Equazioni con il modulo

$|2| = 2$ $\quad |-2| = 2$ non si tiene conto del segno

$$|f(x)| = g(x) \qquad D = D(f) \cap D(g)$$

bisogna assumere che $g(x) \geq 0$ perché altrimenti l'equazione non si verifica

$|x+3| = -2 \quad \not\exists$ NB il modulo non può MAI VALERE un numero NEGATIVO

$$\begin{cases} g(x) \geq 0 \\ f(x) = g(x) \quad \text{o} \quad f(x) = -g(x) \end{cases}$$

tutte le soluzioni di $f(x) = \pm g(x)$ t.c $g(x) \geq 0$

es. $|3x+2| = x-1 \qquad D = \mathbb{R}$

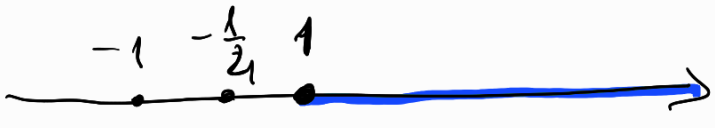
$x-1 \geq 0 \Rightarrow x \geq 1$

1°) $3x+2 = x-1 \Rightarrow f(x) = g(x)$

2°) $3x+2 = -(x-1) \Rightarrow f(x) = -g(x)$

1) $2x = -3 \qquad x = -\frac{3}{2}$

2) $3x+2 = -x+1 \qquad 4x = -1 \qquad x = -\frac{1}{4}$



$-1 \notin [1, +\infty)$

$-\frac{1}{4} \notin [1, +\infty)$

$\nexists$ sol.

- $|x+1| = 1$

$x+1 = 1$ o $x+1 = -1$

$x = 0$ o $x = -2$ $\quad S = \{-2, 0\}$

Più moduli $|f(x)| \pm |g(x)| = h(x)$

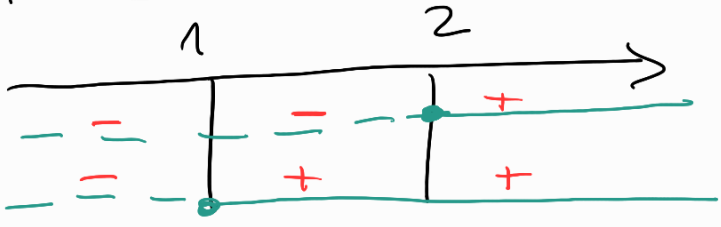
$D = D(f) \cap D(g) \cap D(h)$

es. $|x - x^2| + |x+5| = 3x$

es. $|2x-4| - |x-1| = 3$

$2x - 4 \geq 0 \quad x \geq 2$

$x - 1 \geq 0 \quad x \geq 1$



1) $x < 1$

$-(2x-4) + [+(x-1)] = 3$

$-2x + 4 + x - 1 = 3$

$-x = 0 \quad x = 0 \quad 0 \in (-\infty, 1)$?

Sì, 0 è all'interno $\quad S_1 = \{0\}$

2) $1 \leq x < 2$

$-(2x-4) - (x-1) = 3$

$-2x + 4 - x + 1 = 3$

$-3x = -2 \quad x = \frac{2}{3} \quad \frac{2}{3} \in [1, 2)$?

NO

$S_2 = \{\emptyset\}$

3) $x \geq 2$

$(2x-4) - (x-1) = 3$

$2x - 4 - x + 1 = 3$

$x = 6$ $\quad 6 \in [2, +\infty)$? si

$S_3 = \{6\}$

$S = \{0, 6\}$

prova $x=0$

$|2 \cdot 0 - 4| - |0 - 1| = 3$

$4 - 1 = 3 \quad 3 = 3$ verificata!

$x = 6 \quad |2 \cdot 6 - 4| - |6 - 1| = 3$

$8 - 5 = 3 \quad 3 = 3$ verificata!

- $|3x - 7| = -8 \quad \nexists \quad -8 < 0$

- $|x-1| = -4 + 2x$

$-4 + 2x \geq 0 \iff x \geq 2$

$(x-1) = -4 + 2x$ o $x - 1 = -(-4 + 2x)$

$x - 2x = -4 + 1$

$-x = -3$

$x = 3$

o $x - 1 = 4 - 2x$

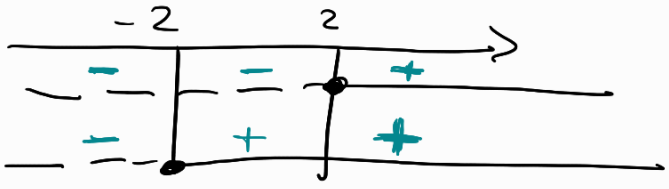
$3x = 5 \quad x = \frac{5}{3}$

$S = \{3\}$

- $2|x-2| + |x+2| = 5x \qquad D = \mathbb{R}$

$x-2 \geq 0 \quad x \geq 2$

$x+2 \geq 0 \quad x \geq -2$



a) se $x \leq -2$

$-2(x-2) - (x+2) = 5x$

$-2x + 4 - x - 2 = 5x$

$-8x + 2 = 0 \qquad x = \frac{1}{4} \qquad S_1 = \{\phi\}$

b) $-2 < x < 2$

$-2(x-2) + x+2 = 5x$

$-2x + 4 + x + 2 = 5x$

$-6x + 6 = 0 \qquad x = 1 \qquad S_2 = \{1\}$

c) $x \geq 2$

$2(x-2) + x + 2 = 5x$

$2x - 4 + x + 2 - 5x = 0$

$-2x - 2 = 0 \qquad x = -1 \qquad S_3 = \{\phi\}$

$S = \{1\}$

Disequazioni con il modulo

$|f(x)| \geq g(x)$ $\quad D = D(f) \cap D(g)$

$|x| \geq 2 \Rightarrow x \leq -2 \vee x \geq 2$

$|x| \leq 2 \Rightarrow -2 \leq x \leq 2$

$|x| \leq -2 \quad \nexists \quad |x| > -2 \quad \forall x \in \mathbb{R}$

es. $\left| \frac{3x+2}{2x-1} \right| \leq 1$ $\quad D: 2x-1 \neq 0$, $x \neq \frac{1}{2}$

$$-1 \leq \frac{3x+2}{2x-1} \leq 1$$

$$\begin{cases} \frac{3x+2}{2x-1} \leq 1 \\ \frac{3x+2}{2x-1} \geq -1 \end{cases}$$

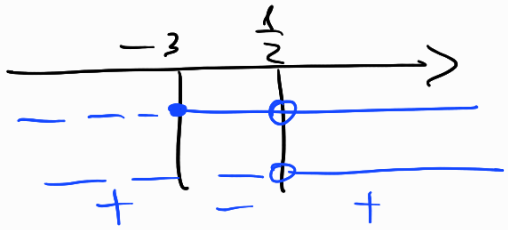
1) $\frac{3x+2}{2x-1} \leq 1$

$$\frac{3x+2}{2x-1} - 1 \leq 0$$

$$\frac{3x+2-2x+1}{2x-1} \leq 0 \qquad \frac{x+3}{2x-1} \leq 0$$

N: $x+3 \geq 0 \quad x \geq -3$

D: $2x-1 > 0 \quad x > \frac{1}{2}$



$S_1 = \left\{ -3 \leq x < \frac{1}{2} \right\} = \left[-3, \frac{1}{2} \right)$

$$\frac{3x+2}{2x-1} \geq -1$$

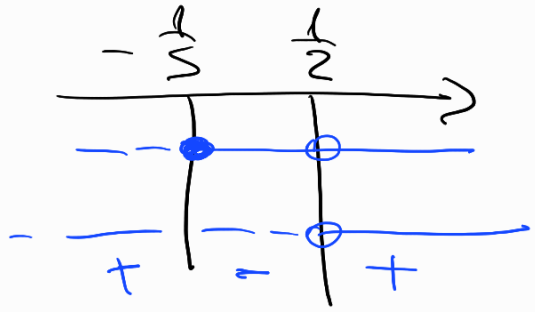
$$\frac{3x+2}{2x-1} + 1 \geq 0$$

$$\frac{3x+2+2x-1}{2x-1} \geq 0$$

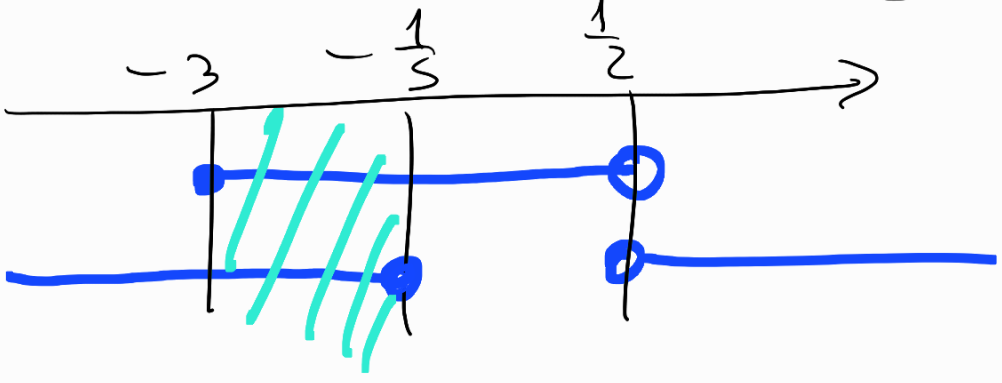
$$\frac{5x+1}{2x-1} \geq 0$$

$N: x \geq -\frac{1}{5}$

$D: x > \frac{1}{2}$



$$S_2 = (-\infty; -\frac{1}{5}] \cup (\frac{1}{2}, +\infty)$$



$$S = [-3, -\frac{1}{5}]$$

- $2x(|x| - 5) \leq 0$

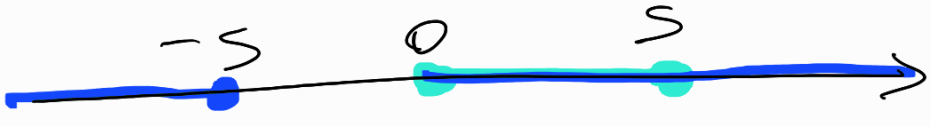
$2x(x-5) \leq 0$ V $2x(-x-5) \leq 0$

$2x^2 - 10x \leq 0$ $-2x(x+5) \leq 0$

$2x(x-5) \leq 0$ $2x(x+5) \geq 0$

$x = 0 \quad x = 5$ $x = 0 \quad x = -5$

$0 \leq x \leq 5$ $x \leq -5 \vee x \geq 0$



$$S = (-\infty, -5] \cup [0, +\infty)$$

• $|3x^2-4x-2|<2$

$-2<3x^2-4x-2<2$

$\begin{cases} 3x^2-4x-2<2 \\ 3x^2-4x-2>-2 \end{cases}$

$\begin{cases} 3x^2-4x-4<0 \\ 3x^2-4x>0 \end{cases}$

a) $3x^2-4x-4<0$

$3x^2-4x-4=0 \qquad \Delta = 16+4\cdot 4\cdot 3 = 64$

$x_{1,2} = \dfrac{4\pm\sqrt{64}}{3\cdot 2} = \dfrac{4\pm 8}{6} = \begin{cases} x_1 = \dfrac{4-8}{6} = \dfrac{-4}{6} = -\dfrac{2}{3} \\ x_2 = 2 \end{cases}$

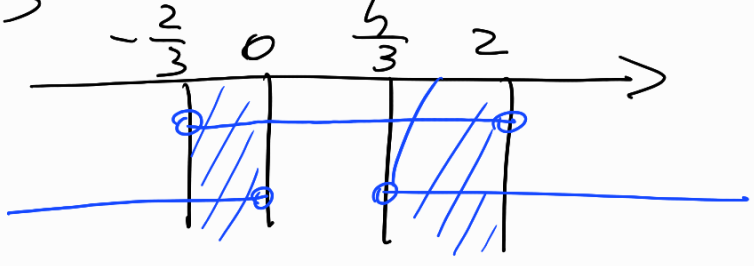
$-\frac{2}{3}<x<2$

b) $3x^2-4x>0$

$x(3x-4)>0 \qquad x=0 \qquad x=\frac{4}{3}$

$x<0 \quad \vee \quad x>\frac{4}{3}$

$\begin{cases} -\frac{2}{3}<x<2 \\ x<0 \vee x>\frac{4}{3} \end{cases}$



$S = \left(-\frac{2}{3}, 0\right) \cup \left(\frac{4}{3}, 2\right)$

• $|x(x-2)|<1$

$\begin{cases} x(x-2)<1 \\ x(x-2)>-1 \end{cases}$

$$\begin{cases} x^2 - 2x - 1 < 0 \\ x^2 - 2x + 1 > 0 \end{cases} \Rightarrow (x-1)^2 > 0$$

a) $x^2 - 2x - 1 < 0$ $\quad \Delta = 4 + 4 = 8 = 2^3$

$$x_{1,2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2} \begin{cases} x_1 = 1 - \sqrt{2} \\ x_2 = 1 + \sqrt{2} \end{cases}$$

$$1 - \sqrt{2} < x < 1 + \sqrt{2}$$

b) $(x-1)^2 > 0 \quad x \neq 1$

$S: (+1 - \sqrt{2}) < x < 1 + \sqrt{2} - \{1\}$

$(+1 - \sqrt{2}, 1) \cup (1 + \sqrt{2})$

- $|x+1| - |x+2| > 0$

$(|x+1|)^2 > (|x+2|)^2$

$x^2 + 2x + 1 > x^2 + 4x + 4$

$-2x > 3$

$2x < -3 \quad x < -\frac{3}{2}$