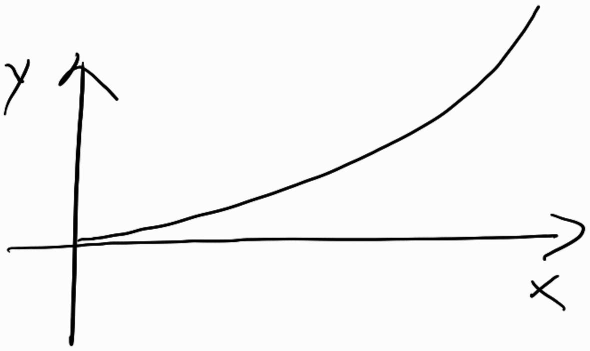
Esponenziali e logaritmi

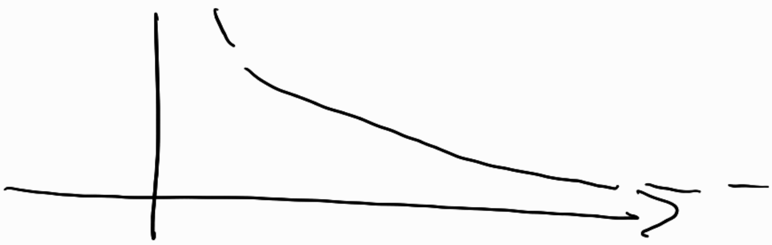
funzione potenza x^q ;

$q > 0$



strettamente crescente

$q < 0$



strettamente decrescente

x^q

x = base

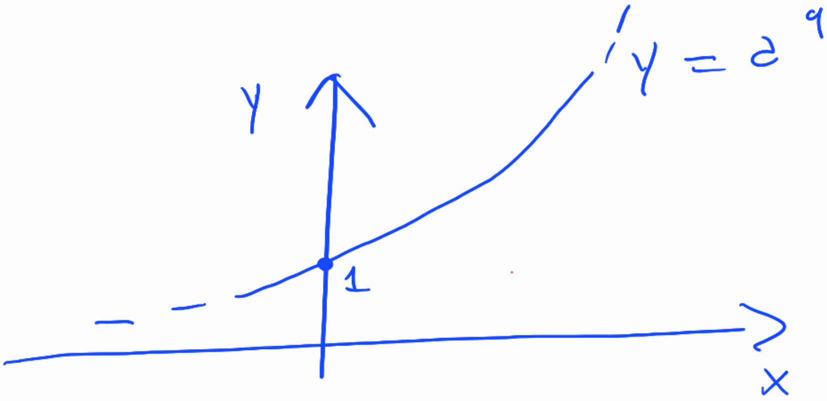
q = esponente

$y = a^q$ condizioni $a > 0$ $a \in \mathbb{R}^+$

D: $\mathbb{R}^+$ $(y > 0)$

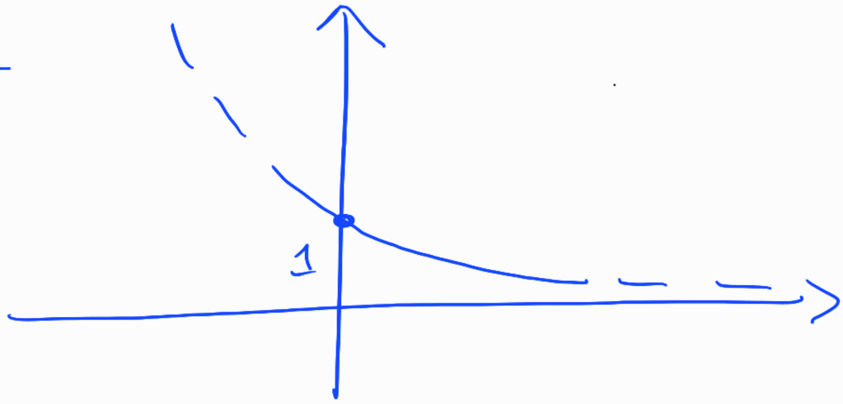
C: $\mathbb{R}$

$a > 1$



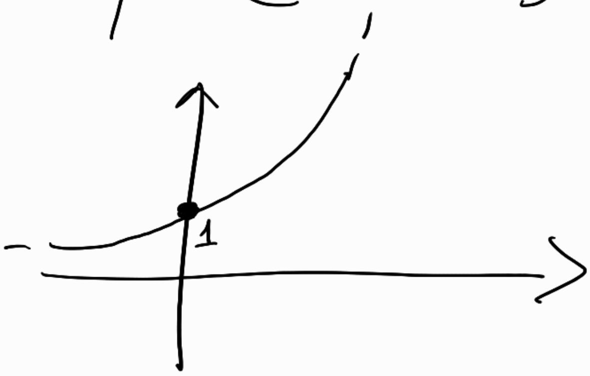
ad es. $y = 2^x$ $x=0$ $y = 2^0 = 1$

se $0 < a < 1$



NUMERO DI NEPLERO $e = 2,711828.....$

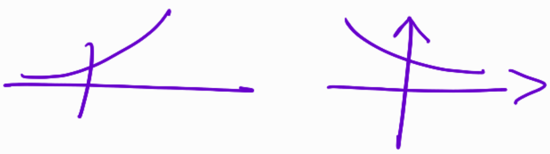
$y = e^x$ si dice esponenziale naturale



Proprietà degli esponenziali

- $a^0 = 1$
- $a^{x+y} = a^x \cdot a^y$
- $(a^x)^y = a^{x \cdot y}$
- $(a \cdot b)^x = a^x \cdot b^x$
- $a^{-x} = \frac{1}{a^x}$
- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

FUNZIONI MONOTONE



$\forall \; x_1 < x_2 \Rightarrow a^{x_1} < a^{x_2}$ monotona crescente

$\forall \; x_1 < x_2 \Rightarrow a^{x_1} > a^{x_2}$ " decrescente

$a^x = b \iff x = \log_a b$ — b = argomento, a = base

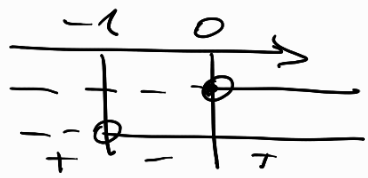
$a > 0$
$b > 0$

NB argomento del logaritmo deve essere sempre positivo

es. $y = \log g(x)$ $\quad$ $\underline{\underline{D: g(x) > 0}}$

$y = \log \left[\frac{x}{1+x}\right]$ per quali valori di x $\exists$?

$\frac{x}{1+x} > 0$ $\quad$ N: $x > 0$ $\quad$ D: $1 + x > 0$ $\quad$ $x > 0$ $\quad$ $x > -1$



S: $x < -1 \lor x > 0$ $\quad$ D: $(-\infty, -1) \cup (0, +\infty)$

Proprietà

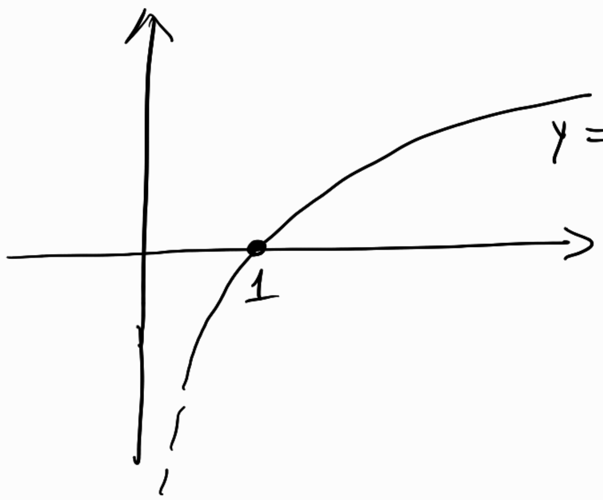
- $\log_a a = 1$
- $\log_a a^c = c \cdot \log_a a = c$
- $\log_a 1 = 0$
- $a^{\log_a b} = b$

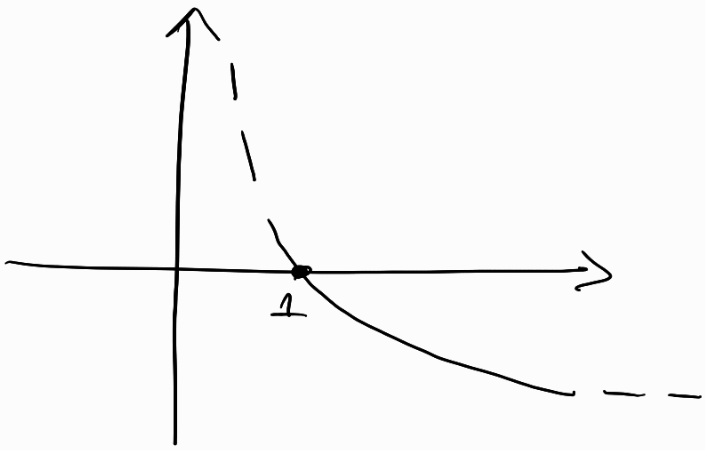
es. $\log_3 27 = \log_3 3^3 = 3 \cdot \log_3 3 = 3$

Graficamente

$y = \log_a x$

se $a > 1$ $\quad$ se $0 < a < 1$





Si tratta di funzioni monotone

- $\log_a (b \cdot c) = \log_a b + \log_a c$
- $\log_a b^c = c \cdot \log_a b$
- $\log_a \dfrac{b}{c} = \log_a b - \log_a c$
- $\log_a b = \dfrac{\log_c b}{\log_c a}$ CAMBIO DI BASE

es. $\log_2 (\sqrt[4]{2} \cdot \sqrt[3]{4}) = \log_2 2^{\frac{1}{4}} \cdot 2^{\frac{2}{3}}$

$= \log_2 2^{\frac{1}{4} + \frac{2}{3}} = \log_2 2^{\frac{11}{12}} = \dfrac{11}{12}$

logaritmo naturale $y = \log_e a$

es. $\log \dfrac{8}{55} = \log 8 - \log 55$

$= \log 2^3 - \log (5 \cdot 11)$

$= 3 \cdot \log 2 - \log 5 - \log 11$

es $\log_4 5 = \dfrac{\log_2 5}{\log_2 4} = \dfrac{\log_2 5}{\log_2 2^2} = \dfrac{\log_2 5}{2}$

EQUAZIONI ESPONENZIALI

$$a^{f(x)} = y \qquad y > 0$$

$$\Updownarrow$$

$$f(x) = \log_a y$$

es. $$\frac{2^{x+1} \cdot 5^{x-1}}{3^x} = 2$$

Facciamo il logaritmo di ambo i membri

$$\log\left(\frac{2^{x+1} \cdot 5^{x-1}}{3^x}\right) = \log 2$$

$$\log 2^{x+1} + \log 5^{x-1} - \log 3^{x} = \log 2$$

$$(x+1) \cdot \log 2 + (x-1) \cdot \log 5 - x \cdot \log 3 = \log 2$$

$$x \cdot \log 2 + \cancel{\log 2} + x \log 5 - \log 5 - x \cdot \log 3 = \cancel{\log 2}$$

$$x(\log 2 + \log 5 - \log 3) = \log 5$$

$$x = \frac{\log 5}{\log 2 + \log 5 - \log 3}$$

es.

Si investono 8000 € ad un tasso annuale del 5%. Quanto tempo ci vuole per accumulare un capitale di 10000 €?

$$10000 = 8000\left(1 + \frac{5}{100}\right)^{n}$$

nostra incognita

$$\frac{10\cancel{0}\cancel{0}\cancel{0}}{8\cancel{0}\cancel{0}\cancel{0}} = (1{,}05)^{n}$$

$$\frac{5}{4} = (1,05)^n$$

$$\log \frac{5}{4} = \log (1,05)^n$$

$$n = \frac{\log \frac{5}{4}}{\log (1,05)} \approx 4,57$$

EQUAZIONI LOGARITMICHE

$$y = \log_a f(x) \qquad a > 0 \quad a \neq 1 \quad y \in \mathbb{R}$$

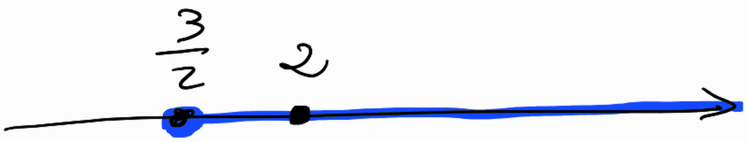
- $\log_2 (2x-3) = 0$ $\quad \to b$

calcolare il dominio

$2x - 3 > 0 \qquad x > \frac{3}{2}$ dominio

$$\cancel{2}^{\cancel{\log_2}(2x-3)} = 2^0$$

$2x - 3 = 1 \qquad 2x = 4 \qquad x = 2$



$2 \in D$ ✓

$x = 2$ è una sol. accettabile

- $\log_3 (x+1) - 2 \log_9 3x = 2$

$D: \begin{cases} x+1 > 0 & x > -1 \\ x > 0 & x > 0 \end{cases}$

$$\log_3 (x+1) - 2 \frac{\log_3 3x}{\log_3 9} = 2$$

$$\log_3 (x+1) - 2 \frac{\log_3 3x}{\log_3 3^2} = 2$$

$$\log_3 (x+1) - \cancel{2} \frac{\log_3 3x}{\cancel{2}} = 2$$

$D: x > 0$

$$\log_3(x+1) - \log_3 3x = 2$$

$$3^{\log_3 \frac{x+1}{3x}} = 3^{2}$$

$$\frac{x+1}{3x} = 9 \qquad x+1 = 9 \cdot 3x$$

$$x+1 = 27x$$

$$26x = 1 \qquad x = \frac{1}{26} > 0$$ quindi $x = \frac{1}{26}$ è una sol. accettabile

DISEQUAZIONI ESPONENZIALI

es. $$\left(\frac{2}{3}\right)^{x-2} \cdot \frac{3}{2} > 1 \qquad \frac{3}{2} = \left(\frac{2}{3}\right)^{-1}$$

$$\left(\frac{2}{3}\right)^{x-2} \cdot \left(\frac{2}{3}\right)^{-1} > 1$$

$$\left(\frac{2}{3}\right)^{x-3} > 1$$

$$\left(\frac{2}{3}\right)^{x-3} > \left(\frac{2}{3}\right)^{0} \iff x-3 < 0 \qquad \boxed{x<3} \text{ Sol.}$$

cambio di segno poiché $\frac{2}{3} < 1$

$$\alpha \; \frac{2}{3} < 1$$

es.

$$25^{x} - 13 \cdot 5^{x} + 30 > 0$$

$$5^{2x} - 13\, 5^{x} + 30 > 0 \qquad 5^{x} = y$$

$$y^2 - 13y + 30 > 0$$

$$\Delta = 13^2 - 4 \cdot 30 = 49$$

$$y_{1,2} = \frac{13 \pm 7}{2} = \begin{cases} \frac{13+7}{2} = \frac{20}{2} = 10 \\ \frac{13-7}{2} = \frac{6}{2} = 3 \end{cases}$$

$$y < 3 \quad \vee \quad y > 10$$

$$5^x < 3 \quad \vee \quad 5^x > 10$$

$$\log_5 5^x < \log_5 3 \quad \vee \quad \log_5 5^x > \log_5 10$$

$$x < \log_5 3 \quad \vee \quad x > \log_5 10$$

$$\log_{\frac{1}{2}} (5+3x) \geq 1$$

$$D: \quad 5+3x > 0 \qquad 3x > -5 \qquad x > -\frac{5}{3}$$

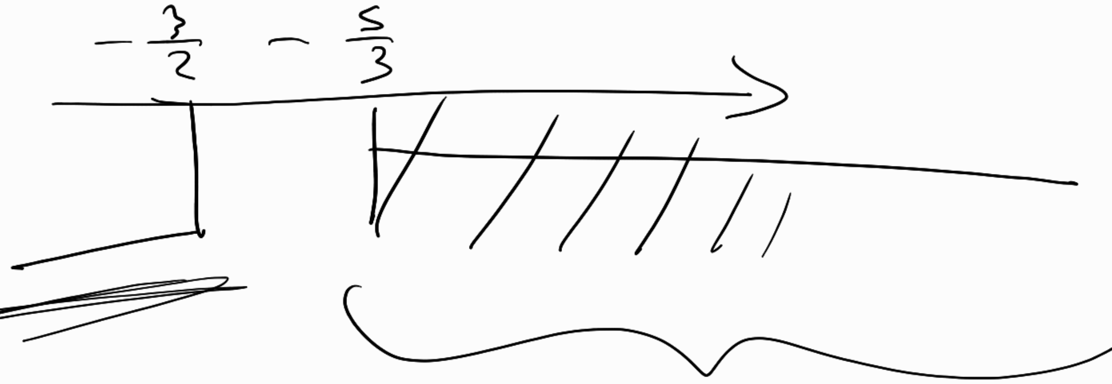
$$\log_{\frac{1}{2}} (5+3x) \geq \log_{\frac{1}{2}} \frac{1}{2}$$

$0 < \frac{1}{2} < 1$ cambio segno

$$5+3x \leq \frac{1}{2}$$

$$\cancel{3}x \leq -\frac{\cancel{9}\,3}{2} \qquad x \leq -\frac{3}{2}$$

$$\begin{cases} x > -\frac{5}{3} \\ x \le -\frac{3}{2} \end{cases}$$



non esiste sol