 \\ 0 & 1/15 & 0 & 0 \\ 0 & 0 & 0 & 1/28 \\ 0 & 0 & 1/20 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1/12 & -1/15 & 0 & 0 \\ -1/12 & 1/15 & 0 & 0 \\ 0 & 1/15 & -1/20 & 0 \\ 0 & -1/15 & 1/20 & 0 \\ 0 & 0 & 1/20 & -1/28 \\ 0 & 0 & -1/20 & 1/28 \end{bmatrix}, \end{aligned}$$

$$J = H_2 - H_1 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

Suppose the control gains on the links are selected as $g_{12} = g_{21} = 3$,$g_{23} = g_{32} = 7$, $g_{34} = g_{43} = 9$. Then $G = \text{diag}[3, 3, 7, 7, 9, 9]$. It follows that

$$M = -J'GH = \begin{bmatrix} -1/2 & 1/2.5 & 0 & 0 \\ 1/2 & -2/1.5 & 7/10 & 0 \\ 0 & 7/7.5 & -4/2.5 & 9/14 \\ 0 & 0 & 9/10 & -9/14 \end{bmatrix},$$

$$W = J'G\widetilde{\Psi} = \begin{bmatrix} 1/5 & -1/4 & 0 & 0 & 0 & 0 \\ -1/5 & 1/4 & 7/20 & -7/15 & 0 & 0 \\ 0 & 0 & -7/20 & 7/15 & 9/28 & -9/20 \\ 0 & 0 & 0 & 0 & -9/28 & 9/20 \end{bmatrix}.$$

Since $\mathbb{1}'J' = \mathbb{1}'(H_2 - H_1)' = 0$, we have $\mathbb{1}'M = 0$ and $\mathbb{1}'W = 0$. We can show that under Assumption (A0), M has rank $r-1$ and is negative semi-definite. The proof uses similar ideas as in [21] and hence is omitted here. Recall that a square matrix $\widetilde{Q} = [\widetilde{q}_{ij}]$ is a generator of a continuous-time Markov chain if $\widetilde{q}_{ij} \ge 0$ for all $i \neq j$ and $\sum_j \widetilde{q}_{ij} = 0$ for each i. Note that a generator of the associated continuous-time Markov chain is irreducible if the system of equations

$$\begin{cases} \nu\widetilde{Q} = 0, \\ \nu\mathbb{1} = C \end{cases} \tag{10}$$

for a given constant $C > 0$ has a unique solution, where $\nu = [\nu_1, \ldots, \nu_r] \in \mathbb{R}^{1\times r}$ with $\nu_i/C > 0$ for each $i = 1, \ldots, r$. When $C = 1$, ν is the associated stationary distribution. Consequently, under Assumption (A0), M is a generator of a continuous-time irreducible Markov chain.

4 Weighted and Constrained Consensus Control Algorithms and Convergence

4.1 Algorithms

We begin by considering the state updating algorithm (9)

$$x_{n+1} = x_n + \mu_n M x_n + \mu_n W \zeta_n, \tag{11}$$

together with the constraint

$$\mathbb{1}'x_n = L, \tag{12}$$

where $\{\mu_n\}$ is a sequence of stepsizes, M is a generator of a continuous-time Markov chain (hence $\mathbb{1}'M = 0$), $\{\zeta_n\}$ is a noise sequence.

Since the algorithm (11) is a stochastic approximation procedure, we can use the general framework in Kushner and Yin [8] to analyze the asymptotic properties. Since $\mathbb{1}'M = 0$ and $\mathbb{1}'W = 0$, starting from the initial condition with $\mathbb{1}'x_0 = L$, the constraint $\mathbb{1}'x_n = L$ is always satisfied by the algorithm structure.

(A1)
1. The stepsize satisfies the following conditions: $\mu_n \ge 0$, $\mu_n \to 0$ as $n \to \infty$, and $\sum_n \mu_n = \infty$.[‡]
2. The noise $\{\zeta_n\}$ is a stationary ϕ-mixing sequence[§] such that $E\zeta_n = 0$, $E|\zeta_n|^{2+\Delta} < \infty$ for some $\Delta > 0$, and that the mixing measure $\widetilde{\phi}_n$ satisfies

$$\sum_{k=0}^{\infty} \widetilde{\phi}_n^{\Delta/(1+\Delta)} < \infty, \tag{13}$$

[‡]Some commonly used stepsize sequences includes $\mu_n = a/n^\alpha$ for $1/2 < \alpha \le 1$. In such cases, $\sum_{n=1}^{\infty} \mu_n = \infty$ but $\sum_{n=1}^{\infty} \mu_n^2 < \infty$.

[§]ϕ-mixing sequences contain independent noises as a special case. However, they can represent a much larger class of noises to accommodate communication uncertainties such as signal interference, signal fading, latency, etc.

where

$$\widetilde{\phi}_n = \sup_{A \in \mathcal{F}^{n+m}} E^{(1+\Delta)/(2+\Delta)} |P(A|\mathcal{F}_m) - P(A)|^{(2+\Delta)/(1+\Delta)},$$
$$\mathcal{F}_n = \sigma\{\zeta_k; k < n\}, \quad \mathcal{F}^n = \sigma\{\zeta_k; k \geq n\}.$$

Under Assumption (A0), M has an eigenvalue 0 of multiplicity 1 and all other eigenvalues are in the left complex plan (i.e., the real parts of the eigenvalues are negative). The null space of M is spanned by the vector $\gamma = [\gamma^1, \ldots, \gamma^r]'$. As a consequence of (A1), the ϕ-mixing implies that the noise sequence $\{\zeta_n\}$ is strongly ergodic [7, p. 488] in that for any integer m

$$\frac{1}{n} \sum_{j=m}^{m+n-1} \zeta_j \to 0, \quad \text{w.p.1 as } n \to \infty. \tag{14}$$

4.2 Convergence Properties

To study the convergence of the algorithm (11), we employ the stochastic approximation methods developed in [8]. All proofs are omitted. Instead of working with the discrete-time iterations, we examine sequences defined in an appropriate function space. This will enable us to get a limit ordinary differential equation (ODE).

It is readily seen that (11) is a stochastic approximation procedure, which can be studied by using the well-known stochastic approximation methods [8]. We define¶

$$t_n = \sum_{j=0}^{n-1} \mu_j, \ \ m(t) = \max\{n : t_n \leq t\}, \tag{15}$$

the piecewise constant interpolation $x^0(t) = x_n$ for $t \in [t_n, t_{n+1})$, and the shift sequence $x^n(t) = x^0(t + t_n)$. Then we can show that $\{x^n(\cdot)\}$ is equicontinuous in the extended sense (see [8, p. 102]) w.p.1. Thus we can extract a convergent subsequence, which will be denoted by $x^{n_\ell}(\cdot)$. Then the Arzela-Ascoli theorem concludes that $x^{n_\ell}(\cdot)$ converges to a function $x(\cdot)$ which is the unique solution (since the recursion is linear in x) of the ordinary differential equation (ODE)

$$\dot{x} = Mx. \tag{16}$$

The significance of the ODE is that the stationary point is exactly the true value of the desired weighted and constrained consensus. Then, convergence becomes a stability issue. For simplicity and clarity, we will only outline the main steps involved in the proof. We can first derive a preliminary estimate on the second moments of x_n.

Lemma 1 *Under Assumption* (A1)*, for any* $0 < T < \infty$*,*

$$\sup_{n \leq m(T)} E|x_n|^2 \leq K \ \ \text{and} \ \ \sup_{0 \leq t \leq T} E|x^n(t)|^2 \leq K, \tag{17}$$

for some $K > 0$*, where* $m(\cdot)$ *is defined in (15).*

¶For convergence analysis, the conditions A1 on μ_n is required. On the other hand, to track time-varying targets, constant step sizes μ can be used. In that case, $t_n = n\mu$.

Theorem 2 *Under Assumption (A1), the iterates generated by the stochastic approximation algorithm (11) satisfies* $\Psi x_n \to \beta \mathbb{1}$ *w.p.1 as* $n \to \infty$.

Furthermore, the algorithm (11) together with $x_n' \mathbb{1} = L$ leads to the desired weighted and constrained consensus. The equilibria of the limit ODE (16) and this constraint lead to the following system of equations

$$\begin{cases} Mx = 0 \\ \mathbb{1}' x = L. \end{cases} \tag{18}$$

The irreducibility of M then implies that (18) has a unique solution $x_* = \beta \Psi^{-1} \mathbb{1} = \beta \gamma$, which is precisely the weighted consensus.

Example 2 We now use the system in Example 1 to demonstrate the weighted consensus control. As in Example 1, the total distance is 53.9 m. Suppose that the initial distance distribution from the three vehicles are $d_0^1 = 12$ m; $d_0^2 = 14$ m; $d_0^3 = 10.9$ m; $d_0^4 = 17$ m. Weighted consensus for vehicle control aims to distribute distances according to the terrain and vehicle conditions defined by $\gamma^1 = 12$, $\gamma^2 = 15$, $\gamma^3 = 20$, $\gamma^4 = 28$, with the total $\mathbb{1}'\gamma = 75$. The target percentage distance distribution over the whole length is $[12/75, 15/75, 20/75, 28/75] = [0.1600, 0.2000, 0.2667, 0.3733]$. From the total length of 53.9 m, the goal of weighted consensus is $d^1 = 8.624$ m; $d^2 = 10.780$ m; $d^3 = 14.373$ m; $d^4 = 20.123$ m.

Suppose that the link observation noises are i.i.d sequences of Gaussian noises with mean zero and variance 1. Figure 4 shows the inter-vehicle distance trajectories. Staring from a large disparity in distance distribution, the top plot shows how distances are gradually distributed according to the terrain and vehicle conditions. The middle plot illustrates that the weighted distances converge to a constant. The weighted consensus error trajectories are plotted in the bottom figure.

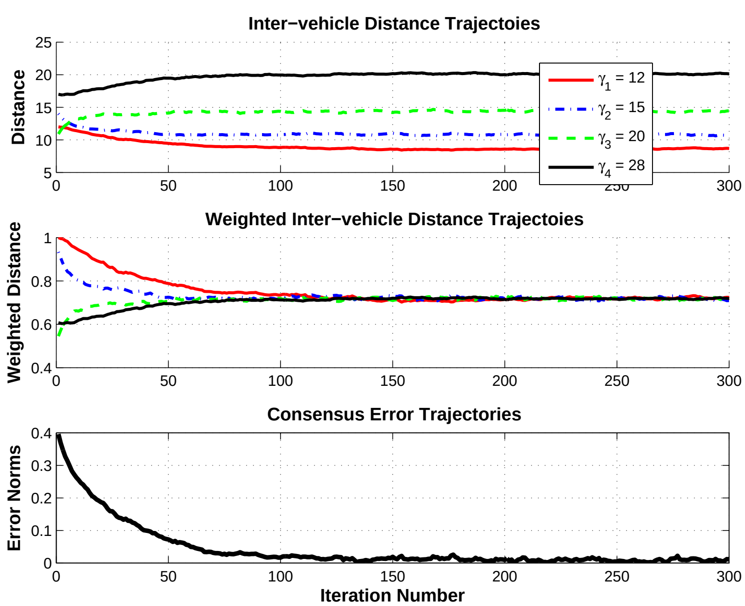

Figure 4: Vehicle distance control with weighted consensus

The capability of the consensus control in attenuating disturbance's impact on the platoon formation can also be evaluated. Suppose that a sudden braking of the leading vehicle results in a sudden distance change in d_1 by 4 m. Consensus control then restores the desired distance distribution, shown in Figure 5.

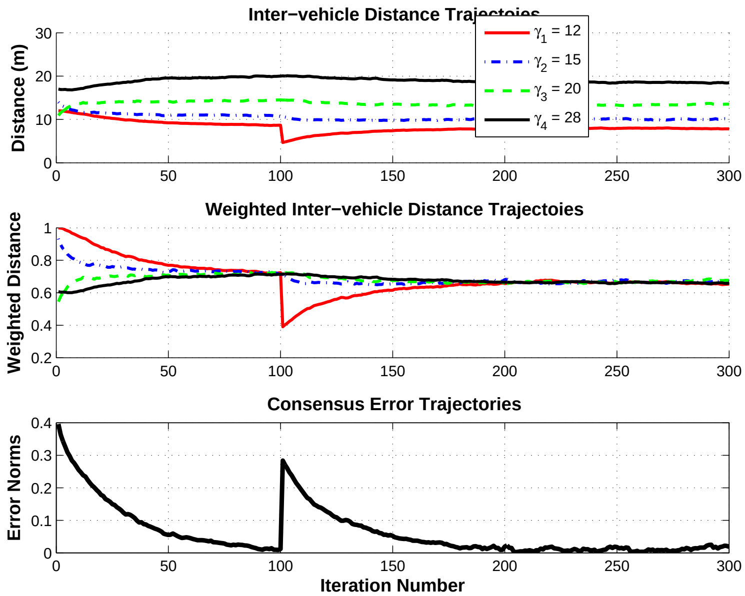

Figure 5: Disturbance rejection in vehicle distance control

5 Observation Noise and Post-Iterate Averaging

The basic stochastic approximation algorithm (11) demonstrates desirable convergence properties under relatively small observation noises. However, its convergence rate is not optimal. Especially when noises are large, its convergence may not be sufficiently fast and its states show fluctuations. For example, for the same system as in Example 2, if the noise standard deviation is increased from 1 to 20, its state trajectories demonstrate large variations, as shown in Figure 6.

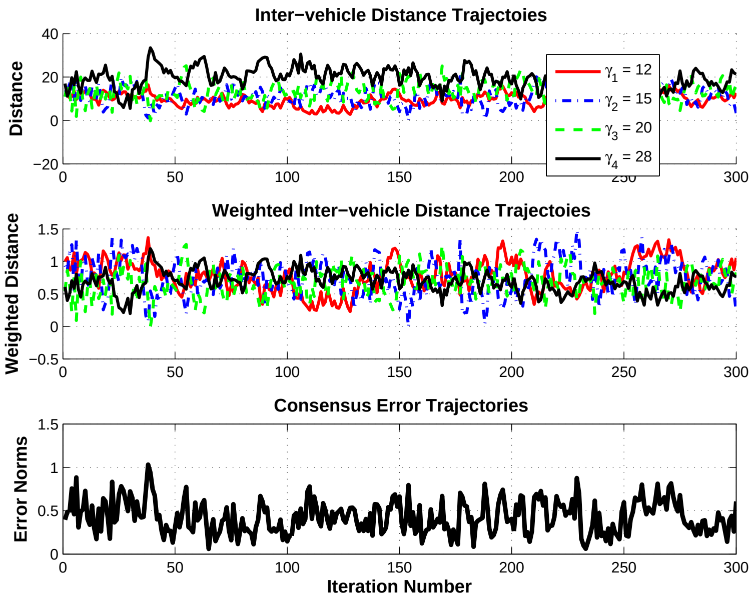

Figure 6: vehicle distance control with weighted consensus under large observation noise

To improve convergence efficiency, we take post-iterate averaging, resulting in a two-stage stochastic approximation algorithm. For definiteness and simplicity, we take $\mu_n = c/n^\alpha$ for some $(1/2) < \alpha < 1$ and $c > 0$. The algorithm is modified to

$$\begin{aligned} x_{n+1} &= x_n + \frac{c}{n^\alpha} M x_n + \frac{c}{n^\alpha} W \zeta_n \\ \overline{x}_{n+1} &= \overline{x}_n - \frac{1}{n+1}\overline{x}_n + \frac{1}{n+1} x_{n+1}. \end{aligned} \tag{19}$$

In what follows, without loss of generality set $c = 1$ for simplicity. In any case, as shown in [8], the constant c does not affect the asymptotic distribution. Since $\mathbb{1}'M = 0$ and $\mathbb{1}'W = 0$, we have $\mathbb{1}'\overline{x}_n = L$. As a result, the constraint (1) remains satisfied after the post-iterate averaging.

5.1 Asymptotic Efficiency

Strong convergence of the averaged $\overline{x}_n$ follows from that of x_n. This is stated in the following theorem with its proof omitted.

Theorem 3 *Suppose the conditions of Theorem* 2 *are satisfied. For iterates generated by algorithm (19)* (*together with the constraint* $\mathbb{1}'x_n = L$), $x_n \to \beta\Psi^{-1}\mathbb{1}$ *w.p.1 as* $n \to \infty$.

We now establish the optimality of the algorithms. For clarity, we will include the dimension of the vector $\mathbb{1}$ in notation in the following derivations when needed. Also, the proofs of the theorems are omitted. The reader is referred to our recent work [21] for details.

Partition the matrix M as

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \tag{20}$$

where $M_{11} \in \mathbb{R}^{(n-1)\times(n-1)}$, $M_{12} \in \mathbb{R}^{(n-1)\times 1}$, $M_{21} \in \mathbb{R}^{(n-1)\times 1}$, and $M_{22} \in \mathbb{R}^{1\times 1}$. Accordingly, we also partition $\overline{x}_n$, x_n, and W as

$$\overline{x}_n = \begin{bmatrix} \widetilde{\overline{x}}_n \\ \overline{x}_n^r \end{bmatrix}; \; x_n = \begin{bmatrix} \widetilde{x}_n \\ x_n^r \end{bmatrix}; \; W = \begin{bmatrix} \widetilde{W} \\ W_1 \end{bmatrix}, \tag{21}$$

respectively, with compatible dimensions with those of M.

Lemma 4 *Under Assumption A0, M_{11} is full rank.*

This result indicates that we can concentrate on $r-1$ components of $\overline{x}_n$. We can show that the asymptotic rate of convergence is independent of the choice of the $r-1$ state variables. To study the rates of convergence of $\overline{x}_n$, without loss of generality we need only examine that of $\widetilde{\overline{x}}_n$. It follows that

$$\begin{cases} \widetilde{x}_{n+1} & = \widetilde{x}_n + \mu_n(M_{11}\widetilde{x}_n + M_{12}x_n^r + \widetilde{W}\zeta_n) \\ & = \widetilde{x}_n + \mu_n(\widetilde{M}\widetilde{x}_n + \widetilde{W}\zeta_n), \\ \widetilde{\overline{x}}_{n+1} & = \widetilde{\overline{x}}_n - \dfrac{1}{n+1}\widetilde{\overline{x}}_n + \dfrac{1}{n+1}\widetilde{x}_{n+1}, \end{cases} \tag{22}$$

where

$$\widetilde{M} = M_{11} - M_{12}\mathbb{1}_{r-1}'.$$

Note that the noise is now reduced also to $\widetilde{W}\zeta_n$, which is $r-1$ dimensional but is a function of l_s dimensional link noise ζ_n.

Let $D = I_{r-1} + \mathbb{1}_{r-1}\mathbb{1}_{r-1}'$.

Lemma 5 *Under Assumption A0, $\widetilde{M} = M_{11}D$ and is full rank.*

For convergence speed analysis, let $e_n = \overline{x}_n - \beta\Psi^{-1}\mathbb{1}_n$. Decompose $e_n = [\widetilde{e}_n', e_n^r]'$.

Theorem 6 *Suppose that $\{\zeta_n\}$ is a sequence of i.i.d. random variables with mean zero and covariance $E\zeta_n\zeta_n' = \Sigma$. Under Assumption A0, the weighted consensus errors $\widetilde{e}_n$ satisfies that $\sqrt{n}\widetilde{e}_n$ converges in distribution to a normal random variable with mean 0 and covariance given by*

$$\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})'.$$

Note that the above result does not require any distributional information on the noise $\{\varepsilon(k)\}$ other than the zero mean and finite second moments. We now state the optimality of the algorithm when the density function is smooth.

Theorem 7 *Suppose that the noise $\{\zeta_n\}$ is a sequence of i.i.d. noise with a density $f(\cdot)$ that is continuously differentiable. Then the recursive sequence $\widetilde{\overline{x}}_n$ is asymptotically efficient in the sense of the Cramér-Rao lower bound on $E\widetilde{e}_n'\widetilde{e}_n$ being asymptotically attained,*

$$nE\widetilde{e}_n'\widetilde{e}_n \to \operatorname{tr}(\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})'). \tag{23}$$

The convergence speed and optimality of e_n is directly related to these of $\widetilde{e}_n$.

Corollary 8 *Under the conditions of Theorem 7, the sequence $\{\overline{x}_n\}$ is asymptotically efficient in the sense of the Cramér-Rao lower bound on $Ee_n'e_n$ being asymptotically attained. The asymptotically optimal convergence speed is*

$$nEe_n'e_n \to \mathrm{tr}(D\widetilde{M}^{-1}\widetilde{W}\Sigma\widetilde{W}'(\widetilde{M}^{-1})') \tag{24}$$

where $D = I_{r-1} + \mathbb{1}_{r-1}\mathbb{1}_{r-1}'$.

Example 3 We now use the system in Example 2 to illustrate the effectiveness of post-iterate averaging. Suppose that the link observation noises are i.i.d sequences of Gaussian noises of mean zero and standard deviation 20. Now, the consensus control is expanded with post-iterate averaging. Figure 7 shows the distance trajectories. The distance distributions converge to the weighted consensus faster with much less fluctuations.

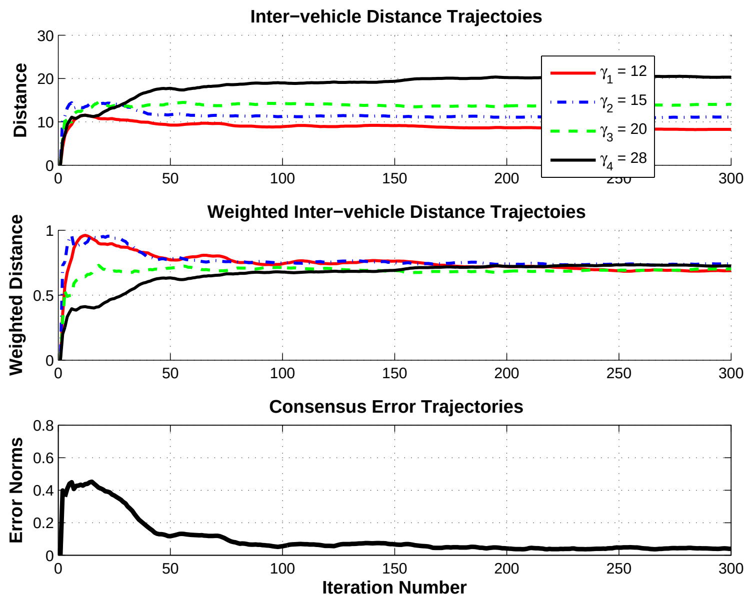

Figure 7: vehicle distance control with post-iterate averaging on weighted consensus algorithms

6 Network Topology and Platoon Control Performance

We investigate now the benefits of using communication systems to enhance platoon control. In a sensor-based information network topology, we assume that the vehicles are equipped with front and rear distance sensors, but control their front distances only. On the other hand, if wireless communications are allowed, inter-vehicle information flows can be further expanded. From the previous analysis, as long as the information topology is connected, convergence of consensus control can be achieved. the main difference is the speed of convergence, which is essential for system robustness, disturbance attenuation, and platoon re-configuration.

Example 4 This example compares the two types of information network topologies. The platoon contains a lead vehicle and 5 other vehicles. In the sensor-based topology, vehicle j

measures d_j by the front distance sensor and d_{j+1} by the rear distance sensor. In this case, the last vehicle will control the overall platoon length L (presumably by using a GPS device and communicating with the lead vehicle or the control tower). On the other hand, if inter-vehicle communications are allowed, d_1 is transmitted to vehicle 3, etc., adding new network branches in the information network topology. Figure 8 depicts these two information network topologies. Under the same consensus control gains and step sizes, Figure 9 illustrates inter-vehicle distance trajectories, starting from the same initial condition. Figure 10 compares the convergence rates under the two topologies. It is clear that the communication network can potentially enhance consensus control by adding new information in the platoon control strategies.

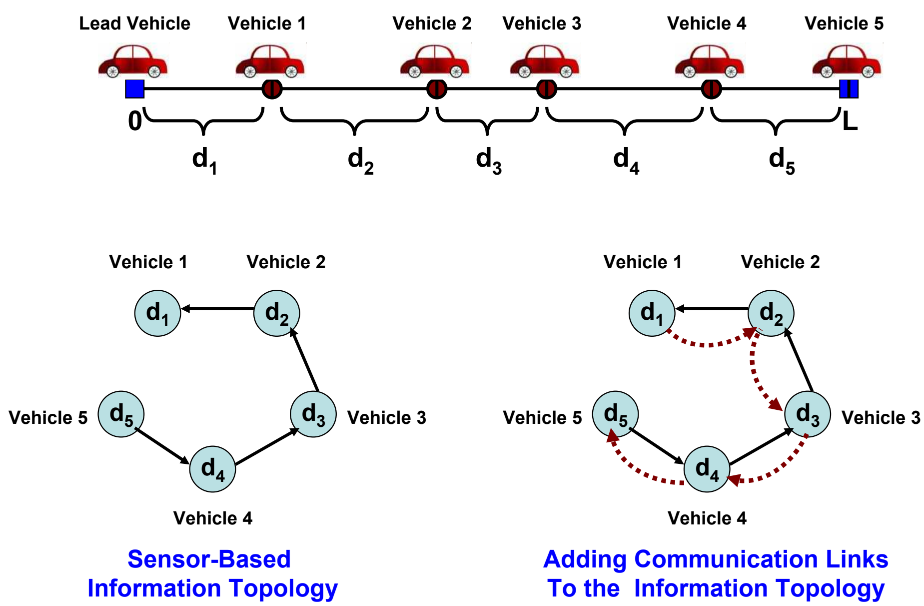

Figure 8: Consensus control using different information topologies

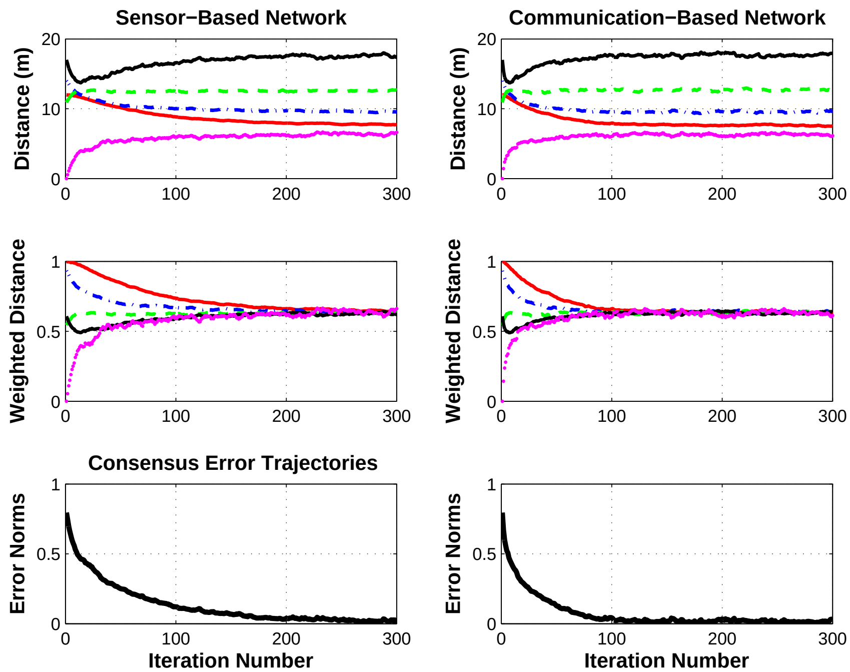

Figure 9: Consensus control using different information topologies

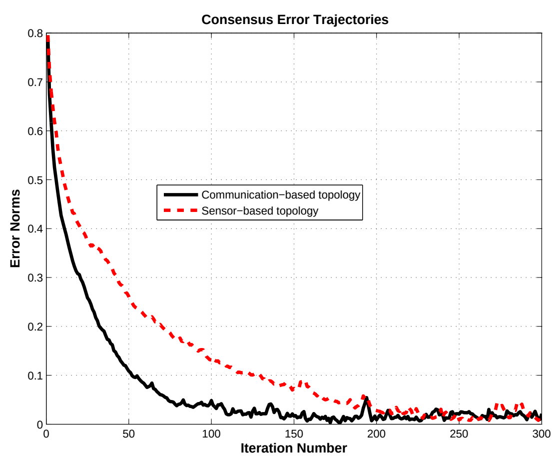

Figure 10: Comparison of convergence speeds between the sensor-based topology and communication-based topology

7 Vehicle Dynamics and Cyber-Physical Interaction

The consensus control generates the *desired* platoon formation based on sensor and communication information by determining the relative distance adjustment of each vehicle at any given decision time. This adjustment decision serves as a command to the local controller for execution. Due to vehicle dynamics and road/traffic conditions, execution of such control actions encounter standard performance limitations such as steady-state errors, overshoot, rising time, delay, and other relevant performance measures. Inter-connection and interaction between information gathering and decision making at the higher level (cyber space) and vehicle control (physical space) are illustrated in Fig. 11.

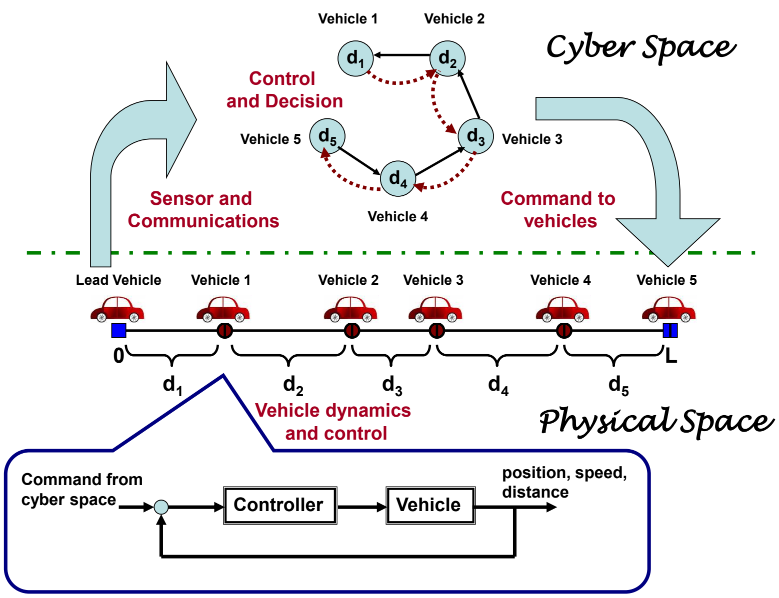

Figure 11: Cyber-physical inter-connection and interaction.

7.1 Vehicle Dynamics and Normalization

The dynamics of the jth vehicle follows the basic law

$$m_j \dot{v}_j = F_j - L_j^0(\dot{v}_j) + \varepsilon_j^0 \tag{25}$$

where m_j is the mass of the vehicle, F_j is the vehicle driving force (when it is positive) or braking force (when it is negative), $L_j^0(\dot{v}_j)$ is the modeled load force (which is known and can be used in control action), and ε_j^0 is the uncertainty term which captures modeling errors, unknown factors on tires, roads, weather conditions, measurement noise, etc.

Normalization of (25) results in

$$\begin{aligned} \dot{v}_j &= \frac{F_j}{m_j} - \frac{L_j^0(\dot{v}_j)}{m_j} + \frac{\varepsilon_j^0}{m_j} \\ &= u_j - L_j(\dot{v}_j) + \varepsilon_j \\ &= w_j + \varepsilon_j. \end{aligned}$$

Here, $u_j = F_j/m_j$ is the control variable, $L_j(\dot{v}_j) = L_j^0(\dot{v}_j)/m_j$ is the normalized drag, $\varepsilon_j = \varepsilon_j^0/m_j$ is the normalized uncertainty, and $w_j = u_j - L_j(\dot{v}_j)$ is a linearized control input.

Together with $p_j(t)$, we have

$$\left\{\begin{array}{lcl} \dot{p}_j & = & v_j \\ \dot{v}_j & = & w_j + \varepsilon_j \quad j = 1, \ldots, r \\ y_j & = & p_j \end{array}\right. \tag{26}$$

Define $x_j = [p_j, v_j]'$. We have

$$\dot{x}_j = Ax_j + Bw_j + B\varepsilon_j; \quad y_j = Cx_j \tag{27}$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1, 0]$$

noting that the matrices A, B, and C are same for all j due to normalization and input linearization.

7.2 Platoon Dynamics and Vehicle Control

The consensus control strategies produce the desired distances $d(t) = [d_1(t), \ldots, d_r(t)]'$ at the decision time t. For the jth vehicle, $d_j(t_k)$ (the distance of the jth vehicle to its front vehicle (the $(j-1)$th vehicle) will be the command to the vehicle's on-board dynamic controller. The actual vehicle distance will be denoted by $\widetilde{d}_j(t)$. The local control is then a tracking control that follows $d_j(t_k)$ during $t \in [t_k, t_{k+1})$. Due to dynamics of the vehicle control systems, the actual inter-vehicle distance trajectories $\widetilde{d}(t_k) = [\widetilde{d}_1(t_k), \ldots, \widetilde{d}_r(t_k)]'$ are different from $d(t) = [d_1(t), \ldots, d_r(t)]'$. As a result, they create a cyber-physical interaction which influences substantially the platoon control performance.

We first build the entire platoon dynamics from the linearized and normalized vehicle dynamics (27)

$$\dot{x}_j = Ax_j + Bw_j + B\varepsilon_j; \quad y_j = Cx_j, j = 1, \ldots, r.$$

Denote $y = [y_1, \ldots, y_r]'$, $u = [w_1, \ldots, w_r]'$, $x = [x_1', \ldots, x_r']'$, $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_r]'$. Let I_r be the r-dimensional identity matrix. Define the block diagonal matrices $\widetilde{A} = I_r \otimes A$, $\widetilde{B} = I_r \otimes B$, $\widetilde{C} = I_r \otimes C$, where $\otimes$ is the Kronecker product [23]. Then the platoon dynamics is

$$\dot{x} = \widetilde{A}x + \widetilde{B}u + \widetilde{B}\varepsilon; \quad y = \widetilde{C}x.$$

For the jth vehicle, the controller F_j will be designed based on the A, B, C, and the tracking error $e_j(t) = d_j(t) - \widetilde{d}_j(t)$ in the following feedback structure [24]

$$\dot{z}_j = e_j; w_j = -Kx_j + k_0 z_j, \tag{28}$$

which includes both the state feedback term $-Kx_j$ for stability and transient performance, and the integral output feedback $k_0 z_j$ for eliminating steady-state tracking errors. Since the linearized and normalized subsystems have the same A, B, and C matrices, the controller matrices k_0 and K will also be uniform over all subsystems.

By denoting $z = [z_1', \ldots, z_r']'$, $e = [e_1, \ldots, e_r]'$, $\widetilde{K} = I_r \otimes K$, the controller for the platoon is

$$\dot{z} = e; u = -\widetilde{K}x + k_0 z.$$

Note that $\widetilde{d}_1 = p_1 - p_0$, ..., $\widetilde{d}_r = p_r - p_{r-1}$, where the leading vehicle's position p_0 is external to the system as the time-varying reference to the platoon. Then, $e_1 = d_1 - (p_1 - p_0)$, ..., $e_r = d_r - (p_r - p_{r-1})$. The platoon dynamics is represented by

$$\begin{cases} \dot{z} &= e \\ \dot{x} &= \widetilde{A}x + \widetilde{B}u + \widetilde{B}\varepsilon \\ u &= -\widetilde{K}x + k_0 z \\ e &= -S\widetilde{C}x + B_1 p_0 + d \end{cases} \tag{29}$$

where

$$S = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & \cdots & 0 & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \cdots & -1 & 1 \end{bmatrix}; B_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

7.3 Platoon Stability

Platoon stability has been studied as a string stability when consecutive vehicles use only the immediate distance information in its cruise control, see [11, 18] and the references therein. Platoon stability is not a trivial issue, depending on how local systems are controlled. Here, we study platoon stability using a unified local control structure by input linearization and scaling, a coordination of desired distances by the weighted and constrained consensus control, and a state-feedback-plus-integral-output-feedback structure for local feedback. The advantage of these structures is that it makes stability analysis simpler.

To illustrate platoon stability, we note that $K = [k_1, k_2]$. This implies $\widetilde{K} = I_r \otimes [k_1, k_2]$. Hence, (29) is

$$\begin{cases} \dot{z} &= e \\ \dot{x} &= \widetilde{A}x + \widetilde{B}u + \widetilde{B}\varepsilon \\ u &= -(I_r \otimes [k_1, k_2])x + k_0 z \\ e &= -S\widetilde{C}x + B_1 p_0 + d \end{cases} \tag{30}$$

Theorem 9 *The jth subsystem has the closed-loop system*

$$\begin{bmatrix} \dot{z}_j \\ \dot{x}_j \end{bmatrix} = \begin{bmatrix} 0 & -C \\ Bk_0 & A - BK \end{bmatrix} \begin{bmatrix} z_j \\ x_j \end{bmatrix} + B_0 p_{j-1} + B_0 d_j + \begin{bmatrix} 0 \\ B \end{bmatrix} \varepsilon_j,$$

where $B_0 = [1, 0, 0]'$.

The closed-loop system for the entire platoon is

$$\begin{bmatrix} \dot{z} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & -S\widetilde{C} \\ \widetilde{B}k_0 & \widetilde{A} - \widetilde{B}(I_r \otimes [k_1, k_2]) \end{bmatrix} \begin{bmatrix} z \\ x \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} p_0 + \begin{bmatrix} I_r \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ \widetilde{B} \end{bmatrix} \varepsilon.$$

Proof: The controller for the jth vehicle is

$$\dot{z}_j = e_j = k_0(d_j - (Cx_j - p_{j-1})); w_j = -[k_1, k_2]x_j + k_0 z_j. \tag{31}$$

p_{j-1} is an external signal to the jth vehicle. Combined with the vehicle dynamics (27), the closed-loop system is

$$\begin{aligned} \dot{z}_j &= -k_0 C x_j + k_0(p_{j-1} + d_j) \\ \dot{x}_j &= Ax_j + Bw_j = (A - BK)x_j + k_0 B z_j + B\varepsilon_j. \end{aligned}$$

k_0, k_1, k_2 are to be designed such that the closed-loop system is stable. Hence, the closed-loop system is

$$\begin{bmatrix} \dot{z}_j \\ \dot{x}_j \end{bmatrix} = \begin{bmatrix} 0 & -C \\ Bk_0 & A - BK \end{bmatrix} \begin{bmatrix} z_j \\ x_j \end{bmatrix} + B_0 p_{j-1} + B_0 d_j + \begin{bmatrix} 0 \\ B \end{bmatrix} \varepsilon_j. \tag{32}$$

From (30), the closed-loop system for the platoon is

$$\begin{cases} \dot{z} &= -S\widetilde{C}x + B_1 p_0 + d \\ \dot{x} &= \widetilde{A}x + \widetilde{B}(k_0 z - (I_r \otimes [k_1, k_2])x) + \widetilde{B}\varepsilon \end{cases} \tag{33}$$

or compactly

$$\begin{bmatrix} \dot{z} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & -S\widetilde{C} \\ \widetilde{B}k_0 & \widetilde{A} - \widetilde{B}(I_r \otimes [k_1, k_2]) \end{bmatrix} \begin{bmatrix} z \\ x \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} p_0 + \begin{bmatrix} I_r \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ \widetilde{B} \end{bmatrix} \varepsilon.$$

□

k_0, k_1, k_2 are designed to achieve local stability. From

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1, 0],$$

we have

$$\phi = \begin{bmatrix} 0 & -C \\ Bk_0 & A - BK \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ k_0 & -k_1 & -k_2 \end{bmatrix}. \tag{34}$$

The subsystem is stable if all eigenvalues of ϕ are in the left-half plane. The characteristic polynomial of ϕ is $\lambda^3 + k_2\lambda^2 + k_1\lambda + k_0$. Apparently, any desired pole assignment can be realized. For example, if the desired characteristis polynomial is $(\lambda + p)^3 = \lambda^3 + 3p\lambda^2 + 3p^2\lambda + p^3$, for some $p > 0$, then $k_0 = p^3$, $k_1 = 3p^2$, $k_2 = 3p$ will be the designed control parameters.

We now establish stability of the entire platoon. Stability of the entire platoon is determined by the eigenvalues of

$$\Phi = \begin{bmatrix} 0 & -S\widetilde{C} \\ \widetilde{B}k_0 & \widetilde{A} - \widetilde{B}(I_r \otimes [k_1, k_2]) \end{bmatrix}. \tag{35}$$

Theorem 10 *The eigenvalues of Φ are the same as the eigenvalues of ϕ, but repeated r times each. As a result, if the subsystems are designed to be stable, then the entire platoon is stable.*

Proof: In consideration of the local control nature, we first re-arrange (30) into a subsystem structure. Let $\xi_j = [z_j, x_j']'$. Then the jth closed-loop system is

$$\left\{ \begin{array}{rcl} \dot{\xi}_j & = & \phi\xi_j + B_0 p_{j-1} + B_0 d_j + \begin{bmatrix} 0 \\ B \end{bmatrix} \varepsilon_j \\ p_j & = & [0, C]\xi_j \end{array} \right. \qquad j = 1, \ldots, r.$$

For stability analysis, we ignore p_0, d_j, and ε_j, arriving at

$$\left\{ \begin{array}{rcl} \dot{\xi}_j & = & \phi\xi_j + B_0 p_{j-1} \\ p_j & = & [0, C]\xi_j \end{array} \right. \qquad j = 1, \ldots, r.$$

Consequently, the platoon closed-loop system is

$$\begin{bmatrix} \dot{\xi}_1 \\ \vdots \\ \dot{\xi}_r \end{bmatrix} = \begin{bmatrix} \phi & 0 & \cdots & 0 & 0 \\ B_0[0, C] & \phi & \cdots & 0 & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \cdots & B_0[0, C] & \phi \end{bmatrix} \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_r \end{bmatrix}. \tag{36}$$

Since this is a block diagonal matrix, its eigenvalues are the collection of the eigenvalues of its diagonal blocks, which are the eigenvalues of ϕ. This re-arrangement of the platoon closed-loop system does not change the eigenvalues of Φ. This completes the proof.

□

Example 5 Select $k_0 = 1$, $k_1 = 2$, $k_2 = 2$. Then

$$\phi = \begin{bmatrix} 0 & -C \\ Bk_0 & A - BK \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & -2 & -2 \end{bmatrix}.$$

Its eigenvalues are -1, $-0.5 \pm \mathrm{j}0.866$. Suppose that the platoon has $r = 3$. It can be verified that

$$\Phi = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -2 & -2 \end{bmatrix}$$

whose eigenvalues are also -1, $-0.5 \pm j0.866$, but repeated 3 times each.

7.4 Case Studies

We will use the system in Example 2 to demonstrate integration of platoon consensus decisions and vehicle control. Since Example 2 does not involve vehicle dynamics, the convergence rate is expressed in terms of the number of iteration steps. When the vehicle dynamics is introduced, the vehicle signal processing sampling time is introduced. The sampling rate is usually quite high. As a result, the actual convergence speed of platoon consensus control is predominantly determined by how fast the vehicles can be controlled to follow the decisions from the cyber space. In the following case studies, the sampling rate is 100 Hz, and consensus control is shown with respect to the clock time.

Example 6 Consider the system in Example 2 with the same initial distance distribution among the five vehicles are $d_0^1 = 12$ m; $d_0^2 = 14$ m; $d_0^3 = 10.9$ m; $d_0^4 = 17$ m; and the same weighting $\gamma^1 = 12$, $\gamma^2 = 15$, $\gamma^3 = 20$, $\gamma^4 = 28$, with the total $\mathbb{1}'\gamma = 75$. At each decision point, the consensus control of Example 2 issues a new distance distribution d_k^1, d_k^2, d_k^3, d_k^4. These desired distances will be communicated to the vehicles as the commend signals. The vehicles' on-board controllers will implement tracking control according to (31). The control parameters are selected as $k_0 = 4.096$, $k_1 = 7.68$, $k_2 = 4.8$, which will place the poles of the local closed-loop systems at $-1.6, -1.6, -1.6$.

Suppose that the link observation noises are i.i.d sequences of Gaussian noises with mean zero and variance 1. Figure 12 shows the inter-vehicle distance trajectories. Staring from a platoon formation, a disturbance occurs that changes d_1 by 4 m. This disturbance causes initially a large deviation of the platoon formation. The top plot shows how distances are gradually distributed according to the desired distributions. The middle plot illustrates that the weighted distances converge to a constant. The weighted consensus error trajectories are plotted in the bottom figure.

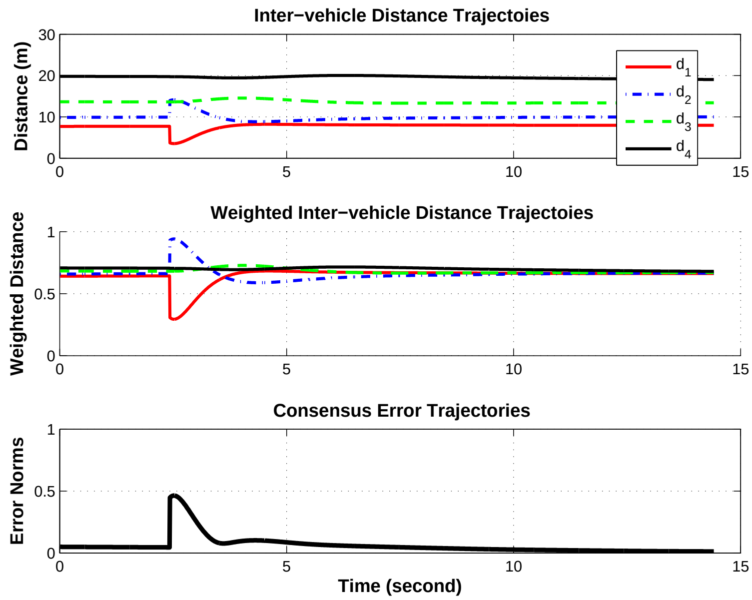

Figure 12: Platoon distance control with vehicle dynamics and relatively slow local controllers

Example 7 The impact of vehicle control can be further studied. Suppose that for the system in Example 6, a more aggressive control action is adopted this time: The vehicle controller is designed to place the poles of the local closed-loop systems at $-2, -2, -2$. The corresponding control parameters are $k_0 = 8$, $k_1 = 12$, $k_2 = 6$. Since the larger control gains are used, more engine torques will be used in vehicle control. Suppose that the link observation noises are i.i.d sequences of Gaussian noises with mean zero and variance 1. Figure 13 shows the inter-vehicle distance trajectories. Staring from a platoon formation, a disturbance occurs that changes d_1 by 4 m. In comparison to Fig. 12, the platoon control achieves a faster convergence.

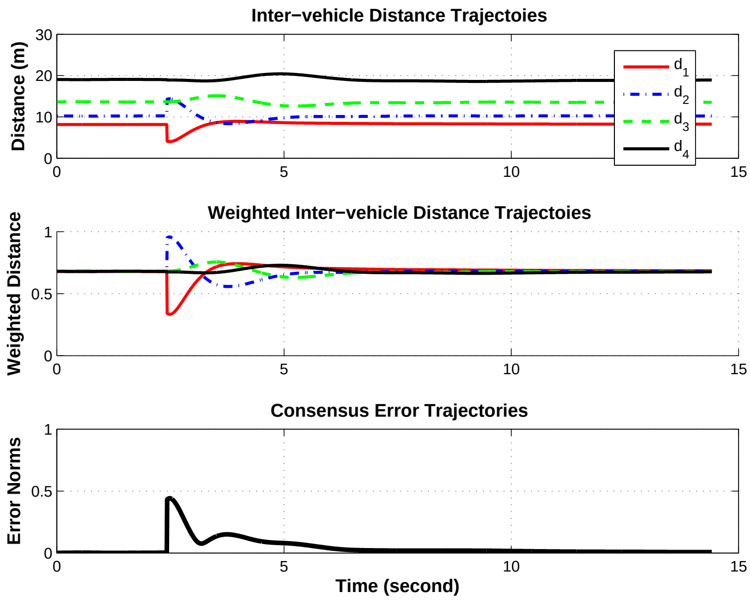

Figure 13: Platoon distance control with vehicle dynamics and more aggressive local controllers

7.5 Remarks on Platoon Coordination with Safety Constraints

Safety concern is the highest priority in highway platoon control. As a result, it is important that safety features are incorporated into platoon control algorithms. In this section, we consider the case of safety boundary constraints on inter-vehicle distances that override other control actions such as consensus algorithms or vehicle local control actions.

Suppose that inter-vehicle distance between the ith and $(i-1)$th vehicles must be bounded below by $d^i_{\min}$ for safety and above by $d^i_{\max}$ for highway utility. For example, 3 m for low speed drive and 5 m for highway cruise are typically used as a safety lower bound in several commercial vehicle safety systems. Mathematically, these constraints amount to a constraint set Ξ, which is assumed to be closed, on the state x_n. Although the constraint set typically changes with operating conditions, such as vehicle speeds, due to similarity in their treatment we will focus on the case that Ξ is given and fixed. In the case of box constraints (i.e., the constrained set being the product of closed intervals), $\Xi = [d^1_{\min}, d^1_{\max}] \times \cdots \times [d^r_{\min}, d^r_{\max}]$. Incorporating the constraint set Ξ into the platoon control involves two levels.

1. *Platoon Coordination.* At the cyber space, the consensus updating algorithm (9)

$$x_{n+1} = x_n + \mu_n(Mx_n + W\zeta_n)$$

is modified to an updating algorithm with projection.

$$x_{n+1} = \Pi_\Xi(x_n + \mu_n(Mx_n + W\zeta_n)) \tag{37}$$

where the projection operator Π_Ξ is defined as

$$\Pi(x) = \begin{cases} x, & x \in \Xi, \\ x^*, & x \notin \Xi. \end{cases}$$

Here x^* is a pre-selected interior point of Ξ. In other words, if the constraint set Ξ is violated, the interative consensus algorithm will be reset to the point x^*.

Note that in the setup of such a box constrained algorithm, all the analysis presented before goes through. Even more complex constrained sets can be incorporated. More details on the analysis of such algorithms can be found in [8].

2. *Vehicle Control.* At the vehicle level, the on-board vehicle control system for the ith vehicle will manipulate its control signal such that if the distance d_i reaches its lower bound d^i_{min}, its control will be confined to the set that can only increase d_i. We will use an example to demonstrate this added feature. This control modification renders a nonlinear feedback structure to the vehicle controller.

8 Concluding Remarks

Platoon control strategies and algorithms introduced in this paper represent a new framework for intelligent highway transportation systems. As a first step in this direction, this paper is focused on establishing the key structure of the framework, the main algorithms, and fundamental interactions between the consensus decision on the cyber space and vehicle control in the physical space. As such the system models are of basic and representative types, rather than detailed physical systems. For the same token, there are many important and intriguing issues left open for further exploration.

First, detailed vehicle dynamics will be of interests. Second, communication systems have experienced great advancement in recent years. For vehicle applications, some communication protocols, such as DSRC, have emerged as potential standards for V2V and V2I communications. As such, it will be important to include detailed features of these protocols in this framework such that relevant evaluations can be conducted on the viability, advantages, and limitations of our framework. Third, enhanced vehicle safety considerations introduce many nonlinear constraints in vehicle control. Integrated consensus decision and vehicle control under nonlinear constraints are of essential importance. These directions will be pursued in our future studies.

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