Microeconomics - Background Knowledge

Introduction

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Theory of Consumption

On the consumer's side, competitive equilibrium - constituting of equilibrium quantities of goods produced at equilibrium prices, optimally satisfying the consumer's demand - is the solution of an optimization problem, that of consumer's utility maximization:

$$\begin{aligned} \underset{x,y}{\text{Max}} \quad & U(X,Y) \\ \text{st} \quad & P_x X + P_y Y \leq I \end{aligned}$$

The Lagrangian for this problem is:

$$\mathcal{L}(X,Y,\lambda) = U(X,Y) + \lambda\,(I - P_x X + P_y Y)$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{\partial U(X,Y)}{\partial X} - \lambda P_x = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial Y} = \frac{\partial U(X,Y)}{\partial Y} - \lambda P_y = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_x X + P_y Y = 0 \quad (3)$$

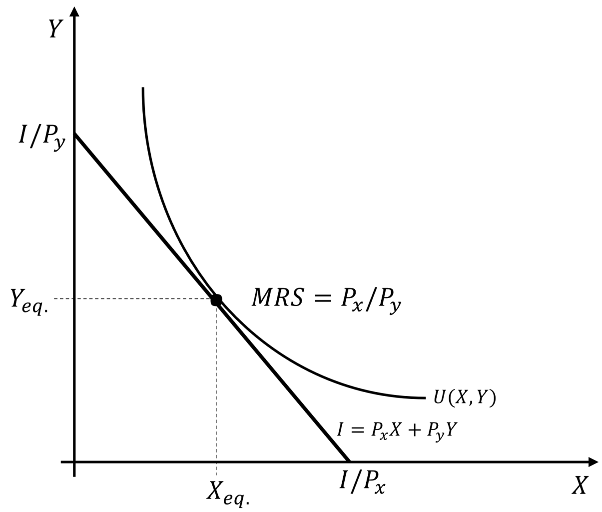
Divide Equation 1 by Equation 2:

$$\frac{\partial U(X,Y)/\partial X}{\partial U(X,Y)/\partial Y} = \frac{\lambda P_x}{\lambda P_y}$$

$$\frac{MU_X}{MU_Y} = \frac{P_x}{P_y}$$

The left-hand side of this equation is the **Marginal Rate of Substitution (MRS)**, the rate at which the consumer is willing to trade good Y for good X. $\frac{P_x}{P_y}$, being the ratio of good prices, is also the slope of consumer's budget constraint. Solving a system of equation that constitutes of this optimizing condition and Equation 3 leads to equilibrium quantities of X and Y.

The consumer thus consumes optimally at the tangency point between their budget constraint and their (highest) utility curve, i.e., equilibrium quantities of X and Y are consumed at a vector of prices that is tangent to the consumer's highest feasible utility function:



Theory of Production

From the perspective of the individual producer, on the other hand, the free market equilibrium can be equivalently derived as the solution to a problem (i) of Costs Minimization, given the specifics of production technology/Output Maximization given their cost function, or (ii) as the solution of a Profit Maximization. For the program of this exam, we will examine case i from the perspective of the producer wishing to maximize their output given production costs.

(i) Output Maximization

A firm can obtain the equilibrium quantity with the objective to produce the highest amount of a certain good Q (that can be either X or Y) given their total cost of production C, i.e., maximizing its output subject to a cost constraint, the sum of the costs of all inputs. Assuming the firm only uses Labor and Capital as inputs, workers will receive their wages w, while the price of Capital (rental rate of capital) is r:

$$\begin{aligned} \underset{L,K}{\text{Max}} \quad & Q = f(L,K) \\ \text{st} \quad & C \geq wL + rK \end{aligned}$$

The Lagrangian function for this optimization problem is:

$$\mathcal{L}(L,K,\lambda) = f(L,K) + \lambda(C - wL + rK)$$

The optimal levels of L and K to produce equilibrium quantity of Q can be derived solving the FOCs:

$$\frac{\partial \mathcal{L}}{\partial L} = \lambda \frac{\partial f(L,K)}{\partial L} - w = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial K} = \lambda \frac{\partial f(L,K)}{\partial K} - r = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = C - wL + rK = 0 \qquad (3)$$

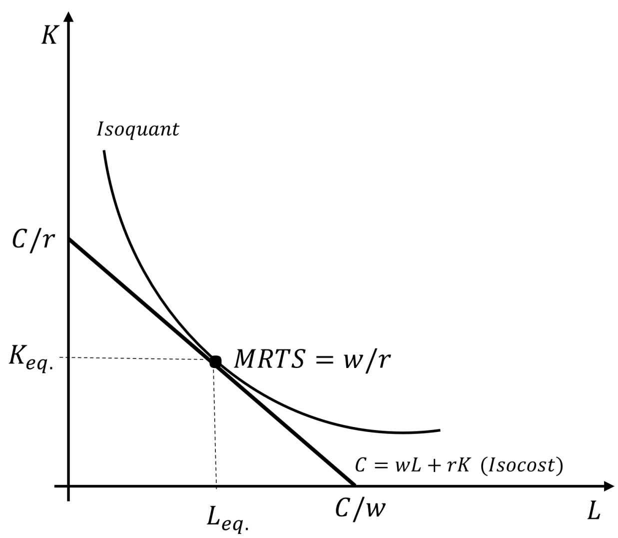
From Equations 1 and 2, you can derive the following optimality condition:

$$\frac{\partial f(L,K)/\partial L}{\partial f(L,K)/\partial K} = \frac{w}{r}$$

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

The left-hand side is the **Marginal Rate of Technical Substitution (MRTS)**, which is the rate at which the firm can substitute labor for capital while keeping output constant.

Firms thus maximize their output subject to their cost constraint when the Marginal rate of Technical Substitution between Labor and Capital equals the wage-rental ration (i.e., the ratio of factor prices). Graphically, the optimality condition states the the optimal quantity of Q produced is the tangency point between the firm's *isocost* and hightest feasible *isoquant* line:



The **Isocost Line** corresponds to all the combinations of L and K that can be purchased at a given cost level, $C = wL + rK$. The **Isoquants** are curves that represent all the possible combinations of the two factors of production that can achieve a certain level of output Q. The equilibrium quantity of Q produced can be obtained by fitting the equilibrium quantity of L and K into the firm's production function. The result of this optimization problem is exactly the same as that of a cost minimization problem, since it is a mirrored optimization problem.

(ii) Profit Maximization

In equilibrium, the optimal quantity of Q is also the quantity that maximizes the producer's profits. The optimization problem of profit maximization, is thus the other side of reaching market equilibrium, in which the producers determine a production regime that maximizes their profits given the revenues they get from selling their output and the costs of producing it:

$$\text{Max} \quad \Pi = PQ - C(Q)$$

The profit function constitutes of **Total Revenue (TR)**, the price of Q multiplied by the quantity produced, minus the **Total Costs (TC)**, the sum of all production costs of K and L :

$$\text{Max} \quad \Pi = TR - TC$$

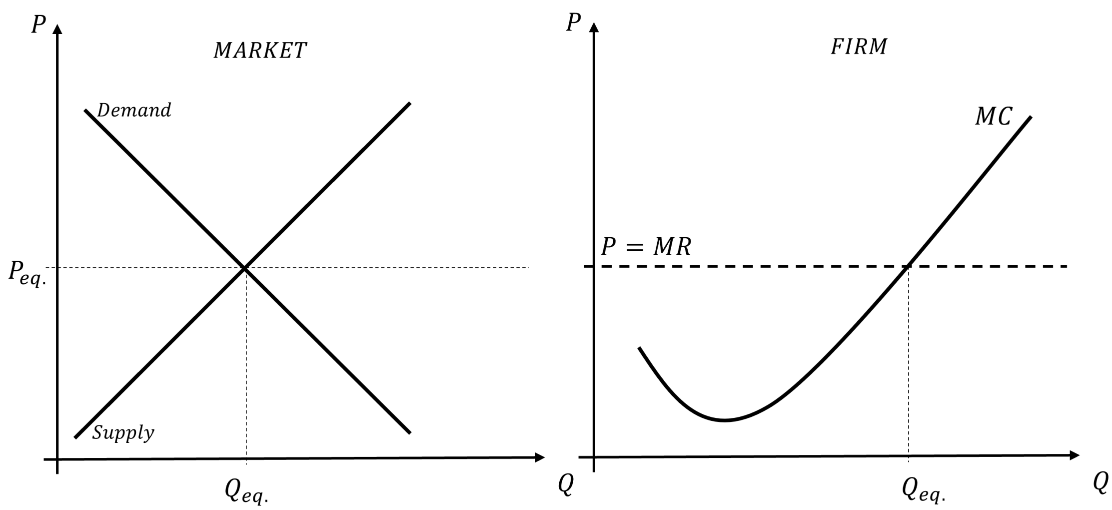
The FOC of this mazimization problem tells us that profits are maximized when $\frac{\partial \Pi}{\partial Q} = 0$, i.e., when the firm's **Marginal Revenues (MR)** are equal to the firm's **Marginal Costs (MC)**

$$\begin{aligned} \frac{\partial \Pi}{\partial Q} = \frac{\partial TR}{\partial Q} - \frac{\partial TC}{\partial Q} &= 0 \\ MR - MC &= 0 \\ MR &= MC \end{aligned}$$

In a competitive market, firms are **Price Takers**, i.e., they can't set Q's price, and sell the output good at a price P that is defined on the market by the intersection of Supply and Demand for Q. That is, in perfect competition, firm's marginal revenue is equal to the selling price of a unit of Q, i.e., $P = MR$. The optimality condition for profit maximization is therefore:

$$P = MR = MC$$

Graphically:



Theory of General Equilibrium

We have previously reasoned about the single producer and single consumer. However, the optimization choices of the individual and the individual firm can be used to represent those of all firms and all consumers in an given economy, assuming that firms all have the same production function, and individuals the same utility curves. In doing so, we use the single consumer/producer as the representative agent.

Combining the production and demand side of the economy, we reach an overall **General Equilibrium**. The theory of general equilibrium states that, in a competitive market where

(a) Consumers maximize their utility subject to a their budget constraint, and

(b) Producers (i) maximize their production given their costs constraints (or, equivalently, minimize their production costs given their level of output) and/or (ii) maximize their profits

Then $\exists$ a vector of equilibrium prices $P^{eq.}$ that clears the market, i.e., a price line (a straight line that has the ratio of prices $\frac{P_x}{P_y}$ as a slope) such that **Aggregate Demand = Aggregate Supply**, $\forall$ commodity in the market. The equilibrium thus obtained is **Pareto Optimal**, there's no other welfare-improving combination of prices and goods (no one can be made better off without making someone else worse off).

References

Markusen J. R., J. R. Melvin, W. H. Kaempfer, and K. E. Maskus, 1995, International Trade. Theory and Evidence - Chapters 2 and 3

Gains from Trade - Notes

Tutorial I

September 24, 2025

General Equilibrium in a Closed Economy

Consider a closed economy, given two goods X, Y, consumers' utility function U and two inputs of production L, K. The boundaries od production of the economy are represented by the **Production Possibility Frontier (PPF)**, i.e., the maximum possible combinations of X and Y that the economy can produce given its factors L and K and its technology (production function).

The slope of the PPF is known as the **Marginal Rate or Transformation (MRT)**, and it illustrates the trade-offs that occur when inputs are allocated between the production of different goods (i.e., opportunity cost of producing one additional unit of X over Y) within the economy:

$$MRT = \frac{\Delta Y}{\Delta X} = \frac{\frac{\delta Y}{\delta L_y}}{\frac{\delta X}{\delta L_x}} = \frac{\frac{\delta Y}{\delta K_y}}{\frac{\delta X}{\delta K_x}} = \frac{MP_{L_y}}{MP_{L_x}} = \frac{MP_{K_y}}{MP_{K_x}}$$

Three conditions determine the **Autarky General Equilibrium**:

(I) **Producer Optimization:** producers' maximize their profits producing amounts of X and Y such that the ratio between producers' prices (i.e., selling prices) is equal to the MRT (i.e., where the price line tangent to PPF):

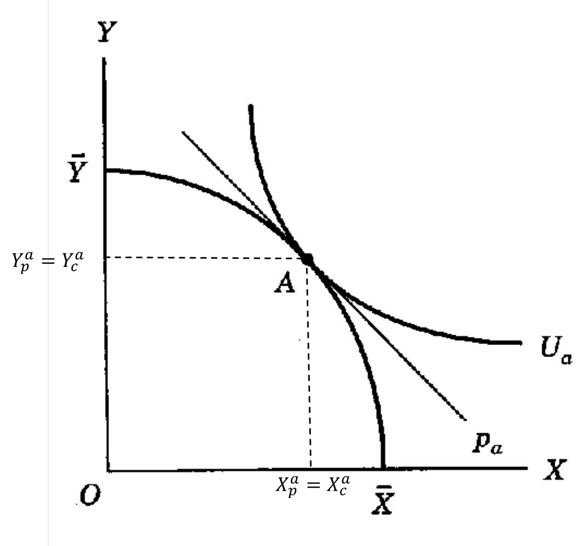
$$\frac{P_x}{P_y} = MRT \tag{1}$$

(II) **Consumer Optimization:** consumers maximize their utility buying amounts of X and Y such that the ratio between consumer prices (i.e., buying prices) is equal to the MRS (i.e., where the price line is tangent to the highest U):

$$\frac{P_x}{P_y} = MRS \tag{2}$$

(III) **Market Clearing:** the quantity of X and Y consumed is equal to the quantity of X and Y produced:

$$X_c = Xp \qquad Y_c = Yp \tag{3}$$



We can represent an equilibrium that satisfies conditions I, II and III as in the diagram below:

Producers produce optimally in A, where the slope of their PPF (MRT) is tangent to autarky price ratio $P_a = P_x/P_y$. Consumers consume optimally in A, where the slope of their indifference curve (MRS) is tangent to P_a. Markets are clear, since the quantities of both goods consumed and produced are equal. the Eq. A is **Pareto Optimal**, the economy consumes at the highest feasible indifference curve given the boundaries of its PPF.

General Equilibrium in an Open Economy

When closed economies opens up to trade, They will trade at a fixed international price ratio P^*. The first two conditions that determine **Trade General Equilibrium** are identical as for the previous case:

(I) **Producer Optimization:**

$$\frac{P_x^*}{P_y^*} = MRT \tag{4}$$

(II) **Consumer Optimization:**

$$\frac{P_x^*}{P_y^*} = MRS \tag{5}$$

However, unlike in the case of autarky, this equilibrium does not achieve market clearing in each country. Countries have now access to international markets, and their consumers desires are not bounded to their domestic production possibility frontiers anymore, allowing for import and exports of goods.

Each country can (and will) now export a certain amount of a goods produced domestically, as well as import a certain amount of the other goods produced abroad. Condition III for an international General Equilibrium is instead:

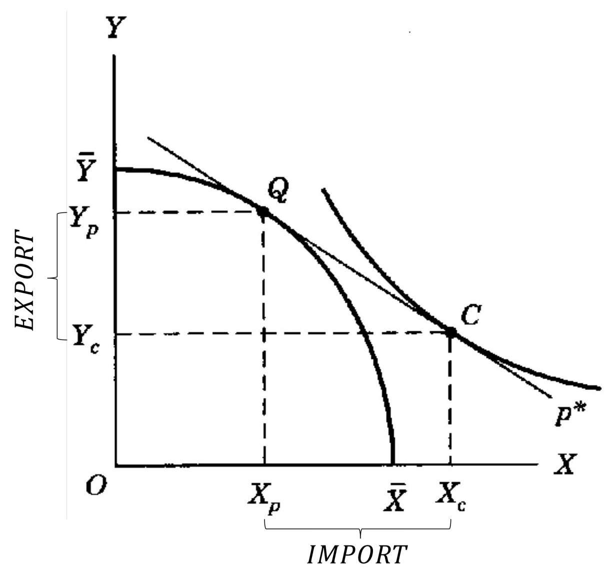
(III) **Trade Balance**, i.e, there is no excess demand of X nor Y on the international market:

$$P_x^* \underbrace{(X_c - Xp)}_{\text{Excess Demand for X}} + P_y^* \underbrace{(Y_c - Yp)}_{\text{Excess Demand for Y}} = 0 \tag{6}$$

Meaning that, at world prices, the value of production must be equal to the value of consumption:

$$P_x^* Xp + P_y^* Yp = P_x^* X_c + P_y^* Y_c \tag{7}$$

When graphically representing this equilibrium, the consumers' indifference curve and the PPF are both tangent to the international price ratio, but the Trade Equilibrium allows for import and export:



Consumers consume optimally in C, where P^* is tangent to their highest feasible utility curve, i.e. $P^* = MRS$. Producers produce optimally in Q, where where P^* is tangent to the PPF of the economy, i.e., $P^* = MRT$. However, this equilibrium price line does not clear the market domestically.

Unlike in autarky, at the international price ratio consumers can afford to purchase more of X than what it is produced domestically, thus they import it from foreign markets. Producers on the other side, produce more of Y than what is sold domestically at world prices, so the export it in other countries.

The sum of the value of X produced and consumed and the value of Y produced and consumed is zero, due to trade balance (NB: you can't see this in the graph, see below).

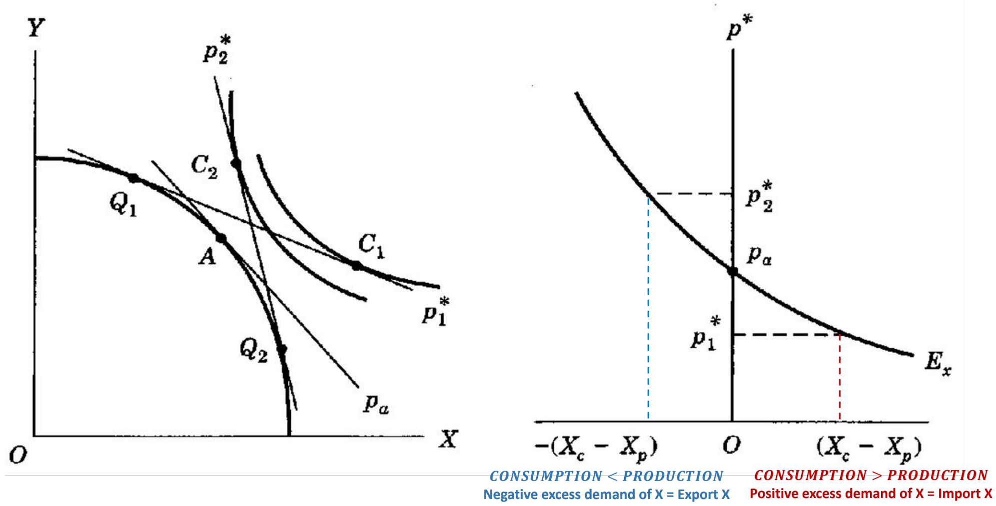
The Excess Demand Function

We can visualize the differences in terms of production and consumption between Autarky Eq. at autarky prices and and different Trade Eqs. at different international price ratios using the excess demand diagram:

- At autarky price ratio P_a the domestic market is clear and there is no excess demand of any good. X nor Y is either imported or exported.
- At international price ratio $P_1^*<P_a$ the relative price of X is lower (relative price of Y is higher than in autarky), therefore economy will import X (export Y).

  X being relatively cheaper, consumer will demand for more X than what is domestically produced and buy X from abroad at a relatively cheaper price than in autarky. Y being relatively more expensive to buy, producers will produce more of it compared to autarky eq. and export it, while consumer will demand less of it.
- At international price ratio $P_2^*>P_a$ the relative price of X is higher (relative price of Y is lower than in autarky), therefore economy will export X (export Y).

  Y being relatively cheaper, consumer will demand for more Y than what is domestically produced. X being relatively more expensive to buy, producers will produce more of it compared to autarky eq. and export it.



International General Equilibrium

Simplifying a general international equilibrium in a model with two countries H and F, producing two goods X and Y with two factors of production L and K, we can predict that the international price ratio in equilibrium when they open to trade with each other will be somewhere in the middle between their autarky prices, P_a^H and P_a^F respectively:

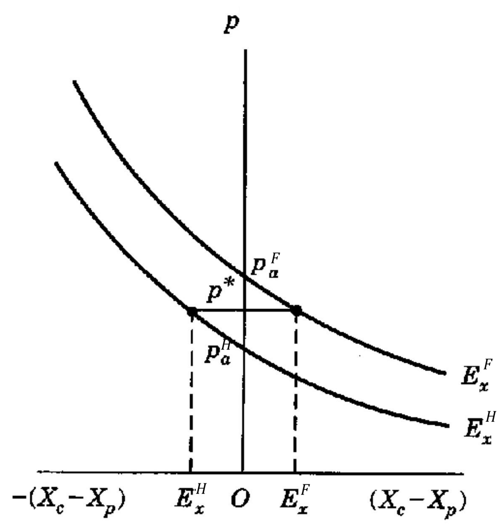
$$P_a^F > P^* > P_a^H$$

So that, for both goods (here we use X as reference, Y is mirrored) the positive excess demand (import) of F is equal to the negative excess demand (export) of H, balancing the trades:

$$E_x^F + E_x^H = 0$$

When H and F engage in trade:

- $\downarrow P$ for F with respect to autarky, $P^{*}<P_a^F$. X is now relatively cheaper for Foreign consumer, and their higher demand (at lower relative price) is supplied by excess X produced in H, since it can't be satisfied by F producers. F has therefore **positive excess demand** X, i.e. will **import** X.
- $\uparrow P$ for H with respect to autarky, $P^{*}>P_a^H$. Since X is now relatively more expensive in H, Home producers will sell it in the foreign market (while Home consumers of X will buy it more cheaply from F). H will therefore have **negative excess demand** for X, i.e. will **export** X.



The excess demand for X of H and F cancel each others out, thus the trade is balanced. Analogously, Y is now relatively more expensive in F, thus Foreign producers will sell it in the home market, i.e, F exports Y. for Home, Y will be relatively cheaper, and they will import it at a lower relative price from F.

Takeaway: Differences in Autarky prices determine the direction of trade.

Gains From Trade

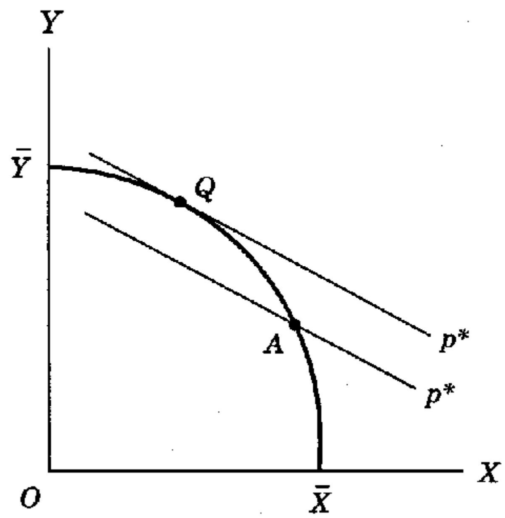
Trade is not a sum-zero game, since it is almost always the case that (i) if two countries are trading at prices different from their autarky price ratio they're both better-off, improving societal wealth, and if (ii) they're trading at an international price ratio that is equal to the autarky one, they are at least not worst-off.

Trade is therefore generally a **positive-sum game** as long as $P^{*} \neq P_a$, regardless the directions of trades themselves.

Gains-from-trade Theorem

Suppose the value of production is maximized at free trade prices. Then the value of free trade consumption at free trade prices exceeds the value of autarky consumption at free trade prices. The free trade consumption bundle must thus be preferred to the autarky bundle, because if it were not, consumers would pick the cheaper autarky bundle.

Graphical representation of the first statement theorem, at free trade prices P^*, Q is tangent to the PPF, while A is not; therefore at free trade prices the value of FT production > value of autarky production:



Formally, we can represent the value of production being maximized at P^* with the following equation:

$$P_x^* X_p^{ft} + P_y^* Y_p^{ft} \geq P_x^* X_p^a + P_y^* Y_p^a \tag{8}$$

Recall that autarky equilibrium requires **Market Clearing** (equation 25), while FT equilibrium requires **Trade Balance** (equation 7).

In you substitute Market Clearing (equation 25) in the right hand side of equation 8, changing production quantities into consumption quantities, and Trade Balance (equation 7) in the left hand side, changing again production quantities into consumption quantities, you obtain:

$$P_x^* X_c^{ft} + P_y^* Y_c^{ft} \geq P_x^* X_c^a + P_y^* Y_c^a \tag{9}$$

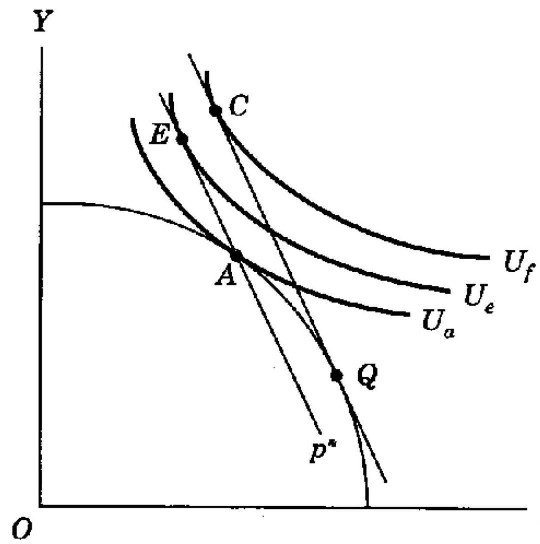
That is a formal proof of the Gains-from-Trade Theorem. **NB:** it only holds if tangency and convexity conditions of the PPF hold.

Gains from Trade Decomposition - Gains from Exchange and Gains from Specialization

Overall gains from trade can be decomposed into the sum of gains from exchange and gains from specialization:

- **Gains from Exchange:** When individuals or countries are endowed with different amounts of goods or have different preferences, they can gain by treading with each others. (e.g., the lest preferred good for i can be the most preferred one for j and vice-versa, and they can exchange their respective goods.)

- **Gains from Specialization:** i and j also achieve additional gains by specializing in the good they can produce more efficiently (i.e., the one good where they have a **Comparative Advantage**)



The (positive) shift in utility from A to E corresponds to *Gains From Exchange* - that is, net of the economy specializing reaching the boundaries (tangency) of its PPF - The shift from E to C is *Gains From Specialization.*

Absolute vs Comparative Advantage

Absolute Advantage: i has an AA in producing X over j if one unit of input$_i$ produces more of X than one unit of of input$_j$.

Comparative Advantage: i has a CA in producing X over j if its *Opportunity Cost* of X in terms of Y is *lower* than j's.

NB: When two countries i and j trade, even if i has an AA on j on both X, Y; they can still (both) gain from trade if they have different opportunity costs of production (i.e., a comparative advantage in producing X or Y).

References

Markusen J. R., J. R. Melvin, W. H. Kaempfer, and K. E. Maskus, 1995, International Trade. Theory and Evidence - Chapters 4 and 5

Ricardian Model - Notes

Tutorial II

October 1, 2025

The Ricardian Model

The model assumes **Labor (L)** is the only factor of production. We also assume **Constant Returns to Scale (CRS)** (and thus Perfect Competition) and **Full Employment**, so that the whole available $\bar{L}$ input is employed in the production process. The model can be summarized as:

$$X = \alpha L_x \tag{10}$$

$$Y = \beta L_y \tag{11}$$

$$\bar{L} = L_x + L_y \tag{12}$$

α, β being the marginal productivity of Labor in producing X and Y respectively (i.e., additional output of X, Y obtained by one additional unit of labor). Considering the usual two country H, F model:

- H has an **Absolute Advantage (AA)** on F on the production of X (Y) if $\alpha_h > \alpha_f$ ($\beta_h > \beta_f$).
- H has an **Comparative Advantage (CA)** on F on the production of X if $\frac{\beta_h}{\alpha_h} < \frac{\beta_f}{\alpha_f}$ (i.e., if H has a lower Opportunity Cost[1] of producing one additional unit of X, compared to F.

Bear in mind that if H has a CA in X, then F must have a CA in Y.

We are also assuming **identical, homogeneous preferences** among consumers, while production functions are **different across countries** - i.e., $\alpha_h \neq \alpha_f,\ \beta_h \neq \beta_f$.

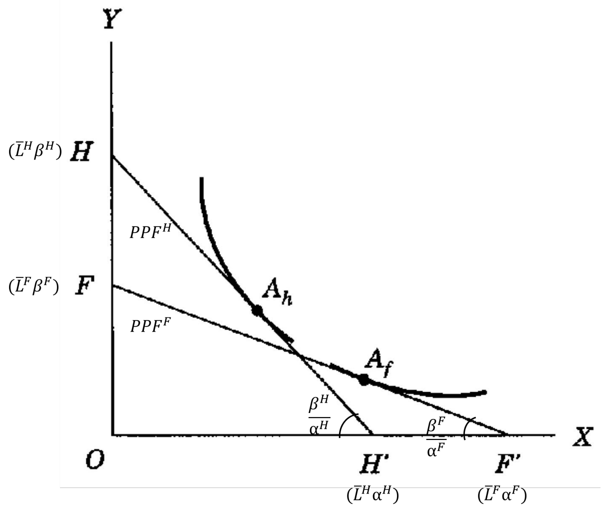
Given all the preivous conditions and assumptions, in this model we have **Linear PPF** which slope is $MRT = \frac{MPL_y}{MPL_x}$ i.e., $\frac{\beta}{\alpha}$, is equal to the equilibrium price ratio in autarky $P_a = \frac{P_x}{P_y}$ - due to the optimality condition of $P_a = MRT$. Autarky price lines are therefore implicit in the diagram representation of the eq. Autarky eq. is therefore found at:

$$MRS = \frac{MU_x}{MU_y} = \frac{\partial U/\partial x}{\partial U/\partial y} = \frac{P_x^a}{P_y^a} = MRT = \frac{MPL_y}{MPL_x} = \frac{\beta}{\alpha} \tag{13}$$

Autarky Equilibrium can be graphically repsented as follows:

[1] You can think of having a lower opportunity cost in producing X as H having to sacrifice to produce less unit of Y to have an additional unit of X, compared to what F would do.

Essentially $OC = \frac{\text{What you sacrifice}}{\text{For an additional unit of this}}$



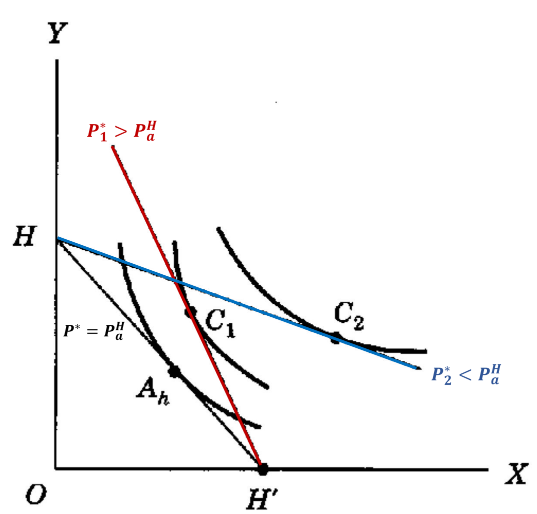
The graph is informative an all the relevant characteristics of the economies:

- The maximum amount of X (Y) Home can porduce allocating all labor $\bar{L}$ in the production of a single good is $\bar{L}^h\alpha^h$ ($\bar{L}^h\beta^h$), i.e., the intercepts of the PPF with the X (Y) axis.
- The distance of PPF from the origin depends on both MPL (α, β) and the total labor endowment $\bar{L}$. Therefore, a county's **AA** can be seen based on the distance of PPF from O. Here, H has an AA in Y, F has an AA in X.
- since P^a overlaps with PPF, the slopes of PPF reflects countries CA. Here, F has a CA in X because $\frac{\beta_f}{\alpha_f}<\frac{\beta_h}{\alpha_h}$. It follows that H has a CA in Y

Last, another property of this model is that, given that $MC = MR$, where $MR = P$ in PC and $MC = w/\beta, \alpha$, nominal wages are equal to $w = \alpha P_x$ and $w = \beta P_y$.

Trade and Complete Specialization

There are three possible cases of Trade Equilibrium depending on the difference between P^* and P^a:



(I) $P^* = P^a \rightarrow$ in FT Eq. H consumes exactly as in autarky, but produces any combination of X and Y on the PPF (including allocating all L in only producing X or Y).

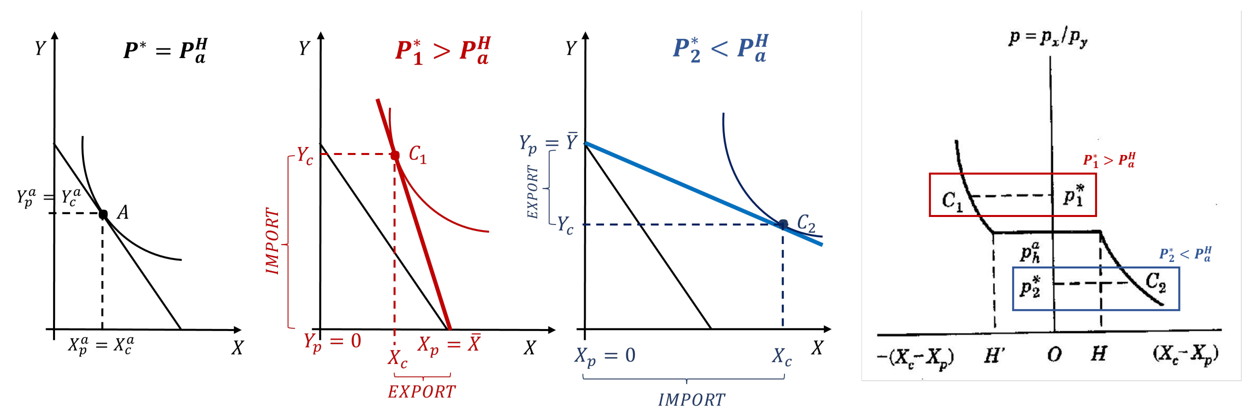
(II) $P_1^* > P^a \rightarrow$ in FT Eq. H consumes in C_1 (i.e., where $MRS = \frac{P_x^*}{P_y^*}$) but **completely specializes** in the production of X (i.e., $X - axis$ corner solution, the tangency between the new P_1^* and the PPF).

(III) $P_2^* < P^a \rightarrow$ in FT Eq. H consumes in C_2 but **completely specializes** in the production of Y (i.e., $Y - axis$ corner solution).

In the Ricardian Model therefore, every time $P^* \neq P^a$, each one of H and F will completely specialize in the production of one of the two goods.

Excess Demand Diagram

Let's represent the three cases above on the excess demand diagram:



(I) $P^* = P^a \rightarrow$ flat line portion of the excess demand function for X. The country is indifferent in producing any quantity of X along the PPF. There can be market clearing, excess demand or excess supply, anywhere from H' to H

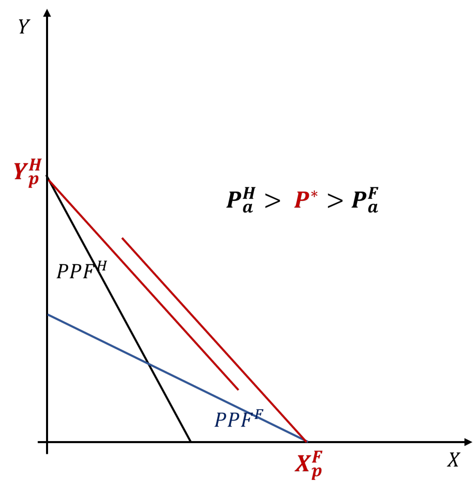
(II) $P_1^* > P^a \rightarrow X$ is relatively more expensive, Y is relatively cheaper. The country completely specializes in X (and does not produce any Y), so it will export X (and import Y).

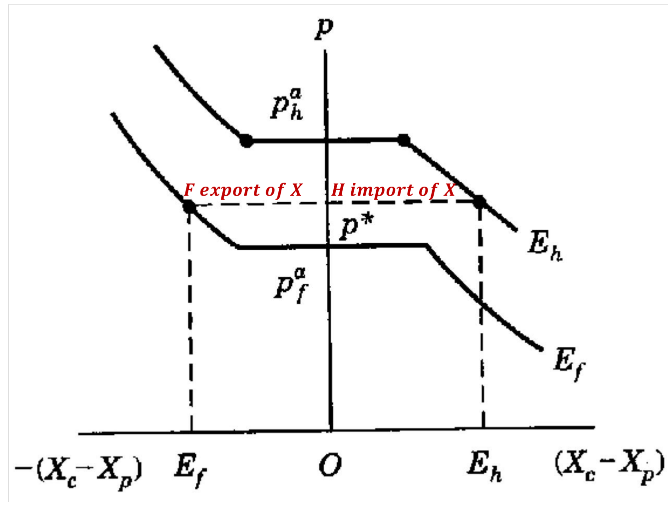
(III) $P_2^* < P^a \rightarrow$ the country does not produce any quantity of X (because it fully specializes in Y), so it will import all the X consumed domestically.

International Equilibrium

Considering a two country model with autarky price ratios P_h^a and P_f^a, the international equilibrium price P^* resulting of H and F trading with each others will fall in between P_h^a and P_f^a, in the exact point where one country's excess demand of a given good equals the other other country's excess supply for the same commodity:

$$P_a^h > P^* > P_f^h \quad (14)$$





There are a few exceptions to this rule, such as one country being considerably bigger than the other or experiencing a significant change in their labor supply, resulting in the excess demand functions to cross. An example of these exceptions can be found in the exercises.

The Role of Wages

In the Ricardian Model, all workers benefit in terms of real wage when their country opens up to trade. However, more productive countries tend to gain more in terms of real wages.

We can show this recalling that in this model:

- **Nominal Wages** for H and F are obtained such that the marginal product of labor is equal to the wage rate in each sector:

$$w_h = P^a_{x_h}\alpha_h \qquad w_h = P^a_{y_h}\beta_h \tag{15}$$

$$w_f = P^a_{x_f}\alpha_f \qquad w_f = P^a_{y_f}\beta_f \tag{16}$$

So nominal wages are a function of both workers productivity and autarky selling prices. However, autarky price ratio P^a_h is completely independent of the wage rate:

$$P^a_h = \frac{P^a_{x_h}}{P^a_{y_h}} = \frac{\beta_h}{\alpha_h} \tag{17}$$

$$P^a_f = \frac{P^a_{x_f}}{P^a_{y_f}} = \frac{\beta_f}{\alpha_f} \tag{18}$$

Countries CA thus only depends on the marginal productivity of their workers (L), and not on their wages (**Labor Theory of Value**).

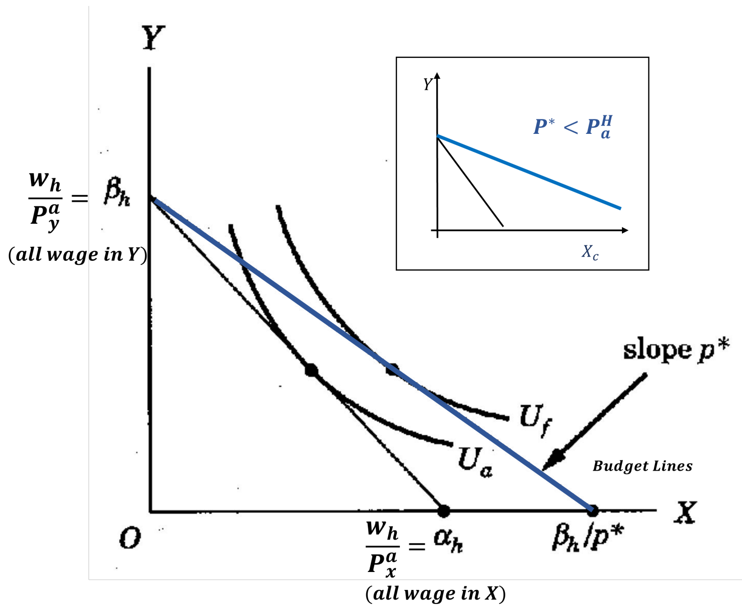
- **Real Wages** instead, are, by definition, nominal wages adjusted by prices, i.e., real wage is identical to the marginal productivity of labor (MPL):

$$\alpha_h = \frac{w_h}{P^a_{x_h}} \qquad \beta_h = \frac{w_h}{P^a_{y_h}} \tag{19}$$

$$\alpha_f = \frac{w_f}{P^a_{x_f}} \qquad \beta_f = \frac{w_f}{P^a_{y_f}} \tag{20}$$

Wage rates therefore determine real wages, that represent workers living standards. Graphically, real wages can be identified as the extremes of workers budget lines. It follows that, if H opens up to trade, as prices shift from P^a_h to P^*, so will change workers' real wages.

Consider the graph of H workers budget line at different price ratios:



Suppose that, as in the graph, because of trade $P* < P^a_h$:

- H completely specializes in Y (importing X)
- **Nominal wages** are altered in both countries ($w^a_h, w^a_f \neq w^*_h, w^*_f$)
- **Real wages** are:
  (i) Constant in terms of the **exported good** (i.e., Y for H) - because, since now H is the only producer of Y, it can only be produced as much as the Y intercept of the PPF, so if all the wage is allocated in Y households can still purchase the same maximum quantity of Y - hence $\beta^a_h = \beta^*_h$

  (ii) Higher in terms of the **imported good** (i.e., X for H) - because:

$$\alpha_h = \frac{w_h}{P^a_{x_h}} = \frac{\beta_h \cdot P^a_{y_h}}{P^a_{x_h}} = \beta_h \cdot \frac{1}{P^a_h} = \frac{\beta_h}{P^a_h} \tag{21}$$

  If $P* < P^a_h$, then

$$\frac{\beta_h}{P^*} > \frac{\beta_h}{P_h^a}; \quad \text{hence} \quad \alpha_h^a > \alpha_h^* \tag{22}$$

Workers therefore enjoy higher real wages in terms of the inported good after opening up to trade. However, one country can have higher real wages in terms of both goods after trade, if the country has AA in both goods (i.e., is more productive in producing both goods).

References

Markusen J. R., J. R. Melvin, W. H. Kaempfer, and K. E. Maskus, 1995, International Trade. Theory and Evidence - Chapter 7

Heckscher-Ohlin Model - Notes

Tutorial III

October 15, 2025

The Heckscher-Ohlin Model

The HO Model is a two-countries (H, F) two-factors (L, K) model. Two goods (X, Y) are produced using Labor and Capital. Workers are paid w wages, while the price of the capital input is r the rental rate of capital. Factors are **mobile** within countries and **immobile** between countries.

We assume identical preferences and CRS production functions, st H and F only differ in **Relative Factor Endowments**.

Be aware of the key difference between factor intensities and factor endowments:

- **Relative Factor Endowments:** refer to the ratio between a country's total stock of Capital and total Labor force:

$$\frac{K_h}{L_h} > \frac{K_f}{L_f} \tag{23}$$

  In this case, we are saying that the country H is **relatively K-abundant (L-scarce)** while F is **relatively L-abundant (K-scarce)**. This has implication on differences in **autarky factor prices**, since it signals that H has relatively cheaper K, while F has relatively cheaper L.

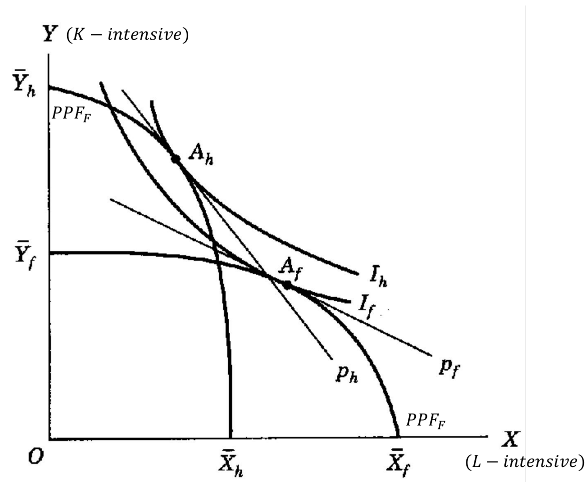
- **Relative Factor Intensities:** refer to the ratio between the necessary units of Capital and Labor to produce a specific commodity:

$$\frac{K_y}{L_y} > \frac{K_x}{L_x} \tag{24}$$

  In this case, we are assuming that the good Y is produced by a **relatively K-intensive** process, while X is a **relatively L-intensive** good.

NB: Factor endowments are different across countries, but we assume the relationship between commodities' relative factor intensities is the same for H and F, e.g., Y is K-intensive in both countries (and vice versa).

Given differences in commodities' production processes, and knowing countries only differ based on their endowments, we can easily represent each country's **Autarky Equilibrium**:



The PPF of the L-abundant (K-abundant) country, i.e., F (H), is biased towards the axis of the relatively L-intensive (K-intensive) good, i.e., X (K).

In autarky, H has a CA in Y (thus Y is relatively cheaper in H), while F has a CA in X (thus X is realtvely cheaper in F):

$$P_h^a = \frac{P_x^h}{P_y^h} \quad > \quad P_f^a = \frac{P_x^f}{P_y^f} \tag{25}$$

Free Trade Equilibrium

Assume now H and F open up to trade, the HO Theorem tells us what will happen to trade flows:

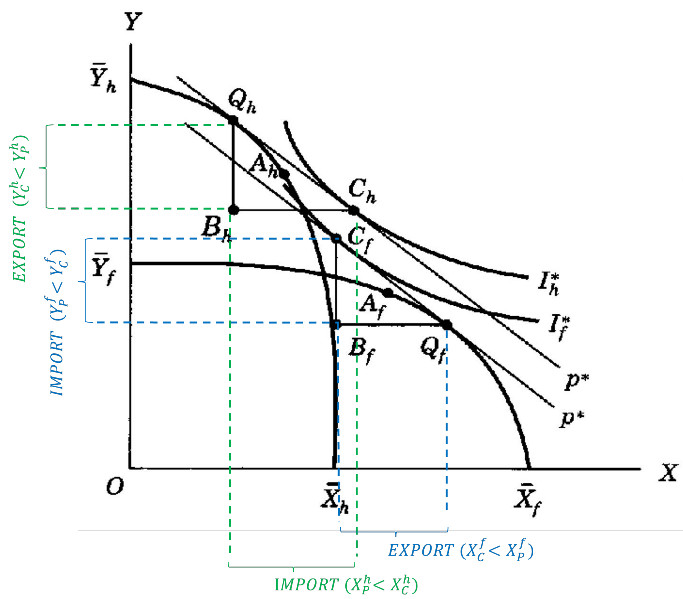
The Heckscher-Ohlin Theorem

Given the assumptions of the model, a country will export the commodity that intensively uses its relatively abundant factor.

Why does that happen? When trades are opened:

- H buys X (L-intensive good) from F (L-abundant country) where it's cheaper
  F buys Y (K-intensive good) from H (K-abundant country) where it's cheaper.
- **Demand** for **domestic production** of X: $\downarrow$ in H, $\uparrow$ in F - consequently $\downarrow P_x^h$, $\uparrow P_x^f$
  Demand for **domestic production** of Y: $\uparrow$ in H, $\downarrow$ in F - consequently $\uparrow P_y^h$, $\downarrow P_y^f$
- However, overall-demand of both X an Y increases, so **Supply** changes accordingly:
  H specializes in Y to satisfy F's demand
  F specializes in X to satisfy H's demand

All until they reach trade balance at $P_h^* = P_f^* = P^*$ (Commodity Price Equalization)



NB: when production is optimized in each country at Trade Eq., they also reallocate their inputs. For instance, specialization in Y means that both inputs L and K are reallocated from industry X to producing more Y. Therefore, specialization in Y means that more of the good Y is produced, and less than the good X, compared to autarky. The opposite is true for specialization in X.

Relative Factor Prices

in HO, wage rental ratio $\omega = \frac{w}{r}$ is a function of the price ratio, $\omega = f(\frac{P_x}{P_y})$. An increase in the relative price of a commodity will increase the relative return to the factor used intensively in that industry:

$$\Uparrow \frac{P_x}{P_y} \quad \rightarrow \Uparrow \frac{w}{r} \tag{26}$$

Since in FT relative commodity prices change in both countries compared to autarky, this has a consequence in relative factor prices too. More specifically, in FT relative factor prices are equalized exactly as relative commodity prices:

The Factor-price-equalization Theorem

Under identical constant-returns-to-scale production technologies, free trade in commodities will equalize relative factor prices through the equalization of relative commodity prices, so long as both countries produces both goods.

Recalling that $P_h > P^* > P_f$; compared to autarky, in free trade:

- **Country** H: $P* < P_h \ \rightarrow \ \omega^* < \omega_h$, in FT H has lower relative returns to labor compared to autarky
- **Country** F: $P* > P_f \ \rightarrow \ \omega^* > \omega_f$, in FT F has higher relative returns to labor compared to autarky

Real Factor Prices

Changes in relative commodity prices also have an impact on real factor prices, i.e., **real wages** and **real return of capital**. Trade are thus expected to affect real factor prices:

The Stolper-Samuelson Theorem

If there are constant returns to scale and both goods continue to be produced, a relative increase in the price of a commodity will increase the real return to the factor used intensively in that industry and reduce the real return of the other factor.

To understand why this happens, first recall that, as for the Ricardian Model, due to producers' optimizing behaviors real factor returns are equivalent to marginal factor productivity:

$$MPL_x = \frac{w}{P_x}; \quad MPK_x = \frac{r}{P_x}$$
$$MPL_y = \frac{w}{P_y}; \quad MPK_y = \frac{r}{P_y}$$

Consider for instance country F, the L-abundant country, where $P* > P_f$.

- When F starts trading with H, the demand for the L-intensive good X grows, causing an increase in the relative price of X.

$$(\uparrow P_x) \quad \uparrow \frac{P_x}{P_y}$$

- Since relative factor prices move accordingly, the relative price of the labor input (relative wages) now grows in F, causing a shift away from labor in both industries.

$$(\uparrow w) \quad \uparrow \frac{w}{r} \qquad \Rightarrow \qquad (\Downarrow L, \uparrow K) \;\text{ resulting in }\; \uparrow \frac{K_x}{L_x}, \frac{K_y}{L_y}$$

- Since less of L is now used, it's used more efficiently, i.e., the factor L becomes more productive. Thus, real wages (real returns of L) grow in F:

$$\uparrow MPL_x, MPL_y \quad = \uparrow \frac{w}{P_x}, \frac{w}{P_y}$$

- The opposite is true for the factor K. Y is now relatively cheaper, causing K to be relatively cheaper as a factor, that is now employed more in both industries and it's hence less productive, i.e., real returns of capital decrease:

$$\downarrow MPK_x, MPK_y \quad = \downarrow \frac{r}{P_x}, \frac{r}{P_y}$$

As the Stolper-Samuelson Theorem says, in F (L-abundant country), following to a relative increase in the price of X (the L-intensive good) has increased the real returns of Labor (the factor used intensively in industry X) and decreased the real returns of Capital (the other factor).

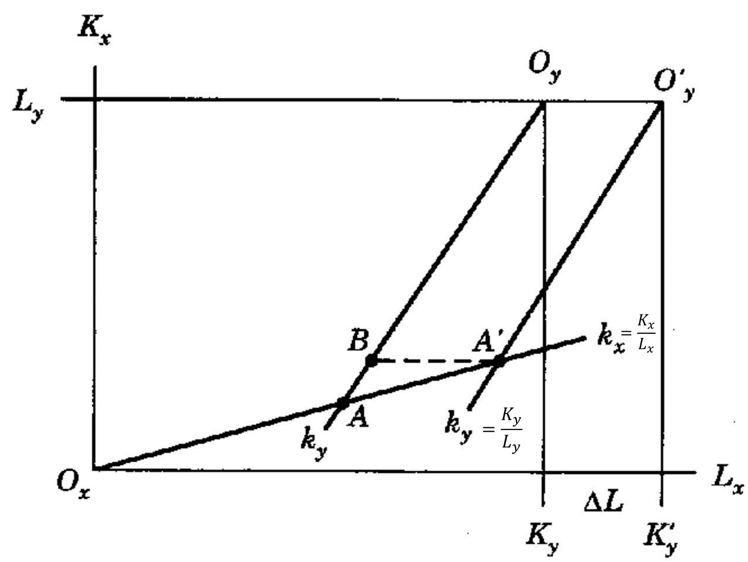
Exactly the opposite will happen in H (K-abundant specializing in the K-intensive good), where an increase in the relative price of Y in FT Eq. would result in decreased real returns of labor (real wages) and an increase in real capital returns (the factor used intensively) [1].

The relationship between changes in $\frac{P_x}{P_y}$ and change in $\frac{w}{r}$, resulting in changes in input allocations and thus MP of factors, has implications in terms of **Uneven Distribution of Gains from Trade**. As a matter of fact, even if each country as a whole gains from trade, the **abundant factor** in each country is made **better off** by FT, while the **scarce factor** is made **worst off**

- H (K-abundant, L-scarce) trades with $F \rightarrow P^{*}<P_h \quad \rightarrow \downarrow \frac{w_h}{P_x}, \frac{w_h}{P_y} \quad \rightarrow$ Labor Force in H (scarce factor) loses real wages.
- F (L-abundant, K-scarce) trades with $F \rightarrow P^{*}>P_f \quad \rightarrow \downarrow \frac{r_f}{P_x}, \frac{r_f}{P_y} \quad \rightarrow$ real return of Capital (scarce factor) in F decreases (e.g., capitalists are worst off).

Rybczynski Theorem

The Rybczynski Theorem allow us to frame how in a given economy output changes due to changes in factor endowments, net of what would be the effect on output of changes (i.e. adjustments) in commodity and factor prices:



The Rybczynski Theorem

If relative commodity prices are constant and if both commodities continue to be produced, an increase in the supply of a factor will lead to an increase in the output of the commodity using that factor intensively and a decrease in the output of the other commodity.

[1] Summing up

- When $P* > P_a$; $(\uparrow P_x) \uparrow \frac{P_x}{P_y} \Rightarrow (\uparrow w) \uparrow \frac{w}{r} \;\Rightarrow\; (\downarrow L) \uparrow \frac{K_{x,y}}{L_{x,y}} \;\Rightarrow \uparrow MPL_{x,y} = \uparrow \frac{w}{P_{x,y}}$ and $\downarrow MPK_{x,y} = \downarrow \frac{r}{P_{x,y}}$
- When $P* < P_a$; $(\uparrow P_y) \downarrow \frac{P_x}{P_y} \Rightarrow (\uparrow r) \downarrow \frac{w}{r} \;\Rightarrow\; (\downarrow K) \downarrow \frac{K_{x,y}}{L_{x,y}} \;\Rightarrow \uparrow MPK_{x,y} = \uparrow \frac{r}{P_{x,y}}$ and $\downarrow MPL_{x,y} = \downarrow \frac{w}{P_{x,y}}$

The theorem is represented in the Edgeworth box above. Every possible equilibrium point in the Edgeworth box is a tangency point between X **and** Y **isoquants** (all the combinations of two factors that produce a given output), and they're called **efficiency locus**. Each one of these efficiency locus is crossed by a straight line, connecting them to the origins O_x, O_y of the K and L inputs axis for good X and Y. Such straight line has as slope X and Y's capital-over-labor ratios (factor intensities).

We are using an Edgeworth box to visualize the production of both commodities **in a single economy**. Consider for instance that only country H is represented in the box, the total labor force available in H grows of a positive quantity $\Delta L>0$. The shifting in the L input is represented moving the origin of the K_y, L_y axis from the original O_y to $O_y^{'}$.

The Equilibrium $A^{'}$, after the shift in the labor force, is on a:

- Higher X isoquant than A, being further away from the origin O_x, thus the output of X increases
- Lower Y isoquant than A, being closer the origin O_y, thus the output of Y decreases

Holding prices constant[2]. **NB:** Since we are holding commodity prices constant, factor prices are constant too (and do not change despite the increase in L input), and therefore the $\frac{K}{L}$ in each industry is held constant. The increase (decrease) in the production of X (Y) is only due to the growth of the Labor input (that leads to an increase in the production of the good that intensively uses labor, and a decrease of the other).

References

Markusen J. R., J. R. Melvin, W. H. Kaempfer, and K. E. Maskus, 1995, International Trade. Theory and Evidence - Chapter 8

[2] You can geometrically see this because $AO_y^{'}$ and BO_y have the same distance from the origin of the Y axis. Being A away from B, and the segment AO_y bigger than BO_y, A must be more distant from the Y-origin than $A^{'}$.

Imperfect Competition - Notes

Tutorial IV

Federico Mutasci - Academic Year 2025/2026

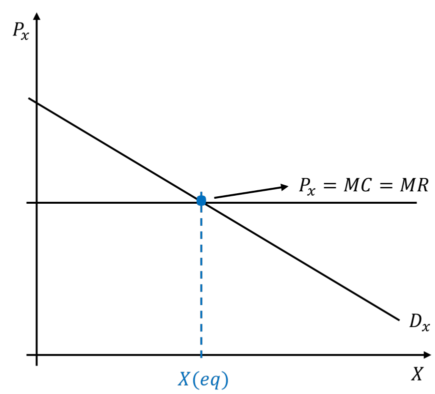
Equilibrium with Monopolized Sector

We previously encountered a two-country (H, F) one factor (L) model with two goods (X, Y) produced with CRS, in **Perfect Competition**. Producers profits π are maximized at:

$$MC = MR$$

As you might recall from the optimality condition from the profit maximization problem. Under these conditions, firms are **Price Takers** (i.e., products are sold at exogenous prices determined by the market).

Let's only consider the market for good X. In Perfect Competition, suppliers of X maximize their profits producing the Eq. quantity of X consumed where the price (defined by market interaction) crosses the consumers' demand. In that point, their $MC = MR$ and they are *breaking even*. Hence, in PC marginal revenue equals the unit selling price of the good ($P_x = MR$), and equilibrium price for X is also $P_x = MC$:



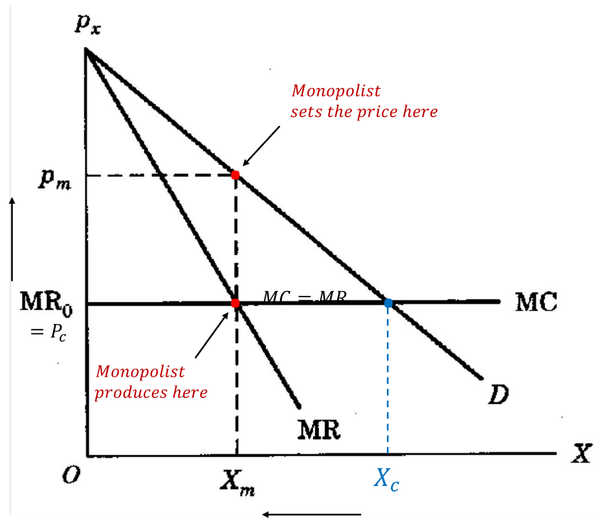
In this single-factor model (Ricardian Model framework), in equilibrium, since $P = MR$, at $MC = MR$:

$$MC_x = w\frac{\Delta L_x}{\Delta X}$$

I.e., the marginal costs are equal to (nominal) wages (w) multiplied by the change in labor input needed for producing an additional unit of X[1] (the reciprocal of the productivity of labor in producing X).

[1] Recall, from here you get the equation for wages in the Ricardian Model, where $P_x = w\frac{\Delta L_x}{\Delta X}$ and thus nominal wage is $w = \alpha P_x$ and real wage $\frac{w}{P_x} = \alpha$

Let's now assume that in H, **good X is supplied by a monopolist**. We also assume there is **no monopsony** (i.e., a single buyer of goods and/or factor inputs):



The monopolist is the **only supplier** of X in H, therefore facing the entire market demand for X. The monopolist chooses the quantity X_m on its MR curve but charges selling prices on D, being now **Price Setter**. Monopolist MR curve is downward-sloping and steeper than D, because, since the monopolist faces the entire demand, it is necessary to lower the price to sell more product (shirking the revenues on each additional unit). Now $P_x^m \neq MC$ and $\neq MR$, indeed, with the monopolist:

$$P_m > MC$$

Because:

$$P_m = MC_x + (P_m \cdot m) \tag{27}$$

Where MC_x is identical to perfect competition price of X, P_x, while m is the monopolist markup $m = \frac{1}{e_x}$:

$$P_m = MC_x + P_m \frac{1}{e_x} \tag{28}$$

Since $MR = MC$, P_m is also higher than MR [2]. Wrt PC, the market now supplies **less X at higher P**.

[2]**Numerical Example:** Recall $TR = P \cdot Q$, $MR = \delta TR/\delta Q$

- **Perfect Competition:** firms are price takers, price exogenously defined by market

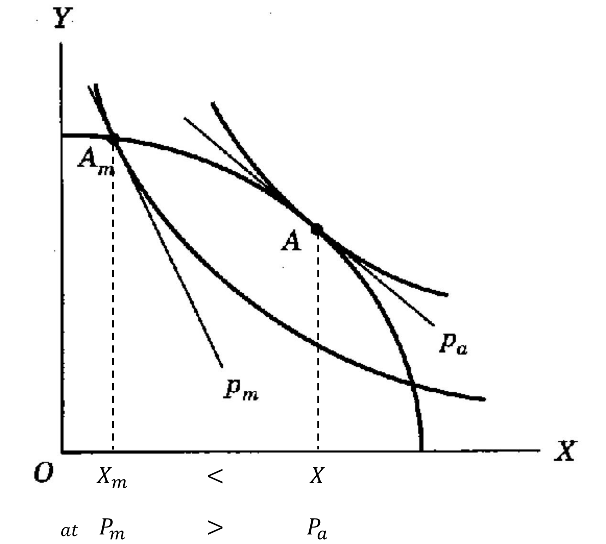
$$P = 30$$
$$TR = P \cdot Q = 30Q, \quad MR = \frac{\delta TR}{\delta Q} = 30, \quad \text{Thus } P = MR$$

- **Monopoly:** firms are price setters, chose what price to charge (function of Q) depending on demand

$$P^m = 100 - Q$$
$$TR^m = (100 - Q) \cdot Q = 100Q - Q^2, \quad MR^m = \frac{\delta TR^m}{\delta Q} = 100 - 2Q, \quad \text{Thus } P^m > MR^m$$

Autarky General Equilibrium

Let's graphically visualize H autarky eq. with perfect competition vs with a monopoly supplier of X:



- **Perfect Competition:** at $MC = P_a = MR$ the tangency condition between P_a, PPF and indifference curve is satisfied
- **Monopoly:** now P_m>MC_x and MR_x, therefore:

$$P_m = \frac{P_x^m}{P_y} \ > \ PPF \text{ Slope}$$

Since we are not in PC, tangency condition between price ratio and PPF is not satisfied, the **Monopoly Eq. is not tangent to the PPF**. in A_m less of X is produced at an higher price, and welfare is reduced U_m<U_a.

PPF is tangent to U when $MRT = MRS$, i.e., when $\frac{MC_x}{MC_y} = \frac{P_x}{P_y}$. With a monopoly for X, however, from Equation 2 you can retrieve that $MC_x = P_x(1 - \frac{1}{e_x})$, and hence:

$$\frac{MC_x^m}{MC_y} = \frac{P_x}{P_y}[1 - \frac{1}{e_x}] \tag{29}$$

Where $\frac{1}{e_x}$ is the markup m, while e_x is the **Elasticity of Demand**:

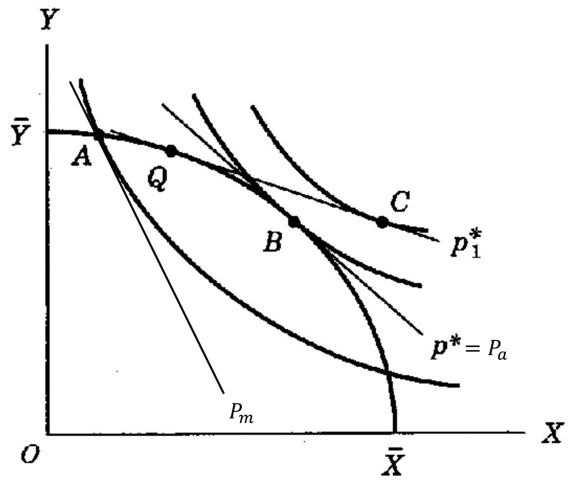
$$e_x = -\frac{\Delta X/X}{\Delta Px/Px} \tag{30}$$

The percentage change in X as a result to a percentage change in P_x.

MRT (PPF) is equal (tangent) to MRS (U) if $m = 0$, i.e, if as $e_x \to \infty$ as in PC, meaning that $\frac{1}{e_x} \to 0$ and consequently $[1 - \frac{1}{e_x}] \to 1$. In perfect competition e_x is indeed **perfectly elastic**.

Free Trade Equilibrium

To visualize trade equilibrium, let's focus on the case of a **small economy:**



- A is **Autarky Eqilibrium** in imperfect competition with a single supplier of X at P_m
- B is a **Trade Equilibrium (1)** where $P^* = P_a$, i.e., the autarky eq. price ratio that the country would have had under perfect competition (B = Autarky Competitive equilibrium)
- C is a **Trade Equilibrium (2)** at a world price ratio $P_1^* \neq P_a$

The **gains from trade** of the economy at price P_1^* can be separated as:

(I) $A \rightarrow B$, **Pro-competitive Gains:** due to increased competition, the monopolist becomes price taker and economy enjoys the same equilibrium overall wealth as in autarky eq. with perfect competition

(II) $B \rightarrow (Q, C)$, **Other Gains From Trade:** gains from specialization that arise from comparative advantage, etc...

Example of Pro-Competitive Gains - Oligopoly

Consider H, F being identical with both having a monopoly for X. Recall:

$$MR_x = P_x[1 - \frac{1}{e_x}] \tag{31}$$

The numerator 1 in $\frac{1}{e_x}$ is the monopolist market share in producing X (100%, the whole market). In other words, in **Autarky**, the share of home firm(s) on total asset for the good X is:

$$S_h^a = \frac{X_h}{X} = 1 \tag{32}$$

Since all X is supplied by the monopolist.

In **Trade**, suppose another monopolist joins the overall market for X. We now have two producers, increased competition, and hence lower markups (and smaller shares of home firms on the market) $\downarrow m \downarrow (S_h)$:

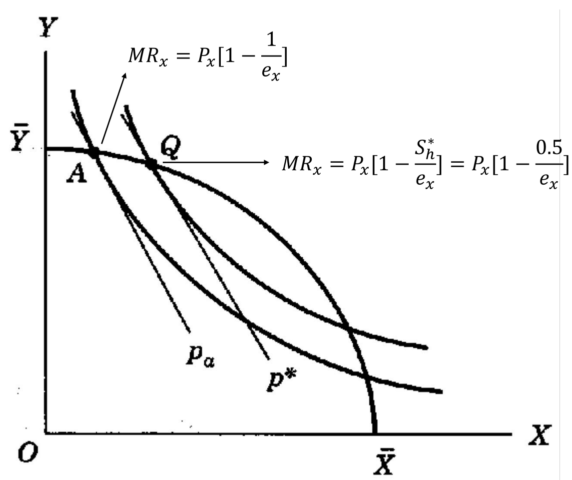
$$S_h^* = \frac{X_h}{X} = \frac{1}{2} = 0.5 \tag{33}$$

I.e., we now have a situation where two identical countries split production in half. Therefore:

$$\frac{S_h^*}{e_x} < \frac{1}{e_x} \tag{34}$$

Both countries are now enjoying lower markups (more X produced at lower P_x) thanks to (little) increased competition.

NB: in this setup, since the two countries are idntical, there is no pattern of comparative advantages. The gains from trade are **pure pro-competitive gains**.



In Free Trade, the equilibrium is reached trough a **Cournout-Nash Competition**, i.e., in FT Eq. each firm produces its best response output given the output of the other firm (and they end up splitting the market in two).

Once H, F open up to trade, $\downarrow m; \uparrow MR$ for both firms st now $MR > MC$. It follows that quantity of $\uparrow X$ to get up to the point where $MR = MC$ (profix maximization).

References

Markusen J. R., J. R. Melvin, W. H. Kaempfer, and K. E. Maskus, 1995, International Trade. Theory and Evidence - Chapter 11

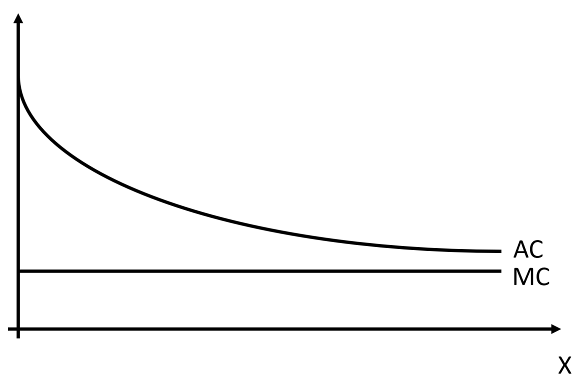
Internal and External Economies of Scale - Notes

Tutorial V

Federico Mutasci - Academic Year 2025/2026

Increasing Returns to Scale (IRS)

Increasing Returns to Scale (IRS) refers to a situation where the **increase in output** is **larger** in proportion than the **increase in inputs**. In economies of scale, Average Costs (AC) decrease as output increases:



Recall that Average Costs (AC) for producing a good are Total Costs (TC) divided by the quantity of good (i.e., per unit cost of Xin this case):

$$TC = FC + MC_x \cdot X \tag{35}$$

$$AC = \frac{FC}{X} + MC_x \tag{36}$$

With $MC_x = \frac{\Delta TC}{\Delta X}$ and are constant. AC are constant for CRS but decrease as X increases with IRS, since production becomes increasingly more efficient and costs are spread out on larger output. With CRS, we usually ignore FC and focus on variable costs only, hence AC = MC. With IRS, production requires huge initial investments (FC) that are later spread out on the production.

External Economies of Scale

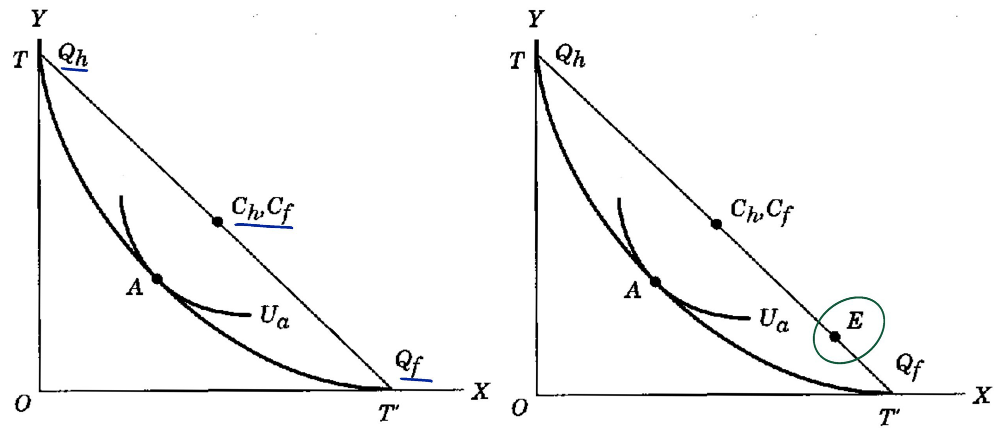
External IRS arise at the **sector or industry level**, meaning that IRS in this case refer to the sum of activities within a sector, due to factors external to individual firms.

General Equilibrium Example

Consider two identical countries, $H$and F, with small identical firms:

- Constant Returns to Scale (CRS) at the firm level.
- IRS at the sector level for both goods X and Y.

IRS translate into a violation of the convexity assumption of the the PPF. This has consequences in terms both of **autarky** and **trade** equilibrium:



Engaging in free trade might still make each country better off. Let's outline the following situation:

- In **Autarky:** both H and F are are assumed to be in equilibrium at point A, where they can enjoy the consumption of both goods. There is no pattern of comparative advantage, as the countries are identical.
- In **Trade:** H can choose to specialize in Y (producing Q_H), and F specializes in X (producing Q_F). They exchange half of their Free Trade production and both consume at points $C_H = C_F$. Both countries benefit from specialization due to IRS, even without comparative advantage.

Both countries are better off, and despite being identical they both **gain from specialization** due to IRS (even if there are no CA). But how likely is (Q, C) as an equilibrium?

As an example, suppose each country wants to **consume at E** instead in FT. There can be **no Trade Eq**. – there is an *excess demand of* X and *Trade Imbalance*. Indeed, **Prices** would have to **adjust** (P_x has to grow and P_y to decrease) in market interactions to reach a feasible Trade Equilibrium…

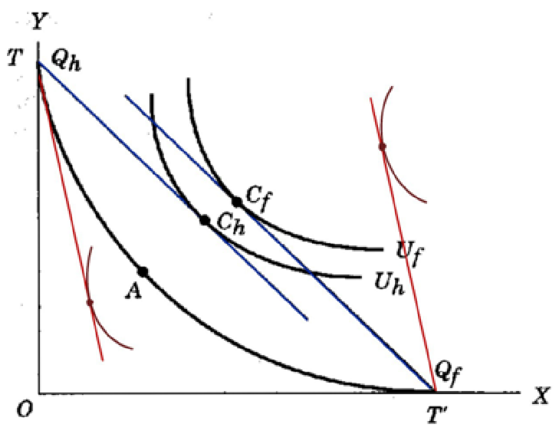
Price Adjustments and Trade Equilibrium

Assume hence each country aims to consume at point E rather than at the Free Trade equilibrium C_h, C_f, as stated above, there will excess demand for X and trade imbalances arise. Commodity prices will eventually change to reach a trade equilibrium that can actually be feasible.

Depending on the international price ratio P^* resulting from market adjustments, with X being relatively more expensive, different scenarios could emerge:

- At $P_1^*\ :\ U_f > U_h$, one country may gain more than the other, leading to **unequal gains** from trade. Still, both countries are still gaining.
- At $P_2^*\ :\ P_y$ fell so much and the new price ratio is so steep that the price line passes below A. This translates in **losses** for H.

For External Economies of Scale, trade might not be beneficial!



Internal Economies of Scale

Internal IRS occur at the **firm level**, where firms benefit from scaling production. These IRS are incompatible with perfect competition equilibria, as demonstrated below. Consider only the market for X as before:

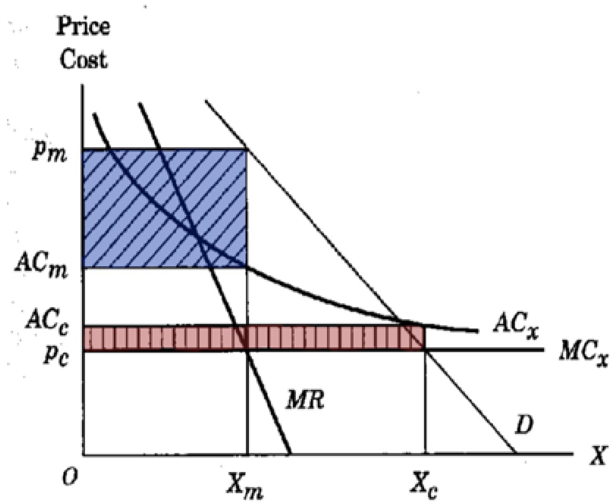
- Suppose $P = P_C$, i.e., P_x with PC, then the price for X would be such that $P = MC$. However, since $AC > MC$ (since AC include Fixed Costs, while MC do not), **Average Costs** of production **exceed prices**, resulting in losses:

$$\pi_C = X_C(P_C - AC)\ <\ 0$$

- Price taker firms would then perish, and a large firm might monopolize the production of X. The market reaches then a **monopoly equilibrium**, and prevent entry of new firms
- Now, with the price setter monopolist, $P = P_m$. The price charged is st $P_m > MC$ and also $P_m > AC$, resulting in profits:

$$\pi_m = X_m(P_m - AC)\ >\ 0$$

Internal IRS result therefore in monopoly production of X and Y. The market for commodity X is represented in the graph:

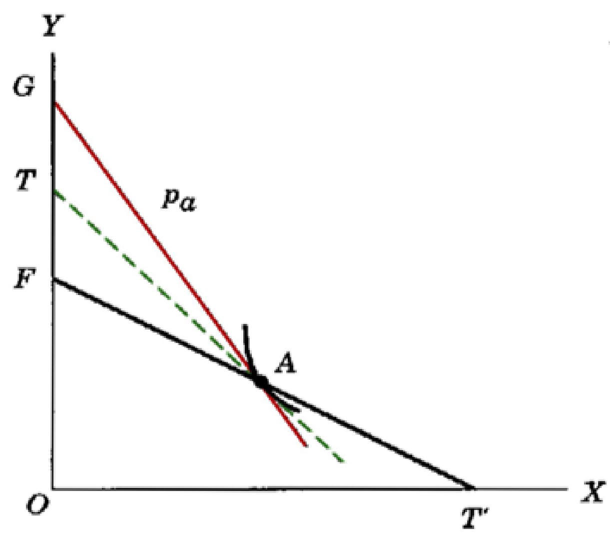


General Equilibrium Model for Internal IRS

Consider two countries, H and F, producing goods X and Y with one factor of production (L). In this model:

- Y is produced with **CRS** and used as a numeraire ($P_y = 1, w = 1$) 1 so the price ratio P will denote the price of X in terms of Y. Moreover, One unit of labor produces one unit of the good, i.e., $Y = L_y$.
- X has internal **IRS** and requires **initial fixed costs** (invested before output is produced), such that the total costs are $TC_x = FC + MC_xX$.

Representing the model graphically:



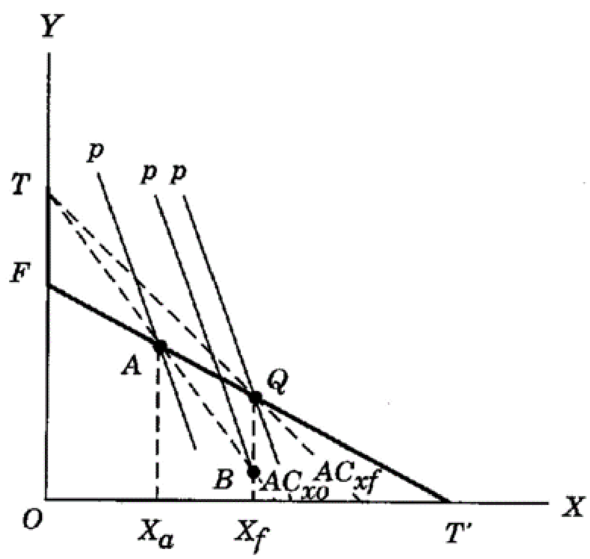
TT' is the PPF - the economy only produces Y until the Fixed Costs (**TF** segment) are invested. After that, producer(s) only bear the constant Marginal Costs MC_x (portion **FT'** of the PPF).

AC (dotted line) is decreasing in the output of X (due to IRS). In **Autarky Eq.**, point A, The producer(s) of X enjoy positive profits at $P_a > AC_x$ (see the slopes). In **Trade**, gains from trade would be similar to Imperfect Competition (and could be disentangled in **different sets of gains**)

Sources of Gains From Trade

A) Pro-Competitive Gains

Gains arise from increased competition as the economy moves from Autarky equilibrium (A) to Trade equilibrium (Q):



They can be decomposed into:

- **Profit Effect:** Movement from A to B - it represents the increase in output caused by the change in profits from π_A to π_{FT} holding AC_{xa} constant.
- **Decreasing AC Effect:** Movement from B to Q - it's the decrease in AC from AC_{xa} to AC_{xFT}, i.e., the fall in AC for producing X when the firm expands its output.

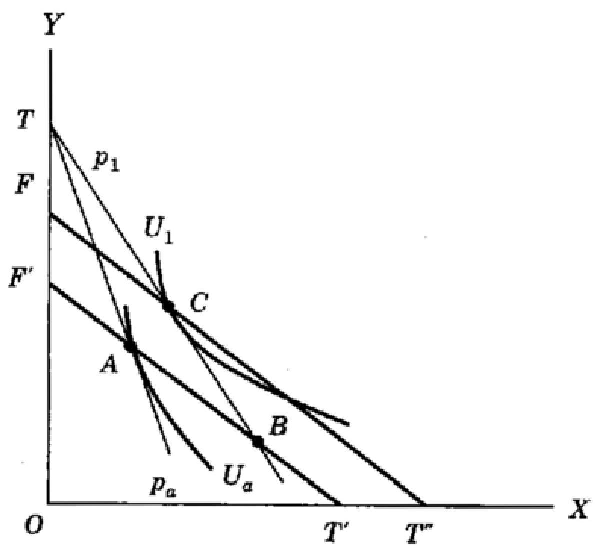
There are some **Special Cases**: (i) If $P = AC$, the movement from A to Q has **no Profit Effect**, and (ii) If X is produced with CRS, there are not FC, $AC = PPF$ and the movement from A to Q is **only** due to **Profit Effect**.

B) Firm Exit Effect

In Free Trade there is an increase in output due to a fall in profits that results from increased competition. This will cause some firms to leave the market due to negative profits:

- Trade can **increase** the **total number of firms** in competition while **reducing** the number of **firms in each country** (this phenomenon is called **exit of redundant firms**)
- **Free Trade Eq.** has a **higher output** (produced by most productive firms to satisfy higher demand) and a **lower price ratio** (lower relative price of X) wrt to Autarky Eq., with fewer firms in each country but more firms in total.

This happens because the firms that didn't exit the market are the most productive ones, and since the total number of firms grew, the market is now more competitive than before. Graphically:



From A to B, **Pro Competitive Gains:** expansion in each firm's output holding the number of firms constant. From B to C, **Firm Exit Effect:** reduction in competing firms, holding each firm's output constant.

Exit of redundant firms has freed the resources previously dedicated to paying fixed costs, shifting the PPF to higher production frontiers, i.e., from TF'T' to TFT".

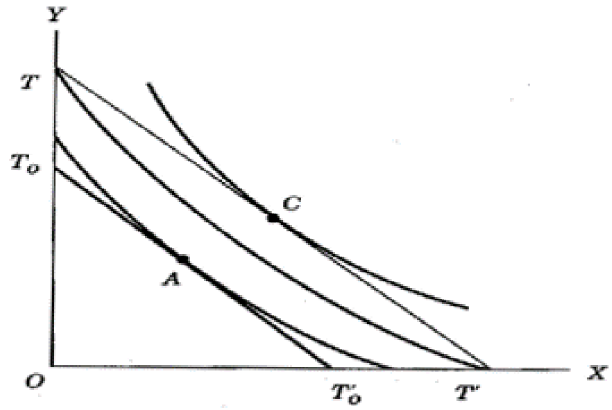
C) Increased Product Diversity

More diversified products can be also interpreted as a form of gains from trade. The theoretical framework we're using as reference is that of a **"Monopolistic competition" model of trade**, i.e., we have a large number of firms that produce somehow different products each.

In this case, assume X, Y are both **IRS** with two **identical production functions** but are **imperfect substitutes** (substitute goods, but consumers would rather have a combination of both than just one – i.e., **love of variety**):

- in **Autarky:** consumers want to consume in A, but it's unlikely as Eq. since firms might want to scale the production and specialize in either X or Y
- in **Trade:** countries can specialize in X or Y and potentially trade half of the output consuming in C; a Pareto Improving solution for consumers

In other words, In autarky, domestic producers woul like to either specialize in Y (producing in T) or in X (producing in T') due to the high fixed costs they have to bear in order to set up their IRS production. Indeed, since both industries have IRS, the PPF of each country is TT'. These corner solutions cross a higher indifference curve than that the economy is able to reach in Autarky Equilibrium in A, but do not yield to any possible equilibrium. In trade, since each country can now completely specialize, the overall output that can be reached is higher if each country sells half of its output for half of the other country's output, making point C reachable:



Consumers might also want the same product but with different ideal characteristics Depending on their difference in taste and income (*ideal variety approach*). In FT ideal tastes are met with wider range of versions of the same products thanks to increased competition and specialization.

References

Markusen J. R., J. R. Melvin, W. H. Kaempfer, and K. E. Maskus, 1995, International Trade. Theory and Evidence - Chapter 12

Trade in Intermediates - Notes

Tutorial VI

November 23, 2025

Introduction

Since the 1980s, **wages of skilled** workers relative to **wages of unskilled** workers have grown in most developed economies. Since it was a period of significant market integration and expansion, economists wondered, did this phenomenon happened **due to trade?**

A possible explanation comes from a Stolper-Samuelson effect on workers wages due to traded goods, using the HO Model framework. This relationship is framed by a model by Deardorff and Staiger (1988):

$$(w^2 - w^1)(F^2 - F^1) \geq 0$$

Where w corresponds to equilibrium **wages** at time 1 and 2, while F is the **factor content of export**, i.e., how much of a factor is used to produce the commodities that are exported.

Using this model, a higher **content of import** for factor k from t 1 to 2- meaning that $F_k^2 < F_k^1$, resulting in $(F_k^2 - F_k^1) < 0$ - is linked to a decrease in wages w for that factor - i.e., $(w_k^2 - w_k^1) < 0$.

Therefore

- Increased factor content of imports from less-developed countries[1] might be linked to a decrease in wages for low-skilled workers (ex. high school dropouts in empirical data).
- Direct import of a factor (e.g., high volumes of low-skilled workers immigration from developing countries) might have the same wage effect (decreasing wages of low-skilled workers in developed countries).

Empirical data from the '80s onward show that decrease in high-school dropouts' wages can be partially explained with increased imports of low-skilled labor intensive commodities from abroad in the US (and low-skilled workers immigration, although to a lesser extent) .

However, this is not an particularly efficient approach to infer the effects of trade on wages, and its applicability and implications are much debated.

[1] The factor intensively used in the production of the imported good becomes comparatively cheaper, is used more and less productively, and loses real returns.

Some Empirical Facts

Why the model from Deardorff and Staiger is limited:

- Looking at **non-production** (imperfect proxy for high-skilled, e.g., supervisors, managers, ...) vs **production** (less-skilled) workers relative wages (figures 4.1, 4.2 in the book):
  - Non-production workers relative wages (wrt to production workers) have developed an **upward trend** after the 1960s.
  - Simultaneously, there has been an **increase in relative employment of skilled workers**.
- Increase in **both** relative employment of skilled workers and their wages contradicts the Stolper-Samuelson theorem. The decrease in relative employment of unskilled workers and the increase in skilled workers' employment conflicts with structural trends of relative increase skilled wages:
  - Higher skilled wages would have led to a shift in employment **away** from skilled labor according to a traditional HO framework—yet it did not happen!

Possible explanation: Increased employments of skilled workers might have been caused by an increase in relative demand for skilled workers **within** industries, and not a shift between industries in the labor force. The root cause may be **trade in intermediate inputs**, which could affect within-industry factor usage and differences in wages.

A Simple Model: Trade in Intermediate Inputs

Trade in intermediate inputs allows firms to spread their production processes across countries, often referred to as **outsourcing**. How can a model of outsourcing represent the link between intermediate input prices and wages?

Structure and Assumptions:

- **Production** in each industry (n) is divided into **three main activities**:

  I Production of **Unskilled Labor Intensive Intermediate Input** [y_1], e.g., raw materials.

  II Production of **Skilled Labor Intensive Intermediate Input** [y_2], e.g., machinery components.

  III Production of the **Finished Product** for each n industry, a byproduct of y_1 and y_2, [y_n].

- **Three Factors of Production** are used:

  I **Unskilled Labor** [L_i]

  II **Skilled Labor** [H_i]

  III **Capital** [K_i].

- Both **Intermediate Inputs** are produced with a combination of all factors, i.e., their **CRS Production Function**:

$$y_i = f_i(L_i, H_i, K_i,), \quad \forall i, i = 1, 2$$

- Some of these activities that are conducted within the factory (the unskilled labor intensive ones) can be **outsourced** (imported), while we assume services and tasks associated with research and development (R&D, skilled labor intensive) are **exported** to support production activities abroad, therefore:

  I A portion of Input (I) is traded $[x_1]$, more specifically **imported**: $x_1<0$

  II A portion of Input (II) is traded $[x_2]$, being instead **exported**: $x_2>0$

  III P_i is the price of each input $i = 1, 2$, st the price vector of trade intermediate inputs is $P = (P_1, P_2)$

- The **Production Function for the Final Good** is given by:

$$y_n = f_n(y_1 - x_1, y_2 - x_2)$$

  And its price is P_n

- The **Total Factor Usage** in producing the final good is therefore:

  I $L_1 + L_2 = L_n$

  II $H_1 + H_2 = H_n$

  III $K_1 + K_2 = K_n$

The model can be solved obtaining an **optimal output level** that uses all three activities.

The solution $G_n(L_n, H_n, K_n, P_n, P)$ is obtained though an optimization problem where, assuming **perfect competition**, the **value of output of the final good**, net of the **value of trade**, is **maximize** subject to a **resource constraint** (production function, total factor usage).

Recall that in Perfect Competition, price = marginal revenues and marginal costs in equilibrium.

Model Solution

$$\underbrace{G_n(L_n, H_n, K_n, P_n, P)}_{Optimal\ Output\ in\ Industry\ n} = \max \overbrace{\underbrace{P_n f_n(y_1 - x_1, y_2 - x_2)}_{Value\ of\ Final\ Good} + \underbrace{P_1 x_1 + P_2 x_2}_{Value\ of\ Traded\ Inputs}}^{Revenue\ Function}$$

$$\text{s.t.} \quad \overbrace{\left.\begin{cases} y_1 = f_1(L_1, H_1, K_1) \\ y_2 = f_2(L_2, H_2, K_2) \\ y_n = f_n(L_n, H_n, K_n) \end{cases}\right\} Production\ Functions}^{Resource\ Constraint}$$

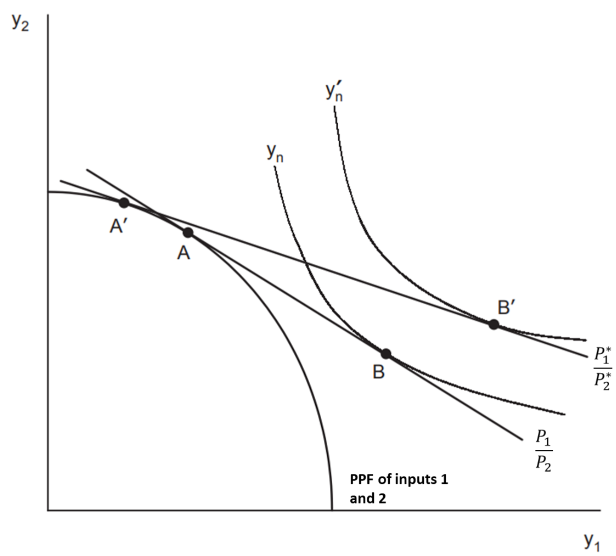
$$\left.\begin{cases} L_1 + L_2 = L_n \\ H_1 + H_2 = H_n \\ K_1 + K_2 = K_n \end{cases}\right\} Total\ Factor\ Usage$$

In the equation x_1 is **imported**, hence X_1<0, x_2 is **exported**, hence X_2>0.

This optimization process do not require to assume trade balance, hence not necessarily $P_1 x_1 + P_2 x_2 = 0$.

1 Intermediate Input Prices vs Output of the Final Goods

To represent graphically how **changes in input prices** relate to **changes in the quantity of the final good produced**, however, we need to assume trade balance to ensure that the output of the final good is maximized on the isoquant that is tangent to the balanced trade price line:



As the picture shows:

- At initial prices, the industry n produces inputs in A, and through trade, at prices P_1, P_2, a point of production of the final good B is reached on the Y_n isoquant.
- If there is a change in input prices, e.g., a drop of the relative price of the imported input (P_1 of x_1); the industry shifts the production towards the production of the skilled labor intensive intermediate input Y_2 at point A'[2].
- From A', at the new price ratio, the industry can reach a production of the final good in B', at the tangency point with a higher Y_n isoquant, Y_n'.

NB: all of this happens within a single manufacturing industry n.

[2]In response to the drop in P_1, firms may choose to substitute less of the skilled labor-intensive processes Y_1 (importing more x_1) with more of the skilled labor-intensive processes Y_2 (exporting more x_2 to support the outsourcing), since drop in the price of x_1 might make it more affordable to allocate resources in higher-skilled labor-intensive processes - affecting thus the substitution between inputs, given both inputs are always employed in the production of both processes. Essentially, firms are importing more X_1 from abroad, affecting substitution between inputs, allowing more Y_2 and thus X_2 to be exported to support the efficient outsourcing process

2 Intermediate Input Prices vs Factor Prices

How would a drop in the relative price of imported inputs ($\downarrow P_1/P_2$ due to a fall in the price x_1) affect **factor prices**?

Let's start from the **zero profit condition** for producing inputs 1 and 2. Since we are in **perfect competition**, firms are breaking even at P=MC, hence:

$$P_i = C_i(w, q, r)$$

Where w is wages for unskilled labor, q is wages for skilled labor, and r is the rental rate of capital. $C_i(w, q, r)$ is a unit cost (average cost) function, and in PC unit costs are equal to marginal costs.

Since we are interested in the relationship between ΔP and $\Delta w, q, r$, we can **totally differentiate**[3] the zero profit condition equation:

$$dP_i = \frac{\partial P_i}{\partial w} dw + \frac{\partial P_i}{\partial q} dq + \frac{\partial P_i}{\partial r} dr$$

Defining $\frac{\partial P_i}{\partial w}$ as ∂_{iL}, $\frac{\partial P_i}{\partial q}$ as ∂_{iH}, $\frac{\partial P_i}{\partial r}$ as ∂_{iK}, we represent the total change in P_i as:

$$\underbrace{dP_i}_{\substack{\text{change in} \\ \text{prices}}} = \underbrace{\partial_{iL} \cdot dw}_{\substack{\text{change in} \\ \text{unskilled wages}}} + \underbrace{\partial_{iH} \cdot dq}_{\substack{\text{change in} \\ \text{skilled wages}}} + \underbrace{\partial_{iK} \cdot dr}_{\substack{\text{change in} \\ \text{rent.rate of K}}}$$

Consider now a transformation of the equation above, with $\hat{P} = \frac{dP}{P}$ being the **percentage change in prices**. $\hat{P}_i$ can thus be written as a function of the percentage change in factor prices $\hat{w}_i, \hat{q}_i, \hat{r}_i$.

[3] **TOTAL DIFFERENTIAL:**

Consider a function $y = f(x_1, X_2)$

- **Partial Derivative:** change in y due to a change in one of the variables, holding the other variable fixed

$$\frac{\partial y}{\partial x_1}; \quad \frac{\partial y}{\partial x_2}$$

- **Total Derivative:** total change in y:

$$dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2$$

i.e., total change in y is equal to the partial change in y due to change in variable 1 (keeping the other fixed) multiplied for the total change in variable 1, plus partial change in y due to change in variable 2 (keeping the other fixed) multiplied for the total change in variable 2.

Since P=MC in perfect competition, it's true that $\hat{P} = \frac{dP}{P} = \frac{dP}{C}$. Therefore dividing dP_i by costs C:

$$\hat{P}_i = \frac{\partial_{iL} \cdot dw}{C} + \frac{\partial_{iH} \cdot dq}{C} + \frac{\partial_{iK} \cdot dr}{C}$$

We can further transform the equation dividing and subtracting by w, q, r (it's possible since it's always 1, nothing is added to the equation):

$$\hat{P}_i = \frac{\partial_{iL} dw}{C} \cdot \frac{w}{w} + \frac{\partial_{iH} dq}{C} \cdot \frac{q}{q} + \frac{\partial_{iK} dr}{C} \cdot \frac{r}{r}$$

Further rearranging:

$$\hat{P}_i = \underbrace{\frac{\partial_{iL} w}{C}}_{\theta_{iL}} \cdot \overbrace{\frac{dw}{w}}^{\hat{w}} + \underbrace{\frac{\partial_{iH} q}{C}}_{\theta_{iH}} \cdot \overbrace{\frac{dq}{q}}^{\hat{q}} + \underbrace{\frac{\partial_{iK} r}{C}}_{\theta_{iK}} \cdot \overbrace{\frac{dr}{r}}^{\hat{r}}$$

We now have an equation that, for each activity i, relates percentage change in input price with percentage changes in factor prices:

$$\hat{P}_i = \theta_{iL}\hat{w} + \theta_{iH}\hat{q} + \theta_{iK}\hat{r}$$

Where θ_{ij} is the **cost share** of factor j in activity i, with $j = L, H, K$. NB: $\sum_j \theta_{ij} = 1$.

We can use this equation to relate changes in intermediate input prices and factor prices between industries i 1 and 2 ($\hat{P}_1 - \hat{P}_2$). We have factor prices ($\hat{w}, \hat{q}, \hat{r}$) as three unknown, but only two equations (one for each industry). We can however assume that **capital has equal cost shares in the two activities**, since we want to focus on skilled and unskilled labor instead, therefore:

$$\theta_{1K} = \theta_{2K}$$

Let's hence take the difference between activity 1 and 2:

$$\begin{aligned}
\hat{P}_1 - \hat{P}_2 &= (\theta_{1L}\hat{w} + \theta_{1H}\hat{q} + \theta_{1K}\hat{r}) - (\theta_{2L}\hat{w} + \theta_{2H}\hat{q} + \theta_{2K}\hat{r}) \\
\hat{P}_1 - \hat{P}_2 &= (\theta_{1L} - \theta_{2L})\hat{w} + (\theta_{1H} - \theta_{2H})\hat{q} + (\theta_{1K} - \theta_{2K})\hat{r}
\end{aligned}$$

With $(\theta_{1K} - \theta_{2K}) = 0$ since $\theta_{1K} = \theta_{2K}$:

$$\hat{P}_1 - \hat{P}_2 = (\theta_{1L} - \theta_{2L})\hat{w} + (\theta_{1H} - \theta_{2H})\hat{q}$$

Notice that, since $\sum_j \theta_{ij} = 1$, it follows that $\theta_{1L} + \theta_{1H} = 1$ and $\theta_{2L} + \theta_{2H} = 1$. Therefore, $\theta_{1H} = 1 - \theta_{1L}$ and $\theta_{2H} = 1 - \theta_{2L}$. Substituting into the equation above:

$$\begin{aligned}
\hat{P}_1 - \hat{P}_2 &= (\theta_{1L} - \theta_{2L})\hat{w} + ((1 - \theta_{1L}) - (1 - \theta_{2L}))\hat{q} \\
\hat{P}_1 - \hat{P}_2 &= (\theta_{1L} - \theta_{2L})\hat{w} + (\cancel{1} - \theta_{1L} - \cancel{1} + \theta_{2L})\hat{q} \\
\hat{P}_1 - \hat{P}_2 &= (\hat{w} - \hat{q})(\theta_{1L} - \theta_{2L})
\end{aligned}$$

To see how the impact of changes in prices of the intermediate inputs affects factor prices, we can rewrite the equation with respect to changes in wages for unskilled and skilled workers:

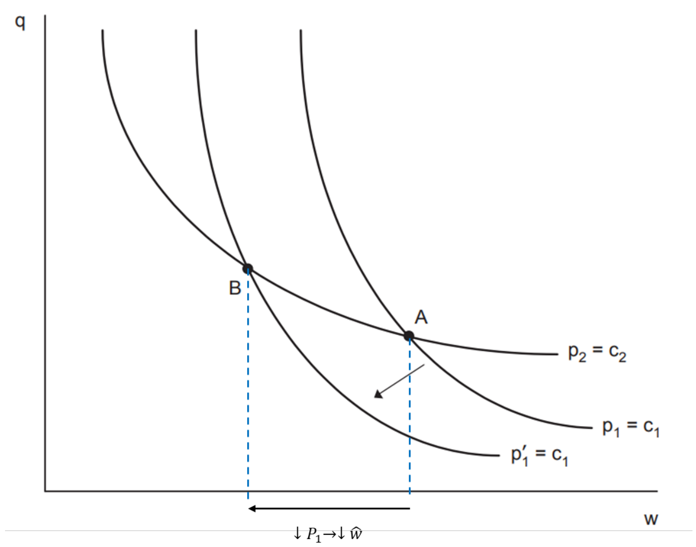
$$(\hat{w} - \hat{q}) = \frac{\hat{P}_1 - \hat{P}_2}{\theta_{1L} - \theta_{2L}}$$

Since activity 1 was assumed to be unskilled labor intensive, we have that $(\theta_{1L} - \theta_{2L}) > 0$ always - i.e., the cost of unskilled labor in activity 1 is higher than the cost of unskilled labor in activity 2.

If there is a **decrease** in the **relative price of the imported input** ($\downarrow P_1$); then $(P_1 - P_2)$<0, and thus, this leads to a **decrease** in the **relative wages of unskilled labor** $\hat{w}$ (q is growing faster than w):

$$(P_1 - P_2) < 0 \quad \rightarrow \quad (\hat{w} - \hat{q}) < 0$$

Graphical Representation:



Let's represent curves $C_i(w, q, r)$ on a graph. r is held constant; w, q are on the each axis. Since we are in perfect competition, we respect the zero profit condition, and P_i corresponds to $C_i(w, q, r)$, i.e., $P_i = C_i(w, q, r)$. Hence, changes in price would shift the isocost curves.

A fall in the price P_1, which is unskilled labor intensive, will shift inwards the isocost line of that activity. Therefore, as it is shown in the picture, a fall in the relative price of the unskilled labor intensive intermediate input determines a decrease in the relative wages of unskilled workers (as data from the US actually show).

3 Intermediate Input Prices vs Final Goods Prices

What happens to the price of the final goods (P_n)?

Denote $C_n(P_1, P_2)$ as a unit cost function for the final good (that is obtained assembling the two intermediate goods/inputs).

Again, we satisfy the **zero profit condition** in perfect competition at:

$$P_n = C_n(P_1, P_2)$$

And as before, it follows that:

$$\hat{P}_n \ = \ \theta_{n1}\hat{P}_1 \ + \ \theta_{n2}\hat{P}_2$$

With θ_{n1}, θ_{n2} being the cost shares of input $i = 1, 2$ respectively in the final product. The change in the price of the final good is a weighted average of the change in intermediate input prices.

If there is a fall in the relative price of an imported intermediate good (intermediate input), $\downarrow \hat{P}_1$ causing $(\hat{P}_1 - \hat{P}_2){<}0$, the price of the final good will be such that $\hat{P}_1{<}\hat{P}_n{<}\hat{P}_2$, i.e., the price of the final good relative to the imported good rises:

$$\hat{P}_n \ - \ \hat{P}_1 > 0$$

Summing up, this model of outsourcing explains the relationship between intermediate inputs (intermediate goods) prices and factor prices (workers' wages and capital returns), exploring the role of intermediate goods' imports and exports. Be aware that the intermediate goods that are imported within industries (e.g., car components) are not the same thing as what is sold at the end of the production process (e.g., finished good, cars).

If the model describes reality, and performs better in explaining the fall in unskilled wages due to trade compared to previous (more simplistic) models, what are its predictions on final goods' real data?

Since the US **imported intermediate inputs** at **lower prices** thanks to **outsourcing**, the model would predict US commodity prices in each industry to rise relative to input prices. This is exactly what happened in the '80s in the US and in many other developed economies, as shown by US data from that period - table 4.2 of the dedicated chapter of the book, page 105.

References

Feenstra R. C., 2004, Advanced International Trade. Theory and Evidence, Princeton University Press (F) - Chapter 4

Heterogeneous Firms and Economics of Multinationals - Notes

Tutorial VII

Federico Mutasci - Academic Year 2025/2026

Introduction

Limitations of the Neoclassical Trade Theory

In the Ricardian and Heckscher-Ohlin models firms are treated as **black boxes**. Neoclassical trade models are limited, since:

- Neoclassical Trade Theory only focuses on explaining how changes in inputs relate to changes in outputs — it is more of a **theory of supply** rather than a theory of firms.
- **Firm size** and **productivity** are **not defined** and never enter the model as variables - general equilibrium is at most determined by the size of the sector to which firms belong, they do not differ in terms of productivity and size.
- **Constant Returns to Scale** production function (and hence **Perfect Competition**) are always assumed.

Therefore, this theory treats **firms** as **homogeneous** and can't be used to study or describe efficiently firm-level dynamics in trade.

New Trade Theory

New Trade Theory first introduced **Increasing Returns to Scale** and **Imperfect Competition** (mainly through the work of Paul Krugman). Although these models allow for violations of neoclassical assumptions, firms are still often treated as homogeneous - the only difference between them being the choice of variety they produce.

However, **empirical evidence** shows firms are **heterogeneous**:

- **Not all firms export**, the vast majority produce and sell only to supply their domestic market.
- **Firms that export** are also usually **larger and more productive** than those that stay in their domestic market.

Moreover, in in New Trade Theory (Helpman-Krugman model) each product variety is produced by a single firm (in one country) and then exported globally. Yet, empirical evidence suggests that most of the exporting firms export only to a single foreign country or, alternatively, to a few countries based on their market size and geographic proximity (e.g., **Gravity Model**).

NTT is also an incomplete theoretical approach, because it only accounts for **transport cost**, and does not account for **fixed costs of export**. Hence every firm that survives the domestic market is automatically assumed to also engage in exports.

Even if New Trade Theory takes a step further from Neoclassical Models, it still fails to provide a solid theoretical foundation to describe what is behind a firm's decision to export. To achieve such objective a new theory was needed that incorporates **firm heterogeneity** in **size** and **productivity**, and provides a theoretical model where only certain firms, i.e., the **most productive firms** engage in **trade**.

Heterogeneous Firms - New "New Trade Theory"

A theoretical framework for studying firms and their decision to export should include **within-industry heterogeneity** in **size**, and **productivity**.

Melitz (2003):

Melitz (2003) Introduced a formal model that explains **firms' decision to export**. Melitz develops a dynamic model with heterogeneous firms where:

- **Exposure to trade** induces only the **most productive firms** to **export**, while some firms continue to supply only the domestic market post-trade exposure.
- The **least productive firms** of all, on the other hand, **exit** the domestic market as well a while after trade flows start.

The Melitz (2003) model therefore analyzes **intra-industry** (firm-level) **effects of international trade**. The models' predictions and mechanisms were consistent with the empirical evidence of the time, allowing the theory to move beyond Krugman's (1980) "Love of Variety" model - which included "monopolistic competition", IRS, but assumed all firms were identical.

Model's Setup

Let's outline a model of international trade based on Melitz (2003):

- Two countries, one sector.
- Intra-industry heterogeneity, with a focus on **productivity**.
- Firms have the possibility to exit or enter the markets.

Consumer-side of the Model: Demand Function

Let's start with a Helpman-Krugman type model. of **monopolistic competition** where each firm produces a different variety. We assume **CES (Constant Elasticity of Substitution) Demand Functions** for the consumers:

$$x(i) = A \cdot p(i)^{-\varepsilon}$$

- $x(i)$ is the quantity of good demanded for each variety (and thus firm) i.
- A is a **Demand Shifter**. i.e., a parameter that affects the "level of demand". It's **exogenous** wrt the single producer i (but it's endogenous to the industry).
- $p(i)$ is the unit price for each variety i.
- ε is the **Elasticity of substitution** - e.g., for goods $[x, y]$

$$\varepsilon_y^x = \frac{\%\ \textit{change in } x}{\%\ \textit{change in } y}$$

Since we are assuming CES, the value of a good ε is given by the following equation:

$$\varepsilon = \frac{1}{1-\alpha}$$

With α being a constant st $0<\alpha<1$ (therefore $\varepsilon>1$)

Here the price elasticity of demand is constant and exogenously determined, which implies that firms' **markups are held constant**.

In this specification, the quantity demanded is an inverse function of the good's price:

$$x(i) = A \cdot \frac{1}{p(i)^{\varepsilon}}$$

Starting from the demand function, we can derive an **Inverse Demand Function** (that is, representing demand with respect to price):

$$\begin{aligned} x(i) = Ap(i)^{-\varepsilon} & \\ p(i)^{-\varepsilon} &= \frac{x(i)}{A} \\ (p(i)^{-\cancel{\varepsilon}})^{-\frac{\cancel{1}}{\cancel{\varepsilon}}} &= \left(\frac{x(i)}{A}\right)^{-\frac{1}{\varepsilon}} \\ p(i) = \left(\frac{x(i)}{A}\right)^{-\frac{1}{\varepsilon}} &= \frac{x(i)^{-\frac{1}{\varepsilon}}}{A^{-\frac{1}{\varepsilon}}} \\ p(i) = A^{\frac{1}{\varepsilon}} \cdot x(i)^{-\frac{1}{\varepsilon}} & \end{aligned}$$

Producer-side of the Model

Assume **labor** is the only factor of production and $w = 1$. When firms enter the market:

- Firms pay an **entry cost** f.
- They observe their **own productivity** $\theta(i)$, and then,
- They decide whether to **produce** or **exit** the market.

Note: The productivity level does not change over time. If firms decide to produce, they will do so facing **unit cost of production** $C = \frac{c}{\theta(i)}$ and paying a **fixed cost** for the **domestic market** cf_D.

NB: as you may notice, costs decrease with an increase in productivity $\uparrow \theta(i)$ (due to IRS).

We can derive a **Profit Function** for the producers:

$$\underbrace{\pi(i)}_{\text{Profit}(i)} = \underbrace{p(i) \cdot x(i)}_{\text{Total Revenue } TR(i)} - \underbrace{C \cdot x(i) - cf_D}_{\text{Total Cost } TC(i)}$$

Recalling: $p(i)$ = price, $x(i)$ = quantity, C = unit cost of production and cf_D = fixed cost for operating in the domestic market.

We then substitute into the profit function the equations for demanded quantities and unit costs:

$$\pi(i) = p(i) \cdot x(i) - C \cdot x(i) - cf_D$$
$$\pi(i) = p(i) \cdot Ap(i)^{-\varepsilon} - \frac{c}{\theta(i)} \cdot Ap(i)^{-\varepsilon} - cf_D$$

Since $p(i) \cdot p(i)^{-\varepsilon} = p(i)^{1-\varepsilon}$, the profit function above becomes:

$$\pi(i) = Ap(i)^{1-\varepsilon} - \frac{c}{\theta(i)} Ap(i)^{-\varepsilon} - cf_D$$

Profit Maximization

Consider the profit function $\pi(i) = Ap(i)^{1-\varepsilon} - \frac{c}{\theta(i)} Ap(i)^{-\varepsilon} - cf_D$, producers want to maximize their profits - i.e., we need to solve the FOCs of a profit maximization problem, setting the first derivative(s) of the objective function = 0.

More specifically, we can maximize the profit function with respect to price ($\frac{\partial \pi(i)}{\partial p(i)} = 0$, making use of Hotelling's Lemma) to get the **Profit Maximizing Price**:

$$
\begin{aligned}
\frac{\partial \pi(i)}{\partial p(i)} &= (1-\varepsilon)Ap(i)^{1-\varepsilon-1} - (-\varepsilon)\frac{c}{\theta(i)}Ap(i)^{-\varepsilon-1} = 0 \\
&= (1-\varepsilon)Ap(i)^{-\varepsilon} + \varepsilon\frac{c}{\theta(i)}Ap(i)^{-\varepsilon-1} = 0 \\
&= (1-\varepsilon)A\frac{\cancel{p(i)^{-\varepsilon}}}{\cancel{p(i)^{-\varepsilon}}} + \varepsilon\frac{c}{\theta(i)}A\frac{p(i)^{-\varepsilon-1}}{p(i)^{-\varepsilon}} = 0
\end{aligned}
$$

Since dividing for $p(i)^{-\varepsilon}$, obtaining $\frac{p(i)^{-\varepsilon-1}}{p(i)^{-\varepsilon}}$, corresponds to subtracting the exponents, we get $p(i)^{-\varepsilon-1-(-\varepsilon)} = p(i)^{-\cancel{\varepsilon}-1+\cancel{\varepsilon}}$. Going on:

$$
= (1-\varepsilon)\frac{\cancel{A}}{\cancel{A}} + \varepsilon\frac{\cancel{A}}{\cancel{A}}\frac{c}{\theta(i)}p(i)^{-1} = 0
$$

And then solving for $p(i)$:

$$
\begin{aligned}
(1-\varepsilon) + \varepsilon\frac{c}{\theta(i)}p(i)^{-1} = 0 \\
p(i)^{-1}\varepsilon\frac{c}{\theta(i)} &= -(1-\varepsilon) \\
p(i)^{-1}\varepsilon\frac{c}{\theta(i)} &= (\varepsilon-1) \\
p(i)^{-1} &= \frac{(\varepsilon-1)}{\frac{\varepsilon c}{\theta(i)}} = \frac{\theta(i)(\varepsilon-1)}{\varepsilon c} = \frac{\varepsilon-1}{\varepsilon}\frac{\theta(i)}{c} \\
p(i) &= \frac{\varepsilon}{\varepsilon-1}\frac{c}{\theta(i)}
\end{aligned}
$$

Hence $p(i) = \frac{\varepsilon}{\varepsilon-1}\frac{c}{\theta(i)}$ is the equation for **Profit-Maximizing Price**.

We can now substitute the profit maximizing price into the demand function for $x(i)$ to obtain **Maximized Demand Quantities**. Recall the demand function is given by:

$$
x(i) = Ap(i)^{-\varepsilon}
$$

Substituting:

$$
x(i) = A\left[\frac{\varepsilon}{\varepsilon-1}\frac{c}{\theta(i)}\right]^{-\varepsilon}
$$

We can then incorporate both profit-maximizing price and demand back into the revenue function for the producer:

$$p(i) \cdot x(i) = \left[\frac{\varepsilon}{\varepsilon-1}\frac{c}{\theta(i)}\right] \cdot A\left[\frac{\varepsilon}{\varepsilon-1}\frac{c}{\theta(i)}\right]^{-\varepsilon}$$
$$p(i) \cdot x(i) = A\left[\frac{\varepsilon}{\varepsilon-1}\frac{c}{\theta(i)}\right]^{1-\varepsilon}$$

The profit maximization is completed by plugging everything back into the profit function:

$$\pi(i) = p(i)x(i) \; - \; \frac{c}{\theta(i)}x(i) \; - \; cf_D$$
$$\pi(i) = A\left[\frac{\varepsilon}{\varepsilon-1}\frac{c}{\theta(i)}\right]^{1-\varepsilon} \; - \; \frac{c}{\theta(i)}A\left[\frac{\varepsilon}{\varepsilon-1}\frac{c}{\theta(i)}\right]^{-\varepsilon} \; - \; cf_D$$
$$\pi(i) = A\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}\left(\frac{c}{\theta(i)}\right)^{1-\varepsilon} \; - \; \frac{c}{\theta(i)}A\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}\left(\frac{c}{\theta(i)}\right)^{-\varepsilon} \; - \; cf_D$$
$$\pi(i) = A\left(\frac{c}{\theta(i)}\right)^{1-\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} \; - \; A\left(\frac{c}{\theta(i)}\right)^{1-\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon} \; - \; cf_D$$

And hence grouping fro $A\left(\frac{c}{\theta(i)}\right)^{1-\varepsilon}$

$$\pi(i) = A\left(\frac{c}{\theta(i)}\right)^{1-\varepsilon} \cdot \left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon} - \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}\right] - \; cf_D$$

Factoring out the common part of the term in square brackets, $\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}$:

$$\pi(i) = A\left(\frac{c}{\theta(i)}\right)^{1-\varepsilon} \cdot \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1} - 1\right) \; - \; cf_D$$

Recalling that $\varepsilon = \frac{1}{1-\alpha}$, and hence $\alpha = \frac{\varepsilon-1}{\varepsilon}$:

$$\pi(i) = A\left(\frac{c}{\theta(i)}\right)^{1-\varepsilon} \cdot \left(\frac{1}{\alpha}\right)^{-\varepsilon}\left(\frac{1-\alpha}{\alpha}\right) \; - \; cf_D$$

Rewriting all in one line:

$$\pi(i) = Ac^{1-\varepsilon}\theta(i)^{\varepsilon-1}\alpha^{\varepsilon}(1-\alpha)\alpha^{-1} \; - \; cf_D$$
$$\pi(i) = \theta(i)^{\varepsilon-1}(1-\alpha)\cdot A\frac{1}{\alpha^{1-\varepsilon}}c^{1-\varepsilon} \; - \; cf_D$$

The equation for **Maximized Operating Profits** - following Melitz (2006) - is hence:

$$\pi(i) = \theta(i)^{\varepsilon-1}\cdot(1-\alpha)A\left(\frac{c}{\alpha}\right)^{1-\varepsilon} \quad - \quad cf_D$$

Graphical Representation of Firm Behavior

I Domestic Market: Firms' Choice to Produce (Intensive Margins)

Firms' decisions to produce in their domestic market is what we refer as **Intensive Margins**. We can graphically analyze this type of decision starting from a linear approximation of the optimal profit function:

$$\underbrace{\pi(i)}_{\text{Maximized Profits }(\pi)} \quad = \quad \underbrace{\theta(i)^{\varepsilon-1}}_{\text{Productivity Threshold }(\Theta)} \quad \cdot \quad \underbrace{(1-\alpha)A\left(\frac{c}{\alpha}\right)^{1-\varepsilon}}_{\text{Slope }(B)} \quad - \quad \underbrace{cf_D}_{\text{Fixed Cost}}$$

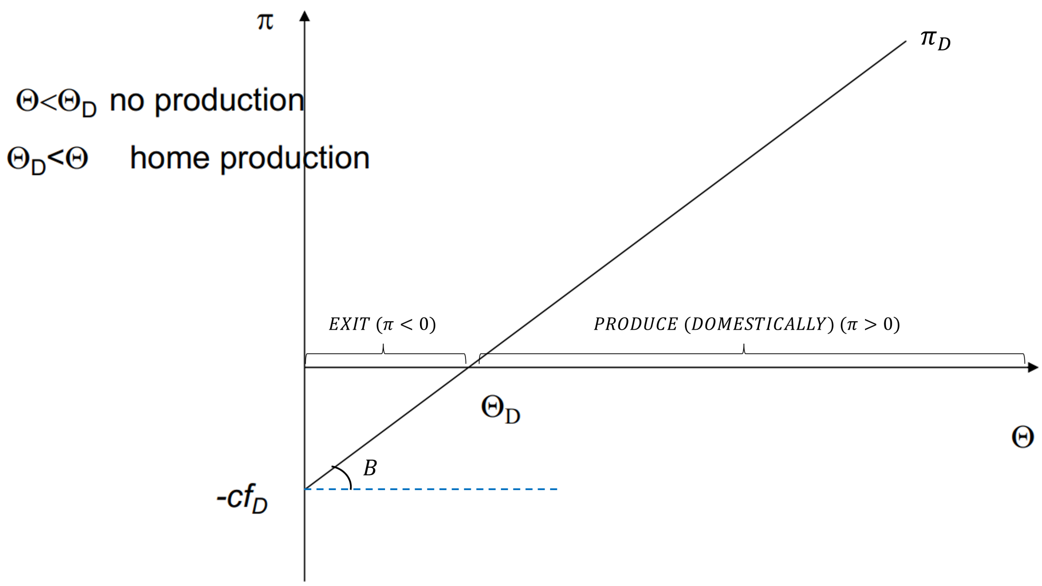
We then proceed to represent the function to optimally produce (maximizing profit) in the **Domestic Market** as a straight line:

$$\pi_D \; = \; \Theta\cdot B \; - \; cf_D$$

Looking at the graph, you can notice how, to produce domestically and stay in the market, firms productivity must be high enough to be above the productivity threshold Θ_D. If this is not true, firm will bear losses due to fixed costs payment - and hence, firms with productivity below Θ_D exit the domestic market.
Evaluating firm productivity:

- if $\Theta < \Theta_D \quad \rightarrow \quad \pi < 0 \; \rightarrow$ EXIT
- if $\Theta > \Theta_D \quad \rightarrow \quad \pi < 0 \; \rightarrow$ PRODUCE DOMESTICALLY

Based on their own (observed) productivity level.



Firms, however, might also choose to export their product in one (or more) foreign markets.

II Foreign Market: Firms' Choice to Export (Extensive Margins)

Let's now introduce a foreign market l, where **Foreign Demand** is given by $x(i)^l = A^l p(i)^{-\varepsilon}$. We also introduce **"Melting Iceberg" Trade Costs** τ>1 - "melting" because they increase with distance - and **Fixed Cost of Export** cf_L (with cf_L>cf_D).

Having to pay trade costs, the unit cost of production now is $\frac{c\tau}{\theta(i)}$:

$$\pi(i) \;=\; \underbrace{\theta(i)^{\varepsilon-1}}_{\Theta} \;\cdot\; \underbrace{(1-\alpha)A^l\left(\frac{c}{\alpha}\right)^{1-\varepsilon}}_{B^l} \;\cdot\; \underbrace{\tau^{1-\varepsilon}}_{\text{Trade Cost}} \;-\; \underbrace{cf_L}_{\text{FC of Export}}$$

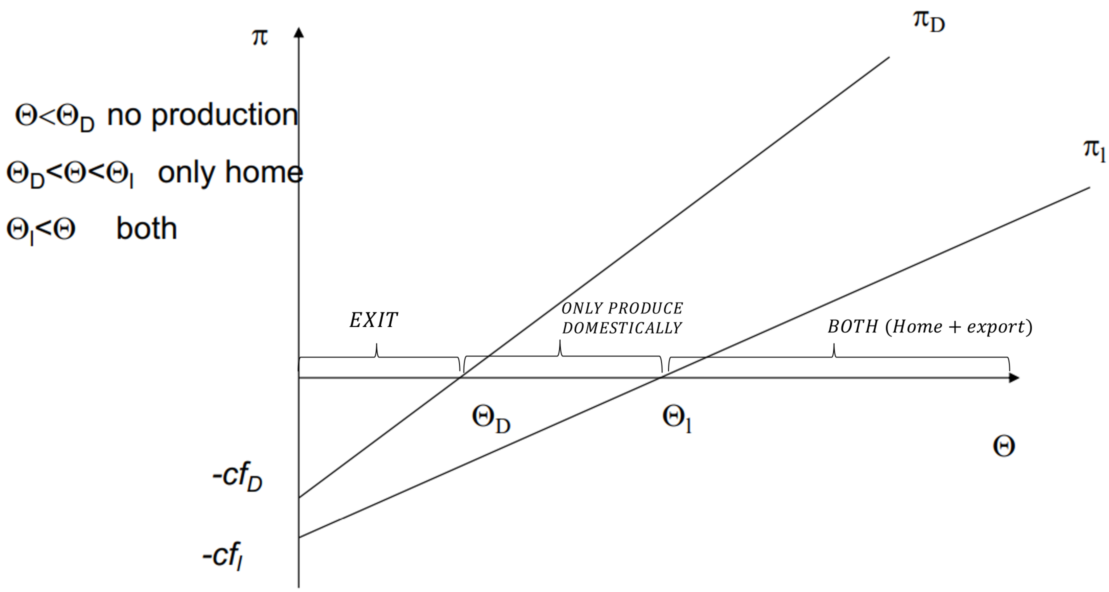
The profit maximizing function for foreign market is therefore:

$$\pi_L \;=\; \tau^{1-\varepsilon}\Theta \cdot B^l \;-\; cf_L$$

Notice that:

- Θ is firm-specific productivity, and it's the same as before (for suplying the domestic market)
- the slope B depends on A, that we defined as a demand shifter. A is a demand modifier related to the share that firms can have in the (foreign) market (hence depending on market size, net of competition).

Both π_D and π_L are graphically represented below. Notice how operative profits are built such that there is a **Trade-Off** between the **size of the market** where firms decide to operate (captured by the parameter A^l - in the form of firms' share in the market) and the **trade cost** (τ). This trade-off corresponds to what is referred in the literature as the **Gravity Model** (size vs proximity trade-off).



Consider the two profit lines in the graph above:

- The profit line π_L is **flatter** because of the trade costs τ.
- A bigger **market size**, keeping competition effects fixed, means firms can take advantage of higher share in the target market (A) thus making the profit line **steeper** - the larger the market, the more a firm that enters it has the opportunity to occupy (supply) a larger slice of the market (larger market share) - considering this is a monopolistic competition model (where each firm produces a different variety).

It follows that higher trade costs require higher productivity levels to operate in foreign markets (rising the productivity threshold), while bigger markets allow for higher profits opportunities (steeper profit curve, lower Θ_L):

$$\uparrow \tau \quad \rightarrow \uparrow \Theta_L$$

$$\uparrow A^l \quad \rightarrow \downarrow \Theta_L$$

As a general rule, however, the productivity threshold needed to export is larger to what it's needed to produce domestically:

$$\Theta_L \ > \ \Theta_D$$

Exporting firms are more productive, **produce more output**. Since we are in IRS, they are, as a consequence, **bigger firms**.

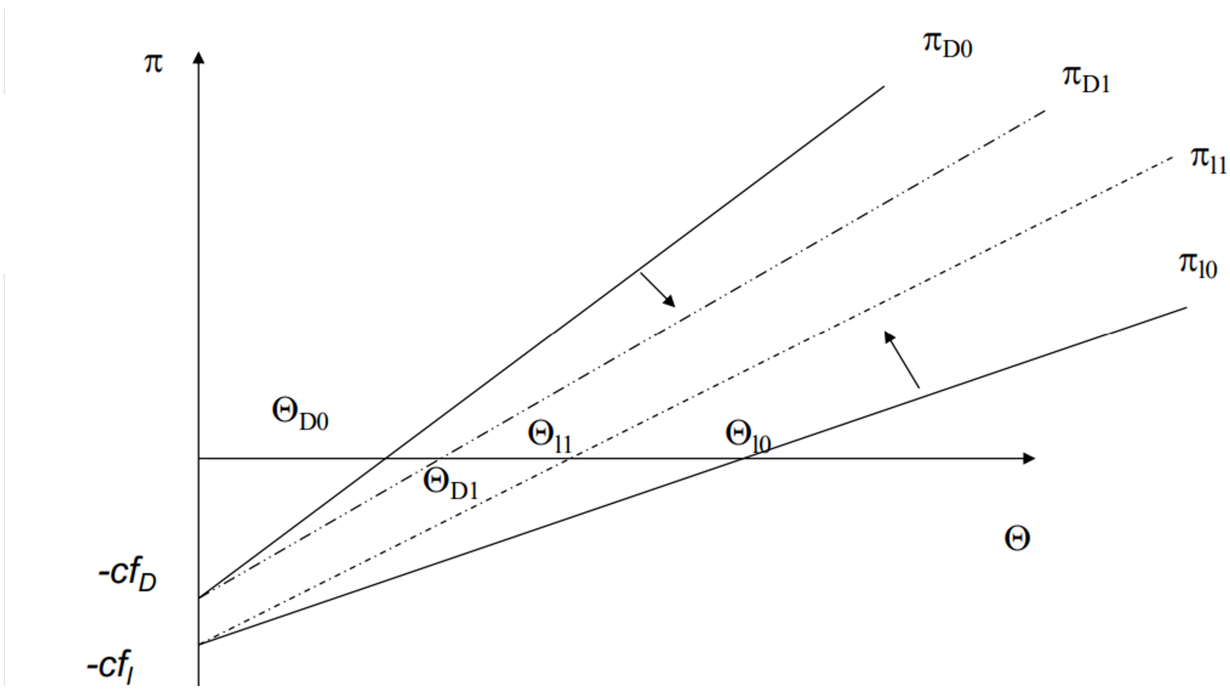
Dynamic Setting

Trade Liberalization

In the static model we kept competition effects as fixed (not considering the effect of increased competition on firms market share, i.e, the effect of the entry of exporting firms in new foreign markets).

On the one hand, if it's true that bigger target markets allow exporting firms to supply bigger shares of such markets, trade liberalization also leads to increased competition, actually causing an increase in the number of firms operating in (joining in) each market, thus leading to a decrease of domestic firms' market share - indeed, firms profit decrease with increase competition, moving towards firms just breaking even in perfect competition.

Let's now evaluate firms' choices to produce domestically and/or export in a dynamic setting - i.e., **before** and **after** trades are liberalized between countries.



Before Trade Liberalization:

- π_{D0} is the maximizing profit line for firms producing domestically and π_{L0} maximizing profit line for exporting firms;
- Θ_{D0} is the productivity thresholds for domestic firms, Θ_{L0} for exporting firms.

After of Trade Liberalization, instead:

- For **Firms in the Domestic Market -** Entry of foreign firms due to trade causes **domestic firms' market shares** A to **shrink**.

  Since $\downarrow A \;\; \rightarrow$ flatter profit line $\;\rightarrow\;$ productivity threshold changes $\Theta_{D0} \;\rightarrow\; \Theta_{D1}$.

  This is the **Competition Effect** that can be captured in the dynamic model. Since increased competition results in a flatter profit lines for domestic firms, the least productive firms of the pre-liberalization regime will now exit the domestic market because their Θ is st $\Theta_{D0} < \Theta < \Theta_{D1}$

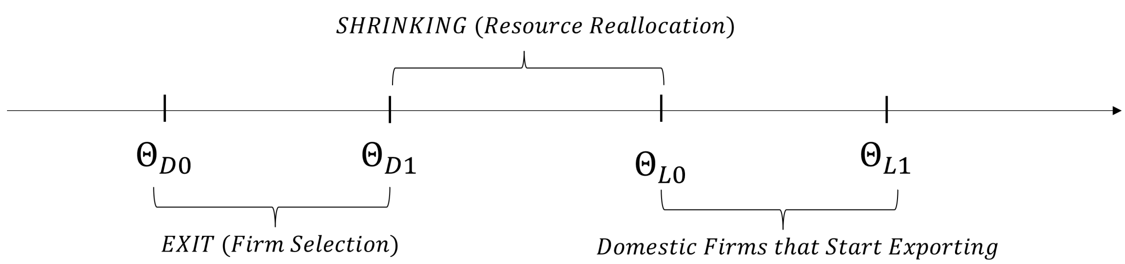
- **Exporting Firms**, that are both selling domestically and abroad, will also face a decline of their market share because of firm entry ($\downarrow A \;\; \rightarrow \downarrow \pi$). However it will now be cheaper to trade due to a **Decline in Trade Cost** ($\downarrow \tau$) thanks to liberalization.

  If, overall, the productivity threshold required to export is $\Theta_{L1} < \Theta_{L0}$, some of the (most productive) domestic firms will also start to export.

Hence, **net effects**, as long as the **reduction in trade costs** outweighs the **competition effect**, trade liberalization will have **positive effects on welfare**.

Firms that continue producing (domestically and internationally) are more productive, produce more, and are larger. Overall, the net impact on exports is positive: more is produced and at higher productivity levels.

Summarizing the effects of trade liberalization in terms of productivity thresholds to export and produce domestically:



As long as $\Delta\tau \;>\; \Delta A$:

- **Only the most productive firms continue to produce** (more productively and at lower average prices), while many domestic unproductive firms exit the market due to increased competition.
- **Each country** produces **less varieties** than before (due to firm exit, having now less firms in the market), but the **total number of varieties** available **globally increases** due to market integration (domestically produced varieties + exported ones in each market).

NB: we are keeping markups as fixed.

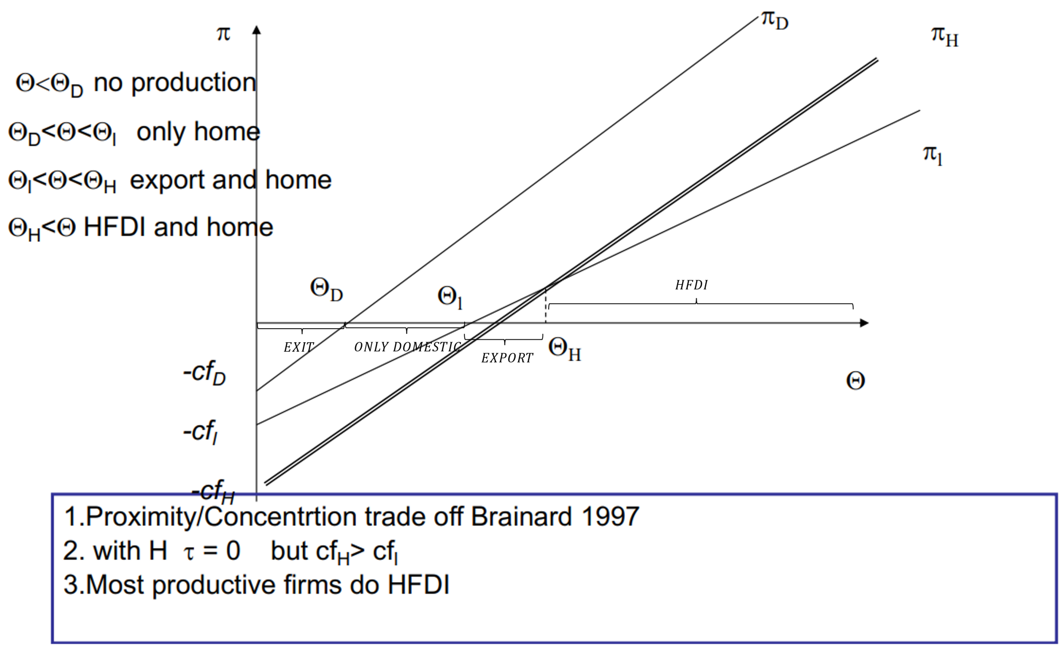
Horizontal FDI

Based on the work of Helpman, Melitz, and Yeaple (2004) and Melitz (2006), **Horizontal FDI (Foreign Direct Investment)** involves a **firm establishing abroad the same type of activity it operates at home**, instead of exporting its products.

In this model, firms that wish to serve a foreign market face two options:

- **Exporting:** paying transportation costs and facing (relatively) lower fixed costs
- **Setting up Production Abroad:** avoiding transportation costs, but incurring in high fixed costs of establishing a plant

Firms that decide to set up plants abroad become **multinationals**.



Firms face a **trade-off** where they must decide between exporting (and paying transportation cost τ) or horizontal FDI (paying high fixed costs of setting up a plant cf_H).

In the graph, π_D, π_L are the same as before, while π_H is the profit line of firms that set up a foreign plant and become multinationals.

In this model, the cost of setting up foreign subsidiaries is bigger than that of producing domestically and exporting, i.e., $cf_H > cf_L$.

π_H is parallel to π_D (same slope), assuming that the market size and share is the same domestically and in the foreign market where the plant is opened (and of course, because HFDI do no pay trade costs τ, $\tau = 0$).

Depending on **Firm Productivity**, there are four main outcomes:

- $\Theta < \Theta_D \rightarrow$ exit the domestic market
- $\Theta_D < \Theta < \Theta_L \rightarrow$ low productivity, only produce domestically
- $\Theta_L < \Theta < \Theta_H \rightarrow$ Firms with medium productivity produce domestically and also export, to enjoy more than just domestic-only profits.
- $\Theta > \Theta_H \rightarrow$ Firms with high productivity choose Horizontal FDI because the profits of establishing a plant abroad (π_H) exceed those from exporting.

Model **Predictions**:

(a) The least productive firms serve only the domestic market.

(b) Exporters are more productive than " domestic-only" firms.

(c) The most productive firms are multinationals.

Achievements And Limitations of the Model

Achievements:

We now have a theoretical framework where (a) **exporting firms** are **larger** and **more productive**, (b) **intensive/extensive margins** and **gravity model are accounted for**, (c) **trade liberalization** affects **reallocation** of production and firms **turnover** (entry/exit).

Limitations:

Assumes Constant Elasticity of Substitution (CES) demand and **Markups are held constant**, which contradicts empirical evidence (which suggests instead that higher demand levels reduces markups and affects prices). Moreover, there seems to be a correlation, empirically, between market size and firm size (that is not accounted for).

Melitz and Ottaviano (2008) Model

Keeps a model of monopolistic competition departing from the CES demand assumption. Price elasticity of demand is not exogenously fixed, it changes with the **"toughness" of the competition** in a market.

The **Effects of Trade Liberalization** in this model are:

- Increased product variety.
- Decrease in prices.
- Decline in markups.

Bigger markets do attract more firms, but the " tougher" competition affects firms markups, making it harder for low-productivity firm to survive.

Trade liberalization rises the productivity cutoff through the channel of **decreased markups** rather than due to higher costs of production (cf_L) or lower demand (smaller market share A) due to other firms' entry as in Melitz (2003) and extensions.

In terms of **Welfare Implications**, trade liberalization leads to a welfare increases due to higher productivity, more product variety, increased competition and lower markups.

References

Helpman E., M. J. Melitz, and S. R. Yeaple, 2004, Export versus FDI with heterogeneous firms, American Economic Review, vol. 94, pp. 300-316;

Helpman, Elhanan. 2006. "Trade, FDI, and the Organization of Firms." Journal of Economic Literature, 44(3):589-630.

Melitz M. J., 2003, The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, Econometrica, vol. 71(6), pp. 1695-1725.

Melitz M., and G. Ottaviano, 2008, Market Size, Trade, and Productivity, Review of Economic Studies, vol.75, pp. 295-316.

Horizontal FDI - Notes

Tutorial VIII

Federico Mutasci - Academic Year 2025/2026

Introduction

Recall: becoming a multinational is considered as an **alternative for export**; i.e., bearing (higher) **fixed costs** of **setting up a plant in a foreign country/market** vs paying **transportation costs**. However, this is not the only reason why Multinational Enterprises (MNE) arise.

When then will exactly a firm choose to become a multinational rather Than just deciding To supply a market through export?

The (more detailed and complete) model in these notes suggests that HDI is more likely if:

a) **Trade costs** are **high**

b) **Firm level scale economies** are **more important** relative to **plant-level scale economies** - i.e., cost advantages or efficiency gains due to IRS are realized at the firm level rather than being confined to individual production plants.

c) **Countries** are relatively **similar** - e.g., between developed economies)

Model Setup

The model is built in the following way:

- Two countries (markets), one industry
- Differentiated k firms each producing its own variety
- Two Types of firms (n, m) supplying two markets $i (i = 1, 2)$, thus yielding to four combinations:

  i) National Firms in $i = 1 \quad \rightarrow \quad n_1$

  ii) National Firms in $i = 2 \quad \rightarrow \quad n_2$

  iii) Multinationals in $i = 1 \quad \rightarrow \quad m_1$

  iv) Multinationals in $i = 2 \quad \rightarrow \quad m_2$

  Where firms (i) and (ii) are the exporters, while (iii) and (iv) represent the horizontal foreign direct investments, whit integrated markets

The decision of **becoming multinationals** is explained with a **two-stage game**:

I) Firms choose the modality of supply (n, m) anticipating future profits (π)

II) Given the chosen mode of supply; firms decide the quantity of produced output and set the price (thus determining actual profits as well)

The game is solved by **backward induction**, so we start from stage II: firms' production decision and profit maximization

Stage II - Profit Maximization

Consider that:

- Each firm k producing in some country will pay **unit cost of production** c_i and set a **price** p_i, having a **market share** $s_i = \frac{p_i x_i}{E_i}$, E_i being the market size
- Exporting firms (nationals n_i) will face **trade costs** τ; with $\tau > 1$
- Firms face **fixed costs**:
  - $c_i H$, cost of **running headquarters**
  - $c_i F$, cost of **setting up a plant**

  Therefore, for Multinational Firms the fixed costs will be $c_i(H+F) + c_j F$ with $i \neq j$

Then **operating profits π for firm k in country i**, i.e., the general profit function common to any firm type, spurious of different FC, will be:

$$\pi_i^k = \left(p_i^k - c_i^k\right) x_i^k \tag{37}$$

Recall the profit function in Equation 1 is obtained from:

$$\pi = TR - TC$$
$$\pi = pq - cq = (p-c)q$$

Each firm will set a price and produce a quantity that maximizes profits, given the demand for its own variety. As you might remember, profit maximization requires marginal revenue to equal marginal cost:

$$MR = MC$$

The condition $MR = MC$ is obtained solving the FOC of the profit maximization problem. Setting the derivative of a general profit function with respect to q as equal to zero:

$$
\begin{aligned}
\frac{\partial \pi}{\partial q} &= 0 \\
\frac{\partial \pi}{\partial q} &= \frac{\partial TR}{\partial q} - \frac{\partial TC}{\partial q} = 0 \\
\frac{\partial TR}{\partial q} &= \frac{\partial TC}{\partial q} \\
\frac{\Delta TR}{\Delta q} &= \frac{\Delta TC}{\Delta q} \\
MR &= MC
\end{aligned}
$$

In this model, **Marginal Cost** is given by:

$$MC = c_i^k \tag{38}$$

While **Marginal Revenue** corresponds to:

$$MR = p_i^k \left(1 - \frac{1}{\varepsilon_i^k}\right) \tag{39}$$

Where ε_i^k is firms' **elasticity of demand**.

You might not be familiar with marginal revenue expressed as in Equation 2. However, we can recover that $\frac{dTR}{dq} = p\left(1 - \frac{1}{\varepsilon}\right)$ simply by totally differentiating the total revenue function:

$$
\begin{aligned}
TR_{(p,\ q)} &= p \cdot q \\
dTR_{(p,\ q)} &= \frac{\partial TR}{\partial p} dp + \frac{\partial TR}{\partial q} dq \\
dTR_{(p,\ q)} &= q \cdot dp + p \cdot dq
\end{aligned}
$$

$$
\begin{aligned}
&\text{Dividing by } dq \\
&\frac{dTR}{dq} = \frac{dp}{dq} q + p
\end{aligned}
$$

$$
\begin{aligned}
&\text{Factoring out } p \\
&\frac{dTR}{dq} = p\left(\frac{dp}{dq}\frac{q}{p} + 1\right)
\end{aligned}
$$

The term $\frac{dp}{dq}\frac{q}{p} = \frac{1}{-\varepsilon}$, since the elasticity of demand $\varepsilon = \frac{dq}{dp}\frac{p}{q} = \frac{\text{\%change in q}}{\text{\%change in p}}$ and it's always negatively sloped. Hence:

$$\frac{dTR}{dq} = MR = p\left(1 - \frac{1}{\varepsilon}\right)$$

We can now use the profit maximization condition $MR = MC$ to equate Equations 2 and 3:

$$p_i^k\left(1 - \frac{1}{\varepsilon_i^k}\right) = c_i^k \tag{40}$$

Substitute Equation 4 (The result of the FOC) in the profit function (Equation 1):

$$\pi_i^k = p_i^k x_i^k - \underbrace{\left[p_i^k\left(1 - \frac{1}{\varepsilon_i^k}\right)\right]}_{= c_i^k \textbf{ due to } MC=MR} x_i^k$$

$$\pi_i^k = \cancel{p_i^k x_i^k} - \cancel{p_i^k x_i^k} + \frac{p_i^k x_i^k}{\varepsilon_i^k}$$

$$\pi_i^k = \frac{p_i^k x_i^k}{\varepsilon_i^k}$$

Consider now that the previously mentioned **market share** is $s_i^k = \frac{p_i^k x_i^k}{E_i}$:

$$s_i^k = \frac{p_i^k x_i^k}{E_i} \quad \rightarrow \quad s_i^k E_i = p_i^k x_i^k$$

Substituting into **maximized operating profits**:

$$\pi_i^k = \frac{s_i^k E_i^k}{\varepsilon_i^k} \tag{41}$$

And hence as $\uparrow s$ and $\downarrow \varepsilon$, $\uparrow \pi$.

Let's now differentiate **Costs** and **Fixed Costs** given Firms **type** and **location**.

- **Production (Unit) Costs** only vary according to where production takes place, therefore:
  - MC of production in county $i = 1 \quad \rightarrow \quad c_1$
  - MC of production in country $i = 2 \quad \rightarrow \quad c_2$

- When national firms **export**, they pay trading costs τ. **Unit cost of production (MC)** for **exports** will Then be $c_1\tau$ and $c_2\tau$

- All firms facing the **same cost** in a market will have the **same price** and hence the **same market share**. We can then drop the k superscript and define market shares as s_i for all firms (nationals and multinationals) in in a country $i = 1, 2$.

- the **unit cost of traded goods is affected by transport costs**, this well also **affect market shares**, but only those of national firms that export their output.

  We will represent this adding a parameter $\varphi \quad (\varphi \leq 1)$ to national firms market shares, st firm i will have marginal cost $c_i\tau$ for supplying country j, and their market share in j will be $s_j\varphi_i$, i.e.,:
  - $c_1\tau \quad \rightarrow \quad s_2\varphi_1$ if the firm produces in 1 to export in 2
  - $c_2\tau \quad \rightarrow \quad s_1\varphi_2$ if the firm produces in 2 to export in 1

  the parameter φ represents the **freeness of trade** and it's a function of trade costs $\varphi = f(\tau)$. the lower the the trade costs, the higher is φ (resulting in higher market shares).

  If follows that **multinationals** have **bigger market shares** (sine they don't have to pay τ): $s_i > s_i\varphi_j$

- **Fixed Costs**, as stated before, vary for national and multinational firms:
  - c_iH = cost of running **headquarters**
  - c_iF = cost of **setting up plants**

  Exporting national firms will have to pay **fixed costs for having an headquarter and plants** in country i : $\quad c_iH + c_iF = c_i(H + F)$

  Multinational firms will have headquarters + plants in country i, but additionally, They will alsp **open plants in the other country** j as well: $c_i(H + F) + c_jF$

We can now summarize all different Types of costs sustained by each type of firm in the model.

Costs and Masker Shares by firm Type

Consider the following table marginal costs, fixed costs and market shares for each firm type:

	Marginal cost of supplying country 1	Marginal cost of supplying country 2	Market share in country 1	Market share in country 2	Fixed costs
National in 1, n_1	c_1	$c_1\tau$	s_1	$s_2\varphi_1$	$c_1(H+F)$
National in 2, n_2	$c_2\tau$	c_2	$s_1\varphi_2$	s_2	$c_2(H+F)$
Multi-national in 1, m_1	c_1	c_2	s_1	s_2	$c_1(H+F)+c_2F$
Multi-national in 2, m_2	c_1	c_2	s_1	s_2	$c_2(H+F)+c_1F$

We can use these information to built profit functions that are specific wrt firms Type

Profit Functions

Profit for multinational firms:

$$\Pi_1^M = \frac{s_1 E_1}{\varepsilon(s_1)} + \frac{s_2 E_2}{\varepsilon(s_2)} - c_1(H+F) - c_2 F \quad \rightarrow \quad \pi \text{ for multinational firms in 1}$$
$$\Pi_2^M = \frac{s_1 E_1}{\varepsilon(s_1)} + \frac{s_2 E_2}{\varepsilon(s_2)} - c_2(H+F) - c_1 F \quad \rightarrow \quad \pi \text{ for multinational firms in 2}$$

Profit for national firms:

$$\Pi_1^N = \frac{s_1 E_1}{\varepsilon(s_1)} + \frac{s_2\varphi_1 E_2}{\varepsilon(s_2\varphi_1)} - c_1(H+F) \quad \rightarrow \quad \pi \text{ for national firms in 1}$$
$$\Pi_2^N = \frac{s_1\varphi_2 E_1}{\varepsilon(s_1\varphi_2)} + \frac{s_2 E_2}{\varepsilon(s_2)} - c_2(H+F) \quad \rightarrow \quad \pi \text{ for national firms in 2}$$

Market Shares

Since profits depend on market shares, we need to define market shares as well to complete the characterization of the game and of how firms choose the modality of supply:

$$\text{Market 1:} \quad 1 = (n_1 + m_1 + m_2)\, s_1 + n_2\varphi_2 s_1$$
$$\text{Market 2:} \quad 1 = (n_2 + m_1 + m_2)\, s_2 + n_1\varphi_1 s_2$$

Market share of firms in each country sum up to 1 (100%). the term $(n_i + m_1 + m_2)$ is the total number of firms (national and HFDI multinationals) supplying the market by producing locally, multiplied by their equal shares s_i + the number of firms supplying the market through trade n_j, multiplied by the market share of exporters (being a function of τ).

Stage I - Mode of Supply (n vs m)

What mode of entry (supply) would firms choose? Suppose there are n_1 and n_2 national firms in each country and m multinationals. Would it be profitable for a firm in country 1 to become multinational?

NB: we are considering a **Greenfield Investment** where one already existing firm (of n_1) transforms its current modality of supply by setting up a new plant in the other market instead. the number of n_2 will be the same. Overall, there will be -1 national and +1 multinational firms, the **total number of firms** (local suppliers, exporters and MNE) $n_1 + n_2 + m_1 + m_2$ is **unchanged**.

We compare (i) **profits of a national firm based in 1** vs (ii) **profits of a firm becoming multinational with headquarter in 1**:

$$\Pi_1^N (n_1, n_2, m) \quad \text{(i)}$$
$$\Pi_1^M (n_1 - 1, n_2, m + 1) \quad \text{(ii)}$$

m being the sum of m_1 and m_2

Going **from** n_1 **to** m will **affect the market share** of the firm **in country 2**:

- **Initial Situation**
$$s_2 = \frac{1}{n_2 + m + n_1\varphi_1}$$
- **Becoming Multinational**
$$s_2' = \frac{1}{n_2 + (m+1) + (n_1 - 1)\varphi_1}$$

Notice that (given $\varphi_1 \leqslant 1$) $s_2' < s_2$; i.e., **as firms become multinationals available market share** s **gets smaller** (m has higher s, lass share available as $n \to m$)

Change in Profits

If one $n_1 \to m$:
$$\Delta\Pi^{M,N} = \Pi_1^M (n_1 - 1, n_2, m + 1) - \Pi_1^N (n_1, n_2, m)$$

Recall: s are assumed To be equal for firms of the same type. Further simplification: assume **constant markups** $\varepsilon(s) = \sigma$

$$\Delta\Pi^{M,N} = \underbrace{\frac{s_1E_1}{\sigma} + \frac{s_2'E_2}{\sigma} - c_1(H+F) - c_2F}_{\Pi_1^M(n_1-1,n_2,m+1)} \underbrace{-\frac{s_1E_1}{\sigma} - \frac{\varphi_1 s_2 E_2}{\sigma} + c_1(H+F)}_{-\Pi_1^N(n_1,n_2,m)}$$
$$\Delta\Pi^{M,N} = \cancel{\frac{s_1E_1}{\sigma}} + \frac{s_2'E_2}{\sigma} - \cancel{c_1(H+F)} - c_2F \quad \cancel{-\frac{s_1E_1}{\sigma}} - \frac{\varphi_1 s_2 E_2}{\sigma} + \cancel{c_1(H+F)}$$
$$\Delta\Pi^{M,N} = \frac{s_2'E_2}{\sigma} - \frac{\varphi_1 s_2 E_2}{\sigma} - c_2F \quad = \quad \frac{E_2}{\sigma}\left(s_2' - \varphi_1 s_2\right) - c_2F$$

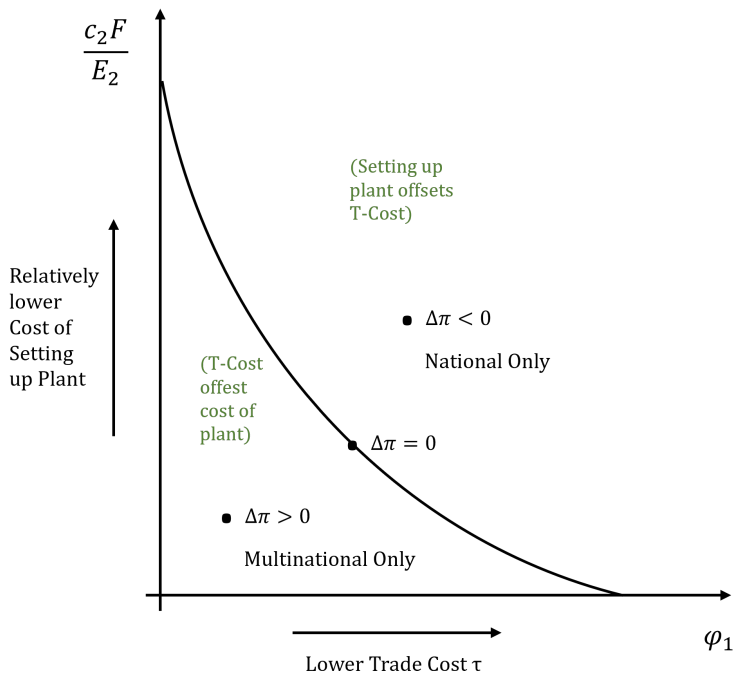
Substituting s_2 and s_2':

$$\Delta\Pi^{M,N} = \frac{E_2}{\sigma}\left[\frac{1}{n_2 + (m+1) + (n_1 - 1)\,\varphi_1} - \frac{\varphi_1}{n_2 + m + n_1\varphi_1}\right] - c_2 F$$

Firms will become multinationals if $\Delta\Pi^{M,N} > 0$

Graphical Representation

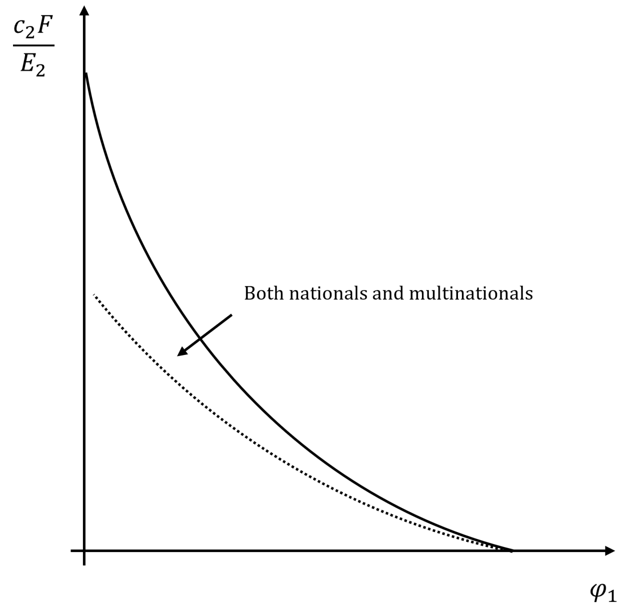
There is a **trade-off** between bearing **plant fixed cost** (given market size) and paying **trade cost** (or transportation cost) for firms:



This trade-off is what's behind firms decision to produce produce nationally and export vs becoming a multinational enterprise (affecting the difference in profits $\Delta\Pi^{M,N}$). **As $\downarrow \varphi$ ($\uparrow \tau$) the incentive to become multinational grows.**

However, remember that in a market as $\uparrow m \;\; \rightarrow \; \downarrow s$:

- As $\downarrow \varphi_1, \uparrow E_2$, and/or $\downarrow c_2 F$, every firm would like to become multinational. However, **as the number of multinationals increases, the market share of each firms shrinks** (since multinationals have bigger market shares).
- Hence, as $\uparrow m$ and $\downarrow n$, the curve in the graph rotates downward.



Thus, as the number of multinational firm rises, we need a bigger decrease in fixed costs or big increases in τ in order to provide enough incentives for national firms to become multinationals.

Equilibrium with Free Entry

The number of firms now is not held constant. Firms entry depends on profit opportunities - the **number of firms is endogenously determined**.

- There are many firms of each type and and each firm decides wether to entry or not given the decision of the other firms
- In **Equilibrium**, non-negative profits for all firms entered and non-positive profits for all firms that have not entered - i.e. there will be a positive n° of firms if they are at least breaking even:

$$\Pi_1^N(n_1, n_2, m) \leq 0, \quad n_1 \geq 0 \qquad \Pi_2^N(n_1, n_2, m) \leq 0, \quad n_2 \geq 0$$
$$\Pi_1^M(n_1, n_2, m) \leq 0, \quad m_1 \geq 0 \qquad \Pi_2^M(n_1, n_2, m) \leq 0, \quad m_2 \geq 0$$

Hence, n_i will be > 0 if $\Pi_i^N = 0$, while n_i will be 0 if $\Pi_i^N < 0$. the same is true for m. We observe a positive number of n, m firms in a market with free entry if Π is non-negative.

Further Assumption: Suppose the Two countries/markets are identical:

$$c_1 = c_2 = c$$

$$E_1 = E_2 = E$$

Same cost and market size for simplicity (and it follows $s_1 = s_2 = s$ since equal s_i for every i).

Profit Function (Free Entry)

Recall: if there is a positive number of active firms, then profits must be at least $\Pi = 0$. If $\Pi < 0$, the number of active firms would be zero:

$$\Pi^N = \frac{sE}{\sigma} + \frac{\varphi sE}{\sigma} - c(H+F) \leq 0, \quad n \geq 0$$

$$\Pi^M = \frac{2sE}{\sigma} - c(H+2F) \leq 0, \quad m \geq 0$$

We assumed the two countries are identical. Multinationals operate in two countries; Π_i^N was $\frac{s_i E_i}{\sigma} + \frac{s_j E_j}{\sigma} - c_i(H+F) - c_j F$, now it has become $2 \cdot \frac{sE}{\sigma}$, and you can group cF costs.

Solve the first equation for s, consider $\Pi^N = 0$

$$\begin{aligned}
&\frac{sE}{\sigma} + \frac{\varphi sE}{\sigma} - c(H+F) = 0 \\
&sE + \varphi sE - \sigma c(H+F) = 0 \\
&s(E + \varphi E) - \sigma c(H+F) = 0 \\
&s(E + \varphi E) = \sigma c(1+F) \quad \rightarrow \quad s = \frac{\sigma c(H+F)}{E(H+\varphi)}
\end{aligned}$$

Substitute s in the multinational profit equation with $\Pi^M = 0$

$$\begin{aligned}
&\frac{2sE}{\sigma} - c(H+2F) = 0 \\
&\frac{2E}{\sigma}\left[\frac{\sigma c(H+F)}{E(1+\varphi)}\right] - c(H+2F) = 0 \\
&\frac{2\cancel{E}}{\cancel{\sigma}}\left[\frac{\cancel{\sigma} c(H+F)}{\cancel{E}(1+\varphi)}\right] - c(H+2F) = 0 \\
&\frac{1}{c} \cdot \frac{2c(H+F)}{(1+\varphi)} - c(H+2F) \cdot \frac{1}{c} = 0 \\
&\cancel{\frac{1}{c}} \cdot \frac{2\cancel{c}(H+F)}{(1+\varphi)} - \cancel{c}(H+2F) \cdot \cancel{\frac{1}{c}} = 0 \\
&\frac{2(H+F)}{(1+\varphi)} - H + 2F = 0 \\
&2H + 2F - (H+2F)(1+\varphi) = 0 \\
&2H + 2F - (H + H\varphi + 2F + 2F\varphi) = 0 \\
&2H + \cancel{2F} - H - H\varphi - \cancel{2F} - 2F\varphi = 0
\end{aligned}$$

$$
\begin{aligned}
&H - H\varphi - 2F\varphi = 0 \\
&H = H\varphi + 2F\varphi \\
&H = \varphi(H + 2F) \\
&\varphi = \frac{H}{H + 2F}
\end{aligned}
$$

Subtract -1 from both sides (it won't affect the outcome, it's a tool to rearrange the equation):

$$
\begin{aligned}
&1 - \varphi = 1 - \frac{H}{H + 2F} \\
&1 - \varphi = \frac{(H + 2F) - H}{H + 2F} \\
&1 - \varphi = \frac{2F}{H + 2F}
\end{aligned}
$$

$$\frac{1}{2}(1 - \varphi) = \frac{F}{H + 2F}$$

This condition tells us when the two types of firms n, m will coexist.

Coexistence of NE and MNE

With free entry of firms and two identical countries/markets; if $\frac{1}{2}(1 - \varphi) = \frac{F}{H+2F}$ both multinationals and nationals coexist. $H + 2F$ represent nationals and multinationals FC (headquarters + plant), and $\varphi = f(\tau)$.

There is a **trade-off** between **trade cost** τ and **firms and plant specific fixed cost** H, F.

More specifically, in the equation:

$$\frac{1}{2}(1 - \varphi) = \frac{F}{H + 2F}$$

$\frac{1}{2}(1 - \varphi)$ is the difference in profits between multinationals and national firms (remember that the total market share is 1, only n_i pay τ cost on which depends the value of φ)

It follows that, if the RHS and the LHS of the equation are not equal:

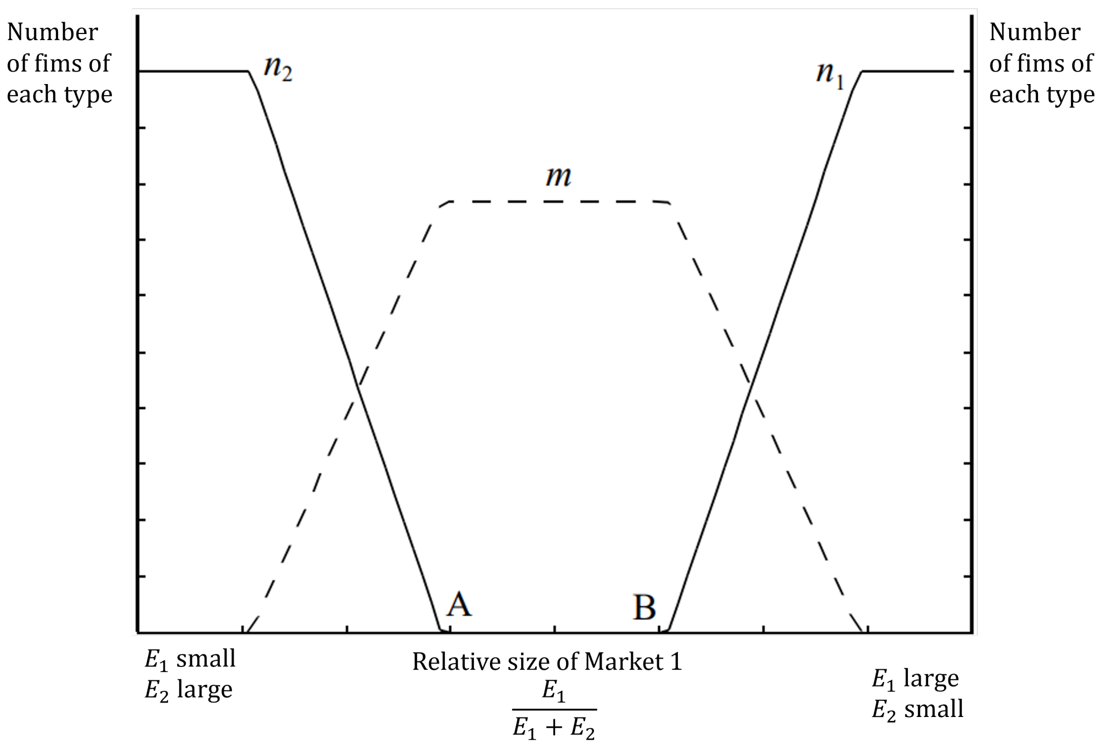
- If $(1 - \varphi)\frac{1}{2} > \frac{F}{H+2F} \quad \Rightarrow \quad \tau$ is high (φ is low) or firm-level fixed cost is high compared to plant cost $\left(\frac{H}{F}\right) \quad \Rightarrow$ ONLY MULTINATIONAL FIRMS.
- If $(1 - \varphi)\frac{1}{2} < \frac{F}{H+2F} \quad \Rightarrow \quad \tau$ is low (φ s high) or firm-level fixes cost is low compared to plant level cost $\left(\frac{H}{F}\right) \quad \Rightarrow$ ONLY NATIONAL FIRMS (exporters).

If the gains in profits (for MNE) would be lower than the fixed cost that multinationals would face we only end up with exporters.

We know how trade costs affects the presence of multinationals, let's focus on the ratio between plant level and firm level fixed costs. **HFDI is more likely if H is bigger compared to F.** the implication of This is that **multinationals will be prevalent in industries where firm level economies of scale are important relative to plant level economies of scale.**

Different Market Size

What happens if the two countries are not equal in terms of market size E?



- On the LHS → E_1 is too small, not worthwhile setting up a plant there, we will have a big number of national firms of country 2 (n_2) that serve market 1 through exports
- On the RHS → again one market is too large and the other is too small, no reason for MNE, to exist. This time n_1 will supply market 2 through exports
- **MNEs are prevalent in countries of similar size** → Markusen and Variables "convergence hypothesis" (mast of this chapter is based on their work)

If countries (markets) have equal size, it's more likely for firms in country i to set up a plant in country j.

Model's prediction on which type od firms are more likely to be tome MNE: firms with high intangible assets in developed markets that are not too liberalized (high trade cost).

Conclusion

Horizontal Foreign Direct Investment arise more lively if:

- τ is high
- Firm level economies of scale is important relative to plant level economies of scale
- Countries are relatively similar

Effect of HDI on Host Economy

The overall effect of an increase in the number of Multinational Enterprises in a given market i is the result of a combination of:

- Their effect on consumer's wealth (on their demand levels, on product availability etc.) passing through **Consumers' Price Index** and **number/level of output of local firms** $\Rightarrow$ PRODUCT MARKET EFFECT
- Their effect on **factors of production** (factor demand, number of employed workers) $\Rightarrow$ LABOR (FACTOR) MARKET EFFECT

Product Market Effect

I - Consumer Price Index

Consider **consumers' utility** depending on all the varieties available in a market i, with CES utility functions:

$$X_i = \left[\sum_k \left(x_i^k\right)^{\frac{(\sigma-1)}{\sigma}} \right]^{\frac{\sigma}{(\sigma-1)}}$$

With σ being the elasticity of substitution. the level of output of each commodity is a function of consumer prices. Hence, we can represent the **consumer price index** as:

$$G_i = \left[\sum_k \left(p_i^k\right)^{(1-\sigma)} \right]^{\frac{1}{(1-\sigma)}}$$

If an additional MNE appear in market i, the consumer price index will **decrease** (leading to an **increase in consumers welfare**) if it's due to a **greenfield investment**. It will **increase** (leading to **decrease in consumers welfare**) if the MNE arises thanks to **Merging and Acquisitions (M&A)**.

II - Number of Firms and Output

The effect on overall output levels and number of participating firms in the market depends on both the characteristics of the firms and the parameter φ.

For instance if **firms are heterogeneous**, the entry of a MNE will force **least efficient firms** to **exit**. At this stage, if the consumer price index stayed the same (see previous section), firms will produce more efficiently at an identical price index (increasing consumers welfare).

One concern related to the entry of a multinational company into the market is that it may cause a **crowding-out effect** against domestic companies. In this model, the total change in the number of national firms in market i given the change in the number of multinationals is represented as follows (see NBV Chapter 3 for the derivation):

$$\frac{dn_i}{dm} = (\varphi - 1)\frac{p_m x_m}{p_i x_i} \;\leq\; 0 \tag{42}$$

The derivative is set $\leq 0 \quad \rightarrow$ if an additional multinational firm enters the market, the number of national firms can either decrease (negative change) or not change (zero change).

The occurrence of crowding-out mechanisms depends on φ. For instance, consider the two extreme cases:

- if $\varphi = 1$, switching from supplying the market through export to HFDI causes **no crowding-out** (the total change is zero).
- if $\varphi = 0$, on the other hand, the activity of an additional MNE in the market leads to a **one-to-one crowding-out**

Intuitively, φ is a function of trade costs, representing **freeness of trade**. If the two markets are fully integrated, the changing of a national firm into a MNE will have effectively no net consequences. If the markets are not communicating at all, a MNE doing Horizontal FDI can leverage its production and selling activities in both markets to push other national firms out of their local market.

Labor Market Effect

Are Foreign Direct Investments a source of employment? Consider **unit cost of production** for both national and multinational firms in market i:

$$c_i = w^{\lambda_i} I^{1-\lambda_i}$$

$$c_m = w^{\lambda_m} I^{\lambda_m}$$

Where w are wages, I is the price of the other inputs, λ_i, λ_m are the shares of labor for national and multinational firms respectively in i.

Consider the parameter L as total employment, the **value of labor demand** for local firms and multinational firms in i is given by:

$$wL = \lambda_i n_i c_i (x_i + H + F) + \lambda_m m c_m (x_m + F)$$

The change in labor demand with the entry of multinational firms is hence given by:

$$w\, dL = \lambda_i c_i (x_i + H + F)\, dn_i + \lambda_m c_m (x_m + F)\, dm$$

Since the change in the number of local firms is determined by crowding out effects, we can use Equation 6 to retreive the following condition:

$$\frac{w}{p_m x_m} \frac{dL}{dm} = \lambda_i (\varphi - 1) \left(\frac{c_i (x_i + H + F)}{p_i x_i} \right) + \lambda_m \left(\frac{c_m (x_m + F)}{p_m x_m} \right). \tag{43}$$

The sign of Equation 7 (and hence increase or decrease in employment due to entry of MNEs) depends on both the share of labor in multinational firms λ_m and the parameter φ. the equation is more likely to be positive if λ_m (higher share of employed labor units by multinationals) and/or φ are high (highly integrated markets with low trade costs).

References

Barba Navaretti G., and T. Venables, 2005, Multinational Firms in the World Economy, Princeton University Press (BNV) - Chapter 3.

Vertical FDI - Notes

Tutorial IX

December 4, 2024

Introduction

- **Horizontal FDI** = Multinational Companies carrying out the **same** type of production activities in countries $i, j, ...$
- **Vertical FDI** = Multinational Companies carrying out **different** types of activities in countries $i, j, ...$, within the same firm's value chain - e.g., a MNE that produces intermediate inputs/goods in country i and assembles the finished good in country j.

Vertical FDI arises in the presence of **factor price differences** across countries. A producer might find it optimal to **fragment the production process** and undertake different parts of the production process in different countries.

Doing so, producers may benefit from lower production costs. However, the choice of the fragmentation of production might also be subject to different trade-offs.

Model Setup

To study the determinants of firms choices to become MNEs with VDI, we make use of the following model:

- **Production** is divided into **two stages**: (1) components, c, (2) assembly, a.
- Two countries i 1 and 2, with two factors, K and L, used in both stages of production - having factor prices w_i (wages for L) and r_i (rental rate of K).
- $c(w_i, r_i)$ = **unit cost of components**, while $a(w_i, r_i)$ = **unit cost of assembly**
- **CRS** and **Perfect competition**: In equilibrium **zero profit condition**: $P = MC$, implying that $P = c(w_i, r_i) + a(w_i, r_i)$ (isocost line)
- Activity a (assembly) is L-intensive
- τ^c, τ^a are **trade costs** of **shipping final products and components**
- Country 2 is L-abundant
- Country 1 is more developed, thus having higher wages ($w_1 > w_2$)

We also make some further key assumptions:

- **Country 1** has a **comparative advantage** in the production as a whole (**Integrated Production**) - meaning that if the two stages are done in the same location, then it's cheaper to produce in 1.
- If **fragmentation** occurs, since **assembly** (a) is L-intensive, it's carried out **in country 2** (L-abundant). Components (c) stay in country 1.

With fragmentation, a firm will choose to split the production process between locations i, j by looking at its **cost-minimizing locations** (i.e., according to cost-minimizing functions).

Cost Functions

Cost of a unit of final output **delivered to country** k (if **components are produced in** i and **final assembly takes place in** j) - NB: It is possible that all three stages take place in the same country.

$$B_{ijk} = \left[\underbrace{c(w_i, r_i) \;\; \tau^c_{ij}}_{\text{Unit cost of components}} + \underbrace{a(w_j, r_j)}_{\text{Unit cost of assembly}} \right] \cdot \tau^a_{jk}$$

i is where components are produced, j where assembly takes place, k where the final good is shipped. τ^c_{ij} is the trade cost of shipping components from i to j, while τ^a_{jk} is the trade cost of shipping final product from the site of assembly (j) to selling market (k). w is wages, r the rental rate of capital, and $i, j, k = 1, 2$.

Hence:

- If production of components and assembly take place in the same country (i), and the final product is sold there as well, then $\tau^c_{ij} = \tau^a_{jk} = 1$.
- If not (fragmentation), then $\tau^c_{ij}, \tau^a_{jk} > 1$ for $i \neq j$ (positive trade cost).

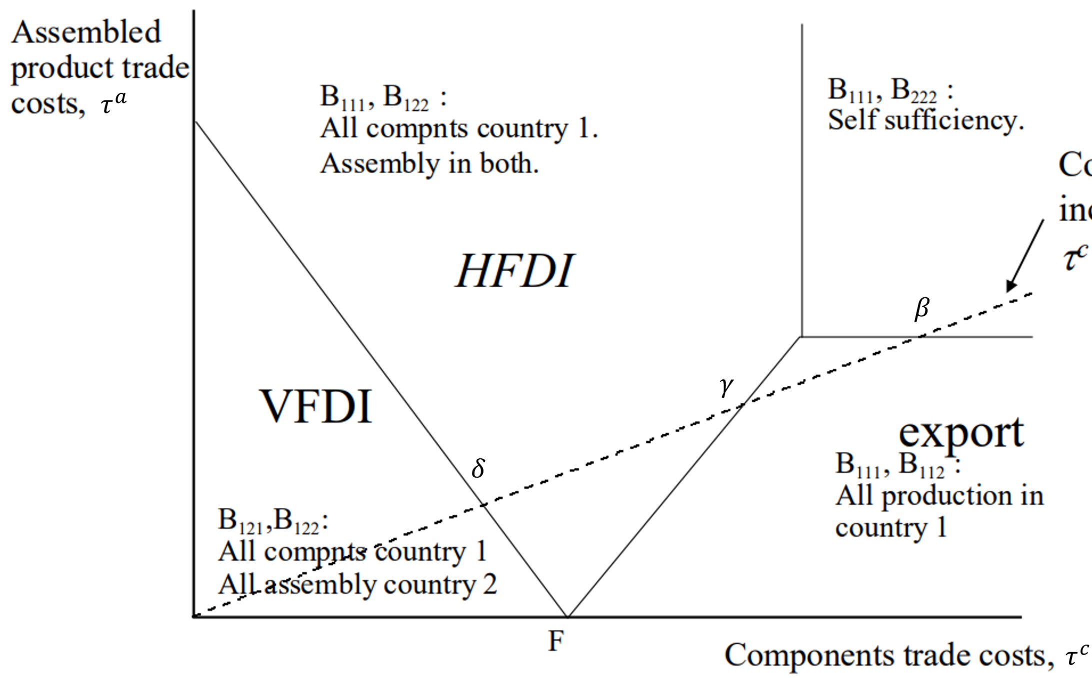
Example: Cost of 1 unit of output when components are produced in 1, assembled in 2, and the final good is shipped back to 1:

$$B_{121} = [c(w_1, r_1)\tau^c_{12} + a(w_2, r_2)] \, \tau^a_{21}$$

Trade Costs and Production Regimes

How do τ^c and τ^a impact fragmentation of production?

Based on the **level of shipping cost of assembled good** τ^a (Y axis) and the **level of trade cost of the components** τ^c (X axis) we can have MNEs organizing as **Vertical FDI** (bottom left), MNEs organizing as **Horizontal FDI** (center of the diagram), **national firms that export** the final good (bottom right) or **national firms that do not export** (all activities are carried out in one country, including selling the final good, top right of the diagram) corresponding to autarky.



Looking at the graph:

- If $\tau^{c,a}$ are too high: **Autarky and self-sufficiency**. No trades nor FDI, components are produced in a country, assembled in the same country, and sold domestically (either country 1 - with B_{111} - or country 2 - with B_{222})
- If τ^a is in the middle but τ^c is high: **Export**, firms in 1 produce everything (since they have a CA in integrated production) and sell both domestically and in country 2 (B_{111}, B_{112}):
  - The line between autarky and export is the indifference line where $B_{112} = B_{222}$, i.e., country 2 can be supplied with the final good at the same cost by either importing from 1 or producing domestically in 2.
- If $\tau^{c,a}$ are too low: **full liberalization** and **Vertical FDI** (B_{121}, B_{222}):
  - All components are produced in country 1, all assembly takes place in country 2, the final product is sold by MNEs in both markets 1 and 2.
  - The change in production mode from export to Vertical FDI can be seen in point F
  - F is the point where **fragmentation** happens, $B_{121} \to B_{111}$ (indeed, F is the indifference point $B_{121} = B_{111}$) where country 1 is supplied by products assembled in 2 rather than in 1. The same for country 2 (now supplied by products assembled in 2).
- If τ^a is high and τ^c is in the middle: **Horizontal DI**; components are all produced in country 1, but the activity of assembly takes place in plants based in both countries. Products are then sold in both countries (B_{111}, B_{122}).

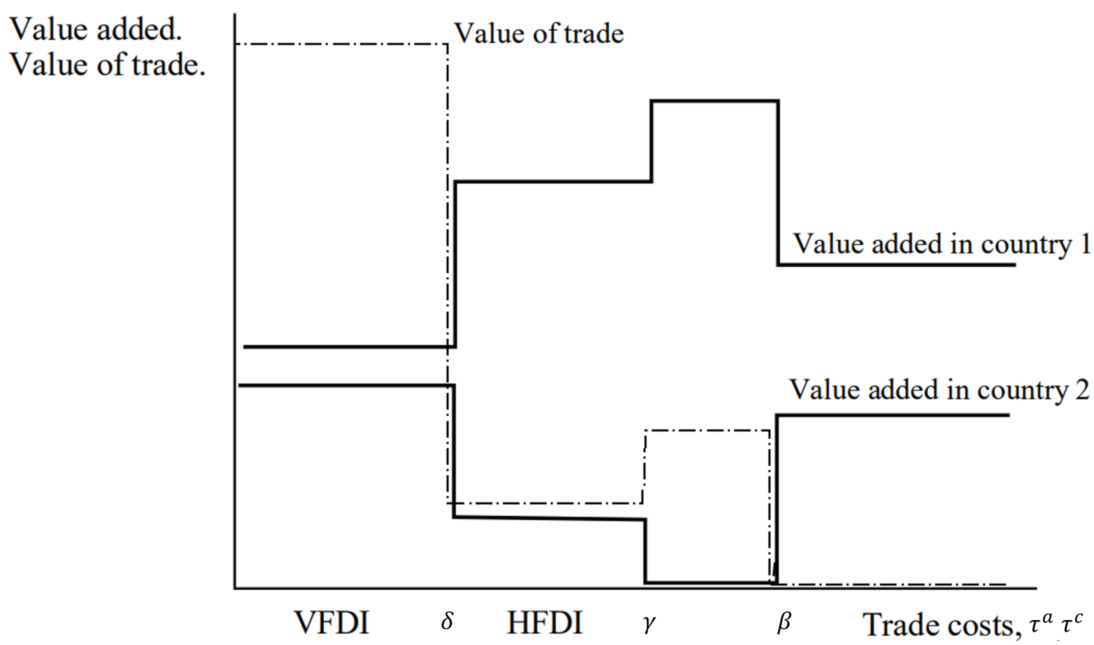
The dotted line in the graph represents the combined increase in τ^c, τ^a.

Hence:

- **When τ^c, τ^a are low**: activities are taken where it's cheaper to carry them out (i.e. components in 1 where it's cheaper to produce them, assembly in 2 where it's cheaper to assemble the final good) $\rightarrow$ Vertical FDI;
- when **both $\tau^{c,a}$ are high** $\rightarrow$ Autarky
- **If $\tau^c \tau^a$ are in the middle**: assembly moves to 1 (**Horizontal FDI**), where it's cheaper to do the whole production
  - **As τ^c rises** $\rightarrow$ Export
  - **As τ^a decreases** $\rightarrow$ Vertical FDI

Trade Costs and Trade Flows

How do τ^a and τ^c impact trade flows? Recall, assembly is L-intnsive and $w_1 > w_2$.



As the graph shows:

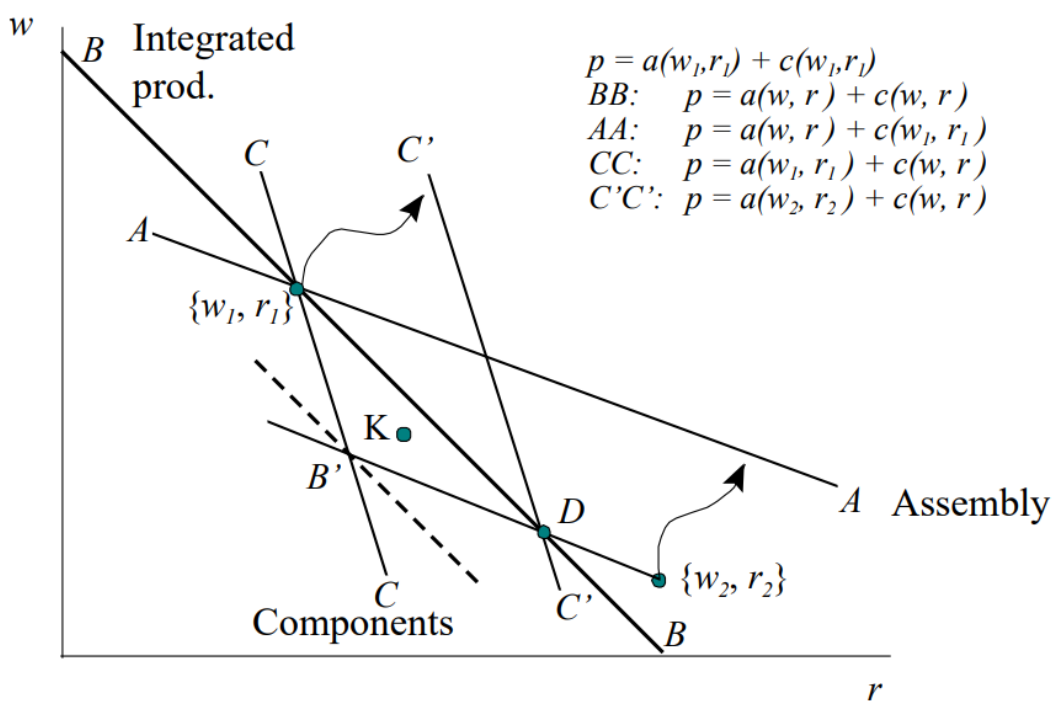
- Again, for **high τ^c, τ^a both countries are almost self-sufficient**.
  - Countries have different levels of Value Added in their production: $VA_1 > VA_2$ as Country 1 is more developed
  - The value of trade is small, or there is no trade at all.

- As τ^c, τ^a **decrease**, there are **changes in the modality of production**:
  - First, a decrease in τ^c, τ^a determines a regime of export from the country with the whole production (Country 1) to the other (Country 2).
  - A further decrease moves the production towards Horizontal FDI, since assembly (from market 1) moves also to country 2 (resulting in a small decrease in trade flows).
- When τ^c, τ^a are **very low** we have **full fragmentation**, i.e. Vertical FDI $\rightarrow$ MNEs locate each step of the production where it's more convenient and trade the products from one country to the other (at low costs). Trade flows are very high, since they support the fragmentation.

Notice that in this model trade flows and value added are not monotonically related.

Fragmentation and Factor Prices

How do differences in factor prices translate in gains and losses from fragmentation?



In the diagram:

- **BB**: Isocost line (all combinations of factors for the same level of cost) for **integrated production** (more distance from O means higher costs):

$$P = a(w, r) + c(w, r)$$

- **AA**: Isocost for **assembly**, holding component cost fixed, while **CC**: Isocost for **components**, keeping assembly cost fixed

With the slopes of the isocost lines reflecting relative factor intensities

We are now assuming $\tau^c, \tau^a = 1$ for simplicity, notice that:

- If we are on the **intersection of CC and AA**, the **integrated production** (corresponding to the point on the BB line) takes place **all in Country 1** (due to Comparative Advantage of country 1) at factor prices w_1, r_1:

$$\text{Initial Situation:} \quad P = C(w_1, r_1) + a(w_1, r_1) \;\; \text{at} \;\; CC = AA$$

- Country 1 has a CA in components. **Fragmentation** will only occur **if some country has a lower cost of assembly** $\rightarrow$ below AA, at a parallel line that we define as A'A' (NB. we are still on the BB isocost line, so the unit cost of the overall production is still P):

$$P = C(w_1, r_1) + a(w_2, r_2) \;\; \text{at} \;\; CC = A'A'$$

Therefore, once fragmentation occurs, assembly moves to Country 2. There are now three possible outcomes:

i) If **factor prices increase in Country 2** $\rightarrow$ shift from A'A' towards AA = **Country 2 gains** from fragmentation (higher w_2 and the level of factors used in equilibrium gets closer to point 1 - country 2 is L-intensive)

ii) If **factor prices in 2 remains constant but factor prices in Country 1 rise** from CC towards C'C' $\rightarrow$ **Country 1 gains** (same unit cost, but higher r_1 is used - country 1 is K-intensive)

iii) If we end up with both C'C' and A'A' (**both factor prices decrease**), we would have a **new equilibrium at a more competitive isocost line (P')**, Output price falls $\rightarrow$ shift BB to B' $\rightarrow$ **Consumers gain** in both countries [$\Delta P < 0$, decrease in prices] and it's cheaper to produce

Addition of Country K: Without fragmentation, consider that Country K would have **lower costs of production than Country 1** $\rightarrow$ Country 1 would lose the whole industry under free trade.

However, **with fragmentation between 1 and 2**, Country 1 could manage to **retain some of the industry**, which could be lost otherwise (e.g., if we end up with B' isocost for the integrated production, lower than K)

Fragmentation and General Equilibrium

Extension of Helpman (1984, 1985) and Helpman and Krugman (1985) of The HO Model That Includes FDI

Model Setup

- 2 countries, 2 goods, 2 factors
- **Trade Costs** such that:
  - $\tau^a = 1 \rightarrow$ is fixed and
  - τ^c can range from 1 (free trade in components) to ∞ (no trade in components)

- **Factor Endowments**: L_1, K_1, L_2, K_2
- One sector of the economy is **manufacturing** (M), divided in **components** and **assembly**, with **fixed factor intensity** $\left(\frac{K}{L}\right)^M$
- The **rest of the economy** is sector Y, that employs the entire endowments available net of the factor endowments used in M (hence it's the REMAINING PART OF THE ECONOMY)

NB: Sector Y can change $\left(\frac{K}{L}\right)^Y$ to absorb any surplus in endowments

Output of Sector Y

The **production function** for the output of Y is given by:

$$Y_i = Y(L_i - L_i^M, K_i - K_i^M)$$

With $L_i - L_i^M$ being the **labor endowment net of the labor force employed in M**, and $K_i - K_i^M$ is the **capital endowment net of the capital used in M**.

The **Market Clearing Factor Prices** (equilibrium wage and rental rate of capital) are:

$$\begin{aligned} w_i = MPL_{i,y} &= \frac{\partial Y(L_i - L_i^M, K_i - K_i^M)}{\partial L_i} \\ r_i = MPR_{i,y} &= \frac{\partial Y(L_i - L_i^M, K_i - K_i^M)}{\partial K_i} \end{aligned}$$

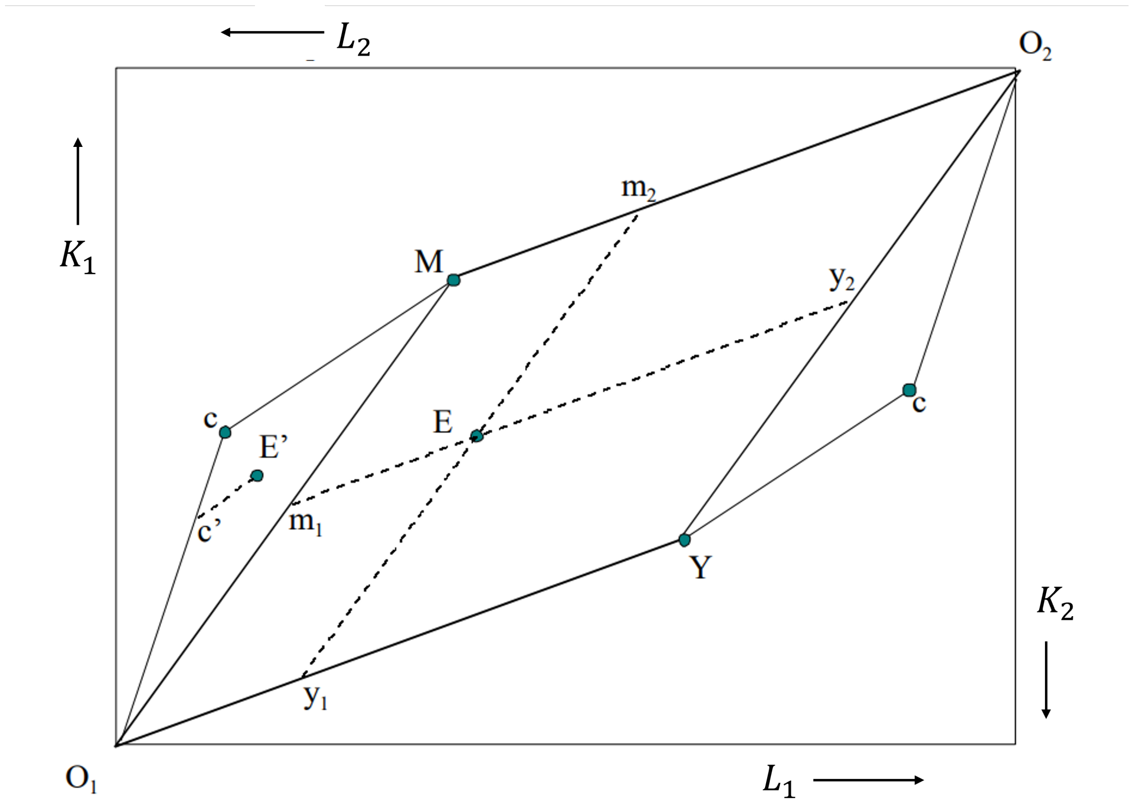
Income in each country is the sum of factor returns multiplied by factor employment:

$$I_i = w_i L_i + \pi_i K_i$$

Consumers have the same utility functions/indifference curves (they have identical preferences) and trade is only driven by differences in the supply-side of production (differences in factor endowments, just like in the standard Heckscher-Ohlin model).

Integrated Equilibrium: Edgeworth Box

We represent the trade equilibrium between two countries (country 1 and country 2) on an Edgeworth Box:



In this model:

- Y_1: L-intensive, M_1: K-intensive
- Y_2: K-intensive, M_2: L-intensive
- If we have initial endowment within **Factor Price Equalization (FPE) Set** $O_1M_1O_1Y_1$ - as an example, where point E lies - **countries have the same factor prices** $\rightarrow$ No incentives to FDI (Fragmentation)
- If endowments are outside the FPE set (point E') $\rightarrow$ we have **differences in factor prices** across countries $\rightarrow$ incentives for FDI

Consider for instance country 1:

- O_1M_1: **Total factor usage in M**, while O_1Y_1: **Total factor usage in Y**
- In order to **absorb a surplus of endowments**, a **change in technology** would be needed, and this is possible through **fragmentation** of the manufacturing O_1M, dividing c and a:
  - O_1c new factor usage in component with fragmentation of the production process, cM assembly
  - the capital-intensive stage of manufacturing was moved to country 1 (components) and the L-intensive process to country 2 (assembly)
  - $\downarrow \frac{w_1}{r_1}$ (L cheaper and K more costly for 1 than before), $\uparrow \frac{w_2}{r_2}$ (L more costly and K cheaper for 2 than before) $\rightarrow$ factor price w, r convergence

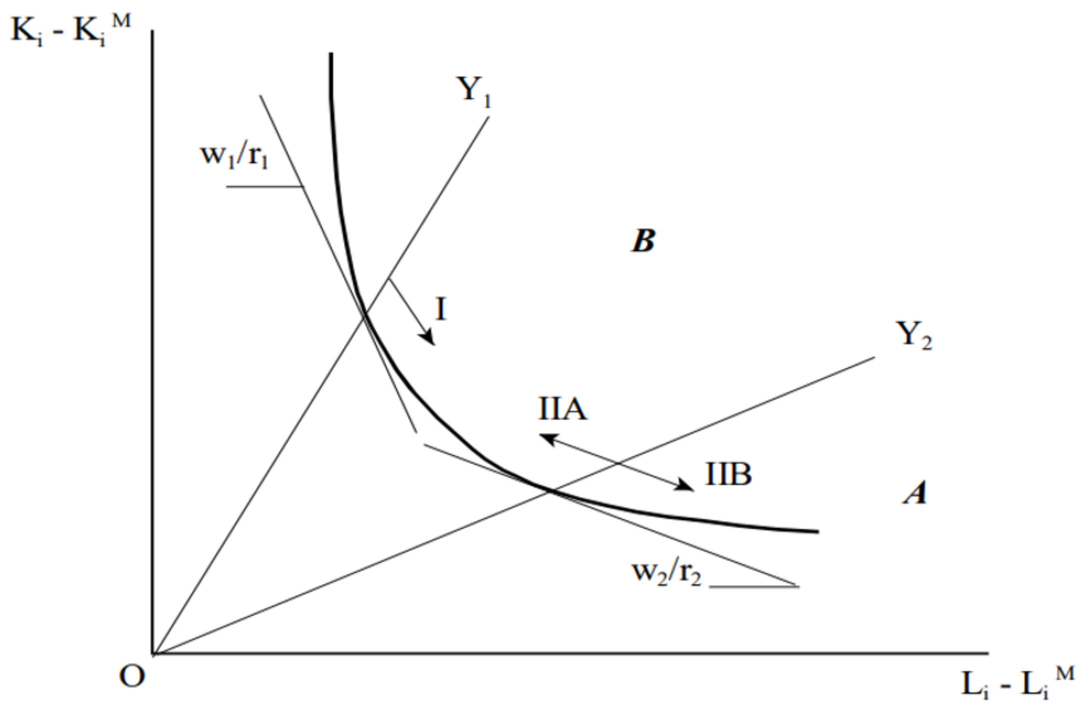
Factor Price Convergence

After fragmentation the K-intensive stage was moved to country 1 and L-intensive in country 2

$$\frac{w_1}{r_1}\downarrow \quad \text{and} \quad \frac{w_2}{r_2}\uparrow \;\rightarrow\; \text{Convergence in factor prices (Factor Price Equalization)}$$

Recall the Assumptions made at the very beginning:

- **Country 1** is the **K-abundant** country (produces Y with relatively more K)
- $\frac{w_1}{r_1} > \frac{w_2}{r_2}$ as reflected by the slopes of the Y sector isoquant when crossed by the OY_1, OY_2 rays (meaning that Country 1 is more developed)



In the diagram (where the thick black line is the Y sector isoquant):

- **Vertical FDI** was **pursued** because there was a **difference in factor prices** (initial situation represented in the graph), and MNEs wanted to displace some of the production to the other country to save some costs.
- However, after a certain amount of time, **factor prices** will start increasing in the country 2, while factor prices in country 1 will fall down, hence **converging towards each other** (OY_1, OY_2 rotating towards the center).

Fragmentation due to of MNE activity (when Vertical FDI arise) has powerful and relevant implication in different countries, since it affects factor prices (including wages) and possibly promotes economic convergence.

References

Barba Navaretti G., and T. Venables, 2005, Multinational Firms in the World Economy, Princeton University Press (BNV) - Chapter 3.